2.3 CHAPTER

Permutation & Combination

Being an area as a measure of students reasoning ability, permutations & combinations becomes one of the important chapter from exams point of view.

Fundamental Principle of Counting

Multiplication Rule

If a task A can be done in 'm' ways & a second task B can be done in 'n' ways, the number of ways of doing both the task one after another is 'm × n' ways.

Additive Rule

If task 'A' can be done in 'm' ways & another task 'B' can be done in 'n' ways then total no. of ways of doing either of task.

= (m + n) ways

These two rules are basic building block to permutations & combinations principles.

Some Basic Examples

1. In a building there are 6 floors. In how many ways 5 persons can get down at different floors such that exactly one person is allowed at any floor. (order in which person are getting down is not important)

Solution.

For 1st person there are 6 choices of floor. Now for 2nd person only 5 choices are left (because one of the floors is occupied by 1st person)

Progressing with same logic

Total no. of ways (Because all five to get down, hence multiplicative rule)

$6 \times 5 \times 4 \times 3 \times 2 = 720$ ways

2. In a party there are 20 couples, 20 boys (single) & 20 girls (single) are invited. If handshake between only two persons of opposite sex is allowed & handshake between spouses is not allowed total how many handshakes took place in the party?

Solution.

	Males	Females
Couples	20	20
Singles	20	20

For each male among couple has a choice of 39 shake hands, because he has not do handshake will his spouse & for each of single boy there is choice of 40 handshakes.

Hence total no. of handshakes.

$= 20 \times 39 + 20 \times 40 = 20 \times 79 = 1580$

Permutations

Number of ways of arranging particular things in certain order is known as permutations. For example for three things (a, b, c) can be arranged in multiple ways in a straight line, total number different arrangements is called permutations.

$$\begin{bmatrix} a, b, c & b, a, c & c, a, b \\ a, c, b & b, c, a & c, b, a \end{bmatrix} \text{Total six ways}$$

$${}^nP_r = \frac{\underline{|n}}{\underline{|n-r}}$$

Distinct permutations of 'r' things taken out of n things taken at timings = ${}^nP_r$.

The terms permutation & arrangement are synonymous and can be used inter-changeably.

Combinations

Combinations is basically selection of a group of certain things out of given set. In combination order of things is not important.

Combinations of 'r' things out of 'n' distinct things is

$${}^nC_r = \frac{{}^nP_r}{\underline{|r}} = \frac{\underline{|n}}{\underline{|n-r} \times \underline{|r}}$$

Special Notes:

At times, question may not explicitly state whether we have find out permutations or combination but nature of question can guide us which has to be use by pure examination of statement in question?

For example

"If I say how many 3 digit number can be made using digit 1, 2, 3 exactly once". I know possible name 123, 132 order is important hence permutation will be used.

If I say out of a group of five friends that I have, in how many ways I can invite three friends, now say out of friend u, v, w, x, y suppose. If I select u, v, w the w, u, v will be same group hence selection will be applicable.

In beginning we will cover all permutation combination formulae with two basic conditions

- If all things are distinct
- Each item is used exactly for once.

Linear Permutation of 'n' distinct things taken 'r' at a time without repetition

$$^nP_r = \frac{\underline{|n}}{\underline{|n-r}}$$

$n! = 1 \times 2 \times 3 \times 4 \times 5 \ldots. \times n$

$0! = 1$

Number of combinations of n distinct things taken r at time

$$= {^nC_r} = \frac{\underline{|n}}{\underline{|n-r} \times \underline{|r}}$$

$^nC_r = {^nC_{n-r}}$

Number of arrangements of 'n' items of which 'P' one of one type, 'q' second type & rest are distinct.

$$= \frac{\underline{|n}}{\underline{|P} \times \underline{|q} \times \underline{|r}}$$

Number of Permutations of 'n' distinct items where each item can be used any number of time (repetition allowed) = n^r

Number of ways of selecting one of more items from n given items = $(2^n - 1)$

$= {^nC_1} + {^nC_2} + {^nC_3} \ldots\ldots {^nC_n} = (2^n - 1)$

Division into groups

Dividing (P + q + r) things into group of P thing, q things & r things.

$$= \frac{(P+q+r)!}{P!q!r!}$$

Special Case.

(i) Dividing 3(P) things into three distinguishable grouping P items each $= \frac{(3P)!}{(P!)^3}$

(ii) Dividing 3(P) things into three identical group of P items each $= \frac{(3P)!}{(P!)^3} \times \frac{1}{3}$

Circular Permutations

Number of circular permutations of n distinct things = (n − 1)! [when clockwise & anticlockwise arrangement are different]

$= \frac{(n-1)!}{2}$ [when clockwise & anticlockwise arrangement are not different]

Partion Rule

No. of ways of distribution of n identical things among r people.

Case-1.

