

CHAPTER 17

Indefinite Integrals

Section-A

JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log(9e^{2x} - 4) + C$, then $A = \dots, B = \dots$ and $C = \dots$ (1990 - 2 Marks)

C MCQs with One Correct Answer

1. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ is (1995S)

- (a) $\sin x - 6 \tan^{-1}(\sin x) + c$
 (b) $\sin x - 2(\sin x)^{-1} + c$
 (c) $\sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x) + c$
 (d) $\sin x - 2(\sin x)^{-1} + 5 \tan^{-1}(\sin x) + c$

2. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is (2005S)

- (a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
 (c) 3 (d) $\sqrt{3}$

3. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx =$ (2006 - 3M, -1)

- (a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$
 (c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

4. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$. Then, for an arbitrary constant C , the value of $J - I$ equals (2008)

(a) $\frac{1}{2} \log \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + C$ (b) $\frac{1}{2} \log \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C$

(c) $\frac{1}{2} \log \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + C$ (d) $\frac{1}{2} \log \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + C$

5. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^2} dx$ equals (for some arbitrary constant K) (2012)

(a) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(b) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(c) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

(d) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

E Subjective Problems

1. Evaluate $\int \frac{\sin x}{\sin x - \cos x} dx$ (1978)

2. Evaluate $\int \frac{x^2 dx}{(a + bx)^2}$ (1979)

3. Evaluate $\int (e^{\log x} + \sin x) \cos x dx$. (1981 - 2 Marks)

4. Evaluate: $\int \frac{(x-1)e^x}{(x+1)^3} dx$ (1983 - 2 Marks)

5. Evaluate the following $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ (1984 - 2 Marks)
6. Evaluate the following $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ (1985 - 2½ Marks)
7. Evaluate: $\int \left[\frac{(\cos 2x)^{1/2}}{\sin x} \right] dx$ (1987 - 6 Marks)
8. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$ (1989 - 3 Marks)
9. Find the indefinite integral $\int \left(\frac{1}{\sqrt[3]{x} + \sqrt[4]{4}} + \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} \right) dx$ (1992 - 4 Marks)
10. Find the indefinite integral $\int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$ (1994 - 5 Marks)
11. Evaluate $\int \frac{(x+1)}{x(1+xe^x)^2} dx$. (1996 - 2 Marks)
12. Integrate $\int \frac{x^3 + 3x + 2}{(x^2 + 1)^2 (x + 1)} dx$. (1999 - 5 Marks)
13. Evaluate $\int \sin^{-1} \left(\frac{2x+2}{\sqrt{4x^2+8x+13}} \right) dx$. (2001 - 5 Marks)
14. For any natural number m, evaluate $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx, x > 0$. (2002 - 5 Marks)

H Assertion & Reason Type Questions

1. Let $F(x)$ be an indefinite integral of $\sin^2 x$.
- STATEMENT-1** : The function $F(x)$ satisfies $F(x + \pi) = F(x)$ for all real x. **because**
- STATEMENT-2** : $\sin^2(x + \pi) = \sin^2 x$ for all real x. (2007 - 3 marks)
- (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True.

Section-B

JEE Main / AIEEE

1. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is [2004]
- (a) $(-\cos \alpha, \sin \alpha)$ (b) $(\cos \alpha, \sin \alpha)$
 (c) $(-\sin \alpha, \cos \alpha)$ (d) $(\sin \alpha, \cos \alpha)$
2. $\int \frac{dx}{\cos x - \sin x}$ is equal to [2004]
- (a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$
 (b) $\frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + C$
 (c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$
 (d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$
3. $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to [2005]
- (a) $\frac{\log x}{(\log x)^2 + 1} + C$ (b) $\frac{x}{x^2 + 1} + C$
 (c) $\frac{xe^x}{1 + x^2} + C$ (d) $\frac{x}{(\log x)^2 + 1} + C$
4. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ equals [2007]
- (a) $\log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$
 (b) $\log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$
 (c) $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$
 (d) $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$
5. The value of $\sqrt{2} \int \frac{\sin x dx}{\sin \left(x - \frac{\pi}{4} \right)}$ is [2008]
- (a) $x + \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$
 (b) $x - \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$
 (c) $x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$
 (d) $x - \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$
6. If the $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$, then a is equal to : [2012]
- (a) -1 (b) -2
 (c) 1 (d) 2
7. If $\int f(x) dx = \psi(x)$, then $\int x^5 f(x^3) dx$ is equal to [JEE M 2013]
- (a) $\frac{1}{3} \left[x^3 \psi(x^3) - \int x^2 \psi(x^3) dx \right] + C$
 (b) $\frac{1}{3} x^3 \psi(x^3) - 3 \int x^3 \psi(x^3) dx + C$
 (c) $\frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$
 (d) $\frac{1}{3} \left[x^3 \psi(x^3) - \int x^3 \psi(x^3) dx \right] + C$
8. The integral $\int \left(1 + x - \frac{1}{x} \right) e^{x + \frac{1}{x}} dx$ is equal to [JEE M 2014]
- (a) $(x+1)e^{x + \frac{1}{x}} + c$ (b) $-xe^{x + \frac{1}{x}} + c$
 (c) $(x-1)e^{x + \frac{1}{x}} + c$ (d) $xe^{x + \frac{1}{x}} + c$

9. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : [JEE M 2015]
10. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3} dx$ is equal to :

[JEE M 2016]

- (a) $-(x^4+1)^{\frac{1}{4}} + c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$
- (c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (d) $(x^4+1)^{\frac{1}{4}} + c$

- (a) $\frac{x^5}{2(x^5+x^3+1)^2} + C$ (b) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$
- (c) $\frac{-x^5}{(x^5+x^3+1)^2} + C$ (d) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$

where C is an arbitrary constant.

Section-A : JEE Advanced/ IIT-JEE

A 1. $\frac{-3}{2}, \frac{35}{36}$, any real value

C 1. (c) 2. (c) 3. (d) 4. (c) 5. (c)

E 1. $\frac{1}{2} \log |\sin x - \cos x| + \frac{x}{2} + C$ 2. $\frac{1}{b^3} \left[a + bx - 2a \log |a + bx| - \frac{a^2}{a + bx} \right] + C$

3. $x \sin x + \cos x - \frac{1}{4} \cos 2x + C$ 4. $\frac{e^x}{(x+1)^2} + C$ 5. $-\left(1 + \frac{1}{x^4}\right)^{\frac{1}{4}} + C$

6. $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x} \sqrt{1-x} + C$ 7. $\frac{1}{\sqrt{2}} \log \left[\frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}} \right] - \log(\cot x + \sqrt{\cot^2 x - 1}) + C$

8. $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + c$

9. $\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2x^{1/2} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12}$
 $+ 12 \log |x^{1/2} + 1| + 6 \left\{ \frac{(1+x^{1/6})^3}{3} - \frac{3}{2}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\}$

10. $\frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \frac{1}{2} \ln \sec 2\theta + C$

11. $\log \left(\frac{1+xe^x}{xe^x} \right) - \frac{1}{1+xe^x} + C$

$\ln(1+x^{1/6}) - \left\{ \frac{(1+x^{1/6})^3}{9} - \frac{3}{4}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\}$

12. $-\frac{1}{2} \log |x+1| + \frac{1}{4} \log(x^2+1) + \frac{3}{2} \tan^{-1} x + \frac{x}{1+x^2} + C$ $+ \frac{(\ln(1+x^{1/6}))^2}{2} + C$

13. $(x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(4x^2+8x+13) + C$ 14. $\frac{1}{6} \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}}}{m+1} + C$

H 1. (d)

Section-B : JEE Main/ AIEEE

1. (b) 2. (a) 3. (d) 4. (c) 5. (c) 6. (d) 7. (c)
 8. (d) 9. (b) 10. (d)

Section-A

JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \ln(9e^{2x} - 4) + C$
- $$\Rightarrow \frac{d}{dx} [Ax + B \ln(9e^{2x} - 4) + C] = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$
- $$\Rightarrow A + \frac{18Be^x}{9e^x - 4e^{-x}} = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$
- $$\Rightarrow \frac{(9A + 18B)e^x - 4Ae^{-x}}{9e^x - 4e^{-x}} = \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}$$
- $$\Rightarrow 9A + 18B = 4; -4A = 6 \Rightarrow A = -\frac{3}{2};$$
- $$B = \left(4 + \frac{27}{2}\right) \frac{1}{18} = \frac{35}{36}; C \text{ can have any real value.}$$

