Chapter - 4

Geometry

Ex 4.1

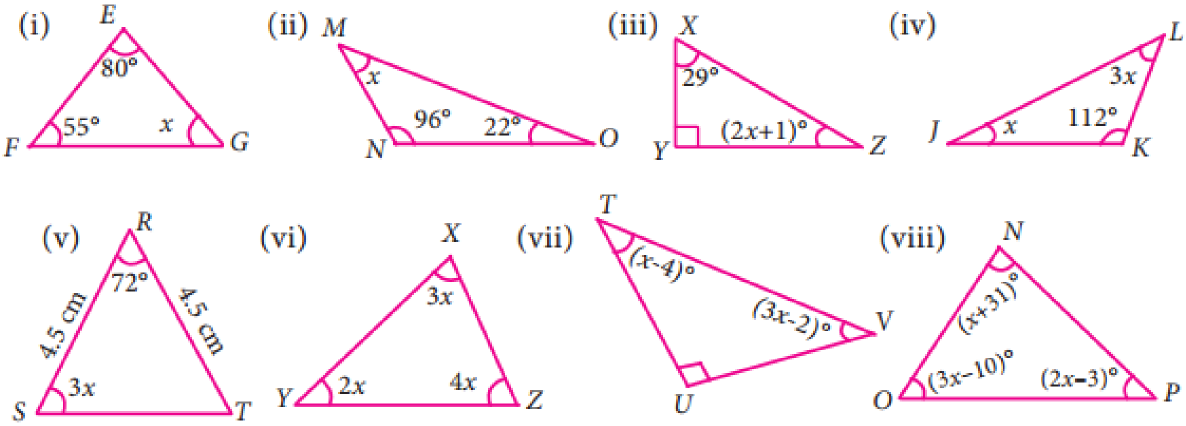
Question 1.
Can 30°, 60° and 90° be the angles of a triangle?

Solution:
Given angles 30°, 60° and 90°
Sum of the angles = 30° + 60° + 90° = 180°
∴ The given angles form a triangle.

Question 2.
Can you draw a triangle with 25°, 65° and 80° as angles?

Solution:
Given angle 25°, 65° and 80°.
Sum of the angles = 25° + 65° + 80° = 170° ≠ 180
∴ We cannot draw a triangle with these measures.

Question 3.
In each of the following triangles, find the value of x.



Solution:
(i) Let ∠G = x
By angle sum property we know that,
∠E + ∠F + ∠G = 180°
80° + 55° + x = 180°

$135^\circ + x = 180^\circ$
$x = 45^\circ$

(ii) Let $\angle M = x$
By angle sum property of triangles we have
$\angle M + \angle M + \angle O = 180^\circ$
$x + 96^\circ + 22^\circ = 180^\circ$
$x + 118^\circ = 180^\circ$
$X = 180^\circ - 118^\circ = 620$

(iii) Let $\angle Z = (2x + 1)^\circ$ and $\angle Y = 90^\circ$
By the sum property of triangles we have
$\angle x + \angle y + \angle z = 180^\circ$
$29^\circ + 90^\circ + (2x + 1)^\circ = 180^\circ$
$119^\circ + (2x + 1)^\circ = 180^\circ$
$(2x + 1)^\circ = 180^\circ - 119^\circ$
$2x + 1^\circ = 61^\circ$
$2x = 61^\circ - 1^\circ$
$2x = 60^\circ$
$x = 60\circ 2$
$x = 30^\circ$

(iv) Let $\angle J = x$ and $\angle L - 3x$.
By angle sum property of triangles we have
$\angle J + \angle K + \angle L = 180^\circ$
$x + 112^\circ + 3x = 180^\circ$
$4x = 180^\circ - 112^\circ$
$x = 68^\circ$
$x = 68\circ 4$
$x = 17^\circ$

(v) Let $\angle S = 3x^\circ$
Given $\overline{RS}$ = Given $\overline{RT} = 4.5$ cm
Given $\angle S = \angle T = 3x^\circ$ [∵ Angles opposite to equal sides are equal]
By angle sum property of a triangle we have,
$\angle R + \angle S + \angle T = 180^\circ$
$72^\circ + 3x + 3x = 180^\circ$
$72^\circ + 6x = 180^\circ$
$x = 108\circ 6$
$x = 18^\circ$

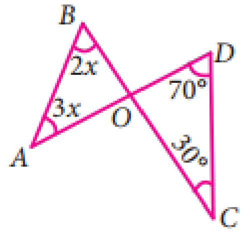
(vi) Given $\angle X = 3x$; $\angle Y = 2x$; $\angle Z = \angle 4x$
By angle sum property of a triangle we have
$\angle X + \angle Y + \angle Z = 180^\circ$

3x + 2x + 4x = 180°
∴ 9x = 180°
x = 180∘9 = 20°

(vii) Given ∠T = (x – 4)°
∠U = 90°
∠V = (3x – 2)°
By angle sum property of a triang we have
∠T + ∠U + ∠V = 180°
(x – 4)° + 90° + (3x – 2)° = 180°
x – 4° + 90° + 3x – 2° = 180°
x + 3x + 90° – 4° – 2° = 180°
4x + 84° = 180°
4x = 180° – 84°
4x = 96°
x = 96∘4 = 24°
x = 24°

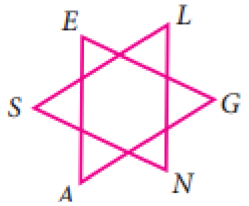
(viii) Given ∠N = (x + 31)°
∠O = (3x – 10)°
∠P = (2x – 3)°
By angle sum property of a triangle we have
∠N + ∠O + ∠P = O
(x + 31)° + (3x – 10)° + (2x – 3)° = 180°
x + 31°+ 3x – 10° + 2x – 3° = 180°
x + 3x + 2x + 31° – 10° – 3° = 180°
6x + 18° = 180°
6x = 180° + 18°
6x = 162°
x = 162∘6 = 27°
x = 27°

Question 4.
Two line segments $\overline{AD}$ and $\overline{BC}$ intersect at O.
Joining $\overline{AB}$ and $\overline{DC}$ we get two triangles, ΔAOB and ΔDOC as shown in the figure. Find the ∠A and ∠B.



Solution:
In ΔAOB and ΔDOC,
$\angle AOB = \angle DOC$ [∵ Vertically opposite angles are equal]
Let $\angle AOB = \angle DOC = y$
By angle sum property of a triangle we have
$\angle A + \angle B + \angle AOB = \angle D + \angle C + \angle DOC = 180°$
$3x + 2x + y = 70° + 30° + y = 180°$
$5x + y = 100° + y = 180°$
Here $5x + y = 100° + y$
$5x = 100° + y - y$
$5x = 100°$
$x = 100∘5 = 20°$
$\angle A = 3x = 3 \times 20 = 60°$
$\angle B = 2x = 2 \times 20 = 40°$
$\angle A = 60°$
$\angle B = 40°$

Question 5.
Observe the figure and find the value of
$\angle A + \angle N + \angle G + \angle L + \angle E + \angle S$.



Solution:
In the figure we have two triangles namely ΔAGE and ΔNLS.
By angle sum property of triangles,
Sum of angles of $\Delta AGE = \angle A + \angle G + \angle E = 180°$ ...(1)
Also sum of angles of $\Delta NLS = \angle N + \angle L + \angle S = 180°$ ... (2)
(1) + (2) $\angle A + \angle G + \angle E + \angle N + \angle L + \angle S = 180° + 180°$
i.e., $\angle A + \angle N + \angle G + \angle L + \angle E + \angle S = 360°$

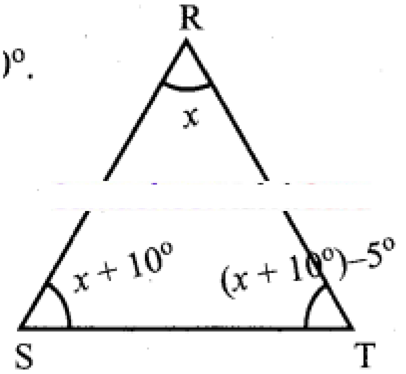
Question 6.
If the three angles of a triangle are in the ratio 3 : 5 : 4, then find them.

Solution:
Given three angles of the triangles are in the ratio 3 : 5 : 4.
Let the three angle be 3x, 5x and 4x.
By angle sum property of a triangle, we have
$3x + 5x + 4x = 180°$
$12x = 180°$

x = 180∘12
x = 15°
∴ The angle are 3x = 3 × 15° = 45°
5x = 5 × 15° = 75°
4x = 4 × 15° = 60°
Three angles of the triangle are 45°, 75°, 60°

Question 7.
In ΔRST, ∠S is 10° greater than ∠R and ∠T is 5° less than ∠S , find the three angles of the triangle.