When few people can have zero things

$= {^{n+r-1}C_{r-1}}$

Case-2.

When each person has to have atleast one thing

$= {^{n-1}C_{r-1}}$

The total number of ways in which a selection can be made by taking some or all out P + q + r + ... things where P are of one kind, q alike of second kind, r of a third kind & so on.

$= [\{(P + 1)(q + 1)(r + 1)....\} - 1]$

Some Additional Points

$^{n+1}C_r = {^nC_r} + {^nC_{r-1}}$

$^nP_r = r \times {^{n-1}P_{r-1}} + {^{n-1}P_r}$

$^nC_0 + {^nC_1} + {^nC_2} + \ldots. {^nC_n} = 2^n$

$^nC_2 + {^nC_4} + {^nC_6} + \ldots. = [2^{(n-1)} - 1]$

$^nC_1 + {^nC_3} + {^nC_5} + \ldots = 2^{(n-1)}$

□□□□

Solved Examples

Direction: (Q.1 to Q.3): How many 5 digit numbers are possible from digits 1, 2, 3, 4, 5, 6, 7.

1. When each digit is distinct?

(a) 2520 (b) 1260

(c) 630 (d) None of these

Ans. (b)

$^7P_5$ or $7 \times 6 \times 5 \times 4 \times 3 = 1260$

2. When repetition of digits is allowed?

(a) 7^4 (b) 5^7

(c) 7^5 (d) None of these

Ans. (c)

$(7)^5 \Rightarrow 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

3. What is sum of all numbers in example 1.

(a) ${}^7P_5 \times 11111 \times 28$ (b) $\frac{{}^7P_5}{7} \times 11111 \times 28$

(c) $\frac{{}^7P_5}{7} \times 11111 \times 24$ (d) None of these

Solution for Example 3.

Total no. possible = $7 \times 6 \times 5 \times 3 \times 4 = 1260$

In these 1260, number at unit plate each digit out of 1, 2, 3, 4, 5, 6, 7 will occur equal no. time that is

each digit at unit place will occur $\frac{1260}{7}$ times $\frac{{}^7P_5}{7}$

same pattern will be applicable 10^{th} place, 100^{th} place, 1000^{th} place, 10000^{th} place digit

Hence sum of all numbers unit place digits will be

$$= \frac{1260}{7}[1+2+3+4+5+6+7]$$

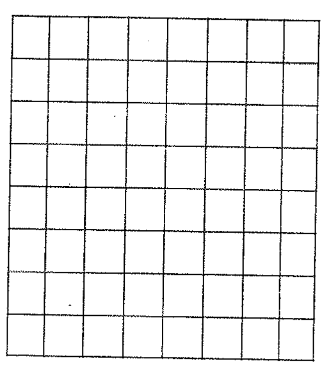
Hence sum of all such number will be

$$= \frac{1260}{7}[1+2+3+4+5+6+7]$$

$$[10000+1000+100+1+10+1]$$

$$= \frac{1260}{7}[28][11111] = \frac{{}^7P_5}{7}[28][11111]$$

Direction: (Q.4 to Q.6): Below is a square grid of 8×8.



Example 4.

What is total No. of squares in the grid?

(a) 204 (b) 285

(c) 140 (d) None of these

Ans. (a)

Square of 1×1 unit $\rightarrow 8^2$

Square of 2×2 unit $\rightarrow 7^2$

Square of 3×3 unit $\rightarrow 6^2$

....

....

Square of 7×7 unit $\rightarrow 2^2$

Square of 8×8 unit $\rightarrow 1^2$

Total number of square

$= 1^2 + 2^2 + 3^2 + ... + 8^2 = 204$

Hence option (a)

Example 6.

What is total number of rectangles in the grid?

(a) 36×36 (b) 35×35

(c) 34×34 (d) 32×32

Ans. (a)

To make one rectangle we need to select two horizontal lines which can selected as = ${}^9C_2$ ways and we need two vertical lines which can be selected as = ${}^9C_2$ ways.

(In a grid of 8×8, there will 9 horizontal lines and 9 vertical lines)

Hence total number of rectangles are

$$= {}^9C_2 \times {}^9C_2 = \left(\frac{9 \times 8}{2}\right) \times \left(\frac{9 \times 8}{2}\right) = 36 \times 36$$

Hence option (a)

Example 6.

How many rectangle of different dimensions are there?

(a) 34 (b) 35

(c) 36 (d) 33

Ans. (c)

Rectangles of

1×1, 1×2, 1×3, 1×4, 1×5, 1×6, 1×7, $1 \times 8 \Rightarrow 8$ rectangles

2×2, 2×3, 2×4, 2×5, 2×6, 2×7, $2 \times 8 \Rightarrow 7$ rectangles

3×3, 3×4, ... $3 \times 8 \Rightarrow 6$ rectangles

4×4 .... $4 \times 8 \Rightarrow 5$ rectangles

5×5 .... $5 \times 8 \Rightarrow 4$ rectangles

6×6, 6×7, $6 \times 8 \Rightarrow 3$ rectangles

7×7, $7 \times 8 \Rightarrow 2$ rectangles

$8 \times 8 = 1$ rectangles

Hence total no. of rectangles

$= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

$= 36$ rectangles

Direction: (Q.7 & Q.8): There are 10 points in a plane & out of which 4 are collinear.