C. MCQs with ONE Correct Answer

1. (c) Let $I = \int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx = \int \frac{(\cos^2 x + \cos^4 x) \cos x}{\sin^2 x (1 + \sin^2 x)} dx$
- $$= \int \frac{[1 - \sin^2 x + (1 - \sin^2 x)^2] \cos x}{\sin^2 x (1 + \sin^2 x)} dx$$
- $$= \int \frac{(2 - 3\sin^2 x + \sin^4 x) \cos x}{\sin^2 x (1 + \sin^2 x)} dx$$
- Put $\sin x = t \Rightarrow \cos x dx = dt$
- $$I = \int \frac{2 - 3t^2 + t^4}{t^4 + t^2} dt, I = \int \left(1 + \frac{2}{t^2} - \frac{6}{t^2 + 1}\right) dt$$
- $$= t - \frac{2}{t} - 6 \tan^{-1}(t) + C$$
- $$= \sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x) + C$$
2. (c) Given that $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$
- $$\Rightarrow \frac{d}{dx} \int_{\sin x}^1 t^2 f(t) dt = \frac{d}{dx} (1 - \sin x)$$
- $$\Rightarrow -\sin^2 x f(\sin x) \cdot \cos x = -\cos x$$
- $$\Rightarrow f(\sin x) = \frac{1}{\sin^2 x} \Rightarrow f(1/\sqrt{3}) = \frac{1}{(1/\sqrt{3})^2} = 3$$

3. (d) $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx = \int \frac{x^2 - 1}{x^5 \sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}} dx$

$$= \frac{1}{4} \int \frac{\frac{4}{x^3} - \frac{4}{x^5}}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}} dx$$

$$\text{Put } 2 - \frac{2}{x^2} + \frac{1}{x^4} = t \Rightarrow \left(\frac{4}{x^3} - \frac{4}{x^5}\right) dx = dt$$

$$= \frac{1}{4} \int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{4} + C = \frac{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}{2} + C$$

$$= \frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$$

4. (c) Given $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx,$

$$J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx = \int \frac{e^{3x}}{e^{4x} + e^{2x} + 1} dx$$

$$\therefore J - I = \int \frac{e^x(e^{2x} - 1)}{e^{4x} + e^{2x} + 1} dx$$

$$\text{Let } e^x = t \Rightarrow e^x dx = dt$$

$$\therefore J - I = \int \frac{t^2 - 1}{t^4 + t^2 + 1} dt = \int \frac{1 - \frac{1}{t^2}}{t^2 + 1 + \frac{1}{t^2}} dt$$

$$\text{Let } t - \frac{1}{t} = u \Rightarrow \left(1 - \frac{1}{t^2}\right) dt = du$$

$$\therefore J - I = \int \frac{du}{u^2 - 1} = \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{\frac{t^2+1}{t} - 1}{\frac{t^2+1}{t} + 1} \right| + C = \frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + C$$

5. (c) $I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$

$$\text{Let } \sec x + \tan x = t \Rightarrow \sec x - \tan x = \frac{1}{t}$$

$$\Rightarrow \sec x = \frac{1}{2} \left(t + \frac{1}{t} \right)$$

$$\text{Also } \sec x (\sec x + \tan x) dx = dt$$

$$\Rightarrow \sec x \, dx = \frac{dt}{t}$$

$$\begin{aligned} \therefore I &= \frac{1}{2} \int \frac{\left(t + \frac{1}{t}\right) dt}{t^{9/2} \cdot t} = \frac{1}{2} \int \left(t^{-9/2} + t^{-13/2}\right) dt \\ &= \frac{1}{2} \left[\frac{t^{-9/2+1}}{-\frac{9}{2}+1} + \frac{t^{-13/2+1}}{-\frac{13}{2}+1} \right] + K \\ &= \frac{-1}{7} t^{-7/2} - \frac{1}{11} t^{-11/2} + K \\ &= -\frac{1}{7t^{7/2}} - \frac{1}{11t^{11/2}} + K = -\frac{1}{t^{11/2}} \left(\frac{1}{11} + \frac{t^2}{7} \right) + K \\ &= \frac{-1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K \end{aligned}$$

E. Subjective Problems

$$\begin{aligned} 1. \quad I &= \int \frac{\sin x}{\sin x - \cos x} dx = \frac{1}{2} \int \frac{2 \sin x}{\sin x - \cos x} dx \\ &= \frac{1}{2} \int \frac{\sin x + \cos x + \sin x - \cos x}{\sin x - \cos x} dx \\ &= \frac{1}{2} \int \frac{\cos x + \sin x}{\sin x - \cos x} dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x - \cos x} dx \\ &= \frac{1}{2} \log |\sin x - \cos x| + \frac{x}{2} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \text{Let } I &= \int \frac{x^2 dx}{(a+bx)^2} \\ \text{Let } a+bx &= t \Rightarrow x = \left(\frac{t-a}{b} \right) \Rightarrow dx = \frac{dt}{b} \\ \therefore I &= \frac{1}{b^3} \int \frac{(t-a)^2}{t^2} dt = \frac{1}{b^3} \int \frac{t^2 - 2at + a^2}{t^2} dt \\ &= \frac{1}{b^3} \int \left(1 - \frac{2a}{t} + \frac{a^2}{t^2} \right) dt = \frac{1}{b^3} \left[t - 2a \log |t| - \frac{a^2}{t} \right] + C \\ &= \frac{1}{b^3} \left[a+bx - 2a \log |a+bx| - \frac{a^2}{a+bx} \right] + C \end{aligned}$$

