GUIDED REVISION

(DIFFERENTIABILITY + MOD)-20 **MATHEMATICS**

SECTION–I(i)

Straight Objective Type (3 Marks each, –1 for wrong answer)

1. Consider the function $f(x)=\begin{cases}\sqrt{[x]+\sqrt{\{x\}}}; & x\geq 0\\ |\sin \pi[x]|; & x<0\end{cases}$ where [.] and {.} denotes greatest integer function and fraction part of function respectively. Then identify the correct option -

(A) $f(x)$ is continuous $\forall\ x\in R-\{0\}$

(B) $f(x)$ is discontinuous for positive integers only

(C) $f(x)$ is discontinuous for negative integers only

(D) $f(x)$ is continuous for all real x

2. If $f(x)$ is a real valued function defined from R to R & $f'(0) = 1$, $f(0) = 1$, then $\lim\limits_{x\to 0}\dfrac{f(2x)+f(4x)+....+f(10x)-5}{x}$

(A) 0 (B) 10 (C) 20 (D) 30

3. If $f\left(\dfrac{\pi}{2}\right)=f'\left(\dfrac{\pi}{2}\right)=0$ and $\dfrac{d^2y}{dx^2}=\cot^2 x$, then the value of $f\left(\dfrac{5\pi}{2}\right)$ is -

(A) $-\pi^2$ (B) $-2\pi^2$ (C) $3\pi^2$ (D) $-4\pi^2$

4. Let A, B and C be square matrices of order 2, such that |A|=|B|=|C|=2. If λ is a scalar for which the relation $\dfrac{|\lambda A^2||(2.B)^3|}{|(2.C)^2|}=32$ holds, then sum of all possible values of λ is -

(A) 0 (B) 1 (C) –1 (D) 2

5. Let $f(x) = x^5 + x + \sin x + 1$ and g(x) be its inverse, then area bounded by g(x), x-axis, ordinate x = 1, and $x = (\pi^5 + \pi + 1)$ is equal to -

(A) $\dfrac{5\pi^6+3\pi^2-12}{6}$ (B) $\dfrac{5\pi^6+3\pi^2-6}{6}$

(C) $\dfrac{5\pi^6+3\pi^2+6\pi+12}{6}$ (D) $\dfrac{5\pi^6+3\pi^2+6\pi+6}{6}$

SECTION–I(ii)

Multiple Correct Answer Type (4 Marks each, –1 for wrong answer)

6. If $f(x) = \begin{cases} \dfrac{ae^x + b}{be^x - a} & x > 0 \\ 2 & x = 0 \\ p\sin x + qx + q & x < 0 \end{cases}$ is differentiable function at x = 0, where a & b are non-zero number, then-

(A) $b = 3a$ (B) $p = -9/2$ (C) $q = 2$ (D) $f'(0) = -\dfrac{5}{2}$

7. Let $f(x) = \begin{cases} e^x - 2 & x < 0 \\ \cos x & 0 \le x < \pi \\ 1-(x-\pi)^2 & x \ge \pi \end{cases}$, then

(A) $f(x)$ is continuous everywhere
(B) $|f(x)|$ is continuous everywhere
(C) $|f(x)|$ is non differentiable at two points
(D) $f(x)$ is non differentiable at two points

8. Consider $f(x) = \begin{cases} -x^3 & , & -1 \le x \le 0 \\ x & , & 0 < x < 1 \\ 2-x & , & 1 \le x \le 2 \\ (x-2)^2 & , & x > 2 \end{cases}$,

$g(x) = \max \{f(t), -1 \le t, \le x\}, -1 \le x < \infty$

Which of the following holds good ?

(A) g(x) is continuous function $\forall x \in [-1,\infty)$
(B) g(x) is derivable function $\forall x \in R$
(C) range of g(x) is $[1,\infty)$
(D) g(x) is many one function

9. For two curves $C_1 : y = 1 - \cot x$, $x \in (0, \pi)$ and $C_2 : y = 2|x| + \lambda$ to touch each other value of λ can be

(A) $-\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{2}$ (C) $2-\dfrac{3\pi}{2}$ (D) $2+\dfrac{\pi}{2}$

10. The points P, Q and R are taken on the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ with eccentric angles θ, $\theta + \alpha$, $\theta + 2\alpha$ then area of triangle PQR is :

(A) Independent of θ
(B) Independent of α
(C) Maximum when $\alpha = \dfrac{2\pi}{3}$
(D) Maximum when $\theta = \dfrac{\pi}{2}$