Example 7.

How different triangles can be mode using these 10 points.

(a) ${}^{10}C_3 - {}^3C_3$ (b) ${}^9C_3 - {}^2C_2$

(c) ${}^{10}C_3 - {}^4C_3$ (d) ${}^{10}C_3 - {}^5C_3$

Ans. (c)

To make a triangle we need 3 non-collinear point

$\Rightarrow$ Out 10, 3 points can be selected in $^{10}C_3$ ways but in this selection $^4C_3$ selection are also included but these 4 points are collinear hence total number of feasible triangles.

$= {}^{10}C_3 - {}^4C_3$ Option (c)

Example 8.

How many different lines can be drawn?

(a) $^{10}C_2 - {}^4C_2$ (b) $^{10}C_2 - {}^4C_2 + 1$

(c) $^{10}C_3 - {}^4C_3 + 1$ (d) None of these

Ans. (b)

To draw a line in a plane we need two points out of 10 points we can select 2 points in $^{10}C_2$ ways, but in this selection 4 points are through which only one line passes.

Hence total number of lines

$= {}^{10}C_2 - {}^4C_2 + 1$

Hence option (b)

Example 9.

How many non-negative integral solution possible for equation below

$x + y + z + w = 20$

(a) $^{23}C_3$ (b) $^{22}C_2$

(c) $^{19}C_3$ (d) None of these

Ans. (a)

(x, y, z, w and integers can take value from 0 to 20). It is application of partion rule. 20 things to be distributed among x, y, z, w such that some values can be zero.

Hence total no. of non-negative integral solution.

$= {}^{20+4-1}C_{4-1} = {}^{23}C_3$

Hence option (a)

Example 10.

How many natural number solutions are possible for equation below?

$x + y + z + w = 20$

(a) $^{23}C_3$ (b) $^{22}C_2$

(c) $^{19}C_3$ (d) None of these

Ans. (c)

In this case again partion rule is applicable, such that each of x, y, z & w should get at least one thing.

Hence no. of ways $= {}^{20-1}C_{4-1} = {}^{19}C_3$

Hence option (c)

Example 11.

There are 15 people to be seated around a round table. In how many ways we can arrange them such that

(I) There is no restriction

(a) 14! (b) 13!

(c) 12! (d) 11!

Ans. (a)

Normal circular permutation = (15 – 1)!

Hence option (a)

(II) Such that 3 particular person A, B, C are all ways together.

(a) 12! (b) 13!

(c) $12! \times 3!$ (d) $13! \times 3!$

Ans. (c)

Consider those three persons as single group now 13 objects (12 people and a group) which can be arrange in (13 – 1)! ways but 3 persons of group can be arranged in 3! ways.

Hence total possible no. of arrangements.

$= 12! \times 3!$

Hence option (C)

(III) Such that A, B, C all three are never together.

(a) $12! \times 3!$ (b) $12! \times [176]$

(c) $12! \times 182$ (d) $12! \times 142$

Ans. (b)

Total no. of required permutations

(Total circular permutations) – (Total circular permutations when all 3 are together.)

$= 14! - 12! \times 3! = 12!\,[14 \times 13 - 6]$

$= 12!\,[182 - 6] = 12! \times [176]$

Hence option (b)

□□□□

Solved Examples

1. In the above question how many numbers can be formed when repetition is allowed?
   Solution: We have to form two digit number i.e., we have to fill two place out of 4 numbers & any number can be used any number of times this can be done in $4 \times 4 = 16$ ways.

2. How many words can be formed by using letters of word MADAM?
   Solution: In MADAM we have

   2 – M's

   2 – A's

So total number of ways of forming words are $\frac{5!}{2! \times 2!} = 30$

3. Of the different words that can be formed from letters of word MOBILE, how many begins with M and ends with E?

Solution: [M] [ ] [ ] [ ] [ ] [E]

M and E are fixed at the start and end positions, Hence, we have to arrange B, I, L, O among themselves (ie in four places), this can be done in 4! ways.

4. Six boys and 4 girls wanted to enjoy a movie. How many ways of sitting arrangement can be possible for them if girls want to sit together?