$$\begin{aligned} 3. \quad \text{To evaluate } \int (e^{\log x} + \sin x) \cos x \, dx \\ &= \int (x + \sin x) \cos x \, dx \quad [\text{Using } e^{\log x} = x] \\ &= \int x \cos x + \frac{1}{2} \int \sin 2x \, dx \end{aligned}$$

$$= x \sin x - \int \sin x \, dx + \frac{1}{2} \left(\frac{-\cos 2x}{2} \right)$$

$$= x \sin x + \cos x - \frac{1}{4} \cos 2x + C$$

$$\begin{aligned} 4. \quad I &= \int \frac{(x-1)e^x}{(x+1)^3} dx = \int \frac{(x+1-2)e^x}{(x+1)^3} dx \\ &= \int \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] e^x dx = \frac{e^x}{(x+1)^2} + C \\ &\quad (\text{Using } \int e^x (f(x) + f'(x)) dx = e^x f(x)) \end{aligned}$$

$$5. \quad \text{Let } I = \int \frac{dx}{x^3 x^2 \left(1 + \frac{1}{x^4} \right)^{3/4}}$$

$$\text{Put } 1 + \frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5} dx = dt \Rightarrow \frac{dx}{x^5} = -\frac{dt}{4}$$

$$\begin{aligned} \therefore I &= \int \frac{-dt}{4t^{3/4}} = \frac{-1}{4} \left(\frac{t^{-3/4+1}}{-\frac{3}{4}+1} \right) + C \\ &= -t^{1/4} + C = -\left(1 + \frac{1}{x^4} \right)^{1/4} + C \end{aligned}$$

$$6. \quad I = \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$\text{Put } x = \cos^2 \theta \Rightarrow dx = -2 \cos \theta \sin \theta d\theta$$

$$\begin{aligned} \therefore I &= -\int \frac{\sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta}} \cdot 2 \sin \theta \cos \theta d\theta \\ &= -\int \frac{\sin \theta / 2}{\cos \theta / 2} \cdot 2 \sin(\theta/2) \cos(\theta/2) \cos \theta d\theta \\ &= -2 \int (1 - \cos \theta) \cos \theta d\theta \\ &= -2 \int (\cos \theta - \cos^2 \theta) d\theta \\ &= -2 \int \left(\cos \theta - \frac{1 + \cos 2\theta}{2} \right) d\theta \\ &= -2 \left[\sin \theta - \frac{1}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) \right] + C \\ &= -2\sqrt{1-x} + \left[\cos^{-1} \sqrt{x} + \sqrt{x} \sqrt{1-x} \right] + C \\ &\quad [\text{Using } \sin \theta = \sqrt{1-x}] \\ &= -2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x} \sqrt{1-x} + C \end{aligned}$$

$$7. \quad I = \int \frac{\sqrt{\cos 2x}}{\sin x} dx = \int \frac{\sqrt{\cos^2 x - \sin^2 x}}{\sin^2 x} dx = \int \frac{\sqrt{\cot^2 x - 1}}{\sin^2 x} dx$$

$$\text{Let } \cot x = \sec \theta \Rightarrow -\operatorname{cosec}^2 x dx = \sec \theta \tan \theta d\theta$$