Solution:
In ΔRST. Let ∠R = x.
Then given S is ∠10° greater than ∠R
∴ ∠S = x + 10°
Also given ∠T is 5° less then ∠S.
So ∠T = ∠S – 5° = (x + 10)° – 5° = x + 10° – 5°
By angle sum property of triangles, sum of three angles = 180°.

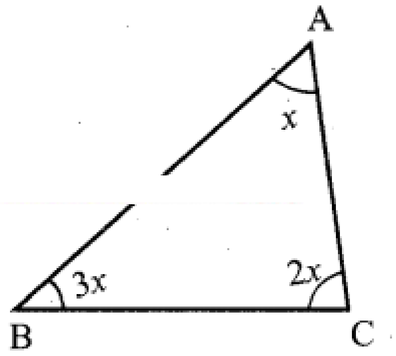


∠R + ∠S + ∠T = 180°
x + x + 10° + x + 5° = 180°
3x + 15° = 180°
3x = 180° – 15°
x = 165∘3 = 55°
∠R = x = 55°
∠S = x + 10° = 55° + 10° = 65°
∠T = x + 5° = 55° + 5° = 60°
∴ ∠R = 55°
∠S = 65°
∠T = 60°

Question 8.
In ΔABC , if ∠B is 3 times ∠A and ∠C is 2 times ∠A, then find the angles.

Solution:
In ABC, Let ∠A = x,

then $\angle B = 3$ times $\angle A = 3x$
$\angle C = 2$ times $\angle A = 2x$
By angle sum property of a triangles,
Sum of three angles of $\Delta ABC = 180°$.
$\angle A + \angle B + \angle C = 180$
$x + 3x + 2x = 180°$
$x (1 + 3 + 2) = 180°$
$6x = 180°$



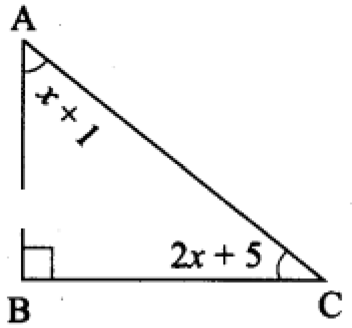
$x = 180∘6 = 30°$
$\angle A = x = 30°$
$\angle B = 3x = 3 \times 30° = 90°$
$\angle C = 2x = 2 \times 30° = 60°$
$\therefore \angle A = 30°$
$\angle B = 90°$
$\angle C = 60°$

Question 9.
In ΔXYZ, if $\angle X : \angle Z$ is 5 : 4 and $\angle Y = 72°$. Find $\angle X$ and $\angle Z$.

Solution:
Given in ΔXYZ, $\angle X : \angle Z = 5 : 4$
Let $\angle X = 5x$; and $\angle Z = 4x$ given $\angle Y = 72°$
By the angle sum property of triangles sum of three angles of a triangles is 180°.
$\angle X + \angle Y + \angle Z = 180°$
$5x + 72 + 4x = 180°$
$5x + 4x = 180° – 72°$
$9x = 108°$
$x = 108∘9 = 12°$
$\angle X = 5x = 5 \times 12° = 60°$
$\angle Z = 4x = 4 \times 12° = 48°$
$\therefore \angle X = 60°$
$\angle Z = 48°$

Question 10.
In a right angled triangle ABC, ∠B is right angle, ∠A is x + 1 and ∠C is 2x + 5. Find ∠A and ∠C.

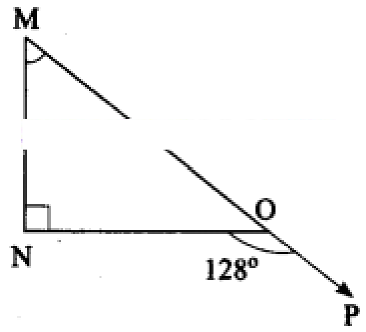
Solution:
Given in ΔABC ∠B = 90°
∠A = x + 1
∠B = 2x + 5



By angle sum property of triangles
Sum of three angles of ΔABC = 180°
∠A + ∠B + ∠C = 180°
(x + 1) + 90° + (2x + 5) = 180°
x + 2x + 1° + 90° + 5° = 180°
3x + 96° = 180°
3x = 180° – 96° = 84°
x = 84∘3 = 28°
∠A = x + 1 = 28 + 1 = 29
∠C = 2x + 5 = 2 (28) + 5 = 56 + 5 = 61
∴ ∠A = 29°
∠C = 61°

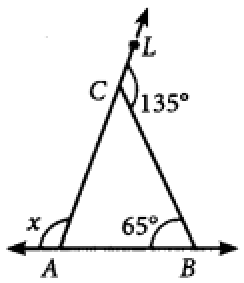
Question 11.
In a right angled triangle MNO, ∠N = 90°, MO is extended to P. If ∠NOP = 128°, find the other two angles of ΔMNO.

Solution:
Given ∠N = 90°
MO is extended to P, the exterior angle ∠NOP = 128°
Exterior angle is equal to the sum of interior opposite angles.

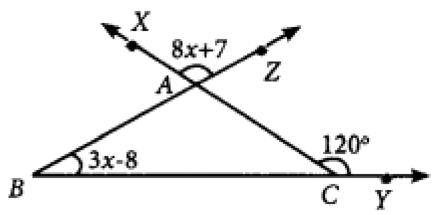


∴ ∠M + ∠N = 128°
∠M + 90° = 128°
∠M = 128° – 90°
∠M = 38°
By angle sum property of triangles,
∴ ∠M + ∠N + ∠O = 180°
38° + 90° + ∠O = 180°
∠O = 180° – 128°
∠O = 52°
∴ ∠M = 38° and ∠O = 52°

Question 12.
Find the value of x in each of the given triangles.



(ii)



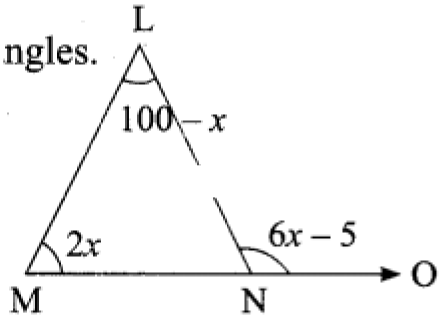
Solution:
(i) In ΔABC, given B = 65°,
AC is extended to L, the exterior angle at C, ∠BCL = 135°
Exterior angle is equal to the sum of opposite interior angles.
∠A + ∠B = ∠BCL
∠A + 65° = 135°
∠A = 135° – 65°
∴ ∠A = 70°
x + ∠A = 180° [∵ linear pair]
x + 70° = 180° [∵ ∠A = 70°]
x = 180° – 70°
∴ x = 110°

(ii) In ΔABC, given B = 3x – 8°
∠XAZ = ∠BAC [∵ vertically opposite angles]

$8x + 7 + \angle BAC$
i.e., In ΔABC, $\angle A = 8x + 7$
Exterior angle $\angle XCY = 120^\circ$
Exterior angle is equal to the sum of the interior opposite angles.
$\angle A + \angle B = 120^\circ$
$8x + 7 + 3x - 8 = 120^\circ$
$8x + 3x = 120^\circ + 8 - 7$
$11x = 121^\circ$
$x = 121\circ 11 = 11^\circ$

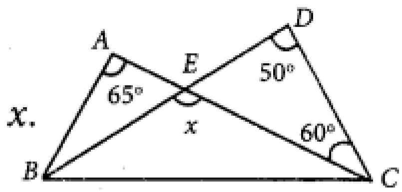
Question 13.
In ΔLMN, MN is extended to O. If ∠MLN = 100 – x, ∠LMN = 2x and ∠LNO = 6x – 5, find the value of x.

Solution:
Exterior angle is equal to the sum of the opposite interior angles.
$\angle LNO = \angle MLN + \angle LMN$



$6x - 5 = 100^\circ - x + 2x$
$6x - 5 + x - 2x = 100^\circ$
$6x + x - 2x = 100^\circ + 5^\circ$
$5x = 105^\circ$
$x = 105\circ 5 = 21^\circ$
$x = 21^\circ$

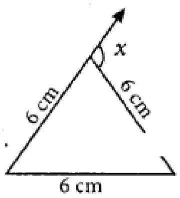
Question 14.
Using the given figure find the value of x.



Solution:
In ΔEDC, side DE is extended to B, to form the exterior angle $\angle CEB = x$.
We know that the exterior angle is equal to the sum of the opposite interior

angles
∠CEB = ∠CDE + ∠ECD
x = 50° + 60°
x = 110°

Question 15.
Using the diagram find the value of x.