Solution: $B_1B_2B_3B_4B_5B_6, G_1G_2G_3G_4$ ...(1)

Now if all girls are sitting together then

$B_1B_2B_3B_4B_5B_6X$ ...(2)

(here X is symbolic notation that all girls are sitting together)

equation (2) Can be arranged in 7! ways

Now girls can be arranged among themselves i.e., 4! ways so total number of sitting arrangement will be $7! \times 4!$ ways

5. In the above question find number of ways of arrangement if no two girls sit together.

Solution: This can be done in following manner

$- B_1 - B_2 - B_3 - B_4 - B_5 - B_6 -$

First of all, boys can be arranged in 6! ways, now vacant seats between them will be filled by girls. This can be done in $7P_4$ ways so total arrangement $= 7P_4 \times 6!$

6. How many 5 digit numbers can be formed by using 0, 1, 2, 3, 4, 5, 6, 7 only once?

Solution: d_1 d_2 d_3 d_4 d_5 [ ] [ ] [ ] [ ] [ ]

box d_1 cannot be filled with zero so only 7 numbers can be filled in first box rest can be done in

$7 \times 7 \times 6 \times 5 \times 4 = 5880$ ways

7. How many different sums can be formed with the following coins, selecting any number of coins from 5 rupees, 1 rupee, 50 paisa, 25 paisa, 10 paisa, 1 paisa.

Solution: A distinct sum will be formed by selecting either 1 or 2 or 3 or 4 or 5 or all 6 coins.

${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$ ways.

By binomial theorem this can be done in $2^6 - 1$ way.

Answer = 63.

8. Out of 15 points in a plane, 5 of which are collinear. How many straight lines can be formed?

Solution: If all 15 points were non collinear then the answer would have been ${}^{15}C_2$. However, in this case, since the 5 collinear points have also been counted. These would have been counted as ${}^5C_2$ whereas they should have been counted as 1. We need to adjust by reducing the count by $({}^5C_2 - 1)$.

Hence, the answer ${}^{15}C_2 - ({}^5C_2 - 1) = 96$

9. How many triangle can be formed by 18 points if no three among them are collinear.

Solution: To form a triangle we have to connect any three points. i.e., in this case select any three points out of 18. This can be ${}^{18}C_3$ done in ways = 816

10. In the above situation how many triangle can be formed if 5 points are collinear.

Solution: The triangles will be given by

${}^{18}C_3 - {}^5C_3 = 806$ ways

Practice Exercise: I

1. How many numbers of 3 digits can be formed with the digits 1, 2, 3, 4, 5 (repetition of digits not allowed)?
   (a) 125 (b) 120
   (c) 60 (d) 150

2. How many numbers between 2000 and 3000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7 (repetition of digits not allowed)?
   (a) 42 (b) 210
   (c) 336 (d) 440

3. In how many ways can a person send invitation cards to 6 of his friends if he has four servants to distribute the cards?
   (a) 6^4 (b) 4^6
   (c) 24 (d) None of these

4. In how many ways can 7 Indians, 5 Pakistanis and 6 Dutch be seated in a row so that all persons of the same nationality sit together?
   (a) 3! (b) 7!5!6!
   (c) 3! 7! 5! 6! (d) 18!

5. How many straight lines can be formed from 8 non-collinear points on the X-Y plane?
   (a) 28 (b) 56
   (c) 18 (d) 19860

6. In how many ways can the letters of the word PATNA be rearranged?
   (a) 60 (b) 120
   (c) 119 (d) 59

7. In the above question, how many words would be there which would start with the letter P?
   (a) 24 (b) 12
   (c) 60 (d) 18

8. A captain and a vice-captain are to be chosen out of a team having 11 players. How many ways are there to achieve this?
   (a) 10×9 (b) $^{11}C_2$
   (c) 110 (d) 109!

9. In how many ways can Ram choose a vowel and a consonant from the letter ALLAHABAD?
   (a) 4 (b) 6
   (c) 9 (d) 5

10. In how many ways can the letters of the word EQUATION be arranged so that all the vowels come together?
   (a) $^9C_4 . ^9C_5$ (b) 4! . 5!
   (c) 9! / 5! (d) 9! – 4!5!

Direction for questions 11-12 : There are 25 points on a plane of which 7 are collinear. Now solve the following:

11. How many straight lines can be formed?
   (a) 7 (b) 300
   (c) 280 (d) None of these

12. How many triangles can be formed from these points?
   (a) 453 (b) 2265
   (c) 755 (d) None of these

13. How many batting orders are possible for the Indian cricket team if there is a squad of 15 to choose from such that Sachin Tendulkar is always chosen?
   (a) 1001.11! (b) 364.11!
   (c) 11! (d) 15.11!