$$\begin{aligned} \text{We get, } I &= \int \sqrt{\sec^2 \theta - 1} \cdot \frac{\sec \theta \tan \theta}{-(1 + \sec^2 \theta)} d\theta \\ &= -\int \frac{\sec \theta \tan^2 \theta}{1 + \sec^2 \theta} d\theta = -\int \frac{\sin^2 \theta}{\cos \theta + \cos^3 \theta} d\theta \\ &= -\int \frac{1 - \cos^2 \theta}{\cos \theta (1 + \cos^2 \theta)} d\theta = -\int \frac{(1 + \cos^2 \theta) - 2\cos^2 \theta}{\cos \theta (1 + \cos^2 \theta)} d\theta \\ &= -\int \sec \theta d\theta + 2 \int \frac{\cos \theta}{1 + \cos^2 \theta} d\theta \\ &= -\log |\sec \theta + \tan \theta| + 2 \int \frac{\cos \theta}{2 - \sin^2 \theta} d\theta \\ &= -\log |\sec \theta + \tan \theta| + 2 \cdot \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + \sin \theta}{\sqrt{2} - \sin \theta} \right| + C \\ &= -\log \left| \cot x + \sqrt{\cot^2 x - 1} \right| + \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \sqrt{1 - \tan^2 x}}{\sqrt{2} - \sqrt{1 - \tan^2 x}} \right| + C \end{aligned}$$

$$\begin{aligned} 8. \quad I &= \int (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} dx \\ &= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx = \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx \\ \text{Put } \sin x - \cos x &= t \Rightarrow (\cos x + \sin x) dx = dt \\ \text{also } (\sin x - \cos x)^2 &= t^2 \Rightarrow 1 - \sin 2x = t^2 \\ \Rightarrow \sin 2x &= 1 - t^2 \quad \therefore I = \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}} \end{aligned}$$

$$= \sqrt{2} \sin^{-1} t + C = \sqrt{2} \sin^{-1} (\sin x - \cos x) + C$$

$$\begin{aligned} 9. \quad \text{Let } I &= \int \left(\frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} + \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} \right) dx \\ &= \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx + \int \frac{\ln(1 + \sqrt[6]{x})}{\sqrt[3]{x} + \sqrt{x}} dx \\ I &= I_1 + I_2 \quad \dots(1) \end{aligned}$$

$$\text{where } I_1 = \int \frac{1}{\sqrt[3]{x} + \sqrt[4]{x}} dx$$

$$\text{Let } x = y^{12} \text{ so that } dx = 12y^{11} dy$$

$$\begin{aligned} \therefore I_1 &= \int \frac{12y^{11}}{y^4 + y^3} dy = 12 \int \frac{y^8}{1 + y} dy \\ &= 12 \int \left(y^7 - y^6 + y^5 - y^4 + y^3 - y^2 + y - 1 + \frac{1}{y+1} \right) dy \end{aligned}$$

$$\begin{aligned} &= 12 \left[\frac{y^8}{8} - \frac{y^7}{7} + \frac{y^6}{6} - \frac{y^5}{5} + \frac{y^4}{4} - \frac{y^3}{3} \right. \\ &\quad \left. + \frac{y^2}{2} - y + \log |y+1| \right] + C_1 \end{aligned}$$

$$\begin{aligned} &= \frac{3}{2} x^{2/3} - \frac{12}{7} x^{7/12} + 2x^{1/2} - \frac{12}{5} x^{5/12} + 3x^{1/3} \\ &\quad - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12 \log |x^{1/12} + 1| + C_1 \quad \dots(2) \end{aligned}$$

$$I_2 = \int \frac{\ln(1 + (x)^{1/6})}{(x)^{1/3} + (x)^{1/2}} dx$$

$$\text{Let } x = z^6 \text{ so that } dx = 6z^5 dz$$

$$= \int \frac{\ln(1 + z)}{z^2 + z^3} \cdot 6z^5 dz = \int \frac{6z^3 \ln(z+1)}{z+1} dz$$

$$\text{Put } z+1 = t \Rightarrow dz = dt$$

$$\begin{aligned} \therefore I_2 &= \int \frac{6(t-1)^3 \ln t}{t} dt = 6 \int (t^2 - 3t + 3 - \frac{1}{t}) \ln t dt \\ &= 6 \left[\int (t^2 - 3t + 3) \ln t dt - \int \frac{1}{t} \ln t dt \right] \\ &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \int \left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \cdot \frac{1}{t} dt - \frac{(\ln t)^2}{2} \right] \\ &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \int \left(\frac{t^2}{3} - \frac{3}{2}t + 3 \right) dt - \frac{(\ln t)^2}{2} \right] \end{aligned}$$

$$\begin{aligned} &= 6 \left[\left(\frac{t^3}{3} - \frac{3t^2}{2} + 3t \right) \ln t - \left(\frac{t^3}{9} - \frac{3t^2}{4} + 3t \right) - \frac{(\ln t)^2}{2} \right] + C_2 \end{aligned}$$

$$\begin{aligned} &= 6 \left[\left\{ \frac{(1+x^{1/6})^3}{3} - \frac{3}{2}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\} \right. \\ &\quad \left. \ln(1+x^{1/6}) - \left\{ \frac{(1+x^{1/6})^3}{9} - \frac{3}{4}(1+x^{1/6})^2 + 3(1+x^{1/6}) \right\} + \frac{[\ln(1+x^{1/6})]^2}{2} \right] + C_2 \quad \dots(3) \end{aligned}$$

Thus we get the value of I on substituting the values of I_1 and I_2 from (2) and (3) in equation (1).