11. If $\vec{a}$, $\vec{b}$, $\vec{c}$ & $\vec{d}$ are unit vectors such that $\vec{a}+2\vec{b}+3\vec{c}+4\vec{d}=0$ and $\vec{b}$ & $\vec{c}$ are perpendicular, then-

(A) $|\vec{a}+\vec{d}|=1$ (B) $|\vec{b}+\vec{c}|=\sqrt{2}$

(C) $|\vec{a}+\vec{b}|$ can be equal to $\sqrt{5}$ (D) $|\vec{a}+\vec{b}+\vec{c}|$ can be equal to $\sqrt{7}$

SECTION–I(iii)

Linked Comprehension Type (3 Marks each, –1 for wrong answer)

Paragraph for Question 12 to 13

A differentiable function f satisfies the relation $f(x + y) = f(x) + f(y) + 3xy(x + y)\forall x,y \in R$ and $f'(0) = -4$.

12. Range of $f(x)$ will be-

(A) R (B) $(-\infty,0)$ (C) $(0,\infty)$ (D) $[0,\infty)$

13. If $\alpha < \beta$ are non negative roots of equation $f(x) = 0$, then value of $\int_{\alpha}^{\beta} f(\alpha+\beta-x)dx$ is-

(A) 2 (B) 4 (C) –2 (D) –4

Paragraph for Question 14 to 16

Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and B_1, B_2,B_3 be row matrices satisfying $B_1A = [1\ 0\ 2]$, $B_2A = [2\ 1\ 1]$, $B_3A = [3\ 2\ 1]$. Let B is a 3×3 matrix whose rows are B_1, B_2,B_3 .

14. The value of |B| is -

(A) 0 (B) 1 (C) –1 (D) 2

15. If $B\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ then quadratic equation $ax^2 + bx + c = 0$ has -

(A) 2 real & equal roots (B) 2 real & distinct roots

(C) non-real roots (D) two real roots with same sign

16. If $3X + 4Y = A$ and $X + 6Y = B$, then trace $(X - Y)$ is -

(A) 0 (B) 5 (C) $\frac{5}{2}$ (D) 10

Paragraph for Question 17 to 19

Consider the function $y = f(x)$ satisfying the differential equation $x\frac{dy}{dx} - y + y\ell n\left(\frac{x}{y}\right) = 0$, $f(1) = e$.

On the basis of above information, answer the following questions :

17. The equation of the solution curve is -
(A) $y = xe^{-x} - e^{-1} + e$
(B) $xy = e^{x}$
(C) $y = xe^{x}$
(D) $xy = e^{-x} - e^{-1} + e$

18. $y = f(x)$ is monotonic decreasing, if x belongs to -
(A) $(-\infty, \infty)$
(B) $(-\infty, 0)$
(C) $(-\infty, -1)$
(D) $(-1, \infty)$

19. The larger area bounded by curve $y = f(x)$, x-axis and the ordinate at the point of extremum of $f(x)$ is-
(A) $2e$
(B) $\frac{2}{e}$
(C) $1+\frac{2}{e}$
(D) $1-\frac{2}{e}$

SECTION–IV

Subjective Type Questions (4 Marks each, –1 for wrong answer)

20. If $f(x) = \tan^{-1}\left(\frac{2.3^x}{1+3.9^x}\right)$, then $e^{5f'(-1)}$ is equal to (where $f'(x)$ denotes the derivative of $f(x)$ w.r.t.x)

21. Let $f(x) = ax^2 + bx + c$ where $a > 0$, $b < 0$, $c > 0$ and $b^2 - 4ac > 0$. If $g(x) = |f(|x|)|$, then number of points where g(x) is NOT differentiable is

22. Let $g(x) = 3x^3 - x^2 + 3$ and $h(x) = 2x^3 + x^2 + x + 1$. If $f(x) = g'(|x|) + h'(|x|)$ and $|x| \le \frac{1}{6}$, then maximum value of $f'(x)$ is equal to (where $f'(x)$, $g'(x)$ & $h'(x)$ are derivatives of $f(x)$, $g(x)$ & $h(x)$ w.r.t. x respectively)

(DIFFERENTIABILITY + MOD)-20 **ANSWER KEY** **MATHEMATICS**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	B	A	A	A,B,C,D	B,D	A,C,D	A,C	A,C
	Q.	11	12	13	14	15	16	17	18	19	
	A.	A,B	A	D	C	C	C	C	C	B	
SECTION-IV	Q.	20	21	22							
	A.	3	5	5							