CAPACITANCE

Contents

JEE (Advance) Syllabus

Capacitance; Parallel plate capacitor with and without dielectrics; Capacitors in series and parallel; Energy stored in a capacitor.

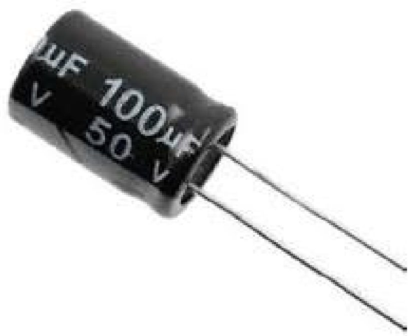
JEE (Main) Syllabus

Conductor and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series an in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy in a capacitor.

CAPACITANCE

INTRODUCTION

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.



CONCEPT OF CAPACITANCE

When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacitance of the conductor.

Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

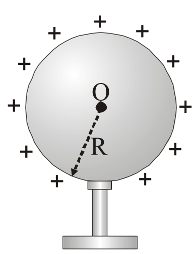
$1\ \mu F = 10^{-6}$ F, $1 nF = 10^{-9}$ F or $1\ pF = 10^{-12}$ F

The capacitance of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor. Theoretically infinite amount of charge can be given to a conductor. But practically the electric field becomes so large that it causes ionisation of medium surrounding it. The charge on conductor leaks reducing its potential.

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$



If conductor is placed in a medium then

$$C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$$

Capacitance depends upon :

(a) Size and Shape of Conductor

(b) Surrounding medium

(c) Presence of other conductors nearby

SOLVED EXAMPLE

Example 1. Find out the capacitance of the earth ? (Radius of the earth = 6400 km)

Solution : $C = 4\pi\epsilon_0 R = \dfrac{6400 \times 10^3}{9 \times 10^9} = 711\ \mu F$

POTENTIAL ENERGY OR SELF ENERGY OF AN ISOLATED CONDUCTOR

Work done in charging the conductor to the charge on it against its own electric field or total energy stored in electric field of conductor is called self energy or self potential energy of conductor.

Electric potential energy (Self Energy) :

Work done in charging the conductor

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$W = U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}.$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

Self energy is stored in the electric field of the conductor with energy density (Energy per unit volume)

$\dfrac{dU}{dV} = \dfrac{1}{2} \epsilon_0 E^2$ [The energy density in a medium is $\dfrac{1}{2} \epsilon_0 \epsilon_r E^2$]

where E is the electric field at that point.

In case of charged conductor energy stored is only out side the conductor but in case of charged insulating material it is outside as well as inside the insulator.

CONDENSER/CAPACITOR

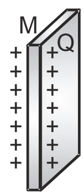
The pair of conductor of opposite charges on which sufficient quantity of charge may be accommodated is defined as condenser.

- ### Principle of a Condenser

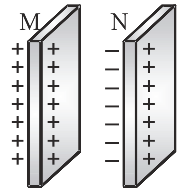
It is based on the fact that capacitance can be increased by reducing potential keeping the charge constant.

Consider a conducting plate M which is given a charge Q such that its potential rises to V then

$$C = \frac{Q}{V}$$

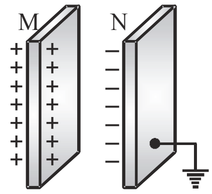


Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then



$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge). $\Rightarrow$ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ ($\because$ the entire positive charge flows to the earth)



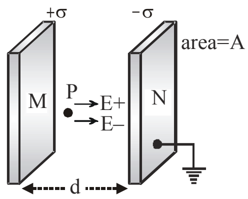
$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C$$

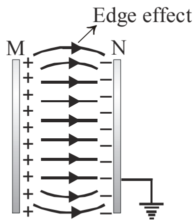
If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

• Parallel Plate Capacitor

(i) Capacitance

It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then





I step : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ $[\varepsilon = \varepsilon_0\varepsilon_r]$

II step : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r}d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ $\left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$

III step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

(ii) Force between the plates

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$

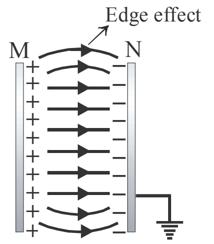
Force on negative charge –Q is $F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \dfrac{Q^2}{2\varepsilon_0 A} = \dfrac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \dfrac{F}{A} = u = p = \dfrac{1}{2}\epsilon_0 E^2$

Note:

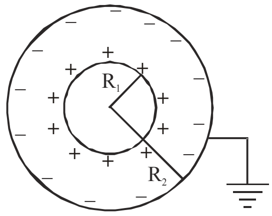
Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



- **Spherical Capacitor**

Outer sphere is earthed

When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.



$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

Potential of inner sphere $V_1 = \dfrac{Q}{4\pi\varepsilon_0 R_1} + \dfrac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{R_2 - R_1}{R_1 R_2}\right)$

As outer surface is earthed so potential $V_2 = 0$

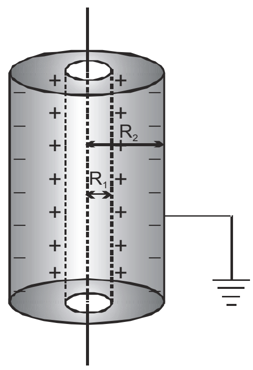
Potential difference between plates $V = V_1 - V_2 = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{(R_2 - R_1)}{R_1 R_2}$

So $C = \dfrac{Q}{V} = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

- **Cylindrical Capacitor**

When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded

Electrical field between cylinders $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{Q/\ell}{2\pi\varepsilon_0 r}$



Potential difference between plates $V = \displaystyle\int_{R_1}^{R_2} \dfrac{Q}{2\pi\varepsilon_0 r\ell}dr = \dfrac{Q}{2\pi\varepsilon_0 \ell}\,\ell n\left(\dfrac{R_2}{R_1}\right)$

Capacitance $C = \dfrac{Q}{V} = \dfrac{2\pi\varepsilon_0 \ell}{\log_e(R_2/R_1)}$

SOLVED EXAMPLE

Example 2. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

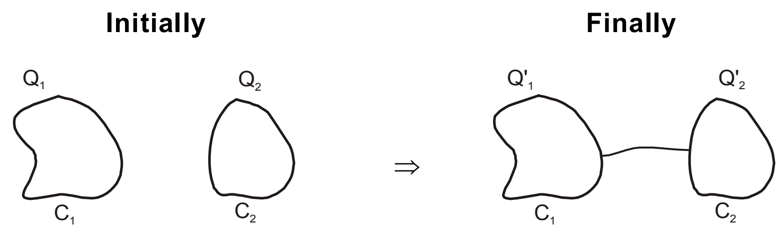
Solution : The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45×10^6 m

a = radius of earth = 6400 km = 6.4×10^6 m

$$\therefore \quad C = \frac{1}{9\times10^9} \times \frac{6.45\times10^6 \times 6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092 \text{ F}$$

SHARING OF CHARGES ON JOINING TWO CONDUCTORS (BY A CONDUCTING WIRE) :



(i) Whenever there is potential difference, there will be movement of charge.

(ii) If released, charge always have tendency to move from **high potential energy** to **low potential energy .**

(iii) If released, positive charge moves from **high potential** to **low potential** [if only electric force act on charge].

(iv) If released, negative charge moves from **low potential** to **high potential** [if only electric force act on charge].

(v) The movement of charge will continue till there is potential difference between the conductors (finally potential difference = 0).

(vi) Formulae related with redistribution of charges :

Before connecting the conductors		
Parameter	**Ist Conductor**	**IInd Conductor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**Ist Conductor**	**IInd Conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \qquad \Rightarrow \qquad \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But, $Q_1' + Q_2' = Q_1 + Q_2$

$\therefore$ $$V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2}$$

$\therefore$ $$Q_1' = \frac{C_1}{C_1+C_2}(Q_1+Q_2)$$

and $$Q_2' = \frac{C_2}{C_1+C_2}(Q_1+Q_2)$$

Heat loss during redistribution :

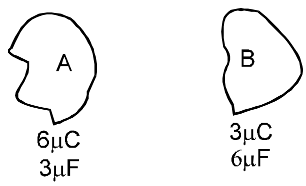
$$\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

Note : Always put Q_1, Q_2, V_1 and V_2 with sign.

SOLVED EXAMPLE

Example 3. A and B are two isolated conductors (that means they are placed at a large distance from each other). When they are joined by a conducting wire:



(i) Find out final charges on A and B ?

(ii) Find out heat produced during the process of flow of charges.

(iii) Find out common potential after joining the conductors by conducting wires?

Solution : (i) $Q_A' = \dfrac{3}{3+6}(6+3) = 3\mu C$

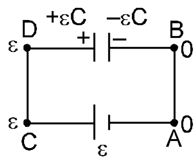
$Q_B' = \dfrac{6}{3+6}(6+3) = 6\mu C$

(ii) $\Delta H = \dfrac{1}{2}\cdot\dfrac{3\mu F.6\mu F}{(3\mu F+6\mu F)}\cdot\left(2-\dfrac{1}{2}\right)^2 = \dfrac{1}{2}.(2\mu F).\left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}\mu J$

(iii) $V_C = \dfrac{3\mu C+6\mu C}{3\mu F+6\mu F} = 1\text{volt.}$

Example 4. A capacitor of capacitance C which is initially uncharged is connected with a battery. Find out heat dissipated in the circuit during the process of charging.

Solution : Final status



Let potential at point A is 0, so at B also 0 and at C and D it is ε. finally, charge on the capacitor

$Q_C = \varepsilon C$

$U_i = 0$

$U_f = \dfrac{1}{2}CV^2 = \dfrac{1}{2}C\varepsilon^2$

work done by battery $= \int Pdt$

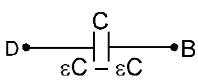
$W = \int \varepsilon i dt = \varepsilon\int i dt = \varepsilon . Q = \varepsilon . \varepsilon C = \varepsilon^2 C$

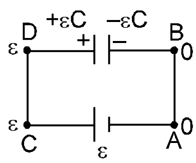
(Now onwards remember that w.d. by battery = εQ if Q has flown out of the cell from high potential and w.d. on battery is εQ if Q has flown into the cell through high potential)

Heat produced $= W - (U_f - U_i) = \varepsilon^2 C - \dfrac{1}{2}\varepsilon^2 C = \dfrac{C\varepsilon^2}{2}$.

Example 5. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε such that the positive terminal of battery is connected with positive plate of capacitor. Find out heat loss in the circuit during the process of charging.

Solution :





Since the initial and final charge on the capacitor is same before and after connection.

Here no charge will flow in the circuit so heat loss = 0

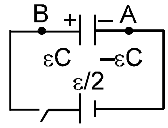
Example 6. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf $\varepsilon/2$ such that the positive terminal of battery is connected with positive plate of capacitor. After a long time

(i) Find out total charge flow through the battery

(ii) Find out total work done by battery

(iii) Find out heat dissipated in the circuit during the process of charging.

Solution : (i) Let potential of A is 0 so at B it is $\frac{\varepsilon}{2}$. So final charge on capacitor = $C\varepsilon/2$

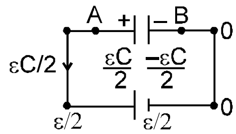


Charge flow through the capacitor = $(C\varepsilon/2 - C\varepsilon) = -C\varepsilon/2$

So charge is entering into battery.

(ii) finally,

Change in energy of capacitor = $U_{final} - U_{initial}$



$$= \frac{1}{2}C\left(\frac{\varepsilon}{2}\right)^2 - \frac{\varepsilon^2 C}{2}$$

$$= \frac{1}{8}\varepsilon^2 C - \frac{1}{2}\varepsilon^2 C = -\frac{3\varepsilon^2 C}{8}$$

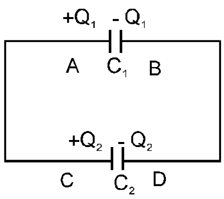
Work done by battery $= \frac{\varepsilon}{2} \times \left(-\frac{\varepsilon C}{2}\right) = -\frac{\varepsilon^2 C}{4}$

(iii) Work done by battery = Change in energy of capacitor + Heat produced

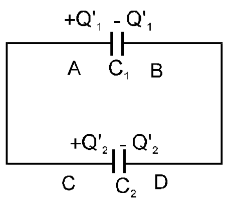
Heat produced = $\frac{3\varepsilon^2 C}{8} - \frac{\varepsilon^2 C}{4} = \frac{\varepsilon^2 C}{8}$

DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are C_1 and C_2 are connected as shown in figure



Initially



Finally

Before connecting the capacitors		
Parameter	**I^{st} Capacitor**	**II^{nd} Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors		
Parameter	**Ist Capacitor**	**IInd Capacitor**
Capacitance	C_1	C_2
Charge	Q'_1	Q'_2
Potential	V	V

(a) Common potential :

By charge conservation of plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$$\Rightarrow \quad V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \frac{C_1}{C_1+C_2}(Q_1 + Q_2) \quad \Rightarrow \quad Q_2' = C_2V = \frac{C_2}{C_1+C_2}(Q_1 + Q_2)$

(c) Heat loss during redistribution :

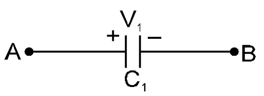
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

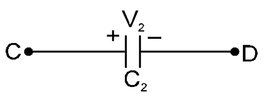
The loss of energy is in the form of Joule heating in the wire.

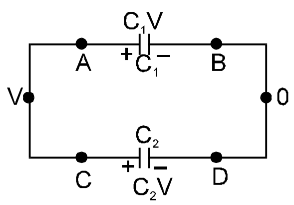
Note : (i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

DERIVATION OF ABOVE FORMULAE :







Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$C_1V + C_2V = C_1V_1 + C_2V_2$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \qquad \Rightarrow \qquad H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

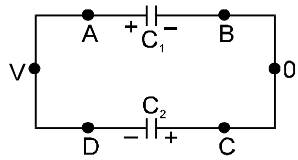
$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

when oppositely charge terminals are connected then



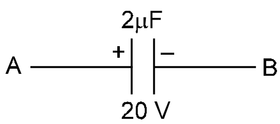
$\therefore \quad C_1V + C_2V = C_1V_1 - C_2V_2$

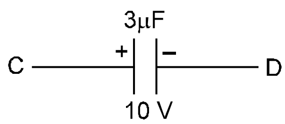
$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

and $\quad H = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$

SOLVED EXAMPLE

Example 7. Find out the following if A is connected with C and B is connected with D.

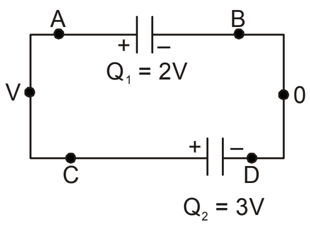




(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.

Solution : **(i)**



Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation,

$3V + 2V = 40 + 30$

$5V = 70$

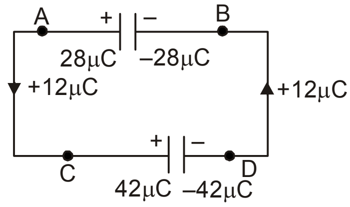
$V = 14$ volt

Charge flow

$= 40 - 28$

$= 12\ \mu C$

Now final charges on each plate is shown in the figure



(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

$= 400 + 150 - 490$

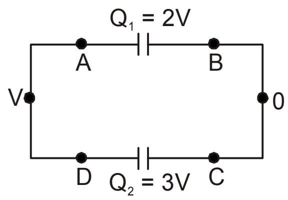
$= 550 - 490$

$= 60\ \mu J$

Note : (i) When capacitor plates are joined then the charge remains conserved.

(ii) We can also use direct formula of redistribution as given above.

Example 8. Repeat above question if A is connected with D and B is connected with C.

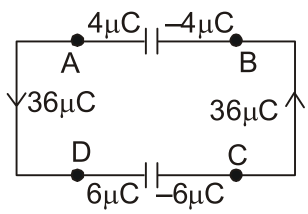


Solution : Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + 3V = 10$

$\Rightarrow$ $V = 2$ volt

Now charge on each plate is shown in the figure



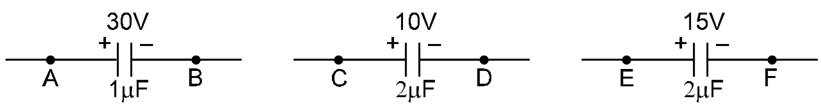
Heat produced $= 400 + 150 - \frac{1}{2} \times 5 \times 4$

$= 550 - 10$

$= 540\ \mu J$

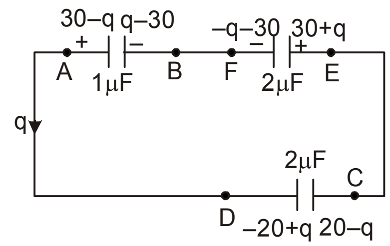
Note : Here heat produced is more. Think why?

Example 9. Three capacitors as shown of capacitance $1\mu F$, $2\mu F$ and $2\mu F$ are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



Solution : Let charge flow is q.

Now applying kirchhoff's voltage low

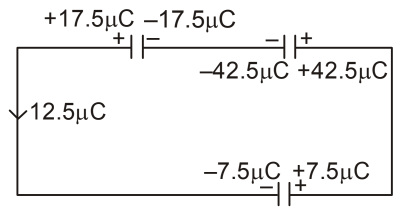


$$-\frac{(q-20)}{2}-\frac{(30+q)}{2}+\frac{30-q}{1}=0$$

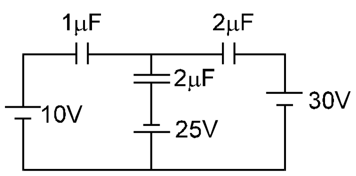
$-2q = -25$

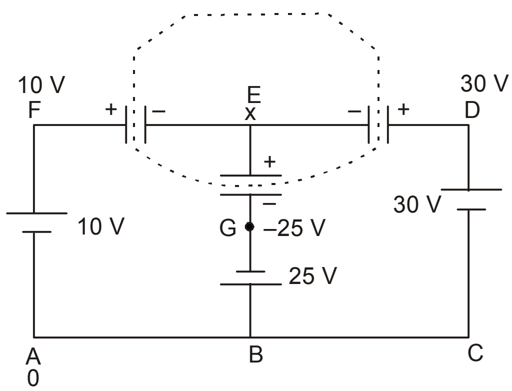
$q = 12.5\ \mu C$

Final charges on plates



Example 10. In the given circuit find out the charge on each capacitor. (Initially they are uncharged)





Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V and let potential at E is x. Now apply kirchhoff's Ist law at point E. (total charge of all the plates connected to 'E' must be same as before i.e. 0)

$\therefore$ $(x - 10) + (x - 30)2 + (x + 25)2 = 0$

$5x = 20$

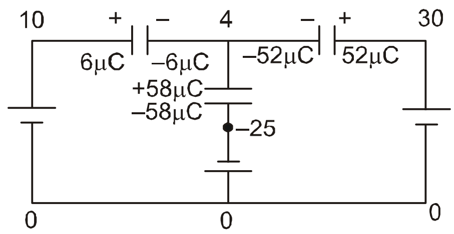
$x = 4$ V

Final charges :

$Q_{2\mu F} = (30 - 4)2 = 52\ \mu C$

$Q_{1\mu F} = (10 - 4) = 6\mu C$

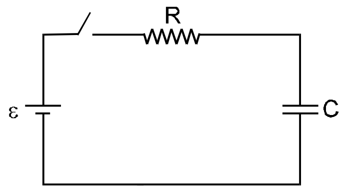
$Q_{2\mu F} = (4 - (-25))2 = 58\ \mu C$



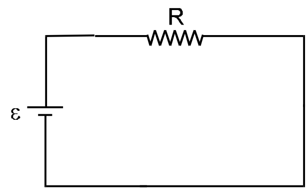
CIRCUIT SOLUTION FOR R–C CIRCUIT AT t = 0 (INITIAL STATE) AND AT t = ∞ (FINAL STATE)

Note : (i) Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.

(ii) When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

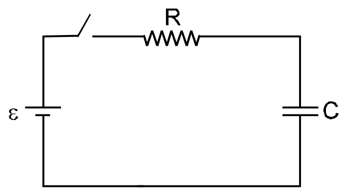


Before connection

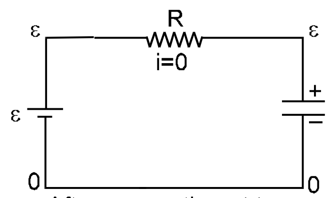


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor.



Before connection



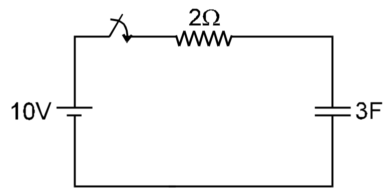
After connection at t = ∞

SOLVED EXAMPLE

Example 11. Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

(a) Just after the switch is closed.

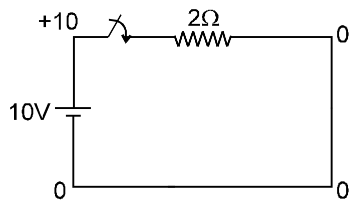
(b) After a long time when switch was closed.



Solution : **(a) For just after closing the switch:**

potential difference across capacitor = 0

$\therefore\ Q_C = 0 \qquad \therefore\ i = \frac{10}{2} = 5A$

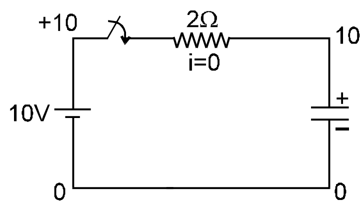


(b) After a long time

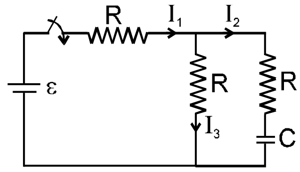
at steady state current i = 0

and potential difference across capacitor = 10 V

$\therefore\ Q_C = 3 \times 10 = 30\ C$



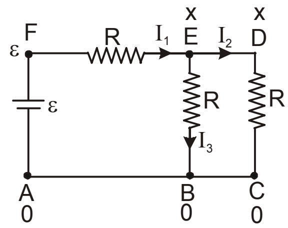
Example 12. Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.



(a) Just after the switch is closed

(b) After a long time when switch is closed.

Solution : **(a)** Initially the capacitor is uncharged so its behaviour is like a conductor Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's Ist law at point E :



$$\frac{x-\varepsilon}{R}+\frac{x-0}{R}+\frac{x-0}{R}=0 \quad \Rightarrow \quad \frac{3x}{R}=\frac{\varepsilon}{R}$$

$$x=\frac{\varepsilon}{3} \quad ; Q_c = 0$$

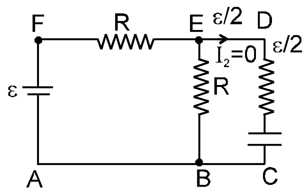
$$\therefore \quad I_1=\frac{-\varepsilon/3+\varepsilon}{R}=\frac{2\varepsilon}{3R} \quad \Rightarrow \quad I_2=\frac{dQ}{dt}=\frac{\varepsilon}{3R} \text{ and } I_3=\frac{\varepsilon}{3R}$$

Alternatively

$$i_1=\frac{\varepsilon}{R_{eq}}=\frac{\varepsilon}{R+\frac{R}{2}}=\frac{2\varepsilon}{3R} \quad \Rightarrow \quad i_2=i_3=\frac{i_1}{2}=\frac{\varepsilon}{3R} \text{ and } \frac{dQ}{dt}=i_2=\frac{\varepsilon}{3R}$$

(b) at t = ∞ (finally)

capacitor completely charged so their will be no current through it.