14. How many distinct words can be formed out of the word PROWLING that start with R and end with W?
   (a) 8!/2! (b) 6!2!
   (c) 6! (d) None of these

15. How many 7-digit numbers are there having the digit 3 (three times) and the digit 0 (four times)?
   (a) 15 (b) $3^3 \times 4^4$
   (c) 30 (d) None of these

❑❑❑❑

Solutions

1. **Ans. (c)**
   Three digits can be formed in $^5P_3$ ways that is
   $$\frac{5!}{(5-3)!} = 60$$

2. **Ans. (b)**
   Number between 2000 and 3000 are four digit numbers in which 2 is the first digit
   d_1 d_2 d_3 d_4
   [2] [ ] [ ] [ ]
   So, 2 must be taken in box d_1
   Now rest box can be filled in
   $^7P_3$ ways
   $$\Rightarrow \frac{7!}{(7-3)!} = 210 \text{ ways}$$

3. **Ans. (b)**
   Person has six invitation cards 1 card can be given to any of 4 servants so six cards can be distributed in
   $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$ ways

4. **Ans. (c)**
   We have
   → 7 Indians
   → 5 Pakistanis and
   → 6 Dutch

   7 Indians can sit in 7! ways
   5 Pakistanis can sit in 5! ways
   6 Dutch can sit in 6! ways
   Also they can sit together in 3! different ways
   $7! \times 5! \times 6! \times 3!$

5. **Ans. (a)**
   To draw a line we have to choose two points
   So, $^8C_2 = 28$ ways

6. Ans. (a)

This can be done in $\frac{5!}{2!}$ ways

$\frac{5!}{2!} = 60$

7. Ans. (b)

In PATNA, if P is fixed so we have only four letter ATNA in which there are two A's.

So, $\frac{4!}{2!} = 12$

8. Ans. (c)

$^{11}C_2 \times 2!$ Ways which is equal to 110.

9. Ans. (a)

In Allahabad we have 4 A's that is only one vowel and 4 consonants (B, D, H, L)

So any four combination.

AB, AD, AH, AL are only 4 possibilities so.

10. Ans. (b)

In EQUATION there are five Vowels A, E, I, O and U and three consonants N,Q, T so the possible arrangements will be $4! \times 5!$ ways

11. Ans. (c)

Total number of straights lines

$= {}^{25}C_2 - {}^{7}C_2 + 1 = 280$

12. Ans. (b)

Total triangle formed will be equal to ${}^{25}C_3 - {}^{7}C_3 = 2265$

13. Ans. (a)

${}^{15-1}C_{11-1} \times 11!$

$\Rightarrow {}^{14}C_{10} \times 11! = 1001 \times 11!$

14. Ans. (c)

$l_1 \; l_2 \; l_3 \; l_4 \; l_5 \; l_6 \; l_7 \; l_8$

l_1 is R & l_8 is W rest boxes can be filled in 6! ways

15. Ans. (a)

$d_1 \; d_2 \; d_3 \; d_4 \; d_5 \; d_6 \; d_7$

d_1 is 3 because we have to form 7 digit number now we have rest six boxes d_2, d_3, d_4, d_5, d_6, d_7 to be filled, this can be done in

$\frac{6!}{2! \times 4!} = 15$ ways

Practice Exercise: II

1. There are 15 buses running between Delhi & Mumbai. In how many ways can a man go to mumbai and return by a different bus?
   (a) 280 (b) 310
   (c) 240 (d) 210

2. A teacher of a class wants to set one question from each of two exercises in a book. If there are 15 and 12 questions in the two exercises respectively, then in how many ways can the two questions be selected?
   (a) 160 (b) 140
   (c) 180 (d) 120

3. A code word is to consist of two distinct English alphabets followed by two distinct numbers between 1 and 9. For example, CA23 is a code word. How many such code words are there?
   (a) 615800 (b) 46800
   (c) 719500 (d) 410800

4. There are 5 letters and 5 directed envelopes. Find the number of ways in which the letters can be put into the envelopes so that all are not put in directed envelopes?
   (a) 129 (b) 119
   (c) 109 (d) 139

5. How many different numbers of two digits can be formed with the digits 1, 2, 3, 4, 5, 6; no digits being repeated?
   (a) 40 (b) 30
   (c) 35 (d) 45

6. How many three-digit odd numbers can be formed from the digits 1, 2, 3, 4, 5, 6 when
   (i) repitition of digits is not allowed
   (ii) repitition of digits is allowed?
   (a) (i) 60, (ii) 108 (b) (i) 50, (ii) 98
   (c) (i) 70, (ii) 118 (d) (i) 80, (ii) 128

7. How many numbers are there between 100 and 1000 in which all the digits are distinct?
   (a) 548 (b) 648
   (c) 748 (d) 756

8. If $(n+1)! = 6[(n-1)!]$, find n
   (a) 6 (b) 4
   (c) 8 (d) 2

9. How many words, with or without meaning, can be formed using all letters of the word EQUATION, using each letter exactily once?

(a) 38320 (b) 39320

(c) 40320 (d) 38400

10. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

(a) 2880 (b) 2480

(c) 3680 (c) 3280

11. Four books, one each in chemistry, Physics, Biology and Mathematics are to be arranged in a shelf. In how many ways can this be done?