Solution:
Given triangle is an equilateral triangle as the three sides are equal. For an equilateral triangle all three angles are equal and is equal to 60° Also exterior angle is equal to sum of opposite interior angles.
x = 60° + 60°.
x = 120°

Objective Type Questions

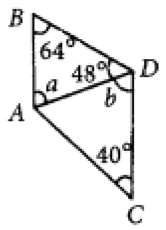
Question 16.
The angles of a triangle are in the ratio 2:3:4. Then the angles are
(i) 20,30,40
(ii) 40, 60, 80
(iii) 80, 20, 80
(iv) 10, 15, 20

Answer:
(ii) 40, 60, 80

Question 17.
One of the angles of a triangle is 65°. If the difference of the other two angles is 45°, then the two angles are
(i) 85°, 40°
(ii) 70°, 25°
(iii) 80°, 35°
(iv) 80° , 135°

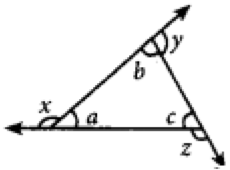
Answer:
(iii) 80°,35°

Question 18.
In the given figure, AB is parallel to CD. Then the value of b is



(i) 112°
(ii) 68°
(iii) 102°
(iv) 62° A
Answer:
(ii) 68°

Question 19.
In the given figure, which of the following statement is true?



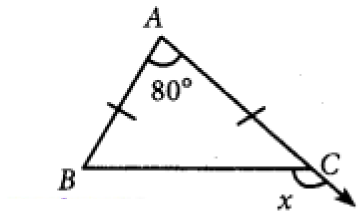
(i) $x + y + z = 180°$
(ii) $x + y + z = a + b + c$
(iii) $x + y + z = 2(a + b + c)$
(iv) $x + y + z = 3(a + b + c)$

Ans :
(iii) $x + y + z = 2(a + b + c)$]

Question 20.
An exterior angle of a triangle is 70° and two interior opposite angles are equal. Then measure of each of these angle will be
(i) 110°
(ii) 120°
(iii) 35°
(iv) 60°

Answer:
(iii) 35°

Question 21.
In a ΔABC, AB = AC. The value of x is _____.



(i) 80°
(ii) 100°
(iii) 130°
(iv) 120°

Answer:
(iii) 130°

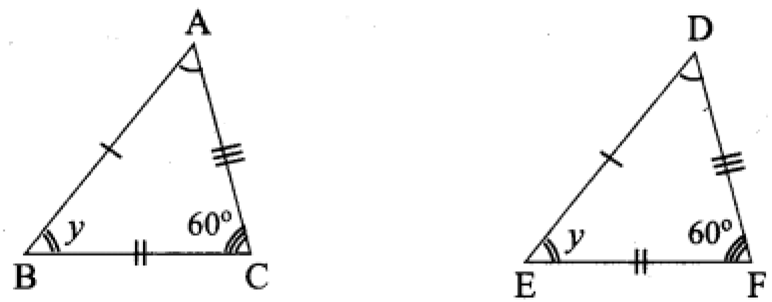
Question 22.
If an exterior angle of a triangle is 115° and one of the interior opposite angles is 35°, then the other two angles of the triangle are
(i) 45°, 60°
(ii) 65°, 80°
(iii) 65°, 70°
(iv) 115°, 60°

Answer:
(ii) 65°, 80°

Ex 4.2

Question 1.
Given that ΔABC = ΔDEF (i) List all the corresponding congruent sides
(ii) List all the corresponding congruent angles.

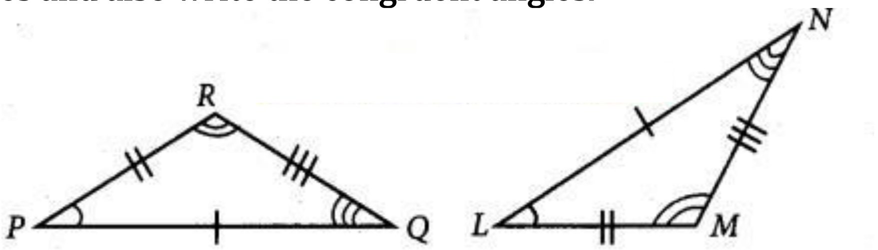
Solution:
Given ΔABC ≅ DEF.

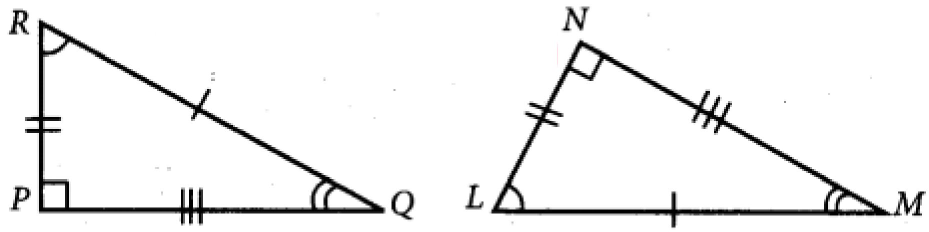


(i) Corresponding congruent sides.
$\overline{AB} = \overline{DE}$; $\overline{BC} = \overline{EF}$; $\overline{AC} = \overline{DF}$

(ii) Corresponding congruent angles.
∠ABC = ∠DEF; ∠BCA = ∠EFD ; ∠CAB = ∠FDE

Question 2.
If the given two triangles are congruent, then identify all the corresponding sides and also write the congruent angles.

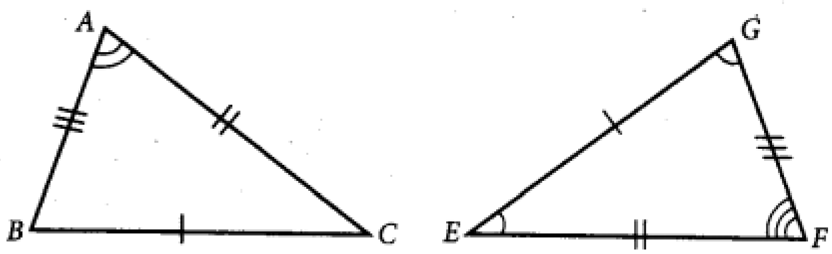
(i)


(ii)


Solution:
Given ΔPQR ≅ ΔLNM
(i) (a) Corresponding sides
$\overline{PQ}$ = $\overline{LN}$; $\overline{PQ}$ = $\overline{LM}$; $\overline{RQ}$ = $\overline{MN}$
(b) Corresponding angles
∠RPQ = ∠NLM; ∠PQR = ∠LNM; ∠PRQ = ∠LMN

(ii) Given ΔPQR ≅ ΔNML
(a) Corresponding angles
$\overline{QR}$ = $\overline{LM}$; $\overline{RP}$ = $\overline{LN}$; $\overline{PQ}$ = $\overline{MN}$
(b) Corresponding angles
∠PQP = ∠NMN; ∠QRP = ∠MLN; ∠RPQ = ∠LNM

Question 3.
Find the unit digit of expanded form.

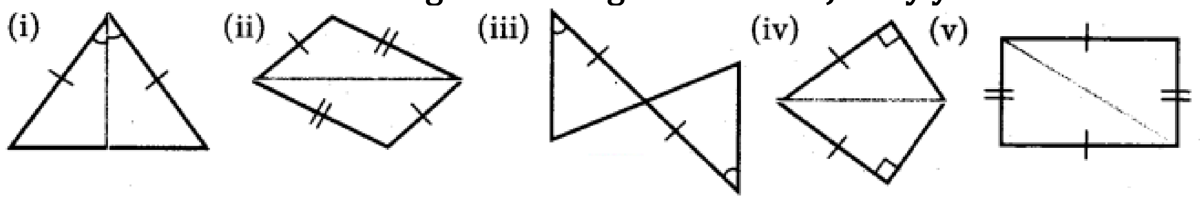


(i) ∠A and ∠G
(ii) ∠B and ∠E
(iii) ∠B and ∠G

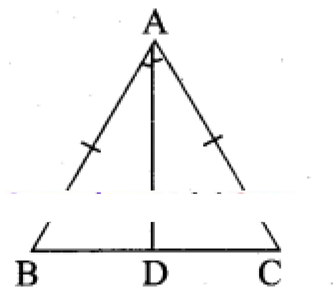
(iv) $\overline{AC}$ and $\overline{GF}$
(v) $\overline{BA}$ and $\overline{FG}$
(vi) $\overline{EF}$ and $\overline{BC}$

Solution:
Given ΔABC ≅ ΔEFG. Also from given triangles.
$\overline{AB} = \overline{FG}$ $\overline{BC} = \overline{GF}$ $\overline{AC} = \overline{EF}$
Also ∠A = ∠F ∠B = ∠G ∠C = ∠E
Answer:
(i) ∠A and ∠G are not corresponding angles.
(ii) ∠B and ∠E are not corresponding angles.
(iii) ∠B and ∠G are corresponding angles.
(iv) $\overline{AC}$ and $\overline{GF}$ are not corresponding sides.
(v) $\overline{BA}$ and $\overline{FG}$ are corresponding sides.
(vi) $\overline{EF}$ and $\overline{BC}$ are not corresponding sides.