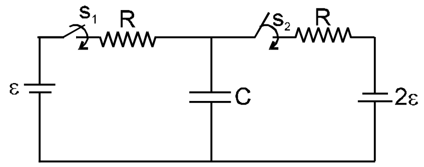
$$I_2 = 0, \quad I_1 = I_3 = \frac{\varepsilon}{2R}$$

$$V_E - V_B = V_D - V_C = (\varepsilon/2R)R = \varepsilon/2$$

$$\Rightarrow Q_C = \frac{\varepsilon C}{2}, \quad \frac{dQ}{dt} = I_2 = 0$$

Time	I_1	I_2	I_3	Q	dQ /dt
Time t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t = ∞	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

Example 13. At t = 0 switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed. Find out

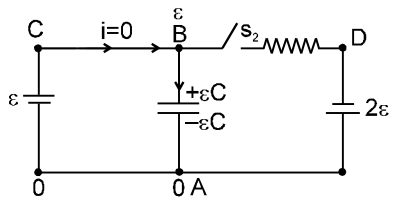


(i) The current through the capacitor immediately after that moment

(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

Solution : (i) Let Potential at point A is zero. Then at point B and C it will be ε (because current through the circuit is zero).

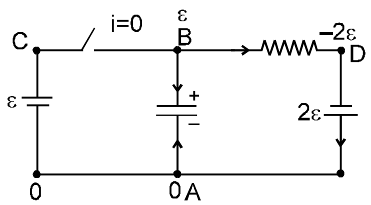


$V_B - V_A = (\varepsilon - 0)$

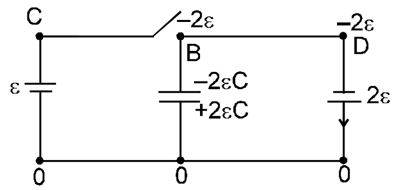
$\therefore$ Charge on capacitor = C(ε – 0) = Cε

Now S_2 is closed and S_1 is open. (p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zero so at D it is – 2ε.



$\therefore$ current through the capacitor = $\dfrac{\varepsilon - (-2\varepsilon)}{R} = \dfrac{3\varepsilon}{R}$ (B to D)



(ii) after a long time, i = 0

$V_B - V_A = V_D - V_A = -2\varepsilon$

$\therefore$ Q = C (–2ε – 0) = –2εC

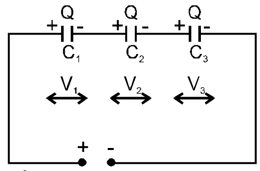
(iii) The charge on the lower plate (which is connected to the battery) changes from –εC to 2εC.

$\therefore$ this charge will come form the battery,

$\therefore$ charge flown from that cell is 3εC downward.

COMBINATION OF CAPACITORS :

Series Combination :



(i) When initially uncharged capacitors are connected as shown then the combination is called series combination.

(ii) All capacitors will have same charge but different potential difference across them.

(iii) We can say that

$$V_1 = \frac{Q}{C_1}$$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2}\ ,\ V_3 = \frac{Q}{C_3}\ ;\ ....$$

$$\text{(iv) } V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$$V \propto \frac{1}{C}$$

Note : In series combination the smallest capacitor gets maximum potential.

(v) $$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......}V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance :

Equivalent capacitance of any combination is that capacitance which when connected in place of the combination, stores same charge and energy that of the combination.

In series :

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

Note : In series combination equivalent capacitance is always less the smallest capacitor of combination.

(vii) Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

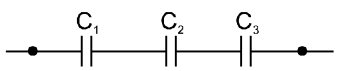
Energy supplied by the battery in charging the combination

$$U_{battery} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

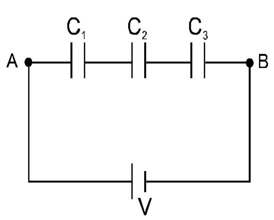
$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Derivation of Formulae :



meaning of equivalent capacitor



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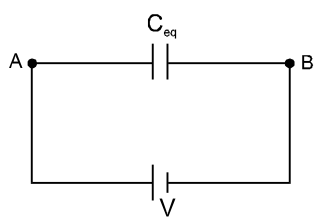
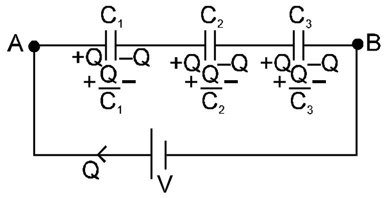


$$C_{eq} = \frac{Q}{V}$$



Now,

Initially, the capacitor has no charge.

Applying kirchhoff's voltage law

$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

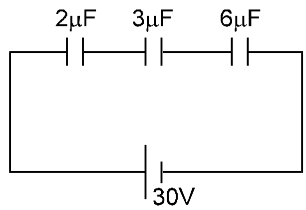
$$V = Q\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]$$

$$\frac{V}{Q} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \quad \Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

in general $\displaystyle \frac{1}{C_{eq}} = \sum_{n=1}^{n}\frac{1}{C_n}$

SOLVED EXAMPLE

Example 14. Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following:-



(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit

Solution : $\displaystyle \frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$

$C_{eq} = 1\mu F$.

(i) $Q = C_{eq} V = 30\mu C$.

(ii) charge on $3\mu F$ capacitor = $30\mu C$

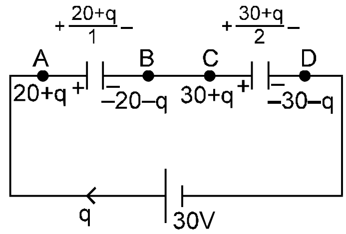
$$\text{energy} = \frac{Q^2}{2C} = \frac{30\times 30}{2\times 3} = 150\mu J$$

(iii) $\displaystyle U_{total} = \frac{30\times 30}{2}\ \mu J = 450\ \mu J$

(iv) Heat produced = $(30\ \mu C)\ (30) - 450\ \mu J = 450\ \mu J$.

Example 15. Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor

Solution : Now applying kirchoff voltage law



$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$-40 - 2q - 30 - q = -60$$

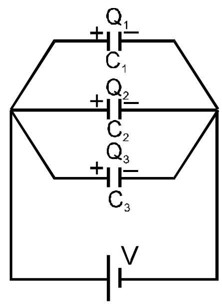
$$3q = -10$$

Charge flow = –10/3 μC.

Charge on capacitor of capacitance 1μF = 20 + q = $\frac{50}{3}$ μC

Charge on capacitor of capacitance 2μF = 30 + q = $\frac{80}{3}$ μC

Parallel Combination :



(i) When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.

(ii) All capacitors have same potential difference but different charges.

(iii) We can say that :

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance

$Q \propto C$

(v) $Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q \quad \Rightarrow \quad Q_2 = \frac{C_2}{C_1 + C_2 + C_3} Q$

$Q_3 = \frac{C_3}{C_1 + C_2 + C_3} Q$

Where $Q = Q_1 + Q_2 + Q_3$ ......

Note : Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination

$C_{eq} = C_1 + C_2 + C_3$

Note : Equivalent capacitance is always greater than the largest capacitor of combination.

(vii) Energy stored in the combination :

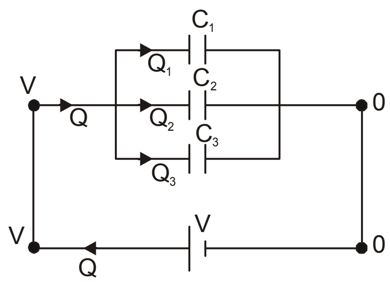
$$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ... = \frac{1}{2}(C_1 + C_2 + C_3 .....)V^2 = \frac{1}{2}C_{eq}V^2$$

$$U_{battery} = QV = CV^2$$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if all capacitors are initially uncharged)

Formulae Derivation for parallel combination :



$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$= V(C_1 + C_2 + C_3)$$

$$\frac{Q}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

In general $\quad C_{eq} = \sum_{n=1}^{n} C_n$

SOLVED EXAMPLE

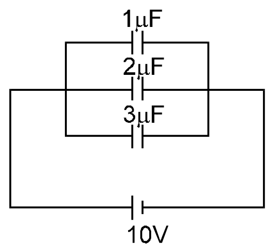
Example 16. Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.



Solution : (i) $Q = (30 + 20 + 10)\mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

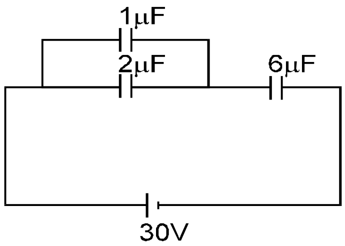
(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

Mixed Combination :

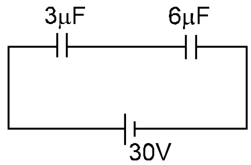
The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category.

SOLVED EXAMPLE

Example 17. In the given circuit find out charge on 6μF and 1μF capacitor.



Solution : It can be simplified as



$C_{eq} = \frac{18}{9} = 2\mu F$

charge flow through the cell = 30 × 2 μC

Q = 60 μC

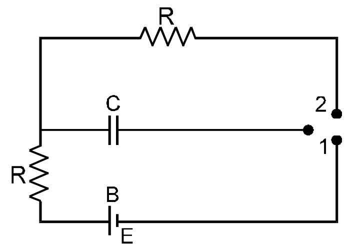
Now charge on 3μF = Charge on 6μF= 60 μC

Potential difference across 3μF = 60/ 3= 20 V

∴ Charge on 1μF = 20 μC.

CHARGING AND DISCHARGING OF A CAPACITOR

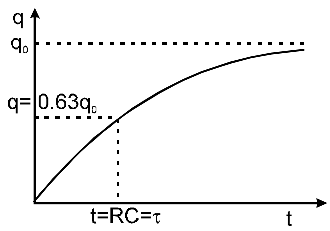
Charging of a condenser :



(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$

Where q_0 = maximum final value of charge at $t = \infty$.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.



(ii) If $t = RC = \tau$ then

$$q = q_0\,[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

or $q = q_0\,(1 - 0.37) = 0.63\,q_0 = 63\%$ of q_0

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

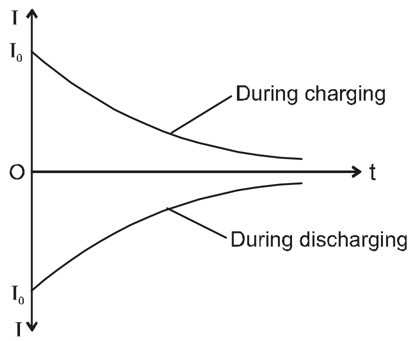
According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If t = RC = τ = Time constant

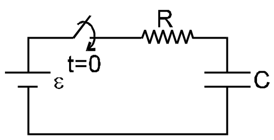
$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\ I_0$$

= 37% of I_0

i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.



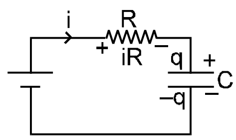
Derivation of formulae for charging of capacitor



it is given that initially capacitor is uncharged.

let at any time charge on capacitor is q

Applying kirchoff voltage law



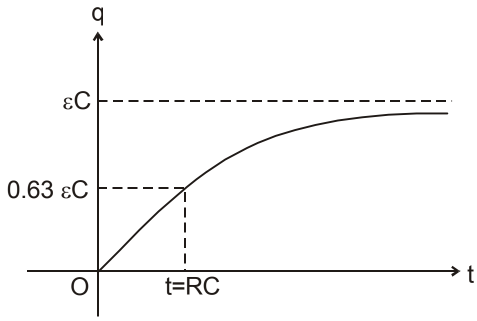
$$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}\ .\ dq = dt.$$

$$\int_O^q \frac{dq}{\varepsilon C - q} = \int_O^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$



$$\varepsilon C - q = \varepsilon C \cdot e^{-t/RC}$$

$$q = \varepsilon C(1 - e^{-t/RC})$$

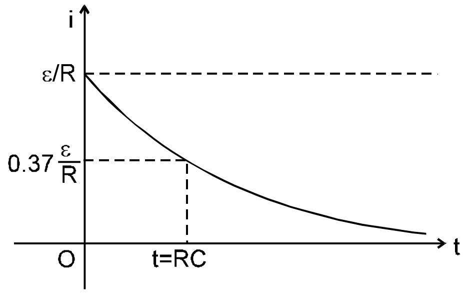
RC = time constant of the RC series circuit.

After one time constant

$$q = \varepsilon C \left(1 - \frac{1}{e}\right)$$

$= \quad \varepsilon C\,(1 - 0.37) = 0.63\,\varepsilon C.$

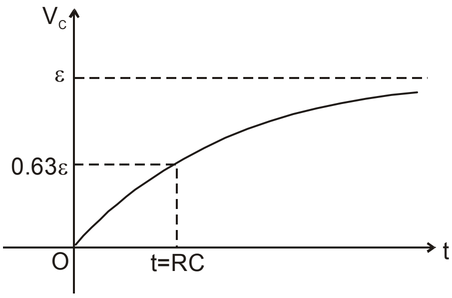
Current at any time t



$$i = \frac{dq}{dt} = \varepsilon C \left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

$$= \frac{\varepsilon}{R} e^{-t/RC}$$

Voltage across capacitor after one time constant V = 0.63 ε



$$Q = CV$$

$$V_C = \varepsilon(1 - e^{-t/RC})$$

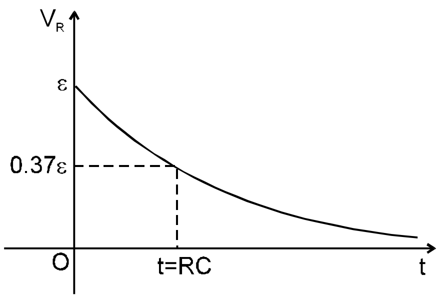
Voltage across the resistor

$$V_R = iR$$

$$= \varepsilon e^{-t/RC}$$

By energy conservation,

Heat dissipated = work done by battery – ΔUcapacitor



$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2} C\varepsilon^2 - 0\right)$$

$$= \frac{1}{2} C\varepsilon^2$$

Alternatively :

$$\text{Heat} = H = \int_0^\infty i^2 R\,dt$$

$$= \int_0^\infty \frac{\varepsilon^2}{R^2}\; e^{-\frac{2t}{RC}}\; R\,dt$$

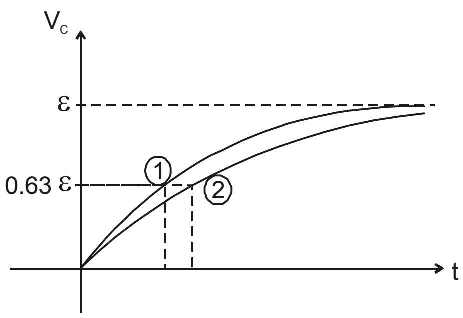
$$= \frac{\varepsilon^2}{R} \int_0^\infty e^{-2t/RC}\, dt$$

$$= \frac{\varepsilon^2}{R} \left[\frac{e^{-\frac{2t}{RC}}}{-2/RC} \right]_0^\infty$$

$$= -\frac{\varepsilon^2 RC}{2R} \left[e^{-\frac{2t}{RC}} \right]_0^\infty$$

$$= \frac{\varepsilon^2 C}{2}$$

Note:-



In the figure time constant of (2) is more than (1)

SOLVED EXAMPLE

Example 18 A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Solution : The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\ (1 - e^{-t/RC})$.

Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is

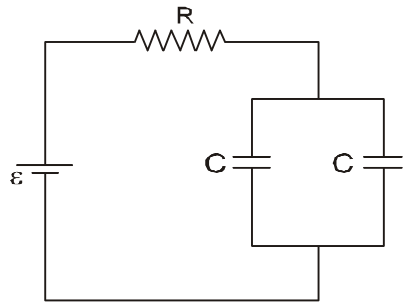
$Q_0/C = 36V$. So, $\Rightarrow$ $12V = 36V(1 - e^{-t/RC})$

or, $1 - e^{-t/RC} = \frac{1}{3}$ or, $e^{-t/RC} = \frac{2}{3}$

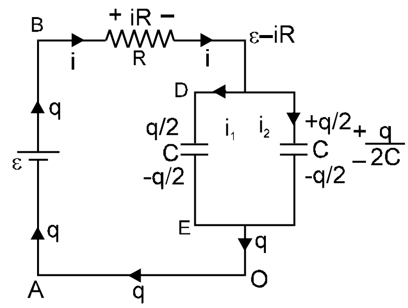
or, $\frac{t}{RC} = \ell n\left(\frac{3}{2}\right) = 0.405$ or, $RC = \frac{t}{0.405} = \frac{2\ \mu s}{0.45} = 4.936\ \mu s$

or, $C = \frac{4.936 \mu s}{20\Omega} = 0.25\ \mu F$.

Example 19 Without using the formula of equivalent. Find out charge on capacitor and current in all the branches as a function of time.



Solution : **Applying KVL in ABDEA**



$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR}$$

$$= \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\; e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\; e^{-t/2RC}$$

Alternate solution

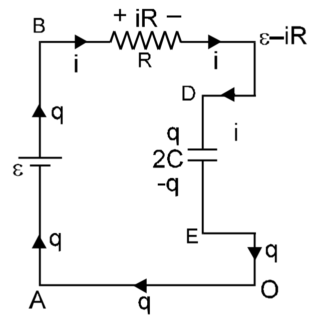
by equivalent

Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below

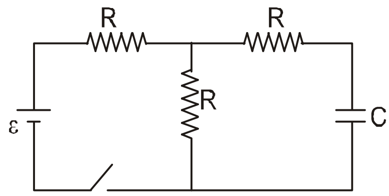
$q = 2\varepsilon C\,(1 - e^{-t/2RC})$



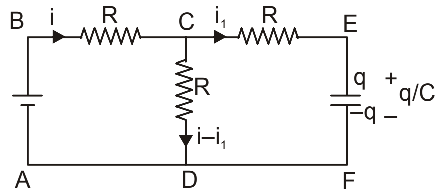
$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\; e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\; e^{-t/2RC}$$

Example 20. Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.



Solution : Applying KVL in loop ABCDA



$$\varepsilon - iR - (i - i_1)R = 0$$

$$\varepsilon - 2iR + i_1R = 0 \qquad \text{...(i)}$$

Applying KVL in loop ABCEFDA

$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

by eq (i) $\dfrac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \dfrac{q}{C} \quad \Rightarrow \quad \varepsilon C - 3i_1RC = 2q$

$$\varepsilon C - 2q = 3\frac{dq}{dt}.RC \quad \Rightarrow \quad \int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$$

$$-\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC} \quad \Rightarrow \quad q = \frac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$$

Method for objective :

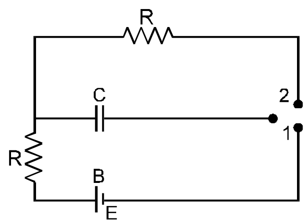
In any circuit when there is only one capacitor then

$q = Q_{st}\left(1 - e^{-t/\tau}\right)$; Q_{st} = steady state charge on capacitor

$\tau = R_{eff}.\,C$

$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

Discharging of a condenser :

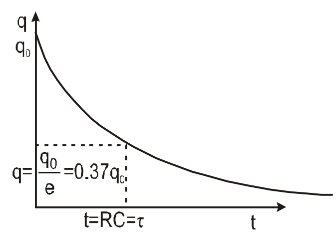


(i) In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\,e^{-(t/RC)}$

i.e. the charge falls exponentially.

here q_0 = initial charge of capacitor



(iii) If t = RC = τ = time constant, then

$$q = \frac{q_0}{e} = 0.37q_0 = 37\% \text{ of } q_0$$

i.e. the time constant is that time during which the charge on condenser plates in discharge process, falls to 37%

(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

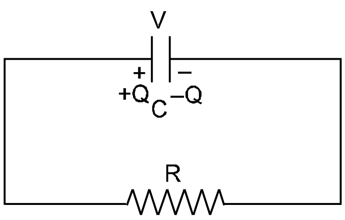
(v) The potential difference across the condenser plates at any instant of time t is given by

$V = V_0e^{-(t/RC)}$ Volt.

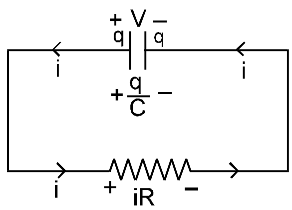
(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.

i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

Derivation of equation of discharging circuit :



Initially



At any time t

Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

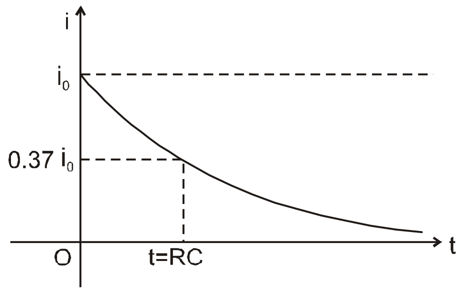
$$i = \frac{q}{CR}$$

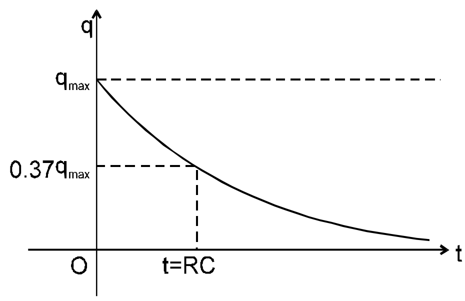
$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$

$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q \, . \, e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC} = i_0e^{-t/RC}$$





SOLVED EXAMPLE

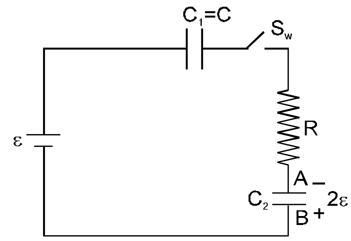
Example 21 At $t = 0$, switch is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following

(Given $C_1 = C_2 = C$)

(a) Charge on C_1 and C_2 as a function of time.

(b) Find out current in the circuit as a function of time.

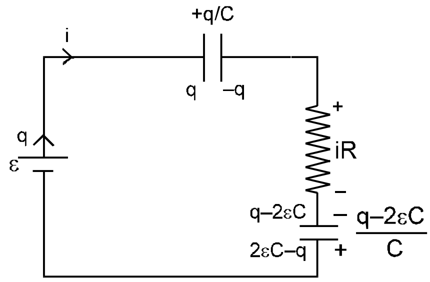
(c) Also plot the graphs for the relations derived in part (a).



Solution : Let q charge flow in time 't' from the battery as shown.

The charge on various plates of the capacitor is as shown in the figure.

Now applying KVL



$$\varepsilon - \frac{q}{C} - iR - \frac{q - 2\varepsilon C}{C} = 0$$

$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR \quad \Rightarrow \quad 3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon C - iRC = 2q \quad \Rightarrow \quad \frac{dq}{dt}RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\frac{1}{2}\ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC} \quad \Rightarrow \quad 3\varepsilon C - 2q = 3\varepsilon C\, e^{-2t/RC}$$

$$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q \quad \Rightarrow \quad q = \frac{3}{2}\varepsilon C\,(1 - e^{-2t/RC})$$

(charge on C, as function of time) **Ans.**

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}\, e^{-2t/RC}$$ **Ans.**

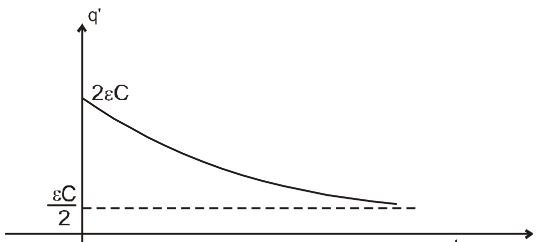
Charge on C_2 as function of time :

$q' = 2\varepsilon C - q$

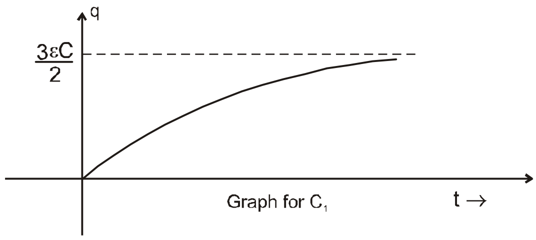
$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C\, e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon C e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$$



Graph for C_2

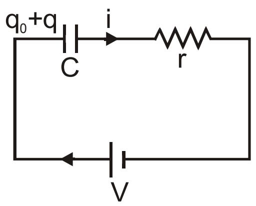


Graph for C_1

Example 22. A capacitor of capacitance C is charged by charge q_0. At t = 0, it is connected to a battery of emf V and internal resistance r. Find the charge on the capacitor at time t (positive plate of capacitor connected with positive plate of battery).

