Chapter 5
Continuity and Differentiability

Exercise 5.2

Q. 1 Differentiate the functions with respect to x. sin $(x^2 + 5)$

Answer:

Given: sin $(x^2 + 5)$

Let y = sin $(x^2 + 5)$

$= \frac{dy}{dx} = \frac{d}{dx}\sin(x^2 + 5)$

$= \cos(x^2 + 5).\ \frac{d}{dx}\sin(x^2 + 5)$

$= \cos(x^2 + 5).\ \left[\frac{d}{dx}(x)^2 + \frac{d}{dx}(5)\right]$

$= \cos(x^2 + 5).\ (2x + 0)$

$= \cos(x^2 + 5).\ (2x)$

$= 2x.\cos(x^2 + 5)$

Q. 2 Differentiate the functions with respect to x. cos (sin x)

Answer:

Given: cos (sin x)

Let y = cos (sin x)

$= \frac{dy}{dx} = \frac{d}{dx}$ (cos (sin x))

$= -\sin(\sin x).\ \frac{d}{dx}(\sin x)$

$= -\sin(\sin x).\ \text{Cos } x$

= -cos x. sin (sin x)

Q. 3 Differentiate the functions with respect to x. sin (ax + b)

Answer:

Given: sin (ax + b)

Let y = sin (ax + b)

$= \frac{dy}{dx} = \frac{d}{dx}$ (sin (ax + b))

= cos (ax + b). $\frac{d}{dx}$ (ax + b)

= cos (ax + b). $\left(\frac{d}{dx}(ax + \frac{d}{dx}(b)\right)$

= cos (ax + b). (a + 0)

= cos (ax + b). (a)

= a. cos (ax + b)

Q. 4

Differentiate the functions with respect to x.

Sec $\left(\tan\left(\sqrt{x}\right)\right)$

Answer:

Given: sec (tan(√x))

Let y= sec (tan(√x))

$= \frac{dy}{dx} = \frac{d}{dx}\left(sec\left(tan\left(\sqrt{x}\right)\right)\right)$

= sec (tan ($\sqrt{x}$)). tan (tan ($\sqrt{x}$)) $\left(\frac{d}{dx}\left(\tan\sqrt{x}\right)\right)$

= sec (tan ($\sqrt{x}$)). tan (tan ($\sqrt{x}$)). Sec^2 ($\sqrt{x}$).$\frac{d}{dx}\left(\sqrt{x}\right)$

$= \sec(\tan(\sqrt{x})).\tan(\tan(\sqrt{x})).\sec^2(\sqrt{x})).\frac{1}{2(\sqrt{x})}$

$= \frac{1}{2(\sqrt{x})}(\sec(\tan(\sqrt{x})).\tan(\tan(\sqrt{x})).\sec^2(\sqrt{x}))$

Q. 5Differentiate the functions with respect to x.

$\frac{\sin(ax+b)}{\cos(cx+d)}$

Answer:

Given: $\frac{\sin(ax+b)}{\cos(cx+d)}$

Let y = $\frac{\sin(ax+b)}{\cos(cx+d)}$

$= \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin(ax+b)}{\cos(cx+d0}\right)$

We know that $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vd(u)-ud(v)}{v^2}$

$= \frac{[\cos(cx+d).d(\sin(ax+b))-\sin(\text{ax+b}).\text{d}(\cos(cx+d))]}{[\cos(cx+d)]^2}$

$= \frac{[\cos(cx+d).(\cos(ax+b)).d(ax+b)-\sin(ax+b).(-\sin(cx+d)d(cx+d)]}{[\cos(cx+d]^2}$

$= \frac{[\cos(cx+d).(\cos(ax+b)).(a)-\sin(ax+b).(-\sin(cx+d)(c)]}{[\cos(cx+d)]^2}$

$= \frac{[\text{a}\cos(cx+d)\cos(ax+b)]}{[\cos(cx+d)]^2} + \frac{[c\sin(cx+d)\sin(ax+b)}{[\cos(cx+d)]^2}$

$= \frac{[a\cos(ax+b)]}{[\cos(cx+d)]} + \frac{[c\sin(cx+d)\sin(ax+b)]}{[\cos(cx+d)]]\cos(cx+d)]}$

= a cos (ax + b) sec (cx + d) + c sin (ax + b) tan (cx + d) sec (cx + d)

Q. 6 Differentiate the functions with respect to x.

$\cos x^3. \sin^2(x^5)$

Answer:

Given: $\cos x^3 . \sin^2 (x^5)$

Let y = $\cos x^3 . \sin^2 (x^5)$

= $\frac{dy}{dx} = \frac{d}{dx} (\cos x^3 . \sin^2 (x^5))$

We know that, $\frac{dy}{dx}$ (u. v) = u. d (v) + v. d (u)

= $\cos x^3 . \frac{d}{dx} \sin^2 (x^5) + \sin^2 (x^5) . \frac{d}{dx} (\cos x^3)$

= $\cos x^3 . 2 \sin (x^5) . \left(\frac{d}{dx} \sin(x^5)\right) + \sin 2 (x^5) . (-\sin x^3) . \left(\frac{d}{dx} x^3\right)$