(a) 12 (b) 36

(c) 24 (d) 48

12. How many different numbers of six digits can be formed with the numbers 3, 1, 7, 0, 9, 5?

(a) 500 (b) 400

(c) 400 (d) 600

13. How many three-digit numbers are there, with no digits repeated?

(a) 648 (b) 548

(c) 848 (d) 748

14. In how many different ways, the letters of the word ALGEBRA can be arranged in a row if

(i) The two As are together?

(ii) The two As are not together?

(a) (i) 720, (ii) 1800

(b) (i) 620, (ii) 1600

(c) (i) 780, (ii) 1860

(d) (i) 720, (ii) 1600

15. How many different words can be formed with the letters of the word 'BHARAT'?

In how many of these B and H are never together?

(a) 240, 180 (b) 360, 240

(c) 320, 200 (c) 380, 260

16. How many different necklaces can be formed with 6 white and 5 red beads?

(a) 18 (b) 24

(c) 21 (d) 27

17. In how many ways can 5 sportmen be selected from a group of 10?

(a) 272 (b) 282

(c) 252 (d) 242

□□□□

Solutions

1. Ans. (d)

The first event of going from Delhi to Mumbai can be performed in 15 ways as he can go by any of the 15 buses. But the event of coming back from Mumbai can be performed in 14 ways (a different bus is to be taken).

Hence, both the events can be performed in $15 \times 14 = 210$ ways.

2. Ans. (c)

Since the first exercise contains 15 questions, the number of ways of choosing the first question in 15. Since the second exercise contains 12 questions, the number of ways of choosing the second question is 12. Hence, by the fundamental principle, two questions can be selected in $15 \times 12 = 180$ ways.

3. Ans. (b)

(i) There are in all 26 English alphabets. We have to choose 2 distinct alphabets.

First alphabet can be selected in 26 ways.

Second alphabet can be selected in 25 ways.

Again, out of 9 digits (1 to 9), first digit can be selected in 9 ways. Second digit can be selected in 8 ways.

Thus, the number of distinct codes

$= 26 \times 25 \times 9 \times 8 = 46800$

4. Ans. (b)

Here, the first letter can be put in any one of the 5 envelopes in 5 ways. Second letter can be put in any one of the 4 remaining evelopes in 4 ways. Continuing in this way, we get the total number of ways in which 5 letters can be put into 5 envelopes

$= 5 \times 4 \times 3 \times 2 \times 1 = 120.$

Since out of the 120 ways, there is only one way for putting each letter in the correct envelope. Hence, the number of ways of putting letters all are not in directed envelopes $= 120 - 1 = 119$ ways.

5. Ans. (b)

We have to fill up two places (since numbers are of two digits.)

The first place can be filled up in 6 ways, as any one of the six digits can be placed in the first place. The 2nd place can be filled up in 5 ways as no digit is to be repeated. Hence, both places can be filled up in $6 \times 5 = 30$ ways

6. Ans. (a)

(i) When repetition of digits is not allowed: Since we have to form a three-digit odd number, thus the digit at unit's place must be odd. Hence, the unit's place can be filled up by 1, 3 or 5, that is, in 3 ways.

Now, the ten's digit can be filled up by any of the remaining 5 digits in 5 ways and then the hundred's place can be filled up by the remaining 4 digits in 4 ways.

Hence, the number of three-digit odd numbers that can be formed $3 \times 4 \times 5 = 60$

(ii) When repetition of digits is allowed: Again, the unit's place can be filled up by 1, 3, 5, that is, in 3 ways. But the ten's and hundred's place can be filled up by any of the six given digits in 6 ways each. (Since repetition is allowed)

Hence, the number of three-digit odd numbers that can be formed

$= 3 \times 6 \times 6 = 108.$

7. Ans. (b)

Any number between 100 and 1000 is of three digits, Since the numbers should have distinct digits, repetition of digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is not allowed.

Also 0 cannot be placed on the extreme left place. Hundredth place can be filled in 9 ways. Tenth place can be filled in 9 ways. Unit's place can be filled in 8 ways.

$\therefore$ The total three-digit numbers

$= 9 \times 9 \times 8 = 648$

8. Ans. (d)

$(n+1)! = 6[(n-1)!]$

$\Rightarrow (n+1).n.[(n-1)!] = 6[(n-1)!]$

$\Rightarrow n^2 + n = 6 \Rightarrow n^2 + n - 6 = 0$

$\Rightarrow (n-2)(n+3) = 0$

$\therefore$ Either $n - 2 = 0$ or $n + 3 = 0$

$\Rightarrow n = 2$ or $n = -3$

n being natural number, so $n \neq -3$

$\therefore n = 2.$

9. Ans. (c)

The word EQUATION has exactly 8 letters which are all different.

$\therefore$ Number of words that can be formed = number of permutations of 8 letters taken all at a time.