Question 4.
State whether the two triangles are congruent or not. Justify your answer.

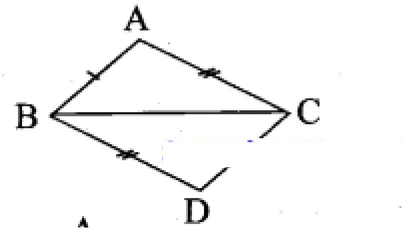


Solution:
(i) Let the given triangle be ΔABC. $\overline{AD}$ divides ΔABC into two parts giving ΔABD and ΔACD.
In ΔABD and ΔACD



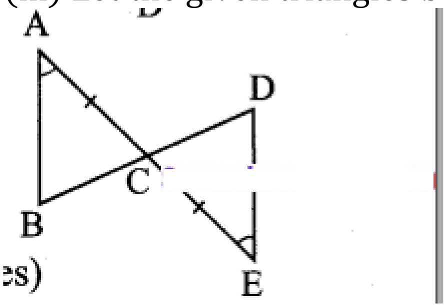
$\overline{AB} = \overline{AC}$ (given)
$\overline{BD} = \overline{AD}$ (common side)
∠BAD = ∠CAD (included angles)
∴ By SAS criterion ΔABD ≅ ΔACD.

(ii) Let the given triangles in the figure be ΔABC and ΔDCB.
In both the triangles



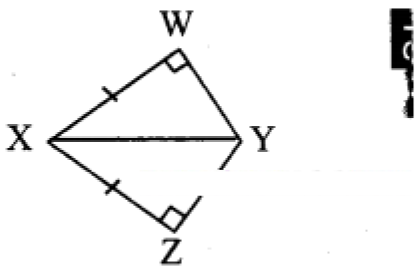
$\overline{BC} = \overline{BC}$ (Common side)
$\overline{AB} = \overline{DC}$
$\overline{AC} = \overline{BD}$
∴ By SSS Criterion ΔABC ≅ ΔDCB

(iii) Let the given triangles be ΔABC and ΔCDE.



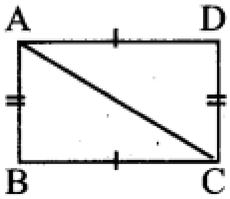
Here $\overline{AC} = \overline{CE}$ (given)
∠BAC = ∠DEC (given)
∠ACB = ∠DCE (vertically opposite angles)
Two angles and the included side are equal.
Therefore by ASA criterion ΔABC ≅ ΔCDE.

(iv) Let the two triangles be ΔXYZ and ΔXYW



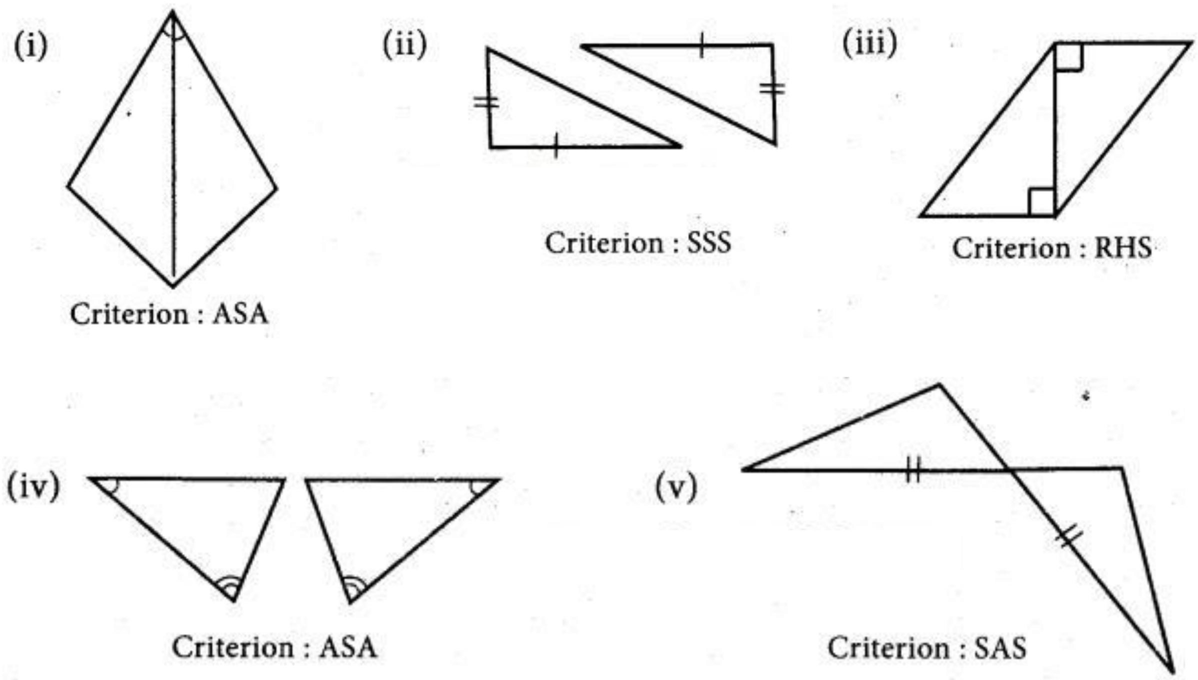
Here ∠W = ∠Z = 90°
$\overline{XY} = \overline{XY}$ (Common Hypothenure)
$\overline{XW} = \overline{XZ}$ (given)
By RHS criterion ΔXYZ ≅ ΔXYW

(v) Let the two triangles be ΔABC and ΔADC

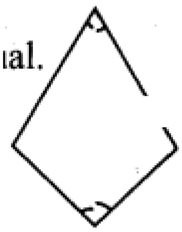


In both the triangles $\overline{AC} = \overline{AC}$ (common sides)
$\overline{AD} = \overline{BC}$ (given)
$\overline{AB} = \overline{DC}$ (given)
By SSS criterion ΔABC ≅ ΔADC.

Question 5.
To conclude the congruency of triangles, mark the required information in the following figures with reference to the given congruency criterion.



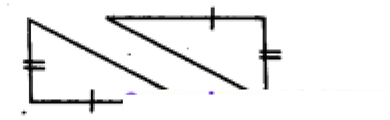
Solution:
(i) In the given triangles one angle is equal and a side is common and so equal.



To satisfy ASA criterion one more angle should be equal such that the

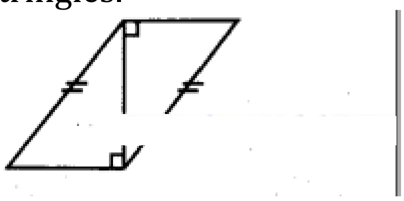
common side is the included side of both angles of a triangle.
The figure will be as follows.

(ii) In the two given triangles two sides of one triangle is equal to two sides of the other triangle.



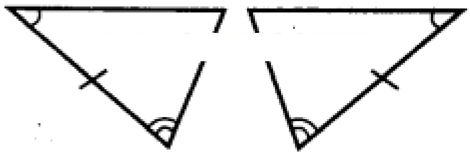
To satisfy SSS criterion the third sides mut be equal.

(iii) The given triangles have one side in common. They are right angled tringles.



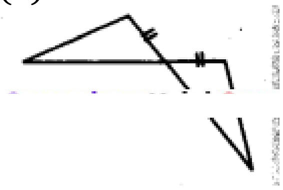
To satisfy RHS criterion their hypotenuse must be equal.

(iv) In the given triangles two angles of one triangle is equal to two angles of the other triangles?



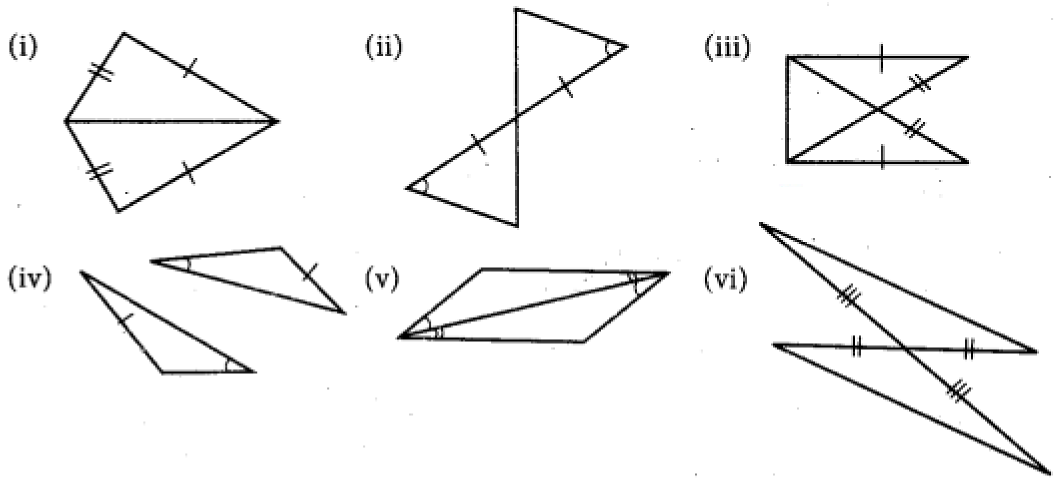
To satisfy ASA criterion included side of two angles must be equal.