Answer: $CV(1-e^{-t/Cr}) + q_0e^{-t/Cr}$

Solution :



at times t using KVL

$$-\frac{q_0+q}{C} - ir + V = 0$$

$$\Rightarrow \quad -\frac{q_0+q}{C} - \frac{dq}{dt}r + V = 0$$

$$\Rightarrow \quad \int_0^q \frac{dq}{(CV-q_0)-q} = \int_0^t \frac{dt}{rC}$$

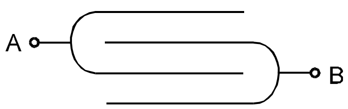
by using integration $\quad q = CV\,(1 - e^{+\infty}) + q_0\,e^{+\infty} - q_0$

$\Rightarrow$ So charge on capacitor = $q_0 + q = CV(1 - e^{+\infty}) + q_0e^{+\infty}$

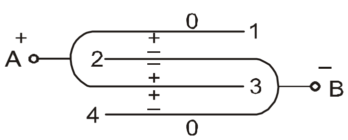
COMBINATION OF PARALLEL PLATES

SOLVED EXAMPLE

Example 23. Find out equivalent capacitance between A and B. (take each plate Area = A and distance between two conjugative plates is d)



Solution : Let numbers on the plates The charges will be as shown in the figure.

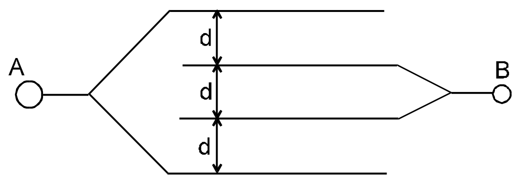


$V_{12} = V_{32} = V_{34}$

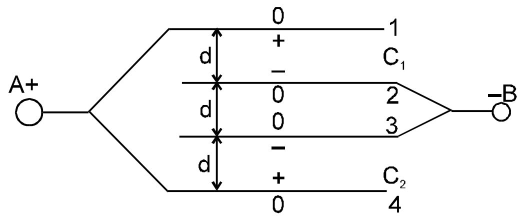
so all the capacitors are in parallel combination.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{3A\,\epsilon_0}{d}$$

Example 24. Find out equivalent capacitance between A and B. (take each plate Area = A)



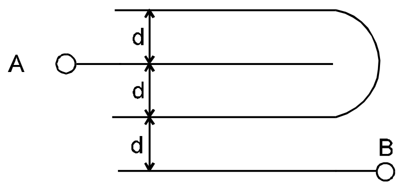
Solution :



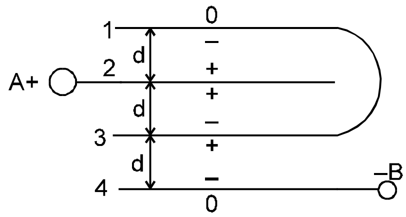
These are only two capacitors.

$$C_{eq} = C_1 + C_2 = \frac{2A \in_0}{d}$$

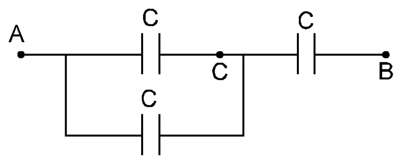
Example 25. Find out equivalent capacitance between A and B. (take each plate Area = A)



Solution :

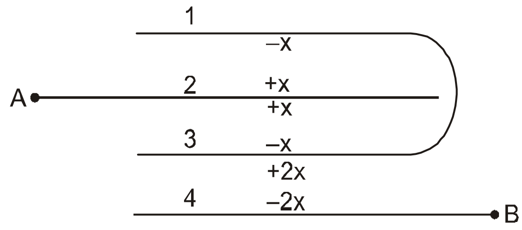


The modified circuit is



$$C_{eq} = \frac{2C}{3} = \frac{2A \in_0}{3d}$$

Other method : Let charge density as shown



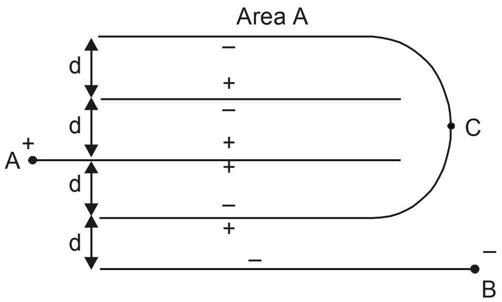
$$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$$

$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4)$$

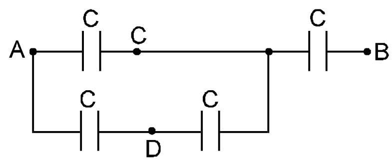
$$= \frac{xd}{\epsilon_0} + \frac{2xd}{\epsilon_0} = \frac{3xd}{\epsilon_0}$$

$$\therefore \quad C_{eq} = \frac{2Ax\epsilon_0}{3xd} = \frac{2A\epsilon_0}{3d} = \frac{2C}{3}.$$

Example 26. Find out equivalent capacitance between A and B.



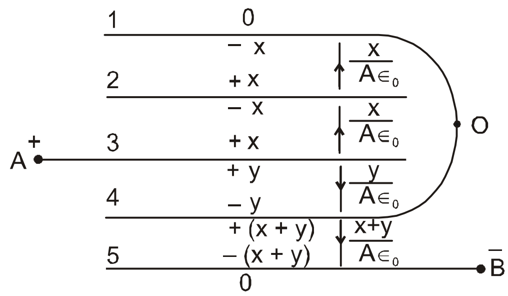
Solution : Let $C = \frac{A\epsilon_0}{d}$ Equivalent circuit : -



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_{eq} = \frac{3C}{5} = \frac{3A\epsilon_0}{5d}$$

Alternative Method : Let charge distribustion on plates as shown :



$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

Potential of 1 and 4 is same

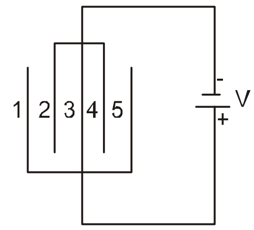
$$\frac{y}{A\epsilon_0} = \frac{2x}{A\epsilon_0}$$

$$y = 2x$$

$$V = \left(\frac{2y+x}{A\varepsilon_0}\right)d$$

$$\Rightarrow \quad C = \frac{(x+2x)A\epsilon_0}{(5x)d} = \frac{3A\epsilon_0}{5d}.$$

Example 27. Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. V as shown in the following diagram. The charge on the plates 1 and 4 will be-



(A) $\frac{\in_0 A}{d}, \frac{-2\in_0 A}{d}$ (B) $\frac{\in_0 AV}{d}, \frac{-2\in_0 AV}{d}$

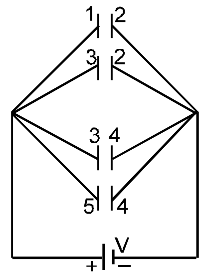
(C) $\frac{-\in_0 AV}{d}, \frac{-3\in_0 AV}{d}$ (D) $\frac{\in_0 AV}{d}, \frac{-4\in_0 AV}{d}$

Answer: **(B)**

Solution : by equivalent circuit diagram Charge on first plate

$Q = CV$

$Q = \frac{\in_0 AV}{d}$



Charge on fourth plate

$Q' = C(-V)$ $Q' = \frac{-\in_0 AV}{d}$

As plate 4 is repeated twice, hence charge on 4 will be $Q'' = 2Q'$

$Q'' = -\frac{2\in_0 AV}{d}$

Hence the correct answer will be (B).

DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.
- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.

Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation vector $\vec{P}$**

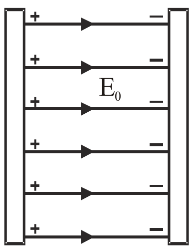
This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

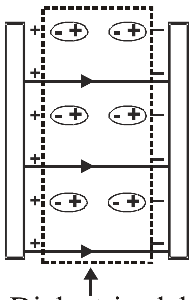
$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

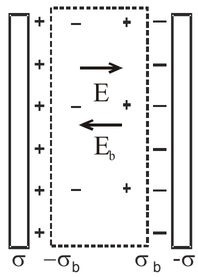
where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$\vec{P} = n\vec{p} = \frac{q_b d}{Ad} = \left(\frac{q_b}{A}\right) = \sigma_b$ = induced (bound) surface charge density.

Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$




Dielectric slab



- **Bound Charge and Bound Charge density:**

Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_b = \frac{\sigma - \sigma_b}{\varepsilon_0} = \frac{Q - Q_b}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric $C_0 = \dfrac{Q}{V_0}$

Capacitance in presence of dielectric $C = \dfrac{Q - Q_b}{V}$

It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

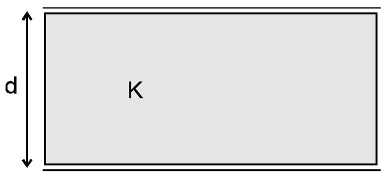
The dielectric constant or relative permittivity K

or $\varepsilon_r = \dfrac{E_0}{E} = \dfrac{V_0}{V} = \dfrac{C}{C_0} = \dfrac{Q}{Q - Q_b} = \dfrac{\sigma}{\sigma - \sigma_b} = \dfrac{\varepsilon}{\varepsilon_0}$

From $K = \dfrac{Q}{Q - Q_b} \Rightarrow Q_b = Q\left(1 - \dfrac{1}{K}\right)$ and $K = \dfrac{\sigma}{\sigma - \sigma_b} \Rightarrow \sigma_b = \sigma\left(1 - \dfrac{1}{K}\right)$

CAPACITANCE IN THE PRESENCE OF DIELECTRIC

If capacitor is completely filled with dielectric



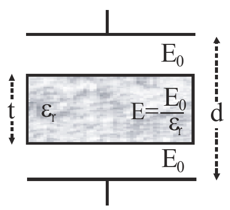
$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K \in_0}.d} = \frac{AK \in_0}{d} = \frac{AK \in_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK \in_0}{d}$$

In case of parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$

If capacitor is partially filled with dielectric



When the dielectric is filed partially between plates, the thickness of dielatric slab is t(t < d).

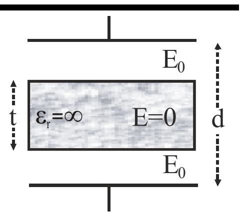
If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \frac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \frac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space (d – t). If V is total potential then $V = E_0(d - t) + E\,t$

$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

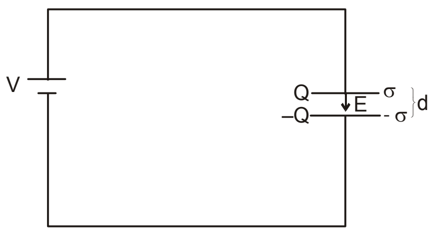
$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \quad ...(i)$$

Note: If capacitor is partialy filled by a conducting slab of thickness (t< d).

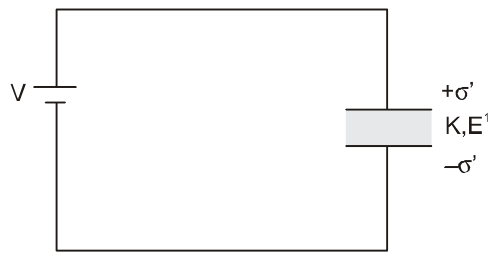


$\because$ $\varepsilon_r = \infty$ for conductor $C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{(d - t)}$

Comparison of E (electric field), σ (surface charges density), Q (charge), C (capacitance) and before and after inserting a dielectric slab between the plates of a parallel plate capacitor.



Case I



Case II

$C = \frac{\epsilon_0 A}{d}$	$C' = \frac{A\epsilon_0 K}{d}$
$Q = CV$	$Q' = C'V$
$E = \frac{\sigma}{\epsilon_0} = \frac{CV}{A\epsilon_0}$	$E' = \frac{\sigma}{K\epsilon_0} = \frac{CV}{A\epsilon_0}$
$= \frac{V}{d}$	$E' = \frac{V}{d}$

Here potential difference between the plates,

$$Ed = V$$

$$E = \frac{V}{d}$$

$$\frac{V}{d} = \frac{\sigma}{\epsilon_0}$$

Here potential difference between the plates

$$E'd = V$$

$$E' = \frac{V}{d}$$

$$\frac{V}{d} = \frac{\sigma'}{K\epsilon_0}$$

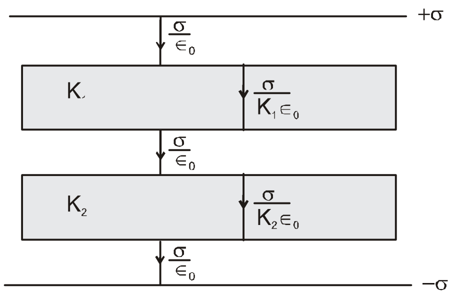
Equating both

$$\frac{\sigma}{\epsilon_0} = \frac{\sigma'}{K\epsilon_0}$$

$$\sigma' = K\sigma$$

In the presence of dielectric, i.e. in case II capacitance of capacitor is more.

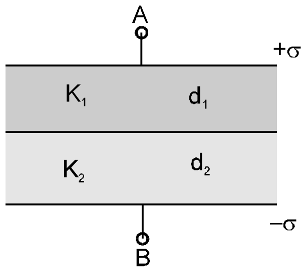
If capacitor is filled with multiple dielectrics



- **Energy density in a dielectric** = $\frac{1}{2}\epsilon_0\epsilon_r E^2$

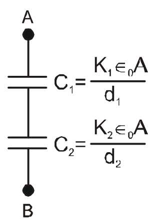
SOLVED EXAMPLE

Example 28 Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Solution : $C = \frac{\sigma A}{V}$; $V = E_1 d_1 + E_2 d_2 = \frac{\sigma d_1}{K_1\epsilon_0} + \frac{\sigma d_2}{K_2\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$

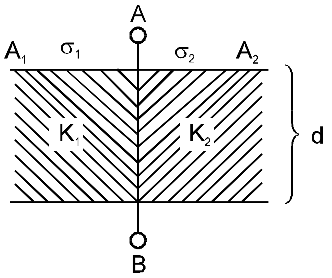
$$\therefore C = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \Rightarrow \frac{1}{C} = \frac{d_1}{AK_1\epsilon_0} + \frac{d_2}{AK_2\epsilon_0}$$



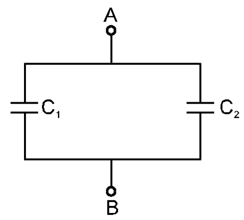
This formula suggests that the system between A and B can be considered as series combination of two capacitors.

Example 29. Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. ($A_1 + A_2 = A$)

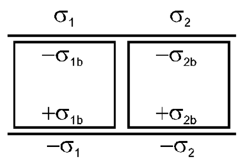
Solution :



$\equiv$

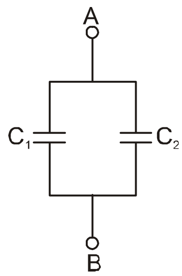


$$C_1 = \frac{A_1K_1 \epsilon_0}{d},\ C_2 = \frac{A_2K_2 \epsilon_0}{d}$$



$$E_1 = \frac{V}{d} = \frac{\sigma_1}{K_1 \epsilon_0},\ E_2 = \frac{V}{d} = \frac{\sigma_2}{K_2 \epsilon_0}$$

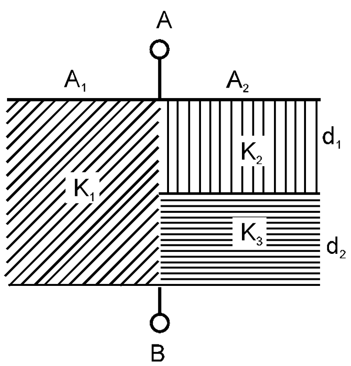
$$\sigma_1 = \frac{K_1 \epsilon_0 V}{d} \qquad \sigma_2 = \frac{K_2 \epsilon_0 V}{d}$$



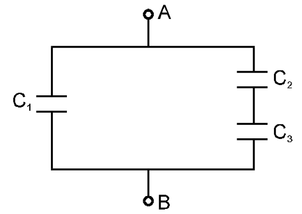
$$C = \frac{Q_1 + Q_2}{V} = \frac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \frac{K_1 \epsilon_0 A_1}{d} + \frac{K_2 \epsilon_0 A_2}{d}$$

The combination is equivalent to : $C = C_1 + C_2$

Example 30. Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)



Solution : It is equivalent to

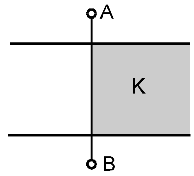


$$C = C_1 + \frac{C_2C_3}{C_2 + C_3}$$

$$C = \frac{A_1K_2 \in_0}{d_1 + d_2} + \frac{\frac{A_2K_2 \in_0}{d_1} \cdot \frac{A_2K_3 \in_0}{d_1}}{\frac{A_2K_2 \in_0}{d_1} + \frac{A_2K_3 \in_0}{d_2}}$$

$$= = \frac{A_1K_1 \in_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \in_0}{A_2K_2 \in_0 d_2 + A_2K_3 \in_0 d_1} = \frac{A_1K_2 \in_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \in_0}{K_2d_2 + K_3d_1}$$

Example 31. A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-



(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$ (C) $\frac{C}{2}[1 + K]$ (D) $\frac{2[1+K]}{C}$

Solution : This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2$$

$$= \frac{\in_0 A/2}{d} + \frac{\in_0 (A/2)K}{d} = \frac{\in_0 A}{2d}[1 + K] = \frac{C}{2}[1 + K]$$

Hence the correct answer will be (C).

Note: **For Variable Dielectric Constant,** If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\frac{1}{C} = \int d\left(\frac{1}{C}\right)$ is solved to get equivalent capacitance C.

SOLVED EXAMPLE

Example 32. A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Solution : If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9 × 10^7 V/m.

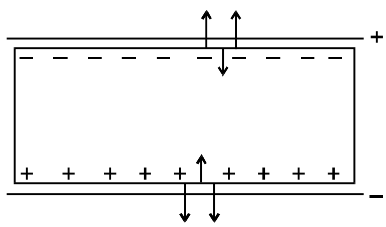
$\therefore$ if the maximum charge which can be given is Q

then $\frac{Q}{KA\varepsilon_0} = 1.9\times10^7$ V/m, $\because$ A = 100 cm^2 = 10^{-2} m^2

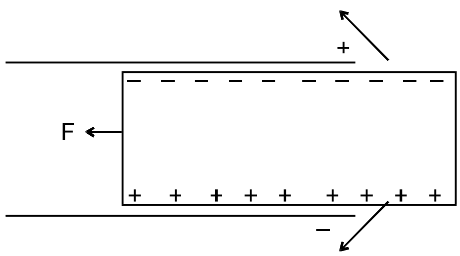
$\Rightarrow$ Q = (5.0) × (10^{-2}) × (8.85 × 10^{-12}) × (1.9 × 10^7) = 8.4 × 10^{-6} C.

Force on a dielectric due to charged capacitor :-

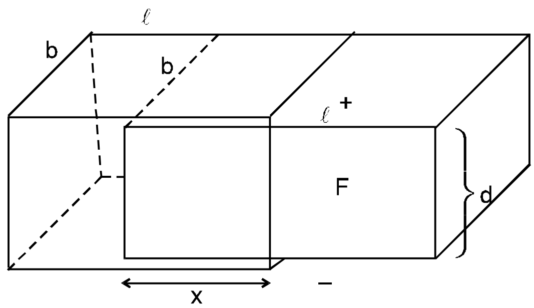
(a) If dielectric is completely inside the capacitor then force is equal to zero.



(b) If dielectric is not completely inside the capacitor.



Case I - Voltage source remains connected



V = constant.

$U = \frac{1}{2}CV^2$

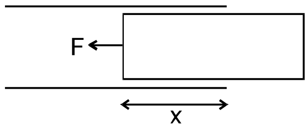
$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx}$

where $C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d}$ $\Rightarrow$ $C = \frac{\epsilon_0 b}{d}[Kx + \ell - x]$

$\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)$

$\therefore F = \frac{\epsilon_0 b(K-1)V^2}{2d}$ = constant (does not depend on x)

Case II : When charge on capacitor is constant



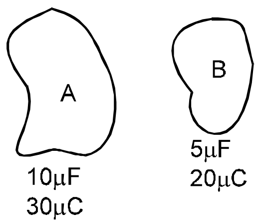
$$C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d}, \qquad U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \qquad \left[\text{where, } \frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)\right]$$

$$= \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \qquad \text{(here force 'F' depends on x)}$$

MISCELLANEOUS SOLVED EXAMPLE

Problem 1. When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.



(A) A to B (B) B to A (C) will not flow (D) can not say.

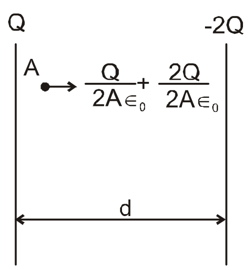
Solution : Charge always flows from higher potential body to lower potential body

Hence, $V_A = \frac{30}{10} = 3V \Rightarrow V_B = \frac{20}{5} = 4\ V$

As $V_B > V_A$ $\therefore$ (B) is correct Answer.

Problem 2. Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Solution : Capacitance = C



Electric field = $\frac{3Q}{2A\epsilon_0}$

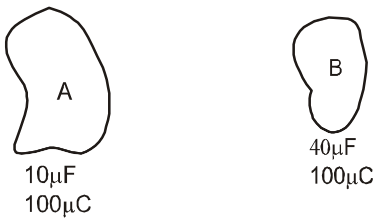
$$V = \frac{3Qd}{2A\epsilon_0}$$

$$\Rightarrow \quad V = \frac{3Q}{2C}$$

Problem 3. A conductor of capacitance $10\mu F$ connected to other conductor of capacitance $40\ \mu F$ having equal charges $100\ \mu C$ initially. Find out final voltage and heat loss during the process?

Answer : (i) V = 4V (ii) H = 225 μJ.

Solution :



$C_1 = 10\mu F$ $\quad$ $C_2 = 40\mu F$

$Q_1 = 100\ \mu C$ $\quad$ $Q_2 = 100\mu C$

$V_1 = Q_1/C_1 = 10\ V$ $\quad$ $V_2 = Q_2/C_2 = 2.5$

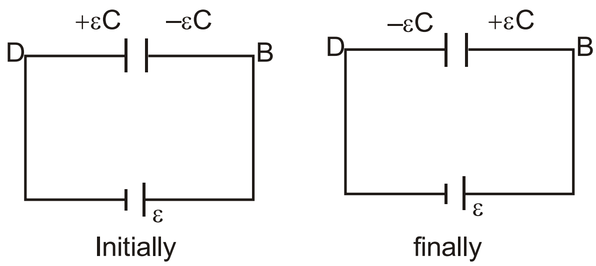
$$\text{Final voltage (V)} = \frac{C_1V_1 + C_2V_2}{(C_1 + C_2)} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{200\,\mu C}{50\,\mu F} = 4V$$

$$\text{Heat loss during the process} = \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2\right] - \frac{1}{2}V^2(C_1 + C_2)$$

$$= \frac{1}{2}\left[Q_1V_1 + Q_2V_2\right] - \frac{1}{2}V^2(C_1 + C_2)$$

$$= \frac{1}{2} \times 100\,\mu[12.5] - \frac{1}{2} \times 16\ (50)\ \mu = 225\ \mu J$$

Problem 4. A capacitor of capacitance C is charged from battery of e.m.f. ε and then disconnected. Now the positive terminal of the battery is connected with negative plate of capacitor. Find out heat loss in the circuit during the process of charging.