Indefinite Integrals

10. Let $I = \int \cos 2\theta \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$

Now we observe that

$$\begin{aligned} & \frac{d}{d\theta} \left\{ \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) \right\} \\ &= \frac{d}{d\theta} [\ln(\cos \theta + \sin \theta) - \ln(\cos \theta - \sin \theta)] \\ &= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} - \frac{-\cos \theta - \sin \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} + \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{2}{\cos 2\theta} \end{aligned}$$

$\therefore$ Integrating I with respect to θ , by parts we get

$$\begin{aligned} I &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \frac{\sin 2\theta}{2} \cdot \frac{2}{\cos 2\theta} d\theta \\ &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \tan 2\theta d\theta \\ &= \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \frac{1}{2} \ln \sec 2\theta + C \end{aligned}$$

11. $I = \int \frac{(x+1)}{x(1+xe^x)^2} dx = \int \frac{e^x(x+1)}{xe^x(1+xe^x)^2} dx$

Put $1+xe^x = t \Rightarrow (xe^x + e^x) dx = dt$

$$\begin{aligned} \therefore I &= \int \frac{dt}{(t-1)t^2} = \int \left(\frac{1}{1-t} + \frac{1}{t} + \frac{1}{t^2} \right) dt \\ &= -\log |1-t| + \log |t| - \frac{1}{t} + C \\ &= \log \left| \frac{t}{1-t} \right| - \frac{1}{t} + C = \log \left| \frac{1+xe^x}{-xe^x} \right| - \frac{1}{1+xe^x} + C \\ &= \log \left(\frac{1+xe^x}{xe^x} \right) - \frac{1}{1+xe^x} + C \end{aligned}$$

12. $I = \int \frac{x^3 + 3x + 2}{(x^2 + 1)^2(x+1)} dx.$

$$\frac{x^3 + 3x + 2}{(x^2 + 1)^2(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

Comparing and solving, we get,

$$A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}, D = 0, E = 2.$$

$$\begin{aligned} \therefore I &= -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{x+1}{x^2+1} dx + 2 \int \frac{dx}{(x^2+1)^2} \\ &= -\frac{1}{2} \log |x+1| + \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + 2I_1 + C \end{aligned}$$

where $I_1 = \int \frac{dx}{(x^2+1)^2}$, putting $x = \tan \theta$,

$$\begin{aligned} I_1 &= \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int (\cos^2 \theta) d\theta = \frac{1}{2} \int (1 + \cos 2\theta) d\theta \\ &= \frac{1}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) = \frac{1}{2} \tan^{-1} x + \frac{1}{4} \cdot \frac{2x}{1+x^2} \end{aligned}$$

$$\begin{aligned} \therefore I &= -\frac{1}{2} \log |x+1| + \frac{1}{4} \log(x^2+1) + \frac{3}{2} \tan^{-1} x \\ &\quad + \frac{x}{1+x^2} + C \end{aligned}$$

where C is constant of integration.

13. $I = \int \sin^{-1} \left(\frac{2x+2}{\sqrt{4x^2+8x+13}} \right) dx$

$$\begin{aligned} &= \int \sin^{-1} \left[\frac{x+1}{\sqrt{x^2+2x+\frac{13}{4}}} \right] dx \\ &= \int \sin^{-1} \left[\frac{x+1}{\sqrt{(x+1)^2 + (3/2)^2}} \right] dx \end{aligned}$$

Put $x+1 = 3/2 \tan \theta$, $dx = \frac{3}{2} \sec^2 \theta d\theta$

$$\begin{aligned} \therefore I &= \int \sin^{-1} \left[\frac{\left(\frac{3}{2} \tan \theta \right)}{\sqrt{\frac{9}{4} \tan^2 \theta + \frac{9}{4}}} \right] \frac{3}{2} \sec^2 \theta d\theta \\ &= \frac{3}{2} \int \sin^{-1} \left[\frac{\sin \theta \cos \theta}{\cos \theta \cdot 1} \right] \sec^2 \theta d\theta \\ &= \frac{3}{2} \int \theta \sec^2 \theta d\theta = \frac{3}{2} \left[\theta \tan \theta - \int \tan \theta d\theta \right] \\ &= \frac{3}{2} [\theta \tan \theta - \log |\sec \theta|] + C \\ I &= \frac{3}{2} \left[\frac{2}{3} (x+1) \tan^{-1} \right] \left[\frac{2}{3} (x+1) \right] \\ &\quad - \log \sqrt{1 + \frac{4}{9} (x+1)^2} + C \end{aligned}$$