(v) In both the triangles one of their sides are equal.



One of their angles are equal or they are vertically opposite angles.
To satisfy SAS criterion, one more side is to be equal such that the angle is the included of the equal sides.

Question 6.
For each pair of triangles state the criterion that can be used to determine the congruency?



Solution:
(i) Given two pair of sides are equal and one side is common to both the triangles.
∴ SSS congruency criterion is used.

(ii) One of the sides and one of the angles are equal.
∴ One more angle is vertically opposite angle and so it is also equal.
ASA criterion is used.

(iii) From the figure hypotenuse and one side are equal in both the triangles.
RHS congruency criterion is used. (∵ Considering ΔABC and ΔBAD)
$\angle A = \angle B = 90°$
$AD = BC$
$AB = AB$ (common)
∴ $AC = BD$ (hypotenuse)

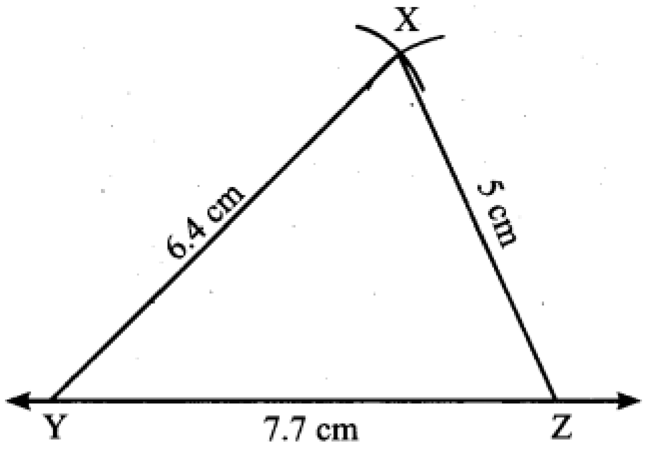
(iv) By ASA criterion both triangles are congruent.

(v) By ASA criterion both triangles are congruent. Since two angles in one triangle are equal to two corresponding angles of the other triangle. Again one side is common to both triangle and the side is the included side of the angles.

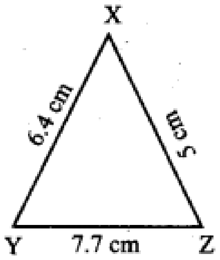
(vi) Two sides are equal. One angle is vertically opposite angles and one equal.
By SAS criterion both triangles are cogruent.

Question 7.
I. Construct a triangle XYZ with the given conditions.

(i) XY = 6.4 cm, ZY = 7.7 cm and XZ = 5 cm
Solution:



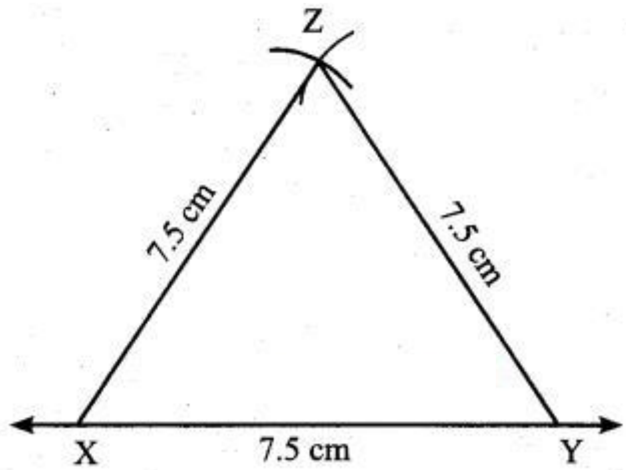
Rough Diagram

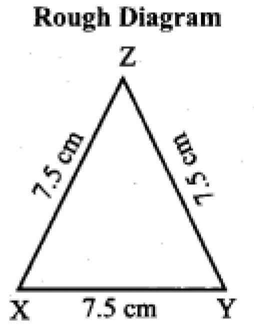


Construction:
Step 1: Draw a line. Marked Y and Z on the line such that YZ 7.7 cm.
Step 2: With Y as centre drawn an arc of radius 6.4 cm above the line YZ.
Step 3: With Z as centre, drwan an arc or radius 5 cm to intersect arc drawn in steps. Marked the point of intersection as X.
Step 3: Joined YX and ZX. Now XYZ is the required triangle.

(ii) An equilateral triangle of side 7.5 cm

Solution:

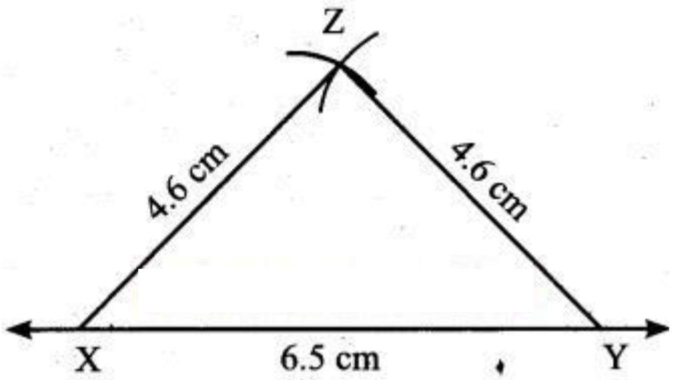


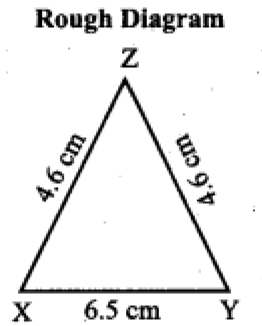


Construction:
Step 1: Drawn a line. Marked X and Y on the line such that XY = 7.5 cm.
Step 2: With X as centre, drawn an arc of radius 7,5 cm above the line XY.
Step 3: With Y as centre, drawn an arc of radius 7.5 cm to intersect arc drawn in steps. Marked the point of intersection as Z.
Step 4: Joined XZ and YZ. Now XYZ in the required triangle.

(iii) An isosceles triangle with equal sides 4.6 cm and third side 6.5 cm

Solution:



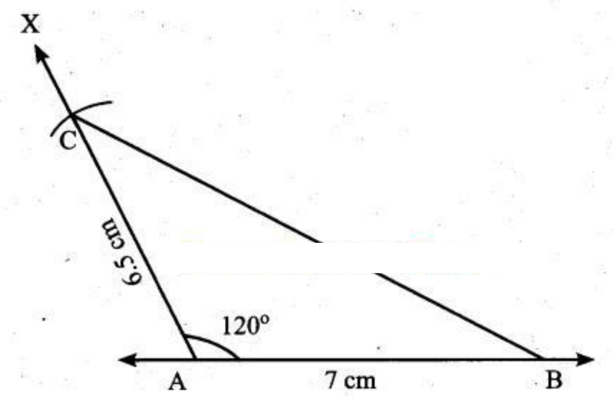


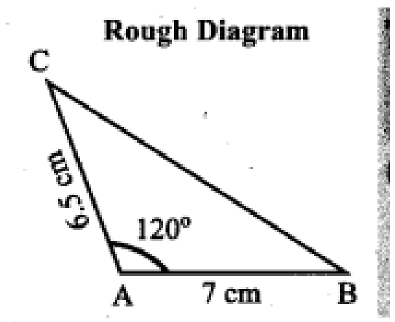
Construction: .
Step 1: Drawn a line. Marked X and Y on the line such that XY = 6.5 cm.
Step 2: With X as centre, drawn an arc of radius 4.6 cm above the line XY
Step 3: with Y as centre, drawn an arc of radius 4.6 cm to intersect arc drawn

in steps. Marked the point of intersection as Z.
Step 4: Joined XZ and YZ. Now XYZ is the required triangle.