Net charge flow through battery = $2\varepsilon C$

Work done by battery = $\varepsilon \times 2\varepsilon C = 2\varepsilon^2 C$

Heat produced = $2\varepsilon^2 C$. **Ans.**

Solution : From figure

Net charge flow through

battery = $Q_{final} - Q_{initial} = \varepsilon C - (-\varepsilon C) = 2\varepsilon C$

$\therefore$ workdone by battery (W) = $Q \times V = 2\varepsilon C \times \varepsilon = 2\varepsilon^2 C$

or Heat produced = $2\varepsilon^2 C$

Problem 5. Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Solution : When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

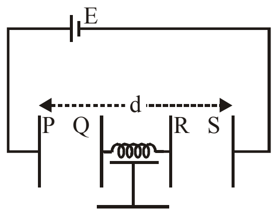
Equating volume we get $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3} C$

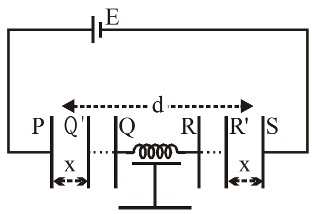
Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3} V$

Problem 6. Two parallel plate capacitors with area A are connected through a conducting spring of natural length ℓ in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



Solution : At any time distance between plates P and Q, R and S is same because force acting on them is same. Let charge on capacitors be q and separation between plates P and Q, R and S be x



Capacitance of capacitor PQ, $C_1 = \frac{\varepsilon_0 A}{x}$

Capacitance of capacitor RS, $C_2 = \frac{\varepsilon_0 A}{x}$ From KVL $\frac{q}{C_1} + \frac{q}{C_2} = E \Rightarrow q = \frac{\varepsilon_0 AE}{2x}$

At this moment extension in spring, $y = d - 2x - \ell$.

Force on plate Q towards P, $F_1 = \frac{q^2}{2A\varepsilon_0} = \frac{\varepsilon_0^2 A^2 E^2}{8Ax^2\varepsilon_0} = \frac{A\varepsilon_0 E^2}{8x^2}$

Spring force on plate Q due to extension in spring, $F_2 = ky$

At equilibrium, separation between plates = extension in spring

Thus x= y = d– 2x – $\ell \Rightarrow x = \frac{d-\ell}{3}$ ...(i) and $F_1 = F_2$ ...(ii)

From eq. (i) and (ii), $\frac{A\varepsilon_0 E^2}{8x^2}$ = ky = kx $\Rightarrow x = \left(\frac{A\varepsilon_0 E^2}{8k}\right)^{1/3}$ ...(iii)

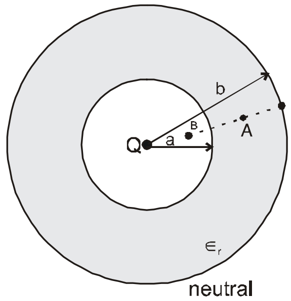
From eq. (i) and (iii), $\left(\frac{d-\ell}{3}\right) = \frac{A\varepsilon_0 E^2}{8k} \Rightarrow k = \frac{A\varepsilon_0 E^2 27}{8(d-\ell)^3}$

Problem 7. Find V and E at : (Q is a point charge kept at the centre of the nonconducting neutral thick sphere of inner radius 'a' and outer radius 'b') (dielectric constant = $\in_r$)

(i) 0 < r < a

(ii) a ≤ r < b

(iii) r ≥ b

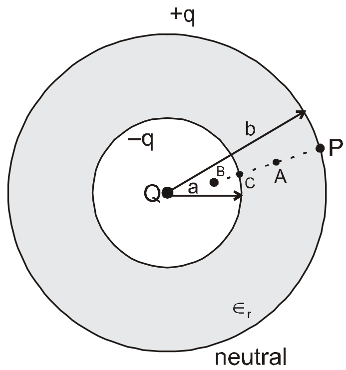


Solution : –q and +q charge will induce on inner and outer surface respectively

$E(0 < r < a) = \frac{KQ}{r^2}$

$E\ (r \geq b) = \frac{KQ}{r^2}$

$E\ (a \leq r < b) = \frac{KQ}{r^2} - \frac{Kq}{r^2} = \frac{KQ}{\in_r r^2}$ **Ans.**



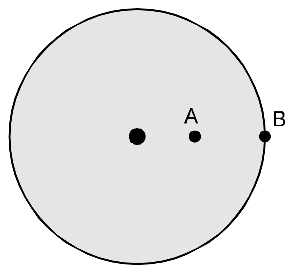
$q = Q\left(1 - \frac{1}{\in_r}\right)$.

$V\ (r \geq b) = \frac{KQ}{r}$

$(a \leq r \leq b)\ V_A = V_P + \int_b^r \frac{KQ}{\in_r r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\in_r}\left(\frac{1}{r} - \frac{1}{b}\right)$

$V\ (r \leq a)\ V_B = V_C + \int_a^r \frac{KQ}{r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\in_r}\left(\frac{1}{a} - \frac{1}{b}\right) + kQ\left(\frac{1}{r} - \frac{1}{a}\right)$

Problem 8. What is potential at a distance r (<R) in a dielectric sphere of uniform charge density ρ , radius R and dielectric constant ε_r..

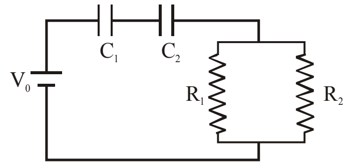


Solution : $V_A = V_B + \frac{W_{B\to A}}{q}$

$V = \frac{Q}{4\pi \in_0 R} + \int_R^r \frac{\rho r}{3 \in_0 \in_r}(-dr) = \frac{Q}{4\pi \in_0 R} + \frac{\rho(R^2 - r^2)}{3 \in_0 \in_r}$

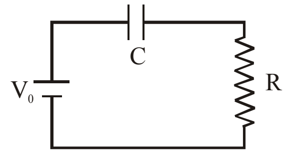
$V_{outside} = \frac{KQ}{r}$

Problem 9. Find the time constant for given circuit if



$R_1 = 4\Omega$, $R_2 = 12\Omega$, $C_1 = 3\mu F$ and $C_2 = 6\mu F$.

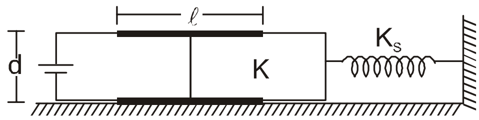
Solution : Given circuit can be reduced to :



$$C = \frac{C_1C_2}{C_1+C_2} = \frac{3\times 6}{3+6} = 2\mu F\ ,\ R = \frac{R_1R_2}{R_1+R_2} = \frac{4\times 12}{4+12} = 3\Omega$$

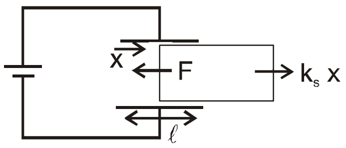
Time constant = RC = (3) (2×10^{-6}) = $6\mu s$

Problem 10. Consider the situation shown in figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf V. All surface are frictionless. Calculate the extension in the spring in equilibrium (spring is nonconducting).



Answer: $\dfrac{\epsilon_0\, bv^2(K-1)}{2dK_S}$

Solution :



Potential energy $= \frac{1}{2}\ (C_1 + C_2)\, V^2$

$$= \frac{1}{2}\left\{\frac{\epsilon_o\, xb}{d} + \frac{\epsilon_o\, K(\ell - x)b}{d}\right\} V^2$$

$$F = -\frac{\partial U}{\partial x} = -\frac{V^2}{2}\frac{dC}{dx}$$

$$F = \frac{-1}{2}\frac{dC}{dx}\ V^2$$

where , $\dfrac{dC}{dx} = \dfrac{\epsilon_o\, b}{d}\ \{1 - K\}$

$$F = \frac{1}{2}\ \frac{V^2 \epsilon_o\, b(K-1)}{d}$$

at equilibrium , $F = K_s x$

$$x = \frac{F}{k_s} = \frac{\epsilon_0\, bv^2(K-1)}{2dK_S}$$

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Definition of capacitance

A-1. Two large parallel conducting plates are 34 mm apart and carry equal but opposite charges on their facing surfaces. An electron placed midway between the plates experiences a force of 3.2×10^{-16} N. What is the potential difference (in volts) between the plates?

A-2. When $30\mu C$ charge is given to an isolated conductor of capacitance $5\mu F$. Find out the following

(i) Potential of the conductor

(ii) Energy stored in the electric field of conductor

(iii) If this conductor is now connected to another isolated conductor by a conducting wire (at very large distance) of total charge 50 μC and capacity 10 μF then

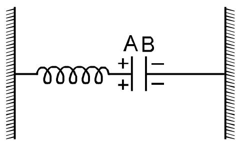
(a) find out the common potential of both the conductors.

(b) Find out the heat dissipated during the process of charge distribution.

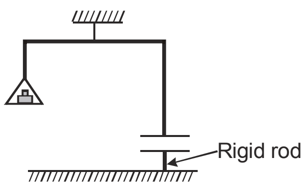
(c) Find out the ratio of final charges on conductors.

(d) Find out the final charges on each conductor.

A-3. Plate A of a parallel air filled capacitor is connected to a nonconducting spring having force constant k and plate B is fixed. If a charge + q is placed on plate A and charge – q on plate B then find out extension in the spring in equilibrium. Assume area of plate is 'A'.



A-4. The lower plate of a parallel plate capacitor is supported on a rigid rod. The upper plate is suspended from one end of a balance. The two plates are joined together by a thin wire and subsequently disconnected. The balance is then counterpoised. Now a voltage V = 5000 volt is applied between the plates. The distance between the plates is d = 5 mm and the area of each plate is A = 100 cm^2. Then find out the additional mass placed to maintain balance. [All the elements other than plates are massless and nonconducting]



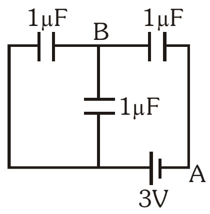
A-5. Each plate of a parallel plate air capacitor has an area S. What amount of work has to be performed by external agent to slowly increase the distance between the plates from x_1 to x_2 if:

(i) the charge of the capacitor, which is equal to q is kept constant in the process.

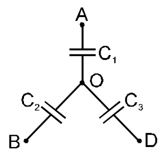
(ii) the voltage across the capacitor, which is equal to V is kept constant in the process.

Section (B) : Circuits with capacitor and use of KCL and KVL

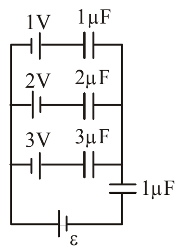
B-1. If potential of A is 5V, then potential of B in volt is



B-2. Three uncharged capacitors of capacitance $C_1 = 1\mu F$, $C_2 = 2\mu F$ and $C_3 = 3\mu F$ are connected as shown in the figure. The potential of point A, B and D are 10 volt, 25 volt and 20 volt respectively. Determine the potential at point O.



B-3. In the figure shown, find the e.m.f. ε for which charge on 2μF capacitor is 4 μC.



B-4. A capacitor of capacitance C, a resistor of resistance R and a battery of emf ε are connected in series at t = 0. What is the maximum value of

(a) the potential difference across the resistor.

(b) the current in the circuit.

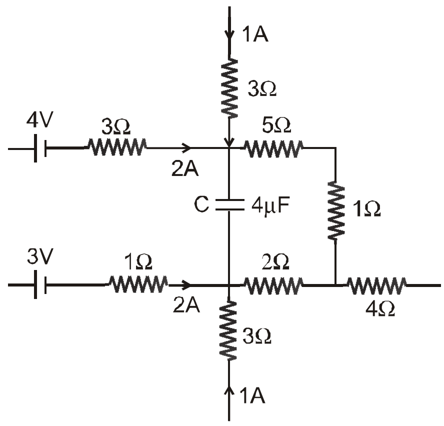
(c) the potential difference across the capacitor.

(d) the energy stored in the capacitor.

(e) the power delivered by the battery.

(f) the power converted into heat.

B-5. A part of circuit in a steady state along with the current flowing in the branches, the values of resistance etc., is shown in the figure. Calculate the energy stored in the capacitor C (4μF) **[1986, 4M]**



B-6. A capacitor having a capacitance of 200 μF is charged to a potential difference of 20V. The charging battery is disconnected and the capacitor is connected to another battery of emf 10V with the positive plate of the capacitor joined with the positive terminal of the battery.

(a) Find the charges on the capacitor before and after the reconnection in steady state.

(b) Find the net charge flown through the 10 V battery

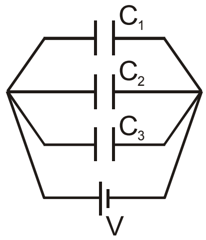
(c) Is work done by the battery or is it done on the battery? Find its magnitude.

(d) Find the decrease in electrostatic field energy.

(e) Find the heat developed during the flow of charge after reconnection.

Section (C) : Combination of capacitors

C-1.



(i) Find out the charges on the three capacitors connected to a battery as shown in figure.

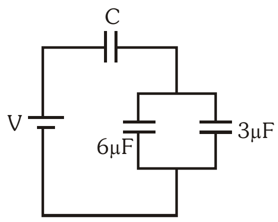
Take C_1= 1.0 μF, C_2 = 2.0 μF, C_3 = 3.0 μF and V = 20 volt.

(ii) Find out the work done by the battery during the process of charging

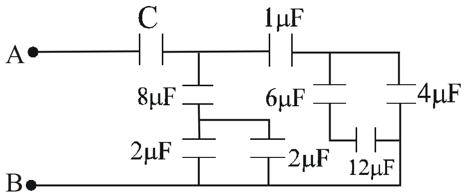
(initially all the capacitors are uncharged)

(iii) Find out the total energy stored in the capacitors .

C-2. If charge on 3μF capacitor is 3μC. Find the charge on capacitor of capacitance C in μC.



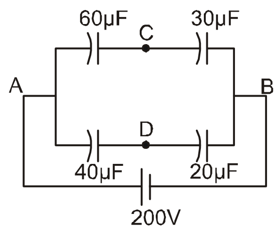
C-3. In the following circuit, the resultant capacitance between A and B is 1 μF. Find the value of C.



C-4. If you have several 2.0 μF capacitors, each capable of withstanding 200 volts without breakdown, how would you assemble a combination having minimum number of capacitors and of given equivalent capacitance which capable of withstanding 1000 volts ;

(a) 0.40 μF (b) 1.2 μF

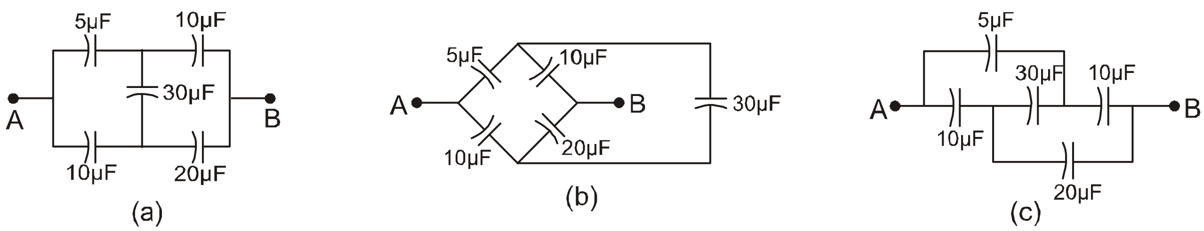
C-5. Take the potential of the point B as shown in the figure to be 100 V.



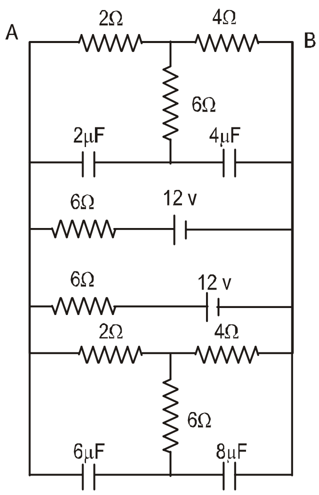
(a) Find the potentials at the point C and D.

(b) If an uncharged capacitor is connected between C and D, then find the amount of charge that will appear on this capacitor

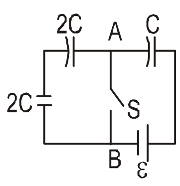
C-6. Find the capacitance between the point A and B of the given assemblies.



C-7. Find the final charges in steady state on the four capacitors of capacitance $2\mu F$, $4\mu F$, $6\mu F$ and $8\mu F$ as shown in figure. (Assuming initially they are uncharged). Also find the current through the wire AB at steady state.



C-8. Consider the situation shown in the figure. The switch S is open for a long time and then closed and again steady state reached then



(a) Find the charge flown through the battery after the switch S is closed.

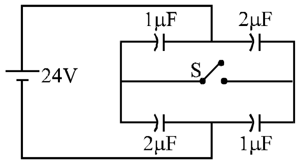
(b) Find the charge flown through the switch S from B to A.

(c) Find the work done by the battery after the switch S is closed.

(d) Find the change in energy stored in the system of capacitors.

(e) Find the heat developed in the system after the switch S is closed.

C-9. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed ?



Section (D) : Equation of charging and discharging

D-1. A capacitor is connected to a 12 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1μs. Find the capacitance of the capacitor.

(Given : $\ell n3 = 1.0986$, $\ell n2 = 0.693$)

D-2. A capacitor of capacitance 200 μF is connected across a battery of emf 10.0 V through a resistance of 40 kΩ for 16.0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 16.0 s after the battery is disconnected ? (Given : $e^{-2} = 0.135$)

D-3. A capacitor of capacity 1μF is connected in a closed series circuit with a resistance of 10^7 ohms, an open key and a cell of 2 V with negligible internal resistance:

(i) When the key is switched on at time t = 0, find;

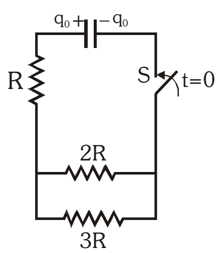
(a) The time constant for the circuit.

(b) The charge on the capacitor at steady state.

(c) Time taken to deposit charge equal to half of charge that will deposit at steady state.

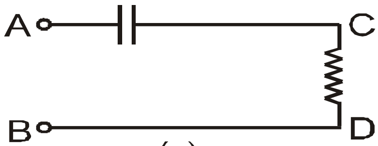
(ii) If after completely charging the capacitor, the cell is shorted by zero resistance at time t = 0, find the charge on the capacitor at t = 50 s. (Given : $e^{-5} = 6.73 \times 10^{-3}$, ln2 = 0.693)

D-4. At t = 0 charge on capacitor is q_0. Now switch S is closed. Heat loss in 3R is $x \times 10^{-6}$ J. Then find the value of x. [Given q_0= 15μC, C = 6/55 μF]

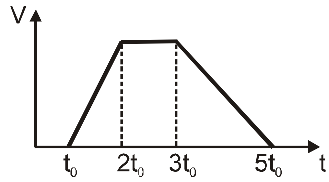


D-5. A 5.0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance 5.0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the connections are made. (Given : $e^{-2} = 0.135$)

D-6. A varying voltage is applied to the clamps AB (figure a) such that the voltage across the capacitor plates varies as shown in figure b.



(a)



(b)

Plot the time dependence of voltage across the clamps CD.

Section (E) : Capacitor with dielectric

E-1. The parallel plates of a capacitor have an area 0.2 m^2 and are 10^{-2} m apart. The original potential difference between them is 3000 V, and it decreases to 1000 V when a sheet of dielectric is inserted between the plates filling the full space. Compute: ($\in_0 = 9 \times 10^{-12}$ S. I. units)

(i) Original capacitance C_0. (ii) The charge Q on each plate.

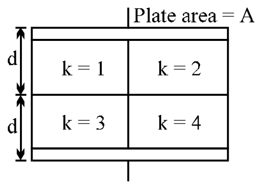
(iii) Capacitance C after insertion of the dielectric. (iv) Dielectric constant K.

(v) Permittivity $\in$ of the dielectric.

(vi) The original field E_0 between the plates.

(vii) The electric field E after insertion of the dielectric.

E-2 Find the capacitance of the system shown in figure.

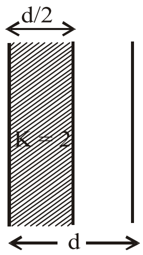


E-3. A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. Find the capacitance of the system. If K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the condenser with air filling the full space? Calculate the ratio of the energy in the two cases and account for the energy change (assuming q charge on the plate to be constant).

E-4. Hard rubber has a dielectric constant of 2.8 and a dielectric strength (maximum electric field)of 18 x 10^6 volt/meter. If it is used as the dielectric material filling the full space in a parallel plate capacitor. What minimum area may the plates of the capacitor have in order that the capacitance be 7.0 x 10^{-2} μF and that the capacitor be able to withstand a potential difference of 4000 volts. ($\in_0 = 8.85 \times 10^{-12}$ S.I unit)

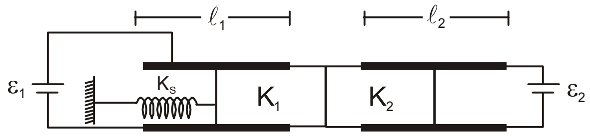
E-5. Two parallel plate air capacitors each of capacitance C were connected in series to a battery with e.m.f. ε. Then one of the capacitors was filled up with uniform dielectric with relative permittivity k. How many times did the electric field strength in that capacitor decrease? What amount of charge flows through the battery?

E-6. A certain series RC circuit is formed using a resistance R, a capacitor without dielectric having a capacitance C = 2F and a battery of emf E = 3V. The circuit is completed and it is allowed to attain the steady state. After this, at t = 0, half the thickness of the capacitor is filled with a dielectric of constant K = 2 as shown in the figure. The system is again allowed to attain a steady state. What will be the heat generated (in joule) in the circuit between t = 0 and t = ∞ ?

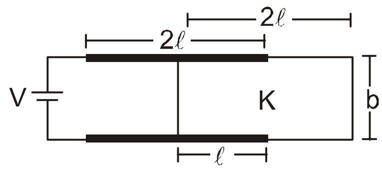


E-7. A capacitor filled with dielectric of permittivity ε = 2.1 loses half the charge acquired during a time interval τ = 3.0 min. Assuming the charge to leak only through the dielectric filler, calculate its resistivity.

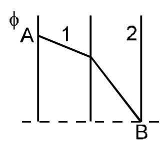
E-8. In figure shown, two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is same for the two capacitors. The plates are rectangular in shape with width b and lengths ℓ_1 and ℓ_2, the separation between plates is d. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2 ($K_2 > K_1$). EMF of the right battery is greater then left battery. Neglecting any friction, find the extension in spring in equilibrium (spring is nonconducting) ($\varepsilon_2 > \varepsilon_1$)



E-9. The plates of the parallel plate capacitor have plate area A and are clamped in the laboratory as shown in figure. The dielectric slab of mass m, length 2ℓ and width 2ℓ is released from rest with length ℓ inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period. (plates of capacitor are square plates of side 2ℓ)



E-10. A parallel plate capacitor is filled with a dielectric up to one half of the distance between the plates. The manner in which the potential between the plates varies with distance is illustrated in the figure. Which half (1 or 2) of the space between the plates is filled with the dielectric and what will be the distribution of the potential after the dielectric is taken out of the capacitor provided that;



(a) The charges on the plates are conserved or

(b) The potential difference across the capacitor is constant.

PART - II : OBJECTIVE QUESTIONS

*** Marked Questions may have more than one correct option.**

Section (A) : Definition of Capacitance

A-1. Choose the CORRECT statement :-

(A) C will increase on increasing Q (B) C will increase on decreasing Q

(C) C will increase on decreasing V (D) C doesnot depend on Q & V

A-2. Two isolated charged metallic spheres of radii R_1 and R_2 having charges Q_1 and Q_2 respectively are connected to each other, then there is:

(A) No change in the electrical energy of the system

(B) An increase in the electrical energy of the system

(C) A decrease in the electrical energy of the system in any case

(D) A decrease in electrical energy of the system if $Q_1 R_2 \neq Q_2 R_1$

A-3. The radii of two metallic spheres are 5 cm and 10 cm and both carry equal charge of 75μC. If the two spheres are shorted then charge will be transferred–

(A) 25 μC from smaller to bigger (B) 25 μC from bigger to smaller

(C) 50 μC from smaller to bigger (D) 50 μC from bigger to smaller

✍**A-4.** A parallel plate capacitor is charged up to a potential of 300 volts. Area of the plates is 100 cm^2 and spacing between them is 2 cm. If the plates are moved apart to a distance of 2.5 cm without disconnecting the power source, then ($\epsilon_0 = 9 \times 10^{-12}\ C^2 N^{-1} m^{-2}$):

(i) Electric field inside the capacitor when distance is 2.5 cm :

(A) 15×10^2 V/m (B) 3×10^3 V/m

(C) 12×10^3 V/m (D) 6×10^3 V/m

(ii) Change in energy of the capacitor is :

(A) 6×10^{-8} J (B) -1215×10^{-10} J

(C) 1215×10^{-10} J (D) -405×10^{-10} J

(iii) If the distance is increased after disconnecting the power source, then electric field inside the capacitor is :

(A) 6×10^3 V/m (B) 3×10^3 V/m

(C) 12×10^3 V/m (D) 15×10^3 V/m

(iv) Change in energy of the capacitor in above case is :

(A) 303.75×10^{-9} J (B) -1215×10^{-10} J

(C) 5.06×10^{-8} J (D) -303.75×10^{-9} J

A-5.* A parallel plate capacitor is charged and the charging battery is then disconnected. The plates of the capacitor are now moved, farther apart. The following things happen :

(A) The charge on the capacitor increases

(B) The electrostatics energy stored in the capacitor increases

(C) The voltage between the plates increases

(D) The capacitance increases.