$$= (x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(9+4x^2+8x+4) \\ + \frac{3}{4} \log 9 + C$$

$$= (x+1) \tan^{-1} \left(\frac{2x+2}{3} \right) - \frac{3}{4} \log(4x^2+8x+13) + C$$

14. I = $\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx$

$$= \int (x^{3m} + x^{2m} + x^m) \left[\frac{2x^{3m} + 3x^{2m} + 6x^m}{x^m} \right]^{1/m} dx$$

$$= \int \left(\frac{x^{3m} + x^{2m} + x^m}{x} \right) (2x^{3m} + 3x^{2m} + 6x^m)^{1/m} dx$$

Put $2x^{3m} + 3x^{2m} + 6x^m = y$

$$\therefore I = \frac{1}{6m} \int y^{1/m} dy = \frac{1}{6m} \left(\frac{y^{1/m+1}}{1/m+1} \right) + C$$

$$= \frac{1}{6} \left(\frac{y^{m+1}}{m+1} \right) + C$$

$$= \frac{1}{6} \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}}}{m+1} + C$$

H. Assertion & Reason Type Questions

1. (d) $F(x) = \int \sin^2 x dx = \frac{1}{2} \int (1 - \cos 2x) dx$

$$= \frac{1}{4} (2x - \sin 2x) + C$$

Now, $F(x + \pi) = \frac{1}{4} (2x + 2\pi - \sin(2x + 2\pi)) + C$

$$= \frac{1}{4} [2x + 2\pi - \sin 2x] + C \neq F(x)$$

$\therefore$ Statement -1 is false.

Also $\sin^2(x + \pi) = \sin^2 x, \forall x \in R$

$\therefore$ Statement -2 is true.

Section-B

JEE Main/ AIEEE

$$1. \quad (b) \quad \int \frac{\sin x}{\sin(x-\alpha)} dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)} dx$$

$$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)} dx$$

$$= \int \{\cos\alpha + \sin\alpha \cot(x-\alpha)\} dx$$

$$= (\cos\alpha)x + (\sin\alpha) \log \sin(x-\alpha) + C$$

$$\therefore A = \cos\alpha, B = \sin\alpha$$

$$2. \quad (a) \quad \int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx = \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right) \right| + C$$

$$\left[\because \int \sec x dx = \log \left| \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \right| \right]$$

$$= \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + C$$

$$3. \quad (d) \quad \int \frac{(\log x - 1)^2}{(1 + (\log x)^2)^2} dx = \int \frac{1 + (\log x)^2 - 2 \log x}{[1 + (\log x)^2]^2} dx$$

$$= \int \left[\frac{1}{(1 + (\log x)^2)} - \frac{2 \log x}{(1 + (\log x)^2)^2} \right] dx$$

$$= \int \left[\frac{e^t}{1+t^2} - \frac{2te^t}{(1+t^2)^2} \right] dt \quad \text{put } \log x = t \Rightarrow dx = e^t dt$$

$$= \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2} \right] dt$$

$$\left[\text{which is of the form } \int e^x (f(x) + f'(x)) dx \right]$$

$$= \frac{e^t}{1+t^2} + c = \frac{x}{1 + (\log x)^2} + c$$

$$4. \quad (c) \quad I = \int \frac{dx}{\cos x + \sqrt{3} \sin x} \Rightarrow I = \int \frac{dx}{2 \left[\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right]}$$

$$= \frac{1}{2} \int \frac{dx}{\left[\sin \frac{\pi}{6} \cos x + \cos \frac{\pi}{6} \sin x \right]} = \frac{1}{2} \int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$$

$$\Rightarrow I = \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{6}\right) dx$$