II. Construct a triangle ABC with given conditions.

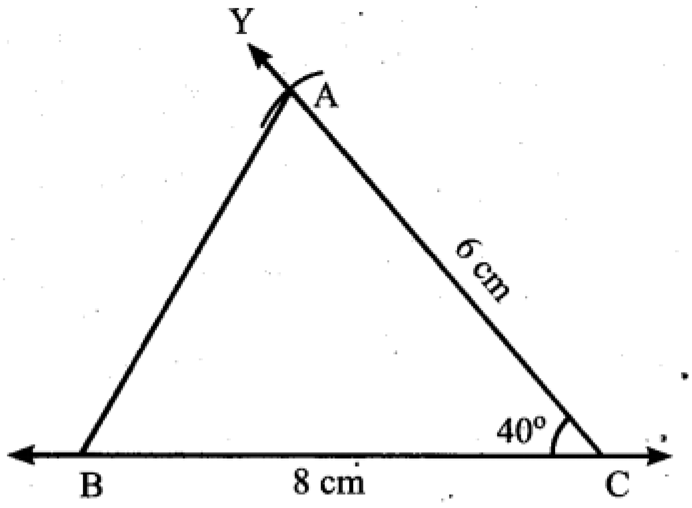
(i) AB = 7 cm, AC = 6.5 cm and ∠A = 120°.
Solution:

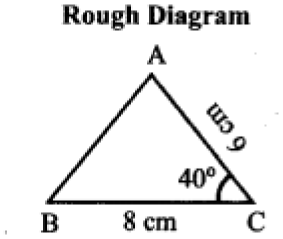




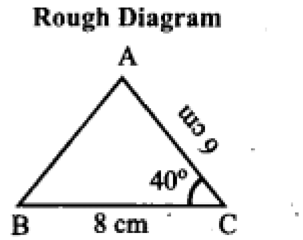
Construction:
Step 1: Drawn a line. Marked A and B on the line such that AB = 7 cm.
Step 2: At A, drawn a ray AX making an angle of 120° with AB.
Step 3: With A as centre, drawn an arc of radius 6.5 cm to cut the ray AX.
Marked the point of intersection as C.
Step 4: Joined BC.
ABC is the required triangle.

(ii) BC = 8 cm, AC = 6 cm and ∠C = 40°.
Solution:



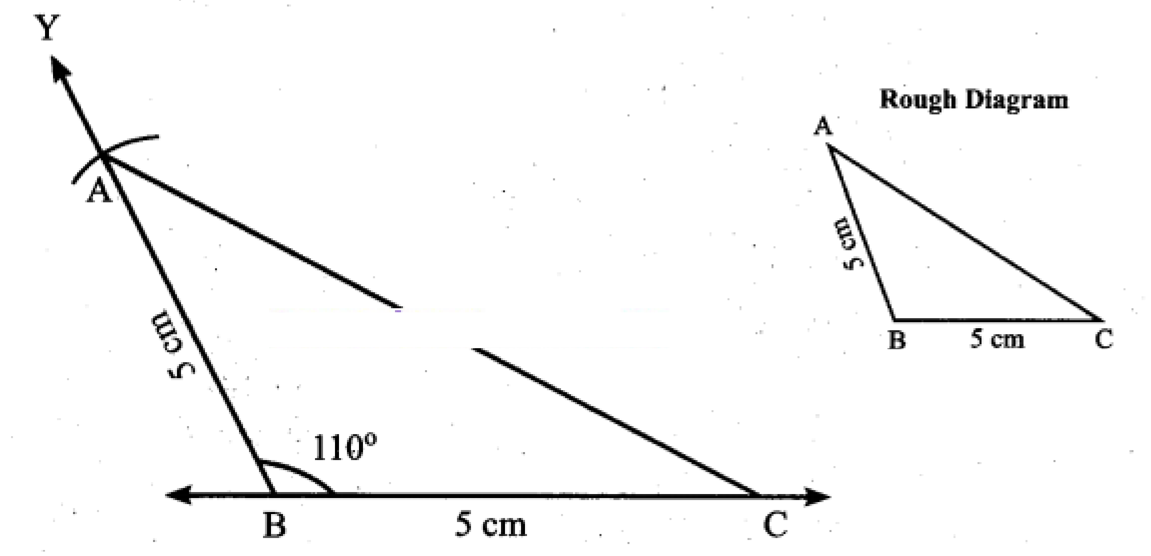


Construction:
Step 1: Drawn a line. Marked B and C on the line such fhat BC = 8 Cm.
Step 2: At C, drawn a ray CY making an angle of 40° with BC.



Step 3: With C as centre, drawn an arc of radius 6 cm to cut the ray CY, marked the point of intersection as A.
Step 4: Joined AB.
AB is the required triangle.

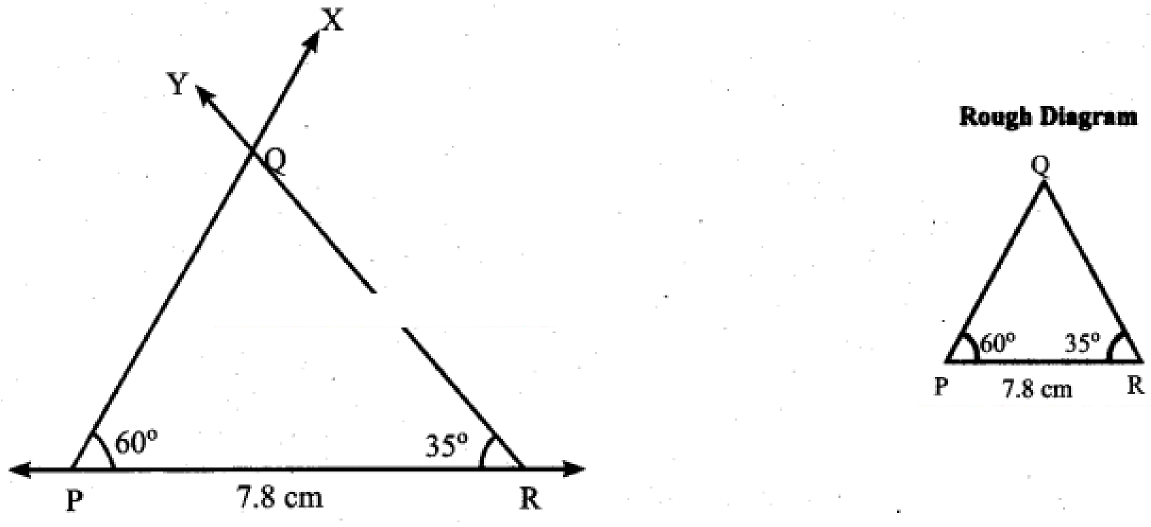
(iii) An isosceles obtuse triangle with equal sides 5 cm
Solution:



Construction:
Step 1: Drawn a line. Marked B and C on the line such that BC = 5 cm.
Step 2: At B drawn a ray BY making on obtuse angle 110° with BC.
Step 3: With B as centre, drawn an arc of radius 5 cm to cut ray BY. Marked the point of intersection as C.
Step 4: Joined BC. ABC is the required triangle.

III. Construct a triangle PQR with given conditions.

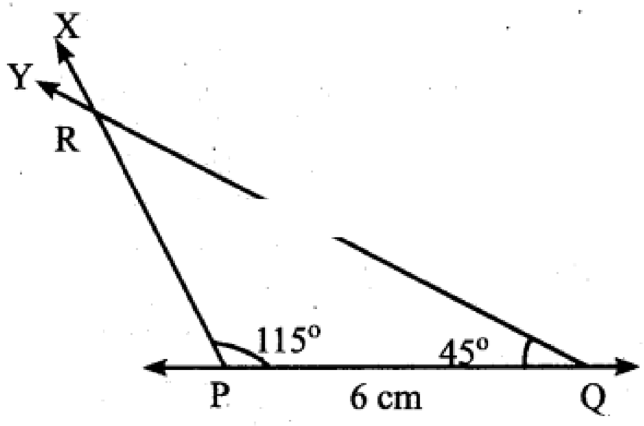
(i) ∠P = 60°, ∠R = 35° and PR = 7.8 cm
Solution:

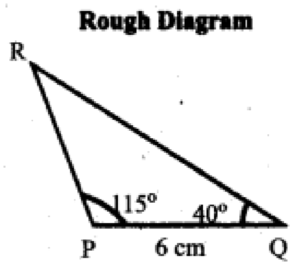


Construction:
Step 1: Drawn a line. Marked P and R on the line such that PR = 7.8 cm.
Step 2: At P, drawn a ray PX making an angle of 60° with PR.
Step 3: At R, drawn another ray RY making an angle of 35° with PR. Mark the point of intersection of the rays PX and RY as Q.
PQR is the required triangle.