A-6. A parallel plate capacitor is charged and then isolated. On increasing the plate separation–

	Charge	Potential	Capacitance
(A)	remains constant	remains constant	decreases
(B)	remains constant	increases	decreases
(C)	remains constant	decreases	increases
(D)	increases	increases	decreases

Section (B) : Circuits with capacitor and use of KCl and KVL

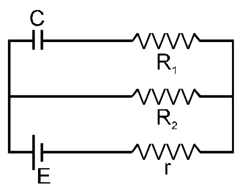
B-1. The work done against electric forces in increasing the potential difference of a condenser from 20V to 40V is W. The work done in increasing its potential difference from 40V to 50V will be (consider capacitance of capacitor remain constant)

(A) 4W (B) $\frac{3W}{4}$ (C) 2W (D) $\frac{W}{2}$

B-2. The plate separation in a parallel plate condenser is d and plate area is A. If it is charged to V volt & battery is disconnected then the work done in increasing the plate separation to 2d will be–

(A) $\frac{3}{2}\frac{\epsilon_0 AV^2}{d}$ (B) $\frac{\epsilon_0 AV^2}{d}$ (C) $\frac{2\epsilon_0 AV^2}{d}$ (D) $\frac{\epsilon_0 AV^2}{2d}$

B-3. The magnitude of charge in steady state on either of the plates of condenser C in the adjoining circuit is-

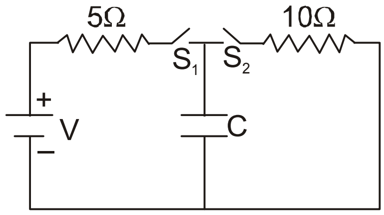


(A) CE (B) $\frac{CER_2}{(R_1+r)}$ (C) $\frac{CER_2}{(R_2+r)}$ (D) $\frac{CER_1}{(R_2+r)}$

B-4. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now :-

(A) V (B) $V+\frac{Q}{C}$ (C) $V+\frac{Q}{2C}$ (D) $V-\frac{Q}{C}$, if V < CV

B-5. In the adjoining diagram, (assuming the battery to be ideal) the condenser C will be charged to potential V if -



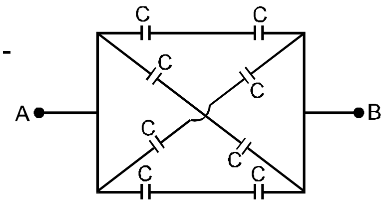
(A) S_1 and S_2 both are open
(B) S_1 and S_2 both are closed
(C) S_1 is closed and S_2 is open
(D) S_1 is open and S_2 is closed.

B-6. A parallel plate condenser of capacity C is connected to a battery and is charged to potential V. Another condenser of capacity 2C is connected to another battery and is charged to potential 2V. The charging batteries are removed and now the condensers are connected in such a way that the positive plate of one is connected to negative plate of another. The final energy of this system is–

(A) zero (B) $\frac{25CV^2}{6}$ (C) $\frac{3CV^2}{2}$ (D) $\frac{9CV^2}{2}$

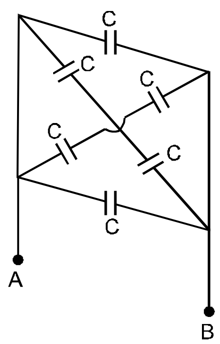
Section (C) : Combination of capacitors

C-1. In the adjoining circuit, the capacity between the points A and B will be -



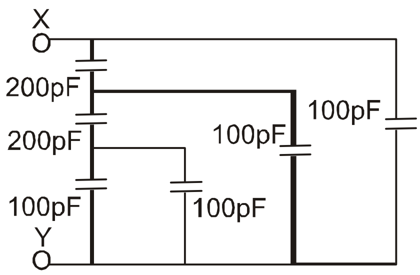
(A) C (B) 2C
(C) 3C (D) 4C

C-2. The resultant capacity between the points A and B in the adjoining circuit will be -



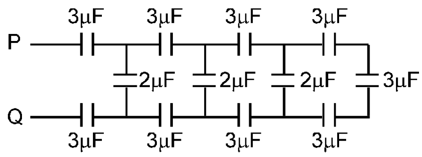
(A) C (B) 2C (C) 3C (D) 4C

C-3. The equivalent capacitance between the terminals X and Y in the figure shown will be–



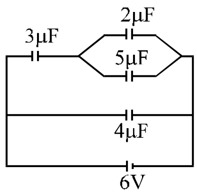
(A) 100 pF (B) 200 pF (C) 300 pF (D) 400 pF

C-4. The effective capacity in the following figure between the points P and Q will be –



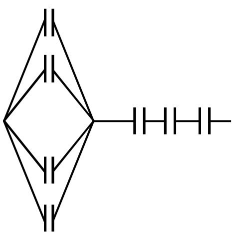
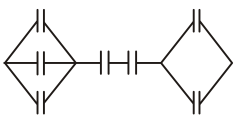
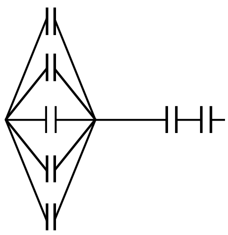
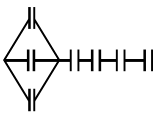
(A) 3μF (B) 5μF (C) 2μF (D) 1μF

C-5 In the circuit shown in figure, the ratio of charges on 5μF and 4μF capacitor is :-



(A) 4/5 (B) 3/5 (C) 3/8 (D) 1/2

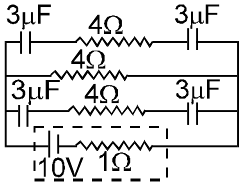
C-6. How the seven condensers, each of capacity 2μF, should be connected in order to obtain a resultant capacitance of $\frac{10}{11}$ μF?

(A)  (B)  (C)  (D) 

C-7. Two parallel plate condensers of capacity of 20μF and 30μF are charged to the potentials of 30V and 20V respectively. If likely charged plates are connected together then the common potential difference will be-

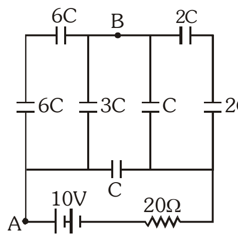
(A) 100 V (B) 50 V (C) 24 V (D) 10 V

C-8. In the following figure, the charge on each condenser in the steady state will be–



(A) 3μC (B) 6μC (C) 9μC (D) 12μC

✍C-9. For the circuit shown here, the potential difference between points A and B is :-



(A) 2.5 V (B) 7.5 V (C) 10 V (D) Zero

Section (D) : Equation of charging and discharging

D-1. An uncharged capacitor of capacitance 8.0 μF is connected to a battery of emf 6.0 V through a resistance of 24 Ω, then

(i) the current in the circuit just after the connections are made is :

(A) 0.25 A (B) 0.5 A (C) 0.4 A (D) 0 A

(ii) the current in the circuit at one time constant after the connections are made is :

(A) 0.25 A (B) 0.09 A (C) 0.4 A (D) 0 A

✍**D-2.** A 3 mega ohm resistor and an uncharged 1 μF capacitor are connected in a single loop circuit with a constant source of 4 volt. At one second after the connection is made what are the rates at which;

(i) the charge on the capacitor is increasing.

(A) $4(1-e^{-1/3})\ \mu$ C/s (B) $4e^{-1/3}\ \mu$ C/s

(C) $\frac{4}{3}e^{-1/3}\ \mu$ C/s (D) $\frac{4}{3}(1-e^{-1/3})\ \mu$ C/s

(ii) energy is being stored in the capacitor.

(A) $\frac{16}{3}(1-e^{-1/3})e^{-1/3}\ \mu$J/s (B) $\frac{16}{3}(1-e^{-2/3})\ \mu$J/s

(C) $\frac{16}{3}e^{-2/3}\ \mu$J/s (D) None of these

(iii) joule heat is appearing in the resistor.

(A) $\frac{16}{3}e^{-1/3}\mu$ J/s (B) $\frac{1}{2}e^{-1/3}\mu$ J/s

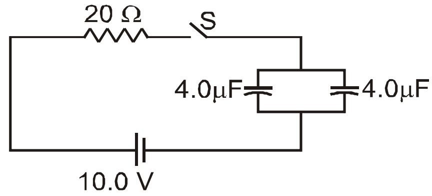
(C) $\frac{16}{3}(e^{-2/3})\ \mu$ J/s (D) $\frac{16}{3}(1-e^{-1/3})^2\ \mu$ J/s

(iv) energy is being delivered by the source.

(A) $16(1-e^{-1/3})\ \mu$ J/s (B) 16μ J/s

(C) $\frac{16}{3}e^{-1/3}\ \mu$ J/s (D) $\frac{16}{3}(1-e^{-1/3})\ \mu$ J/s

D-3. The charge on each of the capacitors 0.16 ms after the switch S is closed in figure is :



(A) 24 μC
(B) 26.8 μC
(C) 25.2 μC
(D) 40 μC

D-4. An uncharged capacitor of capacitance 100μF is connected to a battery of emf 20V at t = 0 through a resistance 10Ω, then

(i) the maximum rate at which energy is stored in the capacitor is :

(A) 10J/s (B) 20 J/s (C) 40J/s (D) 5J/s

(ii) time at which the rate has this maximum value is

(A) (4 ln 2) ms (B) (2 ln 2) ms (C) (ln 2) ms (D) (3 ln 2) ms

D-5. The plates of a capacitor of capacitance 10 μF, charged to 60 μC, are joined together by a wire of resistance 10 Ω at t = 0, then

(i) the charge on the capacitor in the circuit at t = 0 is :

(A) 120 μC (B) 60 μC (C) 30 μC (D) 44 μC

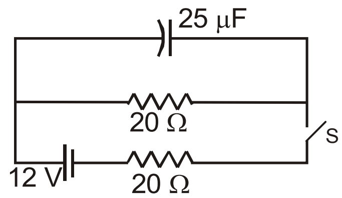
(ii) the charge on the capacitor in the circuit at t = 100 μs is :

(A) 120 μC (B) 60 μC (C) 22 μC (D) 18 μC

(iii) the charge on the capacitor in the circuit at t = 1.0 ms is : (take e^{10} = 20000)

(A) 0.003 μC (B) 60 μC (C) 44 μC (D) 18 μC

D-6. The switch S shown in figure is kept closed for a long time and then opened at t = 0, then the current in the middle 20 Ω resistor at t = 0.25 ms is :



(A) 0.629 A (B) 0.489 A
(C) 0.189 A (D) 23 mA

Section (E) : Capacitor with dielectric

E-1. The distance between the plates of a parallel plate condenser is d. If a copper plate of same area but thickness $\frac{d}{2}$ is placed between the plates then the new capacitance will become-

(A) half (B) double (C) one fourth (D) unchanged

E-2. On placing a dielectric slab between the plates of an isolated charged condenser its–

	Capacitance	Charge	Potential Difference	Energy stored	Electric field
(A)	decreases	remains unchanged	decreases	increases	increases
(B)	increases	remains unchanged	increases	increases	decreases
(C)	increases	remains unchanged	decreases	decreases	decreases
(D)	decreases	remains unchanged	decreases	increases	remains unchanged

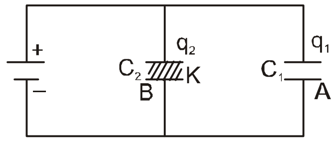
E-3. A parallel plate condenser is connected to a battery of e.m.f. 4 volt. If a plate of dielectric constant 8 is inserted into it, then the potential difference on the condenser will be-

(A) 1/2 V (B) 2V (C) 4V (D) 32V

E-4. In the above problem if the battery is disconnected before inserting the dielectric, then potential difference will be-

(A) 1/2 V (B) 2V (C) 4V (D) 32V

E-5. In the adjoining diagram two geometrically identical capacitors A and B are connected to a battery. Air is filled between the plates of C_1 and a dielectric is filled between the plates of C_2, then -



(A) $q_1 < q_2$ (B) $q_1 > q_2$

(C) $q_1 = q_2$ (D) None of these

E-6. A parallel plate condenser with plate separation d is charged with the help of a battery so that U_0 energy is stored in the system. A plate of dielectric constant K and thickness d is placed between the plates of condenser while battery remains connected. The new energy of the system will be-

(A) KU_0 (B) K^2U_0 (C) $\frac{U_0}{K}$ (D) $\frac{U_0}{K^2}$

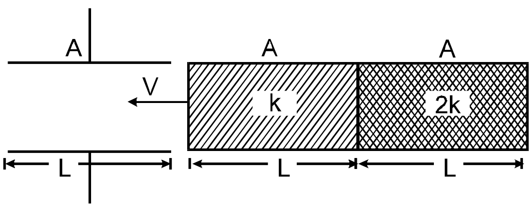
E-7. In the above problem if the battery is disconnected before placing the plate, then new energy will be–

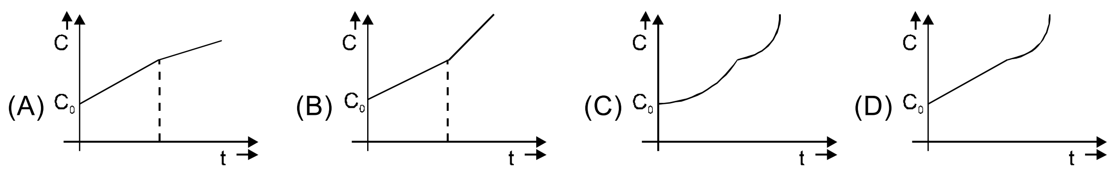
(A) K^2U_0 (B) $\frac{U_0}{K^2}$ (C) $\frac{U_0}{K}$ (D) KU_0

E-8. Three capacitors 2 μF, 3 μF and 5 μF can withstand voltages to 3V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to :-

(A) 5 Volts (B) (31/6) Volts (C) (26/5) Volts (D) None

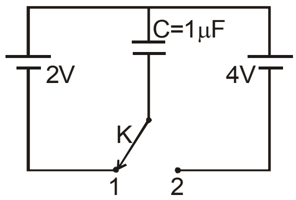
E-9. A parallel plate capacitor without any dielectric has capacitance C_0. A dielectric slab is made up of two dielectric slabs of dielectric constants K and 2K and is of same dimensions as that of capacitor plates and both the parts are of equal dimensions arranged serially as shown. If this dielectric slab is introduced (dielectric K enters first) in between the plates at constant speed, then variation of capacitance with time will be best represented by:





PART - III : MATCH THE COLUMN

1. The circuit involves two ideal cells connected to a 1 μF capacitor via a key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is then pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column II gives corresponding results. Match the statements in Column I with the corresponding values in Column II.

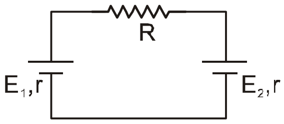
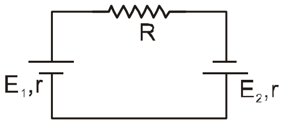
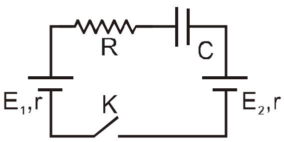
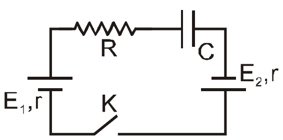


Column I		Column II
(A) The net charge crossing the 4 volt cell in μC is	(p)	2
(B) The magnitude of work done by 4 Volt cell in μJ is	(q)	6
(C) The gain in potential energy of capacitor in μJ is	(r)	8
(D) The net heat produced in circuit in μJ is	(s)	16

2.

	Column-I		Column-II
(A)	Plates of an isolated, charged, parallel plate, air core capacitor are slowly pulled apart.	(p)	Electric energy stored inside capacitor increases in the process.
(B)	A dielectric is slowly inserted inside an isolated and charged parallel plate air cored capacitor to completely fill the space between plates.	(q)	Force between the two plates of the capacitor remain unchanged.
(C)	Plates of a parallel plate capacitor connected across a battery are slowly pulled apart.	(r)	Electric field in the region between plates remain unchanged.
(D)	A dielectric slab is slowly inserted inside a parallel plate capacitor connected across a battery to completely fill the space between plates.	(s)	Total electric energy stored inside capacitor decreases in the process.
		(t)	Electric field in the region decreases.

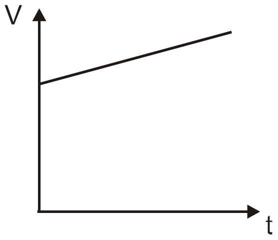
3. In each situation of column-I, a circuit involving two non-ideal cells of unequal emf E_1 and E_2 ($E_1 > E_2$) and equal internal resistance r are given. A resistor of resistance R is connected in all four situations and a capacitor of capacitance C is connected in last two situations as shown. Assume battery can supply infinity charge to the circuit ($r, R \neq 0$, $E_1, E_2 \neq 0$). Four statements are given in column-II. Match the situation of column-I with statements in column-II.

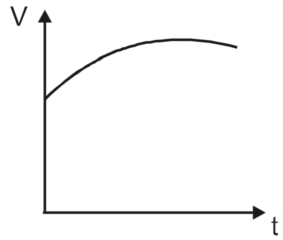
Column-I	Column - II
(A) 	(p) magnitude of potential difference across both cells can never be same.
(B) 	(q) cell of lower emf absorbs energy, that is, it gets charged up as long as current flows in circuit
(C) The capacitor is initially uncharged. After the key K is closed 	(r) potential difference across cell of lower emf may be zero.
(D) The capacitor is initially uncharged. After the key K is closed. 	(s) current in the circuit can never be zero (even after steady state is reached).

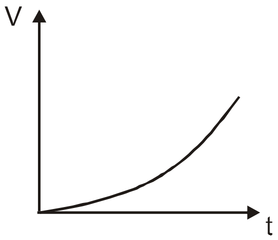
Exercise # 2

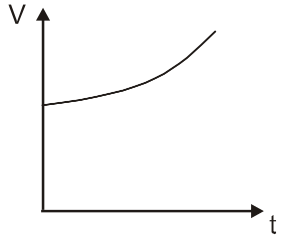
PART - I : OBJECTIVE QUESTIONS

1. Choose Graph between potential and time for an isolated conductor of finite capacitance C, if its charge varies according to the formula $Q = (\alpha t + Q_0)$ coulomb, where Q_0 and α are positive constant.

(A) 

(B) 

(C) 

(D) 

2. The plates of a parallel plate condenser are being moved away with a constant speed v. If the plate separation at any instant of time is d then the rate of change of capacitance with time is proportional to–

(A) $\frac{1}{d}$ (B) $\frac{1}{d^2}$ (C) d^2 (D) d

3. (i) A 3μF capacitor is charged up to 300 volt and 2μF is charged up to 200 volt. The capacitor are connected so that the plates of same polarity are connected together. The final potential difference between the plates of the capacitor after they are connected is :

(A) 220 V (B) 160 V (C) 280 V (D) 260 V

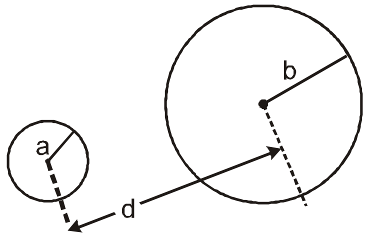
(ii) If instead of this, the plates of opposite polarity were joined together, then amount of charge that flows is :

(A) 6×10^{-4} C (B) 1.5×10^{-4} C (C) 3×10^{-4} C (D) 7.5×10^{-4} C

4. A capacitor of capacitance C_0 is charged to a voltage V_0 and then isolated. A capacitor C is then charged from C_0, discharged and charged again ; the process is repeated n times. Due to this, potential of the C_0 is decreased to V, then value of C is :

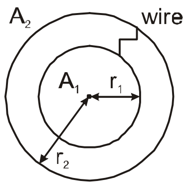
(A) $C_0\,[V_0/V]^{1/n}$ (B) $C_0[(V_0/V)^{1/n} - 1]$ (C) $C_0\,[(V_0/V) - 1]$ (D) $C_0\,[(V/V_0)^n + 1]$

5. There are two conducting spheres of radius a and b (b > a) carrying equal and opposite charges. They are placed at a separation d (>>> a and b). The capacitance of system is



(A) $\frac{4\pi\varepsilon_0}{a-b-d}$ (B) $\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}-\frac{1}{d}}$ (C) $\frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{1}{d}}$ (D) $\frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$

6. Two spherical conductors A_1 and A_2 of radii r_1 and r_2 are placed concentrically in air. The two are connected by a copper wire as shown in figure. Then the equivalent capacitance of the system is



(A) $\frac{4\pi\in_0 k\, r_1.r_2}{r_2-r_1}$ (B) $4\pi\in_0(r_1+r_2)$ (C) $4\pi\in_0 r_2$ (D) $4\pi\in_0 r_1$

7. A spherical condenser has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two spheres is filled with a dielectric of dielectric constant 5. The capacity when;

(i) the outer sphere is earthed.

(A) $\frac{2}{3}\times10^{-10}$ F (B) $\frac{8}{3}\times10^{-10}$ F

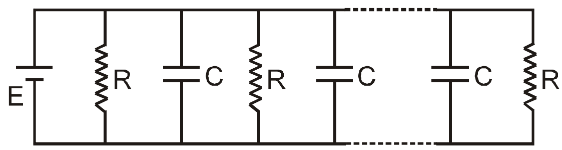
(C) $\frac{10}{3}\times10^{-10}$ F (D) $\frac{16}{3}\times10^{-10}$ F

(ii) the inner sphere is earthed.