$= P(8, 8) = 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 40320$

10. Ans. (a)

Total number of candidates = 5 + 2 = 9

In the row of 9 positions, the even places are 2^{nd}, 4^{th}, 6^{th} and 8^{th}. Now, number of even places = 4.

Number of women to occupy the even places = 4.

$\therefore$ Even places can be filled =P(4, 4) ways

Number of men = 5

$\therefore$ The remaining 5 places can be filled by 5 men = P(5, 5) ways

$\therefore$ By the fundamental principle of counting.

$\therefore$ The required number of seating arrangements

$= P(4, 4) \times P(5, 5) = 4! \times 5!$

$= 24 \times 120 = 2880.$

11. Ans. (c)

4 different books can be arranged among themselves, in a shelf, in P (4, 4)

$= 4 \times 3 \times 2 \times 1 = 24$ ways.

12. Ans. (d)

The numbers that can be formed, by taking all six digits together.

$= {}^6P_6 = 6!$

But we have to neglect the numbers which begin with zero. Now the numbers in which zero comes in the 1^{st} Place= 5!

Hence, the required number=6!-5!

= 720 − 120 = 600.

13. Ans. (a)

The required number of three-digit numbers = The permutations of the 10 objects 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 take 3 at a time, with the condition that 0 is not in the hundred's place.

$= P(10, 3) - P(9, 2)$

$= \frac{10!}{7!} - \frac{9!}{7!}$

$= \frac{10 \times 9 \times 8 \times 7!}{7!} - \frac{9 \times 8 \times 7!}{7!}$

$= 10 \times 9 \times 8 - 9 \times 8$

$= 720 - 72 = 648$

14. Ans. (a)

ALGEBRA has seven letters where 2-A, 1-L, 1-G, 1-E, 1-B and 1-R.

(i) Since two A's are always together, we take both the A's as one letter.

If p is the number of arrangements, then

$p = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

(ii) Total number of permutations

$$q = \frac{7!}{2!} = 7 \times \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$

$= 2520$

In these permutations, in some permutations, two A's are together while in the rest they are not together. Hence, the number of permutations in which two A's are not together in

$q - p = 2520 - 720 = 1800.$

15. Ans. (b)

Out of letters in the word 'BHARAT' two letters, that is, A's are alike.

$\therefore$ Number of permutations $= \frac{6!}{2!} = 360.$

Number of words in which B and H are never together.

= Total number of words – number of words in which B and H are together

$$= 360 - \frac{5!}{2!}.2! = 360 - 120 = 240.$$

16. Ans. (c)

n= total no. of beads = 6 + 5 = 11

$P_1 = 6, P_2 = 5$

$\therefore$ No. of different necklaces

$$= \frac{1(11-1)!}{2 \times 6!\ 5!} = \frac{10!}{2.6!5!}$$

$$= \frac{10.9.8.7.6!}{2.6!5.4.3.2.1} = 3 \times 7 = 21.$$

17. Ans. (c)

The require number of ways = C(10, 5)

$$= \frac{10!}{5!5!} = \frac{10.9.8.7.6}{5.4.3.2} = 252.$$

□□□□

III Practice Exercise: III

1. The letters of word PROMISE are arranged so that no two of the vowels should come together. Find the total number of arrangements.
   (a) 49 (b) 1440
   (c) 7 (d) 1898

2. In an examination paper there are two groups, each containing 4 questions. A candidate is required to attempt 5 questions but not more than 3 questions from any group. In how many ways can 5 questions be selected?
   (a) 24 (b) 48
   (c) 96 (d) None of these

3. A box contains 10 balls out of which 3 are red and the rest are blue. In how many ways can a random sample of 6 balls be drawn from the bag so that at the most 2 red balls are included in the sample and no sample has all the 6 balls of the same colour?
   (a) 105 (b) 168
   (c) 189 (d) 120

4. A cricket team of 11 players is to be formed from 20 players including 6 bowlers and 3 wicket keepers. The number of ways in which a team can be formed having exactly 4 bowlers and 2 wicket keepers is
   (a) 20790 (b) 6930
   (c) 10790 (d) 360

5. In a hockey championship, there were 153 matches played. Every two teams played one match with each other. The number of teams participating in the championship is
   (a) 18 (b) 19
   (c) 17 (d) 16

6. Seven points lie on a circle. How many chords can be drawn by joining these points.
   (a) 22 (b) 21
   (c) 23 (d) 24

7. Ten different letters of an alphabet are given. Words with 5 letters are formed from these given letters. Then the number of words which have at least one letter repeated is
   (a) 69760 (b) 30240
   (c) 99748 (d) None of these

8. Five persons A, B, C, D and E occupy seats in a row such that A and B sit next to each other. In how many possible ways can these five people sit?
(a) 24 (b) 48
(c) 72 (d) None of these