(ii) ∠P = 115°, ∠Q = 40° and PQ = 6 cm

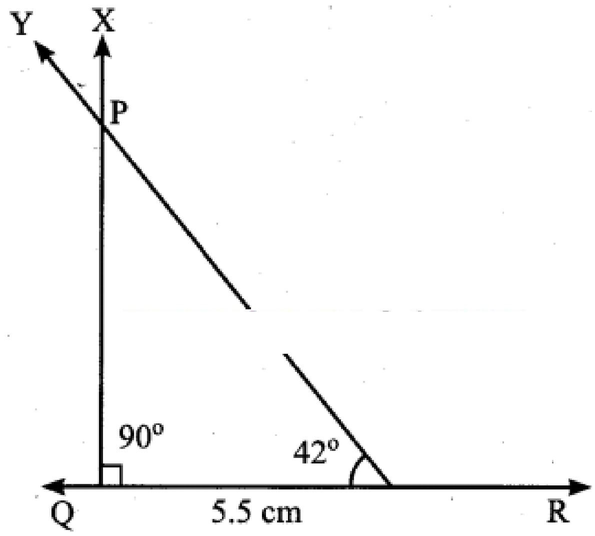
Solution:

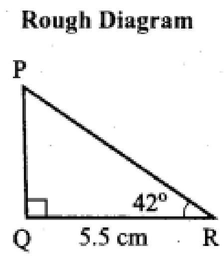




Construction:
Step 1: Drawn a line. Marked P and Q on the line such that PQ = 6 cm.
Step 2: At P, drawn O ray PX making an angle of 115° with PQ.
Step 3: At Q, drawn another ray QY making an angle of 40° with PQ. Marked the point of intersection of the rays PX and Q Y as R.
PQR is the required triangle.

(iii) ∠Q = 90°, ∠R = 42° and QR = 5.5 cm
Solution:





Construction:
Step 1: Drawn a line. Marked Q and R on the line such that QR = 5.5 cm.
Step 2: At Q, drawn a ray QX making an angle of 90° with QR.
Step 3: At R, drawn another ray RY making an angle of 42° QR. Marked the point of intersection of the rays QX and RY as P.
PQR is the required triangle.

Objective Type Questions

Question 8.
If two plans figures are congruent then they have
(i) same size
(ii) same shape
(iii) same angle
(iv) same shape and same size

Answer:
(iv) same shape and same size

Question 9.
Which of the following methods are used to check the congruence of plane figures?
(i) translation method
(ii) superposition method
(iii) substitution method
(iv) transposition method

Answer:
(ii) superposition method

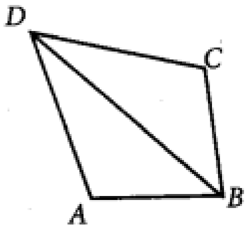
Question 10.
Which of the following rule is not sufficient to verify the congruency of two triangles.
(i) SSS rule
(ii) SAS rule
(iii) SSA rule
(iv) ASA rule

Answer:
(iii) SSA rule

Question 11.
Two students drew a line segment each. What is the condition for them to be congruent?
(i) They should be drawn with a scale.
(ii) They should be drawn on the same sheet of paper.
(iii) They should have different lengths.
(iv) They should have the same length.

Answer:
(iv) They should have the same length.

Question 12.
In the given figure, AD = CD and AB = CB. Identify the other three pairs that are equal.



(i) ∠ADB = ∠CDB, ∠ABD = ∠CBD, BD = BD
(ii) AD = AB, DC = CB, BD = BD
(iii) AB = CD, AD = BC, BD = BD
(iv) ∠ADB = ∠CDB, ∠ABD = ∠CBD, ∠DAB = ∠DBC

Answer:
(i) ∠ADB = ∠CDB, ∠ABD = ∠CBD, BD = BD

Question 13.
In ΔABC and ΔPQR, ∠A = 50° = ∠P, PQ = AB, and PR = AC. By which property ΔABC and ΔPQR are congruent?
(i) SSS property
(ii) SAS property
(iii) ASA property
(iv) RHS property

Answer:
(ii) SAS property

Ex 4.3

Miscellaneous Practice Problems

Question 1.
In an isoscales triangle one angle is 76°. If the other two angles are equal, find them.

Solution:
In an isoscales triangle, angle opposite to equal sides are equal. Let the equal angles be x° and x°.
In a triangle the sum of the three angles is 180°.
x° + x° + 76° = 180°
x° (1 + 1) = 180° – 76° = 104°

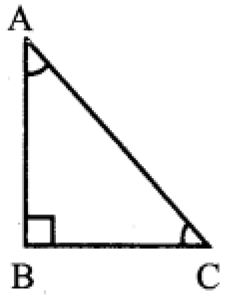
$2x = 104°$
$x = 104 \circ 2 = 52°$
$x = 52°$
∴ Other two angles are 52° and 52°.

Question 2.
If two angles of a triangle are 46° each, how can you classify the triangle?

Solution:
Given two angles of the triangle are same and is equal to 46°. If two angles are equal the sides opposite to equal angles are equal. Therefore it will be an isoscales triangle.

Question 3.
If an angle of a triangle is equal to the sum of the other two angles, find the type of the triangle.

Solution:
Let $\angle B$ is the greater angle then by the given condition $\angle B = \angle A + \angle C$.
Sum of three angle of a triangle = 180°.

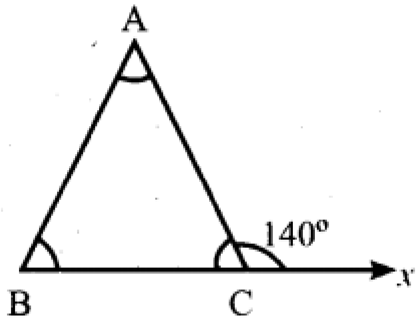


$\angle A + \angle B + \angle C = 180°$.
$\angle A + (\angle A + \angle C) + \angle C) = 180°$.
$2\angle A + 2\angle C = 180°$
$2(\angle A + \angle C) = 180°$
$\angle A + \angle C = 180 \circ 2$
$\angle B = 90°$
∴ One of the angle of the triangle = 90°
It will be a right angled triangle.

Question 4.
If the exterior angle of a triangle is 140° and its interior opposite angles are equal, find all the interior angles of the triangle.

Solution:
Given the exterior angle = 140°

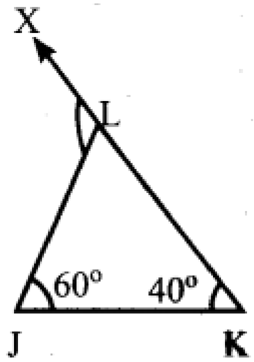
Interior opposite angle are equal.



Let one of the interior opposite angle be x.
Then x + x = 140°.
[∵ Exterior angle = sum of interior opposite angles]
2x = 140°
x = 140∘2 = 70°
x = 70°
Interior opposite angle = 70°, 70°.
Sum of the three angles of a triangle = 180°.
70° + 70° + Third angle = 180°
140° + Third angle = 180°
Third angle = 180° – 140° = 40°
∴ Interior angle are 40°, 70°, 70°.

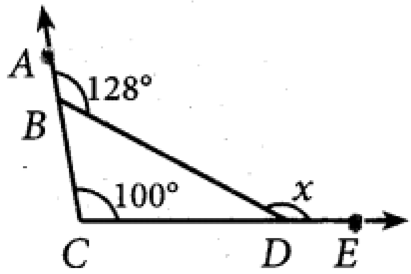
Question 5.
In ΔJKL, if ∠J = 60° and ∠K = 40°, then find the value of exterior angle formed by extending the side KL.

Solution:
When extending the side KL, the exterior angle formed in equal
to the sum of the interior opposite angles.



∠JLX = ∠LJK + ∠LKJ
= 60°+ 40° =100°
Exterior angle formed = 100°

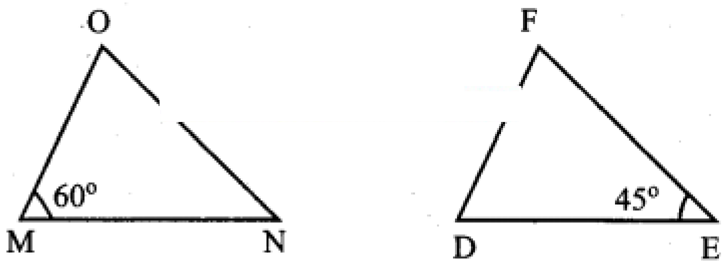
Question 6.
Find the value of 'x' in the given figure.



Solution:
Given ∠DCB = 1000 and ∠DBA = 128°
In the given figure
∠CBD + ∠DBA = 180°
∠CBD + 128° = 180°
∠CBD = 52°
Now exterior angle x = Sum of interior opposite angles.
x = ∠DCB + ∠CBD = 100° + 52° = 152°
x = 152°

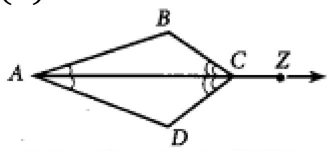
Question 7.
If ΔMNO ≅ ΔDEF, ∠M = 60° and ∠E = 45° then find the value of ∠O.