(A) $\frac{104}{30}\times10^{-10}$ F (B) $\frac{52}{30}\times10^{-10}$ F

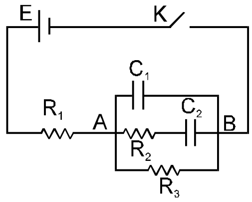
(C) $\frac{26}{30}\times10^{-10}$ F (D) 6×10^{-11} F

8. n resistances each of resistance R are joined with capacitors of capacity C (each) and a battery of emf E as shown in the figure. In steady state condition ratio of charge stored in the first and last capacitor is :



(A) n : 1 (B) (n – 1) : (n + 1) (C) $(n^2+1):(n^2-1)$ (D) 1 : 1

9. A network of uncharged capacitors and resistances is as shown

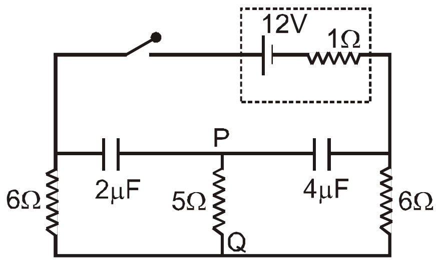


Current through the battery immediately after key K is closed and after a long time interval is :

(A) $\frac{E}{R_1}$, $\frac{E}{R_1+R_3}$ (B) $\frac{E}{R_1+R_3}$, $\frac{E}{R_1+\frac{R_2R_3}{R_2+R_3}}$

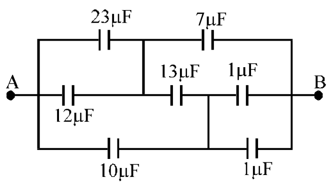
(C) Zero , $\frac{E}{R_1}$ (D) $\frac{E}{R_1+\frac{R_2R_3}{R_2+R_3}}$, $\frac{E}{R_1}$

10. In the circuit shown in figure the capacitors are initially uncharged. The current through resistor PQ just after closing the switch is :



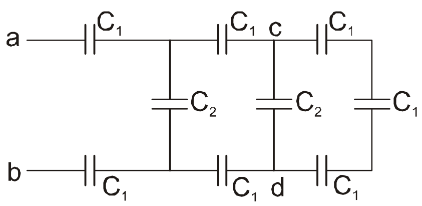
(A) 2A from P to Q (B) 2A from Q to P (C) 6A from P to Q (D) zero

11. Find the equivalent capacitance across A & B :-



(A) $\frac{28}{3}$ μf (B) $\frac{15}{2}$ μF (C) 15 μF (D) none

12. In the arrangement of the capacitors shown in the figure, each C_1 capacitor has capacitance of 3μF and each C_2 capacitor has capacitance of 2μF then,



(i) Equivalent capacitance of the network between the points a and b is :

(A) 1μF (B)2μF (C) 4μF (D) $\frac{3}{2}$ μF

(ii) If V_{ab} = 900 V, the charge on each capacitor nearest to the points 'a' and 'b' is :

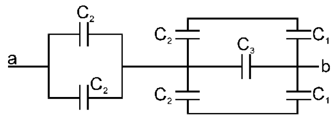
(A) 300 μC (B) 600 μC (C) 450 μC (D) 900 μC

(iii) If V_{ab} = 900 V, then potential difference across points c and d is :

(A) 60 V (B) 100 V (C) 120 V (D) 200 V

13. A combination arrangement of the capacitors is shown in the figure

(i) C_1 = 3 μF, C_2 = 6 μF and C_3 = 2 μF then equivalent capacitance between 'a' and 'b' is :

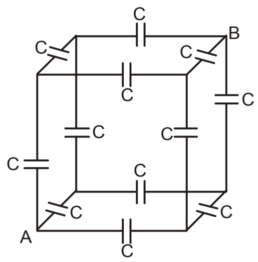


(A) 4 μF (B) 6 μF (C) 1 μF (D) 2 μF

(ii) If a potential difference of 48 V is applied across points a and b, then charge on the capacitor C_3 at steady state condition will be :

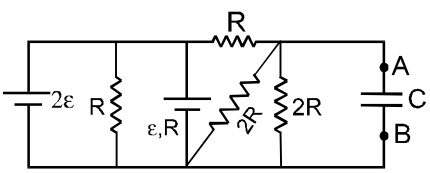
(A) 8 μC (B) 16 μC (C) 32 μC (D) 64 μC

14. Each edge of the cube contains a capacitance C. The equivalent capacitance between the points A and B will be –



(A) $\frac{6C}{5}$ (B) $\frac{5C}{6}$ (C) $\frac{12C}{7}$ (D) $\frac{7C}{12}$

15. The time constant of the circuit shown is :

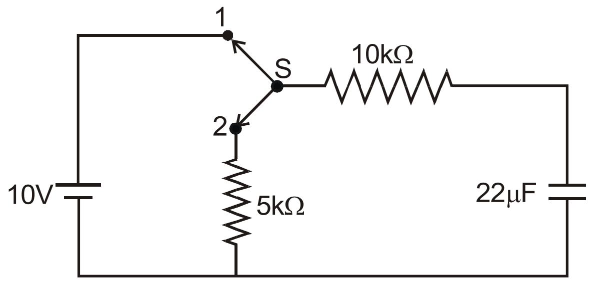


(A) $\frac{RC}{2}$ (B) $\frac{3RC}{5}$ (C) $\frac{RC}{3}$ (D) $\frac{RC}{4}$

16. A fresh dry cell of 1.5 volt and two resistors of 10kΩ each are connected in series. An analog voltmeter measures a voltage of 0.5 volt across each of the resistors. A 100μF capacitor is fully charged using the same source. The same voltmeter is now used to measure the voltage across it. The initial value of the current and the time in which the voltmeter reading falls to 0.5 volt are respectively. **[Olympiad 2014_stage-1]**

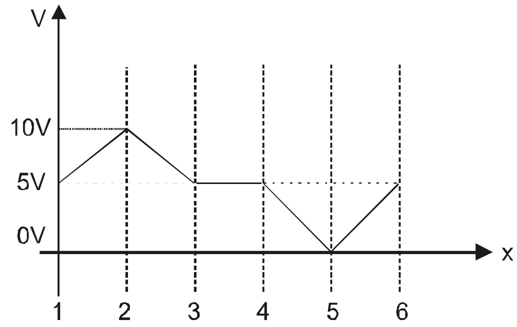
(A) 60μA, 11s (B) 120μA, 15s (C) 150μA, 15s (D) 150μA, 1.1 s

✍17. Refer to the circuit given below. Initially the switch S is in position 1 for 1.5 s. Then the switch is changed to position 2. After a time t (measured from the change over of the switch) the voltage across 5 kΩ resistance is found to be about 1.226 volt. Then, t is **[Olympiad 2014_stage-1]**



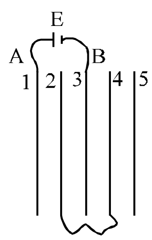
(A) 330 ms (B) 500 ms (C) 33 ms (D) data insufficient

18. The V versus x plot for six identical metal plates of cross-sectional area A is as shown. The equivalent capacitance between 2 and 5 is (Adjacent plates are placed at a separation d) :



(A) $\frac{2\epsilon_0 A}{d}$ (B) $\frac{\epsilon_0 A}{d}$ (C) $\frac{3\epsilon_0 A}{d}$ (D) $\frac{\epsilon_0 A}{2d}$

✍19. Five conducting parallel plates having area A and separation between them d, are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf E is connected. The charge flown through the cell is :-

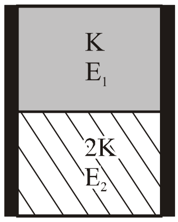


(A) $\frac{3}{4}\frac{\varepsilon_0 AE}{d}$ (B) $\frac{2}{3}\frac{\varepsilon_0 AE}{d}$ (C) $\frac{4\varepsilon_0 AE}{d}$ (D) $\frac{\varepsilon_0 AE}{2d}$

20. The capacitance of a parallel plate condenser is C_0. If a dielectric of relative permittivity ε_r and thickness equal to one fourth the plate separation is placed between the plates, then its capacity becomes C. Then value of $\frac{C}{C_0}$ will be-

(A) $\frac{5\varepsilon_r}{4\varepsilon_r+1}$ (B) $\frac{4\varepsilon_r}{3\varepsilon_r+1}$ (C) $\frac{3\varepsilon_r}{2\varepsilon_r+1}$ (D) $\frac{2\varepsilon_r}{\varepsilon_r+1}$

21. A parallel plate capacitor is connected from a cell and then isolated from it. Two dielectric slabs of dielectric constant K and 2K are now introduce in the region between upper half and lower half of the plate (as shown in figure). The electric field intensity in upper half of dielectric is E_1 and lower half is E_2 then



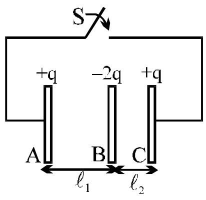
(A) $E_1 = 2E_2$

(B) Electrostatic potential energy of upper half is less than that of lower half

(C) Induced charges on both slabs are same

(D) Charge distribution on the plates remains same after insertion of dielectric

22. Three long conducting plate A, B & C having charges +q, –2q and +q as shown in figure. Here plate A and C are fixed. If the switch S is closed. The middle plate (B) will start moving in



(here $\ell_1 > \ell_2$)

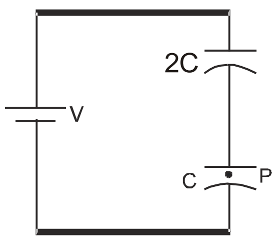
(A) Leftward direction (B) Rightward direction

(C) will not move (D) First move leftward & then rightward

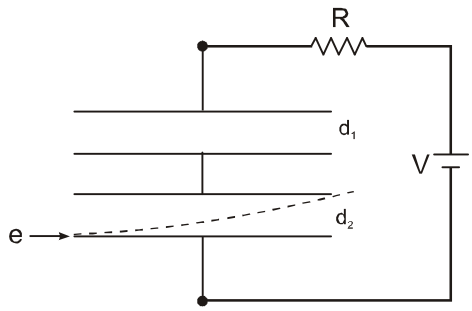
PART - II : INTEGER TYPE QUESTIONS

1. 2 conducting objects one with charge of +Q and another with –Q are kept on x-axis at x = 0 and x = 1 respectively. The electric field on the x-axis is given by $3Q\left(x^2 + \frac{4}{3}\right)$. If the capacitance of this configuration of objects is C. Then find $\frac{1}{C}$ (in F^{-1}).

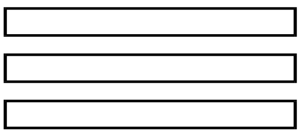
2. The particle P shown in the figure has a mass m and a charge –q. Each horizontal plate has a surface area A. The potential difference $V = \frac{xmg\varepsilon_0 A}{2qc}$ should be applied to the combination to hold the particle P in equilibrium. find x?



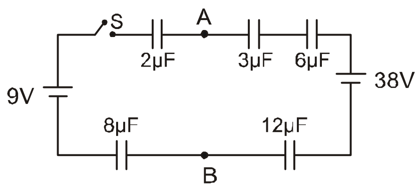
✍**3.** Both the capacitors shown in figure are made of square plates of edge a. The separations between the plates of the capacitors are $d_1 = 2d$ and $d_2 = d$ as shown in the figure. A battery of V volt and a resistance R are connected as shown in figure. At steady state an electron is projected between the plates of the lower capacitor from its lower plate along the plate as shown. Minimum speed should the electron be projected is given by $\frac{1}{\sqrt{n}}\left(\frac{Vea^2}{md^2}\right)^{1/2}$, so that it does not collide with any plate. Find n? Consider only the electric forces.



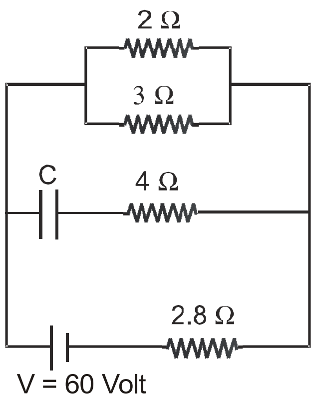
4. Three conducting plates of area 500 cm² area kept fixed as shown. Distance between adjacent plates is 8.85 mm. A charge of 1.0 nC is placed on the middle plate. (a) The charge on the outer surface of the upper plate is given by? $n \times 10^{-10}$. Find n? (b) Find the potential difference developed between the upper and the middle plates.



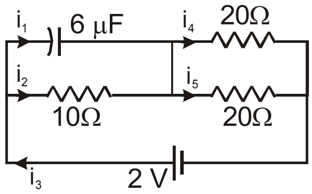
✍**5.** Five capacitors are connected as shown in the figure. Initially S is opened and all the capacitors are uncharged. When S is closed and steady state is obtained. Then find out potential difference between the points A and B.



6. Calculate the steady state current in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF **[JEE-1982; 2M]**



7. In steady state, find the charge on the capacitor shown in figure.

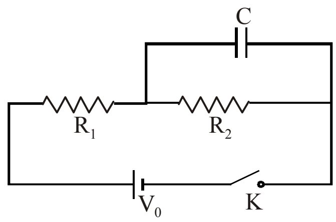


8. In the connection shown in the *figure* the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge up to the maximum and open the switch again. Then (Use the following data : V_0=30 V, R_1=10 kΩ, R_2=5 kΩ.)

(i) the current through R_1 be I_1 immediately after closing the switch

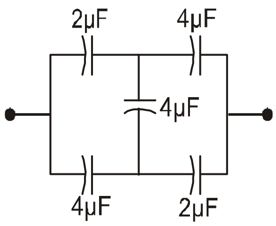
(ii) the current through R_2 be I_2 a long time after the switch was closed

(iii) the current through R_2 be I_3 immediately after reopening the switch

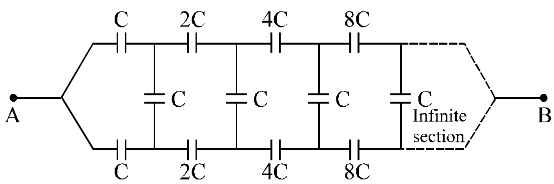


Find the value of $\frac{I_1}{I_2 I_3}$ (in ampere^{-1}).

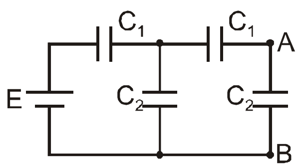
9. The equivalent capacitance of the combination shown in the figure between the indicated points is given by $\left(\frac{n}{7}\right)\mu F$. Find n.



10. The equivalent capacitance of the circuit between point A and B is nC. Find n.



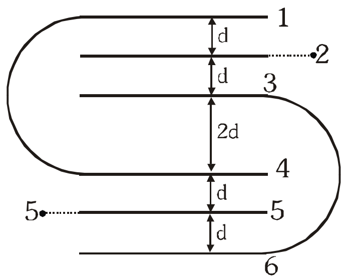
11. Find the potential difference between points A and B of the system shown in figure if the emf is equal to E = 110 V and the capacitance ratio $C_2/C_1 = \eta = 2.0$.



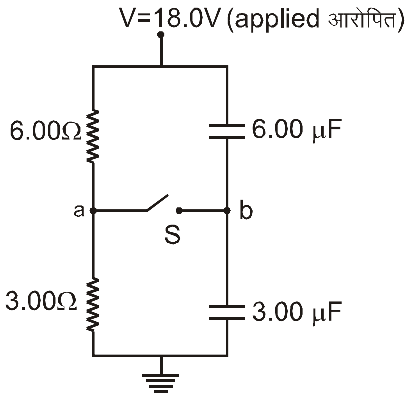
12. The electric field between the plates of a parallel–plate capacitance 2.0 μF drops to one third of its initial value in 4.4 μs when the plates are connected by a thin wire. Find the resistance of the wire.

(Given : ln3 = 1.0986)

13. There are six plates of equal area A and separation between the plates is d (d<<A) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\alpha \frac{\epsilon_0 A}{d}$. Then find the value of α.



14.



(a) What is the potential of point a with respect to point b in figure, when switch S is open ?

(b) What is the final potential of point b with respect to ground in steady state after switch S is closed?

(c) How much charge flows through switch S from a to b after it is closed ?

15. A capacitor of capacitance C charged by battery at V volt and then disconnected. At t = 0, it is connected to an uncharged capacitor of capacitance 2C through a resistance R. The charge on the second capacitor as a function of time is given by q = $\frac{\alpha cv}{3}\left(1-e^{-\frac{3t}{\beta RC}}\right)$. Find the value of $\frac{\alpha}{\beta}$.

16. Two square metal plates of side 1 m are kept 0.01 m apart like a parallel plate capacitor in air in such a way that one of their edges is perpendicular to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of 500 V. The plates are then lowered vertically into the oil at a speed of 0.001 ms^{-1}. The current drawn from the battery during the process is $n \times 10^{-9}$ A. Find n? (Dielectric constant of oil = 11),

($\epsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-1}$) **[JEE- 1994; 6M]**

PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

1. When a charged capacitor is connected with an uncharged capacitor, then which of the following is/are correct option/options.

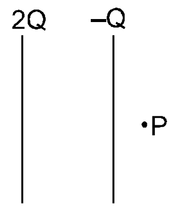
(A) the magnitude of charge on the charged capacitor decreases.

(B) a steady state is obtained after which no further flow of charge occurs.

(C) the total potential energy stored in the capacitors remains conserved.

(D) the charge conservation is always true.

2. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge –Q. The distance between the plates is 'd'.



(A) A point charge at point 'P' will experience electric force due to capacitor

(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi \epsilon_0 d^2}$

3. A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges +Q and –Q respectively. A third plate C, identical to A and B, with charge +Q, is now introduced midway between A and B, parallel to them. Which of the following are correct?

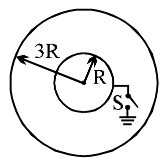
(A) The charge on the inner face of B is now $-\frac{3Q}{2}$

(B) There is no change in the potential difference between A and B.

(C) The potential difference between A and C is one-third of the potential difference between B and C.

(D) The charge on the inner face of A is now Q/2 .

4. Two thin conducting shells of radii R and 3R are shown in the figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of a switch S.



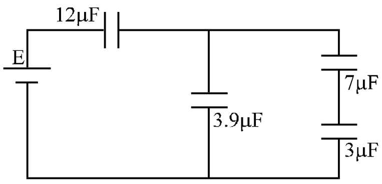
(A) With the switch S open, the potential of the inner sphere is equal to that of the outer.

(B) When the switch S is closed, the potential of the inner sphere becomes zero.

(C) With the switch S closed, the charge attained by the inner sphere is – Q/3.

(D) By closing the switch the capacitance of the system increases.

5. Four capacitors and a battery are connected as shown. The potential drop across the 7 μF capacitor is 6 V. Then the :



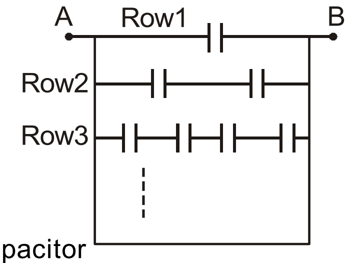
(A) potential difference across the 3 μF capacitor is 10 V

(B) charge on the 3 μF capacitor is 42 μC

(C) e.m.f. of the battery is 30 V

(D) potential difference across the 12 μF capacitor is 10 V.

6. Rows of capacitors containing 1,2,4,8,..........∞ capacitors, each of capacitance 2F, are connected in parallel as shown in figure. The potential difference across AB = 10 volt, then :



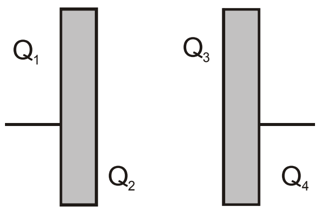
(A) Total capacitance across AB is 4F

(B) Charge of each capacitor will be same

(C) Charge on the capacitor in the first row is more than on any other capacitor

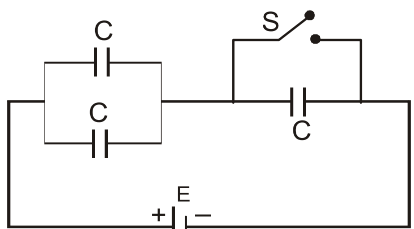
(D) Energy of all the capacitors is 50 J

7. In an isolated parallel plate capacitor of capacitance C the four surfaces have charges Q_1,Q_2,Q_3 and Q_4 as shown in the figure. The potential difference between the plates is :



(A) $\frac{Q_1+Q_2}{C}$ (B) $\left|\frac{Q_2}{C}\right|$ (C) $\left|\frac{Q_3}{C}\right|$ (D) $\frac{1}{C}[(Q_1+Q_2)-(Q_3-Q_4)]$

8. In the circuit shown in figure, each capacitor has a capacitance C. The emf of the cell is E and circuit already in steady state. If the switch S is closed.



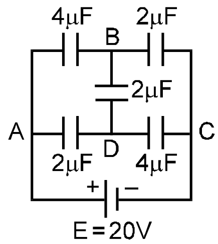
(A) some positive charge will flow out of the positive terminal of the cell

(B) some positive charge will enter the positive terminal of the cell

(C) the amount of charge flowing through the cell will be CE

(D) the amount of charge flowing through the cell will be $\left(\frac{4}{3}\right)$CE

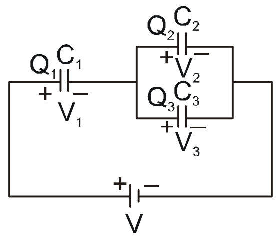
9. The figure shows a diagonal symmetric arrangement of capacitors and a battery. If the potential of C is zero, then (All the capacitors are initially uncharged).



(A) V_A = + 20 V (B) $4(V_A - V_B) + 2(V_D - V_B) = 2V_B$

(C) $2(V_A - V_D) + 2(V_B - V_D) = 4V_D$ (D) $V_A = V_B + V_D$

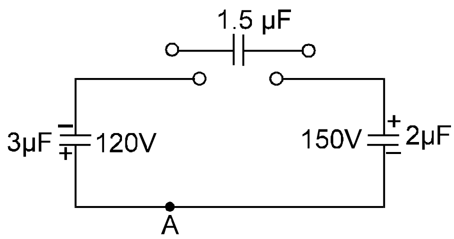
10. In the adjoining diagram all the capacitors are initially uncharged, they are connected with a battery as a shown in figure. Then:



(A) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$ (B) $Q_1 = Q_2 + Q_3$ and $V = V_1 + \frac{V_2 + V_3}{2}$

(C) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_3$ (D) $Q_2 = Q_3$ and $V = V_2 + V_3$

11. Two capacitors of 2 μF & 3 μF are charged to 150 volt & 120 volt respectively. The plates of a capacitor are connected as shown in the fig. A discharged capacitor of capacity 1.5 μF falls to the free ends of the wire and connected through the free ends of the wire, Then :



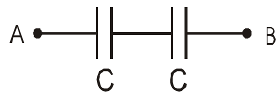
(A) Charge on the 1.5 μF capacitor will become 180 μC at steady state.

(B) Charge on the 2 μF capacitor will become 120 μC at steady state.

(C) Positive charge flows through point A from left to right.

(D) Positive charge flows through point A from right to left.

12. Two similar condensers are connected in parallel and are charged to a potential V. Now these are separated out and are connected in series. Then



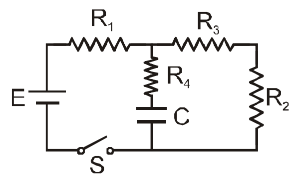
(A) the energy stored in the system increases

(B) the potential difference between end points may becomes zero.

(C) the potential difference between end points may becomes 2V.

(D) the charge on the plates mutually connected nullifies.

13. In the circuit shown in figure the switch S is closed at t = 0.



A long time after closing the switch

(A) voltage drop across the capacitor is E

(B) current through the battery is $\frac{E}{R_1 + R_2 + R_3}$

(C) energy stored in the capacitor is $\frac{1}{2}C\left(\frac{(R_2 + R_3)E}{R_1 + R_2 + R_3}\right)^2$

(D) current through the resistance R_4 becomes zero

14. Capacitor C_1 of the capacitance 1 microfarad and capacitor C_2 of capacitance 2 microfarad are separately charged fully by a common battery. The two capacitors are then separately allowed to discharge through equal resistors at time t = 0.

(A) the current in each of the two discharging circuits is zero at t = 0.

(B) the current in the two discharging circuits at t = 0 are equal but non zero.

(C) the current in the two discharging circuits at t = 0 are unequal

(D) capacitor C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge

15. The terminals of a battery of emf V are connected to the two plates of a parallel plate capacitor. If the space between the plates of the capacitor is filled with an insulator of dielectric constant K, then :

(A) the electric field in the space between the plates does not change

(B) the capacitance of the capacitor increases

(C) the charge stored in the capacitor increases

(D) the electrostatic energy stored in the capacitor decreases

16. The plates of a parallel plate capacitor with no dielectric are connected to a voltage source. Now a dielectric of dielectric constant K is inserted to fill the whole space between the plates with voltage source remaining connected to the capacitor.

(A) the energy stored in the capacitor will become K–times

(B) the electric field inside the capacitor will decrease to K-times

(C) the force of attraction between the plates will increase to K^2 – times

(D) the charge on the capacitor will increase to K-times

17. On a parallel plate capacitor following operations can be performed.

P – connect the capacitor to a battery of emf V

Q – disconnect the battery

R – reconnect the battery with polarity reversed

S – insert a dielectric slab in the capacitor

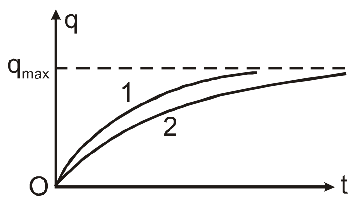
(A) In PQR (perform P, then Q, then R), the stored electric energy remains unchanged and no thermal energy is developed

(B) The charge appearing on the capacitor is greater after the action PSQ then after the action PQS

(C) The electric energy stored in the capacitor is greater after the action SPQ then after the action PQS

(D) The electric field in the capacitor after the action PS is the same as that after SP

18. The charge on capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



Choose the correct statement(s) related to the two circuits.