= $\cos x^3 . 2 \sin (x^5) . \text{Cos} (x^5) \left(\frac{d}{dx} x^5\right) + \sin 2 (x^5) . (-\sin x^3) . (3x^2)$

= $\cos x^3 . 2\sin(x^5) . \cos(x^5) (5x^4) + \sin 2(x^5) . (-\sin x^3) . (3x^2)$

= $10x^4 . \cos x^3 . \sin(x^5) . \cos (x^5) - (3x^2) . \sin 2 (x^5) . (\sin x^3)$

Q. 8 Differentiate the functions with respect to x.

$\cos(\sqrt{x})$

Answer:

Given: $\cos \sqrt{x}$

Let y = $\cos \sqrt{x}$

= $\frac{dy}{dx} = \frac{d}{dx} (\cos \sqrt{x})$

= $-\sin (\sqrt{x}) . \left(\frac{d}{dx} \sqrt{x}\right)$

= $-\sin (\sqrt{x}) . \frac{1}{2} . \left(x^{-\frac{1}{2}}\right)$

= $-\sin (\sqrt{x}) . \frac{1}{2\sqrt{x}}$

= $-\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

Q. 9 Prove that the function f given by f (x) = | x – 1|, x ∈ R is not differentiable at x = 1.

Answer:

Given: f(x)=|x-1|, x ∈R

because a function f is differentiable at a point x=c in its domain if both its limits as:

$\lim_{h\to 0^-} \frac{[f\,(c+h)-f(c)]}{h}$ and $\lim_{h\to 0^+} \frac{[f(c+h)-f(c)]}{h}$ are finite and equal.

Now, to check the differentiability of the given function at x=1,

Let we consider the left hand limit of function f at x=1

$= \lim_{h\to 0^-} \frac{[f(1+h)-f(1)]}{h}$

$= \lim_{h\to 0^-} [|1+h-1|-|1-1|]$

$= \lim_{h\to 0^-} \frac{[|h|-0]}{h}$

$= \lim_{h\to 0^-} \frac{[-h]}{h}$ because, {h < 0 ⇒ |h|= -h}

= -1

Now, let we consider the right hand limit of function f at x=1

$= \lim_{h\to 0^+} [f(1+h)-f(1)]$

$= \lim_{h\to 0^+} [|1+h-1|-|1-1|]$

$= \lim_{h\to 0^+} [|h|-0]$

$= \lim_{h\to 0^+} \frac{[h]}{h}$ because, {h>0 ⇒ |h|= h}

= 1

Because, left hand limit is not equal to right hand limit of function f at x=1, so f is not differentiable at x=1.

Q. 10 Prove that the greatest integer function defined by f (x) = [x], 0 < x < 3 is not differentiable at x = 1 and x = 2.

Answer:

Given: f(x) =[x], 0 < x <3

because a function f is differentiable at a point x=c in its domain if both its limits as:

$\lim_{h \to 0^-} \frac{[f(c+h)-f(c)]}{h}$ and $\lim_{h \to 0^+} \frac{[f\ (c+h)-f(c)]}{h}$ are finite and equal.

Now, to check the differentiability of the given function at x=1,

Let we consider the left-hand limit of function f at x=1

$= \lim_{h \to 0^-} \frac{[f(1+h)-f(1)]}{h}$

$= \lim_{h \to 0^-} \frac{[|1+h|-|1|]}{h}$

$= \lim_{h \to 0^-} \frac{[1+h-1-1]}{h}$

$= \lim_{h \to 0^-} \frac{[h-1]}{h}$ because, {h<0=> |h|= -h}

$= -\frac{1}{0} = \infty$

Let we consider the right hand limit of function f at x=1

$= \lim_{h \to 0^+} \frac{[f(1+h)-f(1)]}{h}$

$= \lim_{h \to 0^+} \frac{[1+h]-[1]}{h}$

$= \lim_{h \to 0^+} \frac{[1-1]}{h}$

$= \lim_{h\to 0^+} \frac{[0]}{h}$

$= 0$

Because, left hand limit is not equal to right hand limit of function f at x=1, so f is not differentiable at x=1.

Let we consider the left hand limit of function f at x=2

$= \lim_{h\to 0^-} \frac{[f(2+h)-f(2)]}{h}$

$= \lim_{h\to 0^-} \frac{[2+h]-[2]}{h}$

$= \lim_{h\to 0^-} \frac{[2+h-1-2]}{h}$

$= \lim_{h\to 0^-} \frac{[h+1-2]}{h}$

$= -\frac{1}{0} = \infty$

Now, let we consider the right hand limit of function f at x=2

$= \lim_{h\to 0^+} \frac{[f(2+h)-f(2)]}{h}$

$= \lim_{h\to 0^+} \frac{[2+h]-[2]}{h}$

$= \lim_{h\to 0^+} \frac{[2-2]}{h}$

$= \lim_{h\to 0^+} \frac{[0]}{h}$

$= 0$

Because, left hand limit is not equal to right hand limit of function f at x=2, so f is not differentiable at x=2.