Solution:
Given ΔMNO ≅ ΔDEF



∴ Corresponding parts of conqruent triangle are congruent.
∠M = ∠D = 60° [given ∠M = 60°]
∠N = ∠E = 45° [given ∠E = 45°]
∠O = ∠F
In triangle MNO, sum of the three angle – 180°.
∠M + ∠N + ∠O = 180°
60° + 45° + ∠O = 180°
105° + ∠O = 180°
∠O = 180° – 105° = 75°
Value of ∠O = 75°

Question 8.
In the given figure ray AZ bisects ∠BAD and ∠DCB, prove that
(i) ΔBAC ≅ ΔDAC
(ii) AB = AD

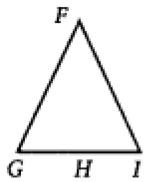


Solution:
(i) In ΔBAC and ΔDAC
∠BAC = ∠DAC [Given AZ‾‾‾‾ bisects ∠BAD]
∠BCA = ∠DCA[AZ‾‾‾‾ bisects ∠DCB]
AC = AC [∵ common side]
∴ Here AC is the included side of the angles. By ASA criterior, ΔBAC ≅ ΔDAC.

(ii) By (i) ΔBAC ≅ ΔDAC
BA = DA [By CPCTC]
i.e., AB = AD

Question 9.
In the given figure FG = FI and H is midpoint of GI, prove that ΔFGH ≅ ΔFHI

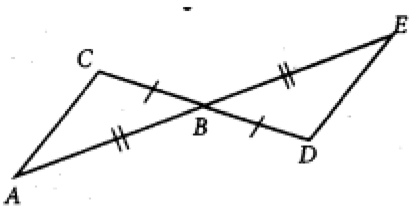
Solution:
In ΔFGH and ΔFHI
Given FG = HI
Also, GH = HI [∵ H is the midpoint of GI]



FH = FH [Common]
∴ By S.SS congruency criteria, ΔFGH ≅ ΔFIH. Hence proved.

Question 10.
Using the given figure, prove that the triangles are congruent. Can you conclude that AC is parallel to DE.

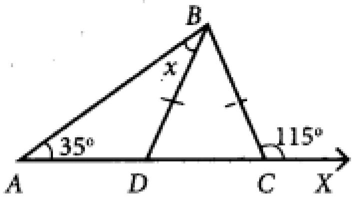
Solution:
In ΔABC and ΔEBD,



AB = EB
BC = BD
∠ABC = ∠EBD [∵ Vertically opposite angles]
By SAS congruency criteria. ΔABC ≅ ΔEBD.
We know that corresponding parts of congruent triangles are congruent.
∴ ∠BCA ≅ ∠BDE
and ∠BAC ≅ ∠BED
∠BCA ≅ ∠BDE means that alternate interior angles are equal if CD is the transversal to lines AC and DE.
Similarly, if AE is the transversal to AC and DE, we have ∠BAC ≅ ∠BED
Again interior opposite angles are equal.
We can conclude that AC is parallel to DE.

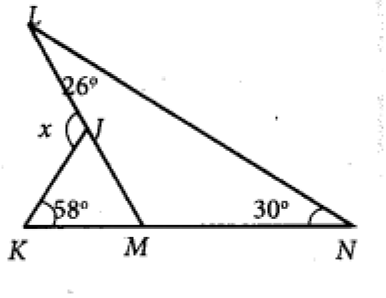
Challenge Problems

Question 11.
In given figure BD = BC, find the value of x.



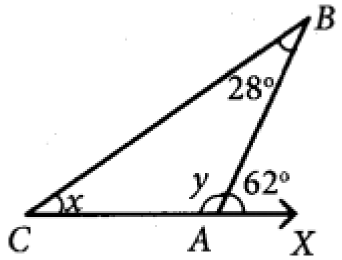
Solution:
Given that BD = BC
ΔBDC is on isoscales triangle.
In isoscales triangle, angles opposite to equal sides are equal.
∠BDC = ∠BCD ……(1)
Also ∠BCD + ∠BCX = 180° [∵ Liner Pair]
∠BCD + 115° = 180°
∠BCD = 180° – 115°
∠BCD = 65° [By (1)]
In ΔADB
∠BAD + ∠ADB = ∠BDC
[∵ BDC is the exterior angle and ∠BAD and ∠ABD are interior opposite angles]
35° + x = 65°
x = 65° – 35°
x = 30°

Question 12.
In the given figure find the value of x.



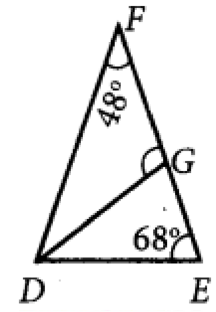
Solution:
For ΔLNM, ∠LMK is the exterior angle at M.
Exterior angle = sum of opposite interior angles
∠LMK = ∠MLN + ∠LNM = 26° + 30° = 56°
∠JMK = 56° [∵ ∠LMK = ∠JMK]
x is the exterior angle at J for ΔJKM.
∴ x = ∠JKM + ∠KMJ [∵ Sum of interior opposite angles]
x = 58° + 56° [∵ ∠JMK = 56°]
x = 114°

Question 13.
In the given figure find the values of x and y.



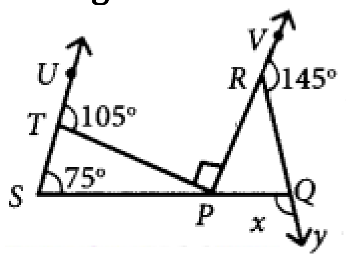
Solution:
In ΔBCA, ∠BAX = 62° is the exterior angle at A.
Exterior angle = sum of interior opposite angles.
∠ABC + ∠ACB = ∠BAX
28°+ x = 62°
x = 62° – 28° = 34°
Also ∠BAC + ∠BAX = 180° [∵ Linear pair]
y + 62° = 180°
y = 180° – 62° = 118°
x = 34°
y = 118°

Question 14.
In ΔDEF, ∠F = 48°, ∠E = 68° and bisector of ∠D meets FE at G. Find ∠FGD.



Solution:
Given ∠F = 48°
∠E = 68°
In ΔDEF,
∠D + ∠F + ∠E = 180° [By angle sum property]
∠D + 68° + 68° = 180°
∠D + 116° = 180°
∠D = 180° – 116° = 64°
Since DG is the angular bisector of ∠D.
∠FDG = ∠GDE
Also ∠FDG + ∠GDE = ∠D
2 ∠FDG = 64°
2 ∠FDG = 64°
∠FDG = 64∘2 = 32°
∠FDG = 32°
In ΔFDG,
∠FDG + ∠GFD = 180° [By angle sum property of triangles]
32° + ∠FDG + 48° = 180°
∠FDG + 80° = 180°
∠FDG = 180° – 80°
∠FDG = 100°

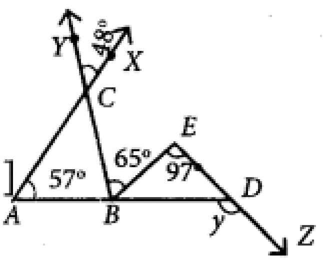
Question 15.
In the figure find the value of x.



Solution:

Exterior angle is equal to the sum of opposite interior angles.
in ΔTSP ∠TSP + ∠SPT = ∠UTP
75° + ∠SPT = 105°
∠SPT = 105° – 75°
∠SPT = 30° ……(1)
∠SPT + ∠TPR + ∠RPQ = 180° [∵ Sum of angles at a point on a line is 180°]
30° + 90° + ∠RPQ = 180°
120° + ∠RPQ = 180°
∠RPQ = 180° – 120°
∠RPQ = 60° …… (2)
∠VRQ + ∠QRP = 180° [∵ linear pair]
145° + ∠QRP = 180°
∠QRP = 180° – 145°
∠QRP = 35°
Now in Δ PQR
∠QRP + ∠RPQ = x [∵ x in the exterior angle]
35° + 60° = x
95° = x

Question 16.
From the given figure find the value of y.



Solution:
From the figure,
∠ACB = ∠XCY [Vertically opposite angles]
∠ACB = 48° …(1)
In ΔABC, ∠CBD is the exterior angle at B.
Exterior angle = Sum of interior opposite angles.
∠CBD = ∠BAC + ∠ACB
∠CBE + ∠EBD = 57° + 48°
65° + ∠EBD = 105°
∠EBD = 105° + 65° = 40° ……… (2)
In ΔEBD, y is the exterior angle at D.
y = ∠EBD + ∠BED
[∵ Exterior angle = Sum of opposite interior angles]

$y = 40° + 97°$ [$\because$ From (2)]
$y = 137°$