9. A department had 8 male and female employees each. A project team involving 3 male and 3 female members needs to be chosen from the department employees. How many different project teams can be be chosen?
(a) 112896 (b) 3136
(c) 720 (d) 112

10. From a group of 7 men and 6 women 5 people are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?
(a) 756 (b) 735
(c) 564 (d) 645

11. A polygon has 44 diagonals, then the number of its side are
(a) 11 (b) 9
(c) 7 (d) 5

12. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is
(a) 211 (b) 216
(c) 221 (d) 311

13. How many parallelograms will be formed if 7 parallel horizontal lines intersected by 6 parallel vertical lines?
(a) 42 (b) 294
(c) 315 (d) None of these

14. A dean must select three students to serve on a committee. If she is considering five students, then from how many different possible threesomes must she choose?
(a) 2 (b) 3
(c) 10 (d) 15

□□□□

Solutions

1. Ans. (b)
The four consonants can be written in 4! ways i.e. 24 ways. The three vowels can be written in 3! ways, i.e. 6 ways. Since no two vowels can come together, therefore vowels can be inserted in any three places out of the five places available, such as,
ⓥP ⓥR ⓥMⓥ Sⓥ, i.e. in $^5C_3$ ways, i.e. 10 ways.
Total number of arrangements required
$= 24 \times 6 \times 10 = 1440$

2. Ans. (b)
$$^4C_3 \times {}^4C_2 + {}^4C_2 \times {}^4C_3 = 4\times6+4\times6 = 48.$$

3. Ans. (b)
The possible ways are as follows:
(i) 1 red ball out of the three and 5 blue balls out the seven.
(ii) 2 red balls out of the three and 4 blue balls out of the seven
∴ Total number of ways in which a random sample of six balls can be drawn.
$$^3C_1 \times {}^7C_5 + {}^3C_2 \times {}^7C_4 = 168$$

4. Ans. (a)
There are 6 bowlers, 3 wicket keepers and 11 batsmen in all. The number of ways in which a team of 4 bowlers, 2 wicket keepers and 5 batsmen can be chosen.
$$= {}^6C_4 \times {}^3C_2 \times {}^{11}C_5$$
$$= {}^6C_2 \times {}^3C_1 \times {}^{11}C_5$$
$$= \frac{6\times5}{2\times1} \times \frac{3}{1} \times \frac{11\times10\times9\times8\times7}{5\times4\times3\times2\times1} = 20790.$$

5. Ans. (a)
$$= {}^{18}C_2 = \frac{18\times17}{2} = 153.$$

6. Ans. (b)
$$= {}^7C_2 = \frac{7!}{2!5!} = \frac{7\times6\times5!}{5!\times2!} = 21.$$

7. Ans. (a)
$$10^5 - {}^{10}P_5 = 69760$$

8. Ans. (b)

$4! \times 2$ ways, i.e. $24 \times 2 = 48$.

9. Ans. (b)

$$= {}^8C_3 \times {}^8C_3 = \frac{8\times7\times6}{3\times2} \times \frac{8\times7\times6}{3\times2}$$

$= 56 \times 56 = 3136$.

10. Ans. (a)

$${}^7C_3 \times {}^6C_2 + {}^7C_4 \times {}^6C_1 + {}^7C_5 \times {}^6C_0$$

$$= {}^7C_3 \times {}^6C_2 + {}^7C_3 \times 6 + {}^7C_2 \times 1$$

$$= \frac{7\times6\times5}{3\times2\times1} \times \frac{6\times5}{2\times1} + \frac{7\times6\times5}{3\times2\times1} \times 6 + \frac{7\times6}{2\times1}$$

$= 525 + 210 + 21 = 756$.

11. Ans. (a)

Let the number of sides be n.

$$\therefore {}^nC_2 - n = 44, n > 0 \Rightarrow \frac{n(n-1)}{2} - n = 44$$

$$\Rightarrow n^2 - 3n - 88 = 0$$

$$\Rightarrow n^2 - 11n + 8n - 88 = 0$$

$$\Rightarrow n(n-11) + 8(n-11) = 0$$

$$\Rightarrow (n-11)(n+8) = 0$$

$$\Rightarrow n = 11.$$

12. Ans. (b)

All permutations formed with 1, 2, 3, 4, 5 (sum = 15) will be divided by 3.

There are $5! = 120$ such permutations. Such numbers can also be formed using 0 and 1, 2, 4, 5. There are $4 \times 4!$ such numbers, i.e. 96. (Factor of 4 for four positions of 0 and 4! for different permutations of these four numbers)

$\therefore$ Total of such numbers = 120 + 96 = 216.

13. Ans. (c)

$${}^7C_2 \times {}^6C_2 = 315.$$

14. Ans. (c)

$${}^5C_3 = {}^5C_2 = \frac{5\times4}{2\times1} = 10.$$