But we know that $\int \operatorname{cosec} x dx = \log |(\tan x/2)| + C$

$$\therefore I = \frac{1}{2} \cdot \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$$

$$5. \quad (c) \quad \text{Let } I = \sqrt{2} \int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)} \quad \text{put } x - \frac{\pi}{4} = t$$

$$\Rightarrow dx = dt \Rightarrow I = \sqrt{2} \int \frac{\sin\left(t + \frac{\pi}{4}\right)}{\sin t} dt$$

$$= \frac{\sqrt{2}}{\sqrt{2}} \int \left(\frac{\sin t + \cos t}{\sin t} \right) dt$$

$$\Rightarrow I = \int (1 + \cot t) dt = t + \log |\sin t| + c_1$$

$$= x - \frac{\pi}{4} + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c_1$$

$$= x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c \quad \left(\text{where } c = c_1 - \frac{\pi}{4} \right)$$

$$6. \quad (d) \quad \int \frac{5 \tan x}{\tan x - 2} dx = \int \frac{5 \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2} dx$$

$$= \int \left(\frac{5 \sin x}{\cos x} \times \frac{\cos x}{\sin x - 2 \cos x} \right) dx$$

$$= \int \frac{5 \sin x dx}{\sin x - 2 \cos x}$$

$$= \int \left(\frac{4 \sin x + \sin x + 2 \cos x - 2 \cos x}{\sin x - 2 \cos x} \right) dx$$

$$= \int \frac{(\sin x - 2 \cos x) + (4 \sin x + 2 \cos x)}{\sin x - 2 \cos x} dx$$

$$= \int \frac{(\sin x - 2 \cos x) + 2(\cos x + 2 \sin x)}{(\sin x - 2 \cos x)} dx$$

$$= \int \frac{\sin x - 2 \cos x}{\sin x - 2 \cos x} dx + 2 \int \left(\frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} \right) dx$$

$$= \int dx + 2 \int \frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} dx$$

$$= I_1 + I_2 \quad \text{where } I_1 = \int dx \text{ and } I_2 = 2 \int \frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} dx$$

$$\begin{aligned} \text{put } \sin x - 2\cos x &= t \\ \Rightarrow (\cos x + 2\sin x) dx &= dt \end{aligned}$$

$$\therefore I_2 = 2 \int \frac{dt}{t} = 2 \ln t + C = 2 \ln (\sin x - 2\cos x) + C$$

$$\begin{aligned} \text{Hence, } I_1 + I_2 &= \int dx + 2 \ln (\sin x - 2\cos x) + C \\ &= x + 2 \ln |(\sin x - 2\cos x)| + C \Rightarrow a=2 \end{aligned}$$

7. (c) Let $\int f(x)dx = \psi(x)$

$$\text{Let } I = \int x^5 f(x^3) dx$$

$$\text{put } x^3 = t \Rightarrow 3x^2 dx = dt$$

$$I = \frac{1}{3} \int 3 \cdot x^2 \cdot x^3 \cdot f(x^3) \cdot dx$$

$$= \frac{1}{3} \int t f(t) dt = \frac{1}{3} \left[t \int f(t) dt - \int f(t) dt \right]$$

$$= \frac{1}{3} \left[t \psi(t) - \int \psi(t) dt \right]$$

$$= \frac{1}{3} \left[x^3 \psi(x^3) - 3 \int x^2 \psi(x^3) dx \right] + C$$

$$= \frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$$

8. (d) Let $I = \int \left(1 + x - \frac{1}{x} \right) e^{x+1/x} dx$

$$= \int e^{x+1/x} dx + \int \left(x - \frac{1}{x} \right) e^{x+1/x} dx$$

$$= x e^{x+1/x} - \int x \left(1 - \frac{1}{x^2} \right) e^{x+1/x} dx + \int \left(x - \frac{1}{x} \right) e^{x+1/x} dx$$

$$= x e^{x+1/x} - \int \left(x - \frac{1}{x} \right) e^{x+1/x} dx + \int \left(x - \frac{1}{x} \right) e^{x+1/x} dx$$

$$= x e^{x+1/x} + C$$

9. (b) $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^3(1+x^{-4})^{3/4}}$

$$\text{Let } x^{-4} = y$$

$$\Rightarrow -4x^{-5} dx = dy \Rightarrow dx = \frac{-1}{4} x^5 dy$$

$$\therefore I = \frac{-1}{4} \int \frac{x^5 dy}{x^3(1+y)^{3/4}} = \frac{-1}{4} \int \frac{dy}{(1+y)^{3/4}}$$

$$= \frac{-1}{4} \times 4(1+y)^{1/4} = -(1+x^{-4})^{1/4} + C$$

$$= -\left(\frac{x^4+1}{x^4} \right)^{1/4} + C$$

10. (d) $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^3}$$

$$\text{Substitute } 1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6} \right) dx = dt$$

$$\Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6} \right) dx = -dt$$

This gives,

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2 \left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^2} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$