QUADRATIC EQUATIONS

FACTORISATION METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers α and β such that sum of α and β is equal to b and product of α and β is equal to ac.
- Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term and let factors are (x + p) and (x + q) i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate reach factor to zero and find the values of x.
- These values of x are the required roots/solutions of the given quadratic equation.

IMPORTANT QUESTIONS

Solve the quadratic equation by using factorization method: $x^2 + 2x – 8 = 0$

Solution: $x^2 + 2x – 8 = 0$

$\Rightarrow x^2 + 4x – 2x – 8 = 0 \Rightarrow x(x + 4) – 2(x + 4) = 0$

$\Rightarrow (x + 4)(x – 2) = 0 \Rightarrow x + 4 = 0, x – 2 = 0 \Rightarrow x = – 4, 2$

Questions for practice

1. Solve the quadratic equation using factorization method: $x^2 + 7x – 18 = 0$

2. Solve the quadratic equation using factorization method: $x^2 + 5x – 6 = 0$

3. Solve the quadratic equation using factorization method: $y^2 – 4y + 3 = 0$

4. Solve the quadratic equation using factorization method: $x^2 – 21x + 108 = 0$

5. Solve the quadratic equation using factorization method: $x^2 – 11x – 80 = 0$

6. Solve the quadratic equation using factorization method: $x^2 – x – 156 = 0$

7. Solve the following for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.

8. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where $D = b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D. There are three cases –

Case – I

When $D > 0$ i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

i.e. $x = \frac{-b + \sqrt{D}}{2a}$ *and* $\frac{-b - \sqrt{D}}{2a}$

Case – II

When $D = 0$, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ *and* $\frac{-b}{2a}$

Case – III

When $D < 0$ then there is no real roots exist.

IMPORTANT QUESTIONS

Find the discriminant of the quadratic equation 2x2 – 4x + 3 = 0, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where a = 2, b = – 4 and c = 3.
Therefore, the discriminant, $D = b^2 – 4ac = (– 4)^2 – (4 \times 2 \times 3) = 16 – 24 = – 8 < 0$
So, the given equation has no real roots.

Questions for Practice

1. Find the discriminant and the nature of the roots of quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 2x2 + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 – 6x – 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 – 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

IMPORTANT QUESTIONS

Solve the quadratic equation by using quadratic formula: $x^2 + x – 6 = 0$

Solution: Here, a = 1, b = 1, c = –6

$\Rightarrow D = b^2 – 4ac = 1 – 4(1)(–6) = 1 + 24 = 25 > 0$

Now, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2(1)} = \frac{-1 \pm 5}{2} \Rightarrow x = \frac{-1-5}{2} or \frac{-1+5}{2} \Rightarrow x = \frac{-6}{2} or \frac{4}{2} \Rightarrow x = -3 \ or \ 2$

Questions for practice

1. Solve the quadratic equation by using quadratic formula: $x^2 – 7x + 18 = 0$
2. Solve the quadratic equation by using quadratic formula: $x^2 – 5x + 6 = 0$
3. Solve the quadratic equation by using quadratic formula: $y^2 + 4y + 3 = 0$
4. Solve the quadratic equation by using quadratic formula: $x^2 + 11x – 80 = 0$
5. Solve the quadratic equation by using quadratic formula: $x^2 + x – 156 = 0$
6. Solve for x by using quadratic formula : $9x^2 – 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

MCQ (1 MARK)

1. The roots of the equation $x^2 + 7x + 10 = 0$ are
   (a) 2 and 5 (b) –2 and 5 (c) –2 and –5 (d) 2 and –5

2. If α, β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$
   (a) 0 (b) 1 (c) –1 (d) none of these

3. If the equation $x^2 + 4x + k = 0$ has real and distinct roots then

(a) $k < 4$ (b) $k > 4$ (c) $k \leq 4$ (d) $k \geq 4$

4. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then the roots are both equal to

(a) $\pm\frac{2}{3}$ (b) $\pm\frac{3}{2}$ (c) 0 (d) ± 3

5. If the equation $x^2 – bx + 1 = 0$ has two distinct roots then

(a) $–3 < b < 3$ (b) $–2 < b < 2$ (c) $b > 2$ (d) $b < –2$

6. If $x = 1$ is a common root of the equations $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$ then $ab =$

(a) 6 (b) 3 (c) –3 (d) $\frac{7}{2}$

7. If p and q are the roots of the equation $x^2 – px + q = 0$, then

(a) $p = 1, q = –2$ (b) $p = –2, q = 0$ (c) $b = 0, q = 1$ (d) $p = –2, q = 1$

8. If the equation $ax^2 + bx + c = 0$ has equal roots then $c =$

(a) $\frac{-b}{2a}$ (b) $\frac{b}{2a}$ (c) $\frac{-b^2}{4a}$ (d) $\frac{b^2}{4a}$

9. If the equation $ax^2 + 2x + a = 0$ has two distinct roots if

(a) $a = \pm 1$ (b) $a = 0$ (c) $a = 0, 1$ (d) $a = –1, 0$

10. Find the values of k for which the quadratic equation $2x^2 + kx + 3 = 0$ has real equal roots.

(a) $\pm 2\sqrt{6}$ (b) $2\sqrt{6}$ (c) 0 (d) ± 2

11. Find the values of k for which the quadratic equation $kx(x – 3) + 9 = 0$ has real equal roots.

(a) $k = 0$ or $k = 4$ (b) $k = 1$ or $k = 4$ (c) $k = –3$ or $k = 3$ (d) $k = –4$ or $k = 4$

12. Find the values of k for which the quadratic equation $4x^2 – 3kx + 1 = 0$ has real and equal roots.

(a) $\pm\frac{4}{3}$ (b) $\pm\frac{2}{3}$ (c) ± 2 (d) none of these

13. The value of k for which equation $9x^2 + 8xk + 8 = 0$ has equal roots is:

(a) only 3 (b) only –3 (c) ± 3 (d) 9

14. Which of the following is not a quadratic equation?

(a) $x - \frac{3}{x} = 4$ (b) $3x - \frac{5}{x} = x^2$ (c) $x + \frac{1}{x} = 3$ (d) $x^2 - 3 = 4x^2 - 4x$