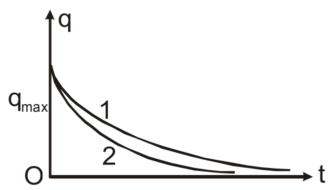
(A) Both the capacitors are charged to the same magnitude of charge

(B) The emf's of cells in both the circuits are equal.

(C) The emf's of the cells may be different

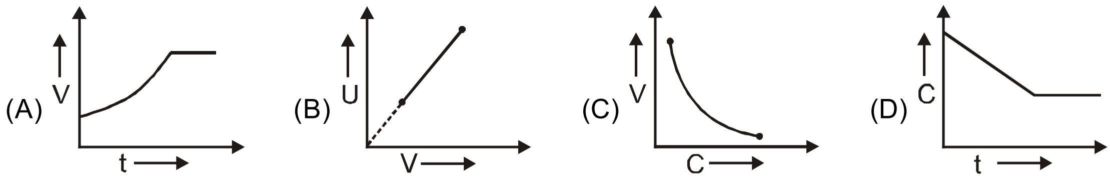
(D) The emf E_1 is more than E_2

19. The instantaneous charge on capacitor in two discharging RC circuits is plotted with respect to time in figure. Choose the correct statement(s) (where E_1 and E_2 are emfs of two DC sources in two different charging circuits and capacitors are fully charged).



(A) $R_1C_1 > R_2C_2$ (B) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$ (C) $R_1 > R_2$ if $E_1 = E_2$ (D) $C_2 > C_1$ if $E_1 = E_2$

20. A parallel plate capacitor has a dielectric slab in it. The slab just fills the space inside the capacitor. The capacitor is charged by a battery and the battery is disconnected. Now the slab is started to pull out uniformly at t = 0. If at time t, capacitance of the capacitor is C, potential difference across plate is V, and energy stored in it is U, then which of the following graphs are correct ?



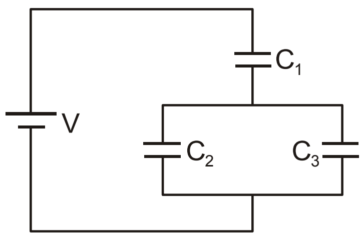
21. A parallel plate capacitor of plate area A and plate separation d is charged to a potential difference V & then the battery disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of the charge on each plate, the magnitude of the electric field between the plates (after the slab is inserted) & the magnitude of the work done on the system, in the process of inserting the slab, then : **[JEE 1997, 2M]**

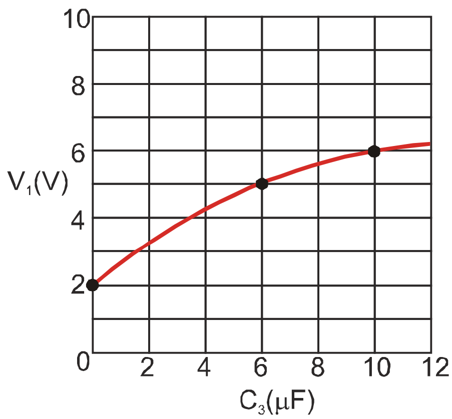
(A) $Q = \frac{\epsilon_0 AV}{d}$ (B) $Q = \frac{\epsilon_0 KAV}{d}$ (C) $E = \frac{V}{Kd}$ (D) $W = \frac{\epsilon_0 AV^2}{2d}\left(1 - \frac{1}{K}\right)$

PART - IV : COMPREHENSION

Comprehension # 1

Capacitor C_3 in the circuit is a variable capacitor (its capacitance can be varied). Graph is plotted between potential difference V_1 (across capacitor C_1) versus C_3. Electric potential V_1 approaches on asymptote of 10 V as $C_3 \to \infty$.





1. EMF of the battery is equal to :

(A) 10 V (B) 12 V (C) 16 V (D) 20 V

2. The capacitance of the capacitor C_1 has value :

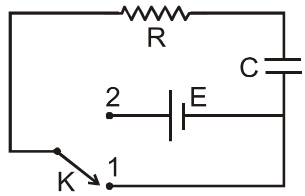
(A) 2 μF (B) 6 μF (C) 8 μF (D) 12 μF

3. The capacitance of C_2 is equal to :

(A) 2 μF (B) 6 μF (C) 8 μF (D) 12 μF

Comprehension # 2

In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volts, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed back to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.



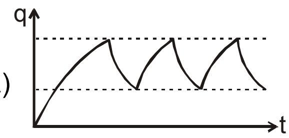
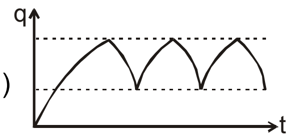
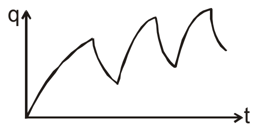
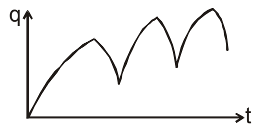
4. The charge on capacitor at t = 2RC second is

(A) CE (B) $CE\left(1-\frac{1}{e}\right)$ (C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$ (D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

5. The current through the resistance at t = 1.5 RC seconds is

(A) $\frac{E}{e^2R}(1-\frac{1}{e})$ (B) $\frac{E}{eR}(1-\frac{1}{e})$ (C) $\frac{E}{R}(1-\frac{1}{e})$ (D) $\frac{E}{\sqrt{e}R}(1-\frac{1}{e})$

6. Then the variation of charge on capacitor with time is best represented by

(A)  (B)  (C)  (D) 

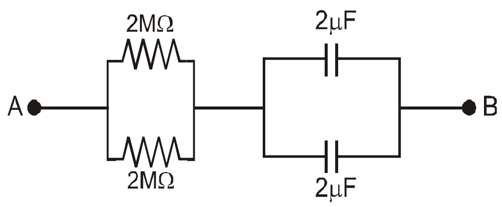
Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

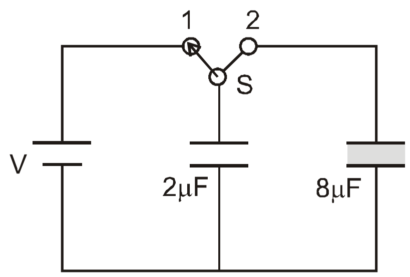
*** Marked Questions are having more than one correct option.**

1. At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4 V?

[Take : ℓn 5 = 1.6, ℓn 3 = 1.1] **[JEE' 2010 ; 3/163]**

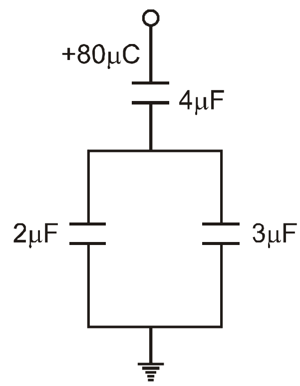


2. A 2μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is **[JEE' 2010 ; 3/160, –1]**



(A) 0% (B) 20% (C) 75% (D) 80%

3. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is : **[IIT-JEE-2012, Paper-2; 3/66, –1]**



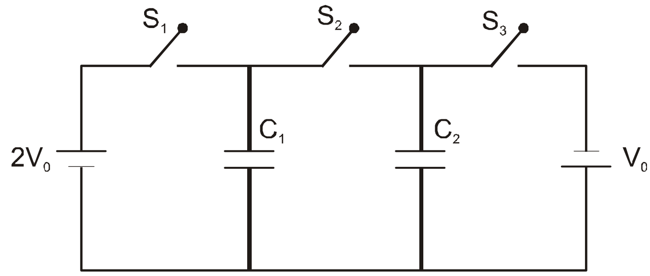
(A) +32 μC

(B) +40 μC

(C) +48 μC

(D) +80 μC

4.* In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time.**[JEE Advanced 2013]**



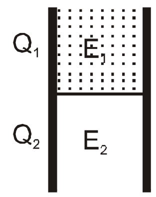
(A) the charge on the upper plate of C_1 is $2CV_0$

(B) the charge on the upper plate of C_1 is CV_0

(C) the charge on the upper plate of C_2 is 0

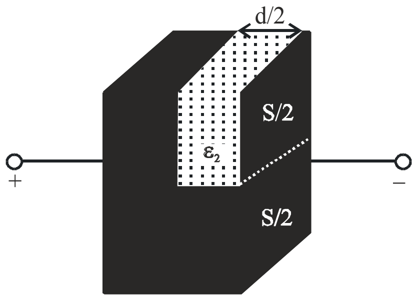
(D) the charge on the upper plate of C_2 is $-CV_0$

5.* A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. Choose the correct option/options, igonoring edge effects. **[JEE Advanced 2014, P-1; 3/60]**



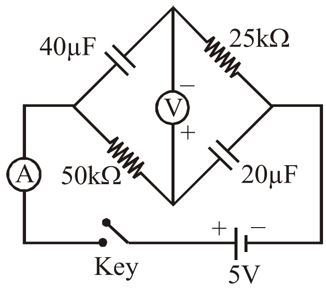
(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

6. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is :- **[JEE Advanced 2015]**



(A) $\frac{6}{5}$ (B) $\frac{5}{3}$ (C) $\frac{7}{5}$ (D) $\frac{7}{3}$

7. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true? **[JEE Advanced 2016]**



(A) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(B) The voltmeter will display 0 V at time t = ln 2 seconds

(C) The current in the ammeter becomes 1/e of the initial value after 1 second

(D) The current in the ammeter becomes zero after a long time

PARAGRAPH–1

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

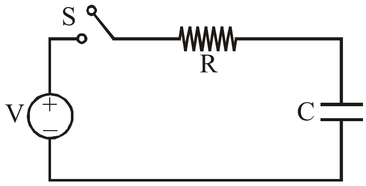


Figure 1

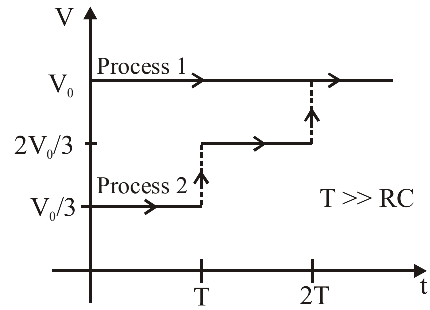


Figure 2

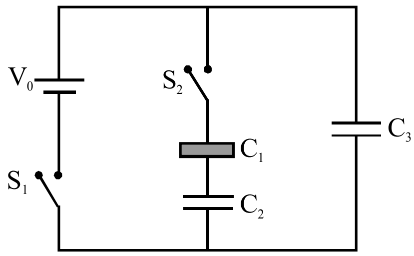
8. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by:-

(A) $E_C = E_D$ (B) $E_C = 2E_D$ (C) $E_C = \frac{1}{2}E_D$ (D) $E_C = E_D \ln 2$

9. In Process 2, total energy dissipated across the resistance E_D is :-

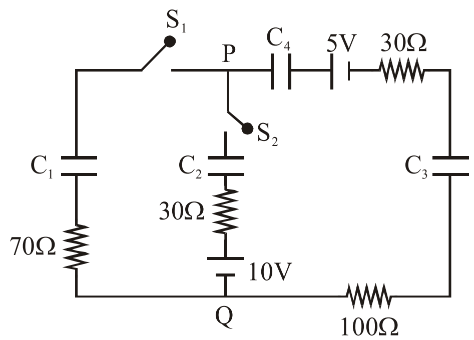
(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = \frac{1}{2}CV_0^2$ (D) $E_D = 3CV_0^2$

10. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ϵ_r. The cell electromotive force (emf) V_0 = 8V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ϵ_r. **[JEE Advanced 2018]**



11. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are C_1 = 10 μF, C_2 = 30 μF and $C_3 = C_4$ = 80 μF.

[JEE Advanced 2019]



Which of the statement(s) is/are correct ?

(A) The keys S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

(B) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(C) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(D) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

12. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\ell n2}\right)$. The value of α will be _____. [$\in_0$ is the permittivity of free space]

[JEE Advanced 2019]

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be **[AIEEE-2010; 4/144, –1]**

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

2. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10}2.5 = 0.4$) **[AIEEE 2011; 4/120, –1]**

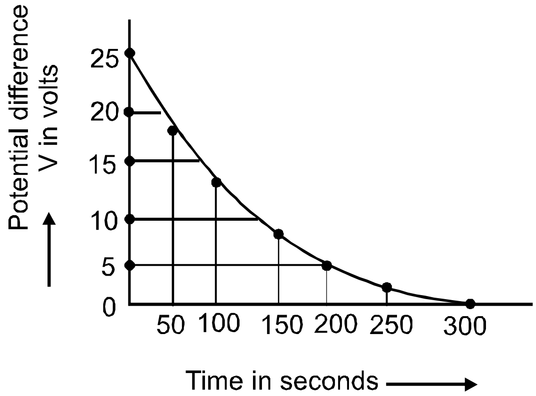
(1) $1.3 \times 10^4 \Omega$ (2) $1.7 \times 10^5 \Omega$ (3) $2.7 \times 10^6 \Omega$ (4) $3.3 \times 10^7 \Omega$

3. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on a (C – R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is :

[AIEEE 2011, 11 May; 4/120, –1]

(1) 10 second (2) 5 second (3) 2.5 second (4) 20 second

4. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between : **[AIEEE 2012 ; 4/120, –1]**



(1) 150 sec and 200 sec

(2) 0 and 50 sec

(3) 50 sec and 100 sec

(4) 100 sec and 150 sec

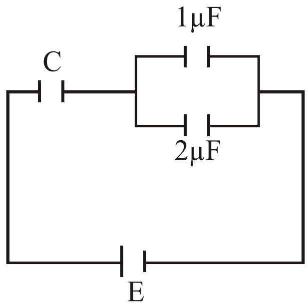
5. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by coing them together the potential on each one can be made zero. Then : **[JEE Main 2013; 4/120, –1]**

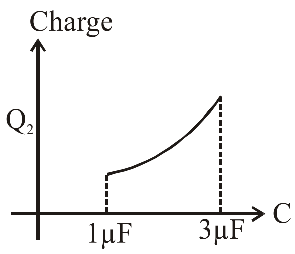
(1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

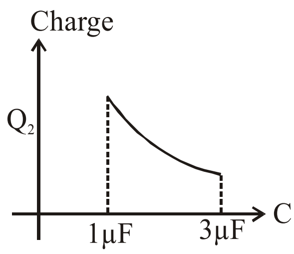
6. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE Main 2014; 4/120, –1]**

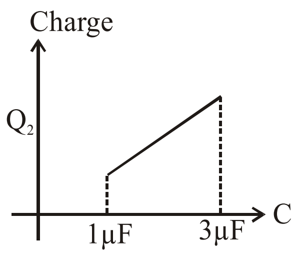
(1) 6×10^{-7}C/m^2 (2) 3×10^{-7}C/m^2 (3) 3×10^4C/m^2 (4) 6×10^4C/m^2

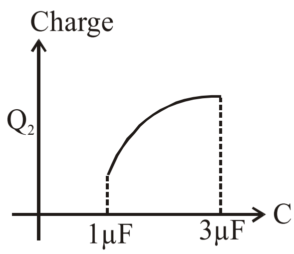
7. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale) :- **[JEE Main 2015; 4/120, –1]**



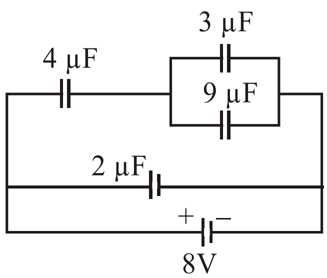
(1)


(2)


(3)


(4)


8. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point 30 m from it , would equal: **[JEE Main 2016; 4/120, –1]**



(1) 480 N/C (2) 240 N/C

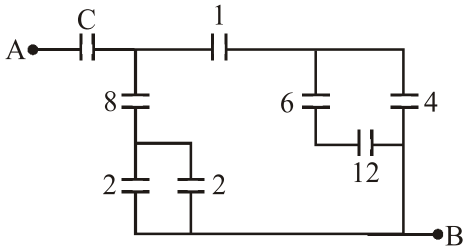
(3) 360 N/C (4) 420 N/C

9. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them : **[JEE Main online 2016]**

(1) two in parallel and one in series (2) all in parallel

(3) two in series and one in parallel (4) all in series

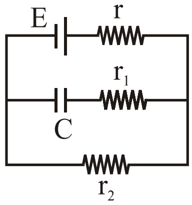
10. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is :- **[JEE Main online 2016]**



(1) $\frac{33}{23}\mu F$ (2) $\frac{34}{23}\mu F$

(3) $\frac{31}{23}\mu F$ (4) $\frac{32}{23}\mu F$

11. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[JEE Main 2017; 4/120, –1]**



(1) $CE\frac{r_2}{(r+r_2)}$ (2) $CE\frac{r_1}{(r_1+r)}$ (3) CE (4) $CE\frac{r_1}{(r_2+r)}$

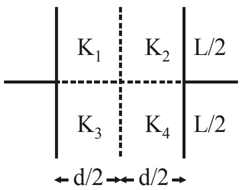
12. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is : **[JEE Main 2017; 4/120, –1]**

(1) 24 (2) 32 (3) 2 (4) 16

13. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant K = $\frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :- **[JEE Main 2018; 4/120, –1]**

(1) 0.3 n C (2) 2.4 n C (3) 0.9 n C (4) 1.2 n C

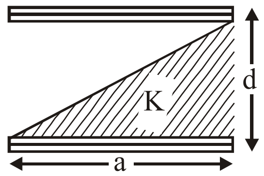
14. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be : **[JEE Main 2019; 4/120, –1]**



(1) $K = \frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$ (2) $K = \frac{(K_1+K_2)(K_3+K_4)}{(K_1+K_2+K_3+K_4)}$

(3) $K = \frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$ (4) None of these

15. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure.



Capacitance of this capacitor is : **[JEE Main 2019,Jan.; 4/120, –1]**

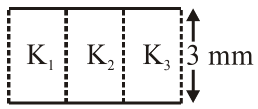
(1) $\frac{1}{2}\frac{k \in_0 a^2}{d}$ (2) $\frac{k \in_0 a^2}{d}\ln K$ (3) $\frac{k \in_0 a^2}{d(K-1)}\ln K$ (4) $\frac{k \in_0 a^2}{2d(K+1)}$

16. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

[JEE Main 2019,Jan.; 4/120, –1]

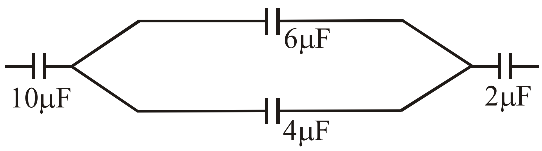
(1) 692 pJ (2) 60 pJ (3) 508 pJ (4) 560 pJ

17. A parallel plate capacitor is of area 6 cm² and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants K_1, = 10, K_2 = 12 and K_3 = 14. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :
[JEE Main 2019,Jan.; 4/120, –1]



(1) 12 (2) 4 (3) 36 (4) 14

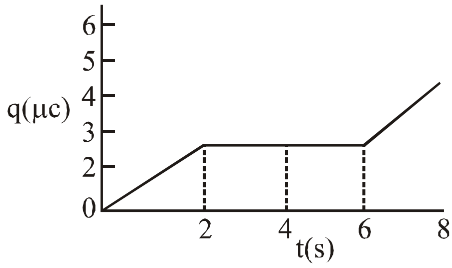
18. In the figure shown below, the charge on the left plate of the 10 μF capacitor is –30 μC. ? The charge on the right plate of the 6 μF capacitor is : **[JEE Main 2019,Jan.; 4/120, –1]**



(1) –18 μC (2) –12 μC (3) +12 μC (4) +18 μC

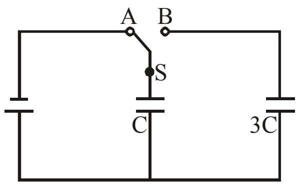
19. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure:

What is the value of current at t = 4 s ? **[JEE Main 2019 Jan.; 4/120, –1]**



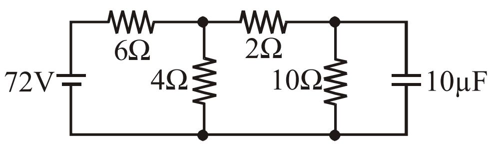
(1) 3μA (2) 2μA (3) zero (4) 1.5μA

20. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is: **[JEE Main 2019 Jan.; 4/120, –1]**



(1) $\frac{3}{8}\frac{Q^2}{C}$ (2) $\frac{3}{4}\frac{Q^2}{C}$ (3) $\frac{1}{8}\frac{Q^2}{C}$ (4) $\frac{5}{8}\frac{Q^2}{C}$

21. Determine the charge on the capacitor in the following circuit : **[JEE Main 2019, April.; 4/120, –1]**



(1) 2μC (2) 60μC (3) 200μC (4) 10μC

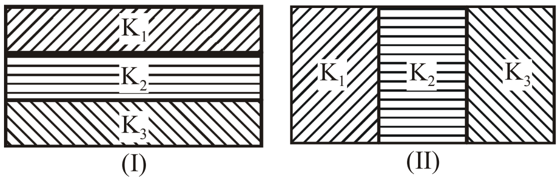
22. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :- **[JEE Main 2019, April.; 4/120, –1]**

(1) $\frac{V}{K+n}$ (2) V (3) $\frac{(n+1)V}{(K+n)}$ (4) $\frac{nV}{K+n}$

23. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II.

If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :

[JEE Main 2019, April.; 4/120, –1]



(1) $\dfrac{E_1}{E_2} = \dfrac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(2) $\dfrac{E_1}{E_2} = \dfrac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(3) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(4) $\dfrac{E_1}{E_2} = \dfrac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

Answers

Exercise # 1

PART - I

Section (A) :

A-1. 068

A-2. **(i)** 6 V **(ii)** 90 μJ

(iii) (a) $\frac{16}{3}$ V (b) $\frac{5}{3}$ μJ (c) $\frac{Q_{5\mu F}}{Q_{10\mu F}} = \frac{1}{2}$

(d) $Q_{5\mu F} = \frac{80}{3}\mu C$ $Q_{10\mu F} = \frac{160}{3}\mu C$

A-3. $\frac{q^2}{2k\in_0 A}$

A-4. $\frac{\in_0}{2g}\times 10^{10}$ kg = 4.425 g

A-5. **(i)** $\frac{q^2(x_2-x_1)}{2\in_0 S}$ **(ii)** $-\frac{\in_0 SV^2\left(\frac{1}{x_2}-\frac{1}{x_1}\right)}{2}$

Section (B) :

B-1. 7 **B-2.** V_o = 20 V

B-3. 6V, 34V

B-4. (a) ε (b) $\frac{\varepsilon}{R}$ (c) ε (d) $\frac{1}{2}$ Cε^2

(e) $\frac{\varepsilon^2}{R}$ (f) $\frac{\varepsilon^2}{R}$

B-5. $\frac{1}{2}(4 \times 10^{-6})(12)^2$ J = 0.288 mJ

B-6. (a) 4000 μC, 2000 μC

(b) 2000 μC

(c) work is done on the battery, 20 mJ

(d) 30 mJ

(e) 10 mJ

Section (C) :

C-1. (i) 20 μC, 40 μC, 60 μC

(ii) 2400 μJ

(iii) 1200 μJ

C-2. 9 **C-3.** $\frac{32}{23}$ μF

C-4. (a) five 2 μF capacitors in series

(b) 3 parallel rows; each consisting of five 2.0 μF capacitors in series

C-5. (a) 700/3 V at each point (b) zero

C-6. (a) 10 μF, (b) 10μF, (c) 10μF

C-7. 4 μC, 16 μC, 12 μC and 32 μC, 1 A.

C-8. (a) Cε/2, (b) –Cε, (c) Cε^2/2

(d) Cε^2/4 (e) Cε^2/4

C-9. 12μC

Section (D) :

D-1. $\frac{10^{-7}}{\ln(3/2)}$ F = 0.25 μF

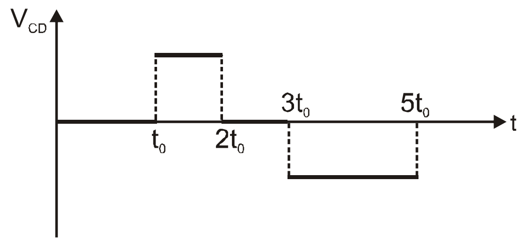
D-2. q = 20 × 10^{-4} (1 – e^{-2})e^{-2} = 233.55 μC

D-3. **(i)** (a) 10 s (b) 2 μC (c) 10 ln2 = 6.93 sec.

(ii) q = (2 e^{-5})μC = 1.348 x 10^{-8} C

D-4. 225

D-5. $40\,(1 - e^{-2})e^{-2}\ \mu J = 4.7\ \mu J$.

D-6.


Section (E) :

E-1. (i) $20\epsilon_0 = 180$ pF (ii) 5.4×10^{-7} C

(iii) 540 pF (iv) 3

(v) $27 \times 10^{-12}\ C^2 N^{-1} m^{-2}$ (vi) 3×10^5 V/m

(vii) 1×10^5 V/m

E-2 $\dfrac{25}{24}\dfrac{\varepsilon_0 A}{d}$

E-3. $C = \dfrac{\epsilon_0 A}{d - t + \dfrac{t}{k}}$, $\dfrac{t}{d} = \dfrac{2}{3}$, $\dfrac{U_i}{U_F} = \dfrac{3}{2}$, $\Delta U = -\dfrac{q^2 d}{6 \epsilon_0 A}$

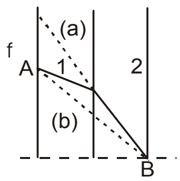
E-4. $\dfrac{CV}{E\,\epsilon_r \epsilon_0} = \dfrac{\pi}{5} = 0.62\ m^2$

E-5. $\dfrac{E_i}{E_f} = \dfrac{1}{2}(1 + k)$, $\Delta q = \dfrac{1}{2} C\varepsilon \dfrac{k-1}{k+1}$

E-6. $\dfrac{3}{4}J$ **E-7.** $\rho = \tau/\varepsilon_0 \varepsilon \ln 2 = 1.4 \times 10^{13}\ \Omega.m.$

E-8. $\dfrac{\epsilon_0 b}{2dK_S}\left[(K_2 - 1)\varepsilon_2^{\,2} - (K_1 - 1)\varepsilon_1^{\,2}\right]$

E-9. $4\sqrt{\dfrac{2bm}{\epsilon_0 V^2 (K-1)}}$.

E-10.


PART - II

Section (A) :

A-1. (D) **A-2.** (D) **A-3.** (A)

A-4. **(i)** (C) **(ii)** (D) **(iii)** (D) **(iv)** (C)

A-5. (B,C) **A-6.** (B)

Section (B) :

B-1. (B) **B-2.** (D) **B-3.** (C)

B-4. (C) **B-5.** (C) **B-6.** (C)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (B)

C-4. (D) **C-5** (C) **C-6.** (C)

C-7. (C) **C-8.** (D) **C-9.** (A)

Section (D) :

D-1. **(i)** (A) **(ii)** (B)

D-2. **(i)** (C) **(ii)** (A)

(iii) (C) **(iv)** (C)

D-3. (C) **D-4.** **(i)** (A) **(ii)** (C)

D-5. **(i)** (B) **(ii)** (C) **(iii)** (A)

D-6. (C)

Section (E) :

E-1. (B) **E-2.** (C) **E-3.** (C)

E-4. (A) **E-5.** (A) **E-6.** (A)

E-7. (C) **E-8.** (B) **E-9.** (B)

PART - III

1. (A) $\rightarrow$ p ; (B) $\rightarrow$ r ; (C) $\rightarrow$ q ; (D) $\rightarrow$ p

2. (A) $\rightarrow$P,Q,R; (B) $\rightarrow$ Q,S,T ; (C) $\rightarrow$ S,T; (D) $\rightarrow$P,R

3. (A) $\rightarrow$ p,q,s (B) $\rightarrow$ p,r,s (C) $\rightarrow$ p,q (D) $\rightarrow$ p,r

Exercise # 2

PART - I

1. (A) **2.** (B) **3.** **(i)** (D) **(ii)** (A)

4. (B) **5.** (D) **6.** (C)

7. **(i)** (C) **(ii)** (A) **8.** (D)

9. (A) **10.** (D) **11.** (B)

12. **(i)** (A) **(ii)** (D) **(iii)** (B)

13. **(i)** (A) **(ii)** (D)

14. (A) **15.** (A) **16.** (D) **17.** (A)

18. (B) **19.** (B) **20.** (B) **21.** (B)

22. (B)

PART - II

1. 5 **2.** 3 **3.** 2

4. (a) 5 (b) 10 V **5.** 24 V

6. 9 **7.** 6 μC **8.** 750

9. 20 **10.** 1

11. $U = E/(1 + 3\eta + \eta^2) = 10$ V.

12. $\frac{11}{5\ell n3} = 2.0\ \Omega$. **13.** 1

14. (a) – 6.00 V; (b) 6.00 V; (c) –54 μC

15. 1 **16.** 4

PART - III

1. (A, B, D) **2.** (A, B, C) **3.** (A,B,C,D)

4. (A,B,C,D) **5.** (B, C, D) **6.** (A, C)

7. (B, C) **8.** (A, D) **9.** (A,B,C,D)

10. (A, B, C) **11.** (A, B, C) **12.** (B,C)

13. (B, C, D) **14.** (B, D) **15.** (A, B, C)

16. (A, C, D) **17.** (B, C, D) **18.** (A, C)

19. (A, C) **20.** (A,B,C,D) **21.** (A, C, D)

PART - IV

1. (A) **2.** (C) **3.** (A)

4. (C) **5.** (D) **6.** (C)

Exercise # 3

PART - I :

1. t = 2 sec **2.** (D) **3.** (C)

4. (B, D) **5.** (A, D) **6.** (D)

7. (A,B,C,D) **8.** (A) **9.** (A)

10. 1.50 **11.** (C, D)

12. (0.99 to1.01)

PART - II

1. (3) **2.** (3) **3.** (3)

4. (4) **5.** (2) **6.** (1)

7. (4) **8.** (4) **9.** (3)

10. (4) **11.** (1) **12.** (2)

13. (4) **14.** (4) **15.** (3)

16. (3) **17.** (1) **18.** (4)

19. (3) **20.** (1) **21.** (3)

22. (3) **23.** (1)

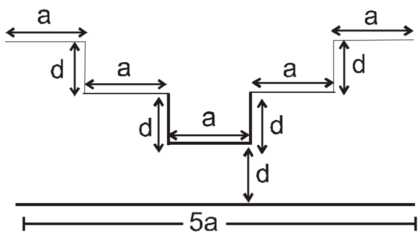
RANKER PROBLEMS

SUBJECTIVE QUESTIONS

1. Three long concentric conducting cylindrical shells have radii R, 2R and $2\sqrt{2}$ R. Inner and outer shells are connected to each other. Find the capacitance across middle and inner shells per unit length?

2. Six 1 μF capacitors are so arranged that their equivalent capacitance is 0.70 μF. If a potential difference of 600 volt is applied to the combination, what charge will appear on each capacitor?

3. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. The width of each stair is a and the height is d. Both plates have the same width perpendicular to plane of paper. Find the capacitance of the assembly.



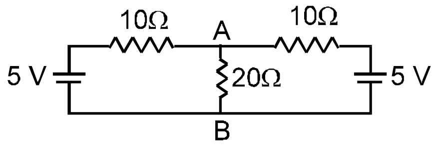
4. The circular plates A and B of a parallel plate air capacitor have a diameter of 0.1 m and are 2 x 10^{-3} m apart. The plates C and D of a similar capacitor have a diameter of 0.12 m and are 3 x 10^{-3} m apart. Plate A is earthed. Plates B and D are connected together. Plate C is connected to the positive pole of a 120 volt battery whose negative is earthed. Calculate **[REE - 98,5]**

(i) The combined capacitance of the arrangement and

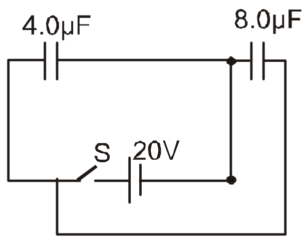
(ii) The energy stored in it.

5. (a) Find the current in the 20 Ω resistor shown in figure.

(b) If a capacitor of capacitance 4μF is joined between the points A and B, what would be the electrostatic energy stored in it in steady state ?



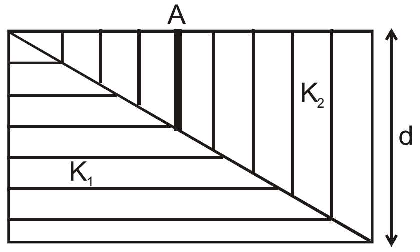
6.



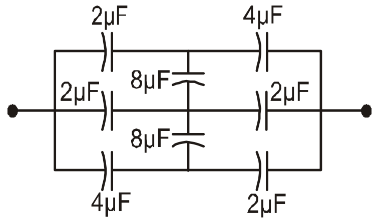
(i) Find the total charge flown through the battery in the arrangement shown in figure after switch S is closed (initially all the capacitors are uncharged).

(ii) Find out final charge on each capacitor.

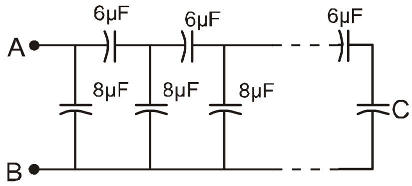
7. The capacitance of a parallel plate capacitor with plate area A and separation d, is C. The space between the plates is filled with two wedges of dielectric constant K_1 and K_2 respectively (figure). Find the capacitance of the resulting capacitor. **[JEE-1996; 2M/100]**



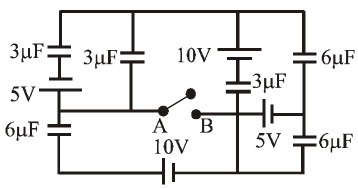
8. Find the equivalent capacitance of the combinations shown in the figure between the indicated points.



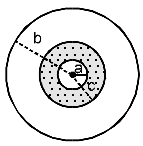
9. A finite ladder circuit is constructed by connecting several sections of 6 μF, 8 μF capacitor combinations as shown in the figure. Circuit is terminated by a capacitor of capacitance C. Find the value of C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between?



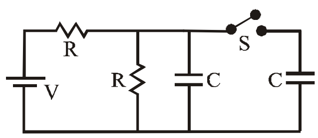
10. Find the charge flown through the switch from A to B when it is closed.



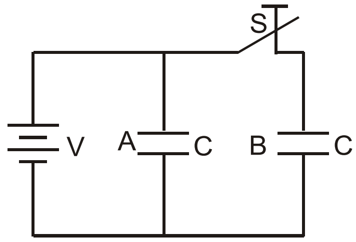
11. A spherical capacitor is made of two conducting spherical shells of radii a and b = 3a. The space between the shells is filled with a dielectric of dielectric constant K = 3 upto a radius c = 2a as shown. If the capacitance of given arrangement is n times the capacitance of an isolated spherical conducting shell of radius a. Then find value of n.



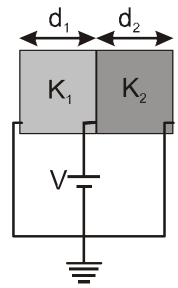
12. In the figure shown initially switch is open for a long time. Now the switch is closed at t = 0. Find the charge on the rightmost capacitor as a function of time given that it was intially uncharged.



13. The figure shows two identical parallel plate capacitors connected to a battery with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant (or relative permittivity) 3. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric. **[JEE 1983; 6M]**



14. A capacitor is composed of three parallel conducting plates. All three plates are of the same area A. The first pair of plates are kept a distance d_1 apart, and the space between them is filled with a medium of a dielectric K_1. The corresponding data for the second pair are d_2 and K_2, respectively. The middle plate is connected to the positive terminal of a constant voltage source V and the external plates are connected the other terminal of V.



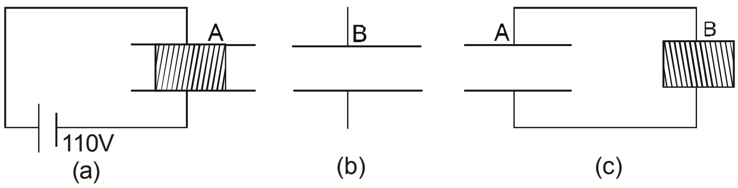
(a) Find the capacitance of the system.

(b) What is the surface charge density on the middle plate ?

(c) Compute the energy density in the medium K_1.

15. Two parallel plate capacitors A and B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A and B are 0.04 m^2 and 0.02 m^2 respectively. A slab of dielectric constant (relative permittivity K = 9) has dimensions such that it can exactly fill the space between the plates of capacitor B.

[JEE 1993, 2+3+2=7 Marks]



(i) The dielectric slab is placed inside A as shown in the figure (a). A is charged to potential difference of 110 V. Calculate the capacitance of A and energy stored in it :

(ii) The battery is disconnected and then the dielectric also slab is moved from A. Find the work done by the external agency in removing the slab from A.

(iii) The same dielectric slab is now placed inside B, filling it completely. The two capacitors A and B are then connected as shown in the figure (c) . Calculate the energy stored in the system.

16. A potential difference of 300 V is applied between the plates of a parallel plate capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find :

(i) Intensity of electric field in each layer.

(ii) The drop of potential in each layer.

(iii) Surface charge density on the capacitor. Given that : $k_{glass} = 6$, $k_{paraffin} = 2$

17. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d, when filled with a dielectric whose dielectric constant varies as; **[REE 2000,6]**

$$K(x) = 1 + \frac{\beta x}{\epsilon_0} \qquad 0 < x < \frac{d}{2}\,; \qquad K(x) = 1 + \frac{\beta}{\epsilon_0}(d - x) \qquad \frac{d}{2} < x < d.$$

For what value of β would the capacity of the condenser be twice that when it is without any dielectric?

Answers

RANKER PROBLEMS

1. $\dfrac{6\pi \in_0}{\ln 2}$

2. 420 μC on one, 180 μC on two, 60 μC on remaining 3 capacitors

3. $\dfrac{8 \in_o A}{15\,d}$

4. (i) $\dfrac{30\pi \in_0}{49} \approx 17\text{pF}$ (ii) 122.4 nJ

5. (a) $\dfrac{1}{5}$A (b) 32 μJ

6. (i) 240 μC (ii) $Q_{4\mu F} = 80\ \mu C$, $Q_{8\mu F} = 160\ \mu C$

7. $C_R = \dfrac{CK_1K_2}{K_2 - K_1} \ln \dfrac{K_2}{K_1}$ where $C = \dfrac{\in_0 A}{d}$

8. $\dfrac{27}{7}\mu F$

9. 12 μF

10. 69 mC

11. n = 3

12. $q = \dfrac{CV}{2}\left(1 - \dfrac{1}{2}e^{-t/RC}\right)$

13. $\dfrac{3}{5}$

14. (a) $\in_0 A\left(\dfrac{K_1}{d_1} + \dfrac{K_2}{d_2}\right)$

(b) $\dfrac{Q_1}{A} = \left(\dfrac{K_1 \in_0 A}{d_1}\right)\dfrac{V}{A} = \dfrac{K_1 \in_0}{d_1}V$ and $\dfrac{Q_2}{A} = \left(\dfrac{K_2 \in_0 A}{d_2}\right)\dfrac{V}{A} = \dfrac{K_2 \in_0}{d_2}V$ (c) $\dfrac{\in_0 K_1 V^2}{2d_1^2}$

15. (i) 2nF, 12.1μJ (ii) 48.4μJ (iii) 11μJ

16. (i) 1.5×10^4 V/m, 4.5×10^4 V/m, (ii) 75 V, 225 V, (iii) 8×10^{-7} C/m^2

17. $C = \dfrac{A\beta}{2\ell n\left(1 + \dfrac{\beta d}{2 \in_0}\right)}$, $\beta d = 4\in_0 \ell n\left(1 + \dfrac{\beta d}{2 \in_0}\right)$

Solution of this equation gives required value of β .

SELF ASSESSMENT PAPER

JEE (ADVANCED) PAPER-1

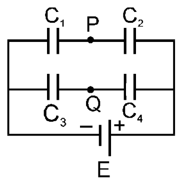
SECTION-1 : ONE OPTION CORRECT (Maximum Marks - 12)

1. Two identical capacitors have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is:

(A) $\frac{1}{4}C(V_1^2 - V_2^2)$ (B) $\frac{1}{4}C(V_1^2 + V_2^2)$

(C) $\frac{1}{4}C(V_1 - V_2)^2$ (D) $\frac{1}{4}C(V_1 + V_2)^2$

2. The potential difference between the points P and Q in the adjoining circuit will be-

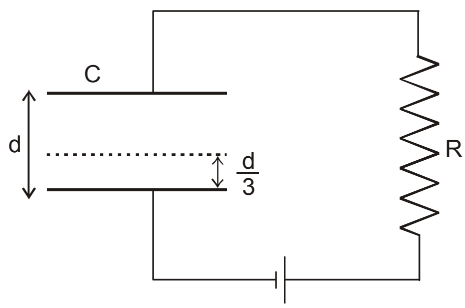


(A) $\frac{(C_1C_4 - C_2C_3)E}{(C_1+C_3)(C_2+C_4)}$ (B) $\frac{C_2C_3E}{C_1C_2(C_3+C_4)}$

(C) $\frac{(C_2C_3 - C_1C_4)E}{(C_1+C_2)(C_3+C_4)}$ (D) $\frac{(C_2C_3 - C_1C_4)E}{(C_1+C_2+C_3+C_4)}$

3. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is

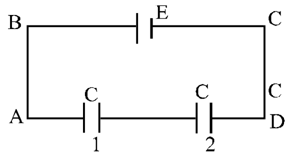
Figure :



(A) $\frac{6\varepsilon_0 R}{5d+3Vt}$ (B) $\frac{(15d+9Vt)\,\varepsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$

(C) $\frac{6\varepsilon_0 R}{5d-3Vt}$ (D) $\frac{(15d-9Vt)\,\varepsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

4. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric consatnt K is inserted between the plates of one of the capacitor, the total charge flowing through battery is :-



(A) $\dfrac{KCE}{K+1}$ from B to C

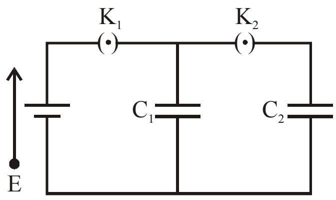
(B) $\dfrac{KCE}{K+1}$ from C to B

(C) $\dfrac{(K-1)CE}{2(K+1)}$ from B to C

(D) $\dfrac{(K-1)CE}{2(K+1)}$ from C to B

SECTION-2 : ONE OR MORE THAN ONE CORRECT (MAXIMUM MARKS - 32)

5. In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q'_1 and Q'_2 as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively. Then



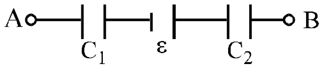
(A) charge on C_1 gets redistributed such that $V_1 = V_2$

(B) charge on C_1 gets redistributed such that $Q'_1 = Q'_2$

(C) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1 E$

(D) charge on C_1 gets redistributed such that $Q'_1 + Q'_2 = Q$

6. A circuit shown in the figure consists of a battery of emf 10 V and two capacitance C_1 and C_2 of capacitances 1.0 μF and 2.0 μF respectively. The potential difference $V_A - V_B$ is 5V



(A) charge on capacitor C_1 is equal to charge on capacitor C_2

(B) Voltage across capacitor C_1 is 5V.

(C) Voltage across capacitor C_2 is 10 V

(D) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2.

7. A parallel plate capacitor has a parallel slab of copper inserted between and parallel to the two plates, without touching the plates. The capacity of the capacitor after the introduction of the copper sheet is:

(A) minimum when the copper slab touches one of the plates.

(B) maximum when the copper slab touches one of the plates.

(C) invariant for all positions of the slab between the plates.

(D) greater than that before introducing the slab.

8. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then :

(A) some charge from the capacitor will flow back into the source.

(B) some extra charge from the source will flow back into the capacitor.

(C) the electric field intensity between the two plate does not change.

(D) the electric field intensity between the two plates will decrease.

9. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant k. The capacitor is connected to a cell of emf E, and the slab is taken out

(A) charge CE(k – 1) flows through the cell

(B) energy $E^2C(k - 1)$ is absorbed by the cell.

(C) the energy stored in the capacitor is reduced by $E^2C(k - 1)$

(D) the external agent has to do $\frac{1}{2}E^2C(k-1)$ amount of work to take the slab out.

10. A capacitor of capacity C_0 is connected to a battery of emf V_0. When steady state is attained a dielectric slab of dielectric constant K is slowly introduced in the capacitor. Mark the Correct statement(s), in final steady state :-

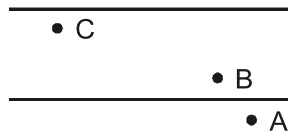
(A) Magnitude of induced charge on the each surface of slab is $C_0V_0(K - 1)$

(B) Net electric force due to induced charges on the plate is zero.

(C) Force of attraction between plates of capacitor is $\frac{K(C_0V_0)^2}{2\in_0 A}$

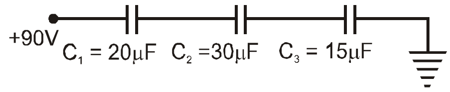
(D) Net field due to induced charges in dielectric slab is $\frac{8V_0(k-1)^2}{K\in_0 A}$

11. For a charged parallel plate capacitor shown in the figure, the force experienced by an alpha particle will be :



(A) maximum at C (B) zero at A (C) same at B and C (D) zero at C

12. We have a combination as shown in following figure. Choose the correct options :



(A) The charge on each capacitor is 600 μC

(B) The potential difference between the plates of C_1 is 30 V

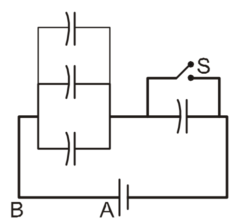
(C) The potential difference between the plates of C_2 is 20 V

(D) The potential difference between the plates of C_3 is 40 V

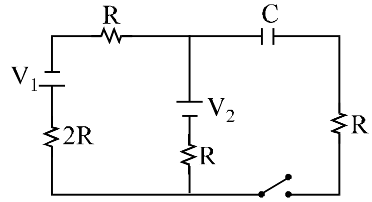
SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. A capacitor of capacitance 10 μF is connected to a battery of emf 2V.It is found that it takes 50 ms for the charge on the capacitor to become 12.6 μC. Then the resistance of the circuit in kΩ is : (Take 1/e = 0.37)

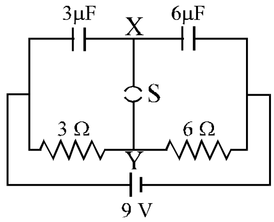
14. Four capacitors of capacitance 10 μF and a battery of 200V are arranged as shown. If the charge flows through AB after the switch S is closed is n × 10^{-3} C. Find n?



15. In the transient circuit shown, if the time constant of the circuit is xRC. Find x:

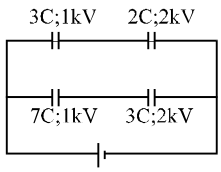


16. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X in μ C is :-



17. A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of 2V Volt with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be :-

18. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source in kV such that no capacitor breaks down?



Answers

ANSWERS TO SELF ASSESSMENT PAPER

1.	(C)	**2.**	(C)	**3.**	(A)	**4.**	(D)	**5.**	(A, C, D)
6.	(A, D)	**7.**	(C, D)	**8.**	(B, C)	**9.**	(A, B, D)	**10.**	(A, B)
11.	(B, C)	**12.**	(A, B, C, D)	**13.**	05.00	**14.**	04.50	**15.**	01.75
16.	27.00	**17.**	02.25	**18.**	2.5				