Congruence of Triangles

Exercise – 3.1

Solution 1:

Congruent sides:
side AB ≅ side MN
side BC ≅ side NP
side AC ≅ side MP
Congruent angles:
∠A ≅ ∠M
∠B ≅ ∠N
∠C ≅ ∠P

Solution 2:

Seg PQ ≅ seg QR
∴∠R ≅ ∠P ….(Isosceles triangle theorem)
m∠P = 70° …(Given)
∴m∠R = 70° ….(1)
Now, the sum of the measures of the angles of a triangle is 180°.
∴ m∠P + m∠Q + m∠R = 180°
∴70° + m∠Q + 70° = 180° …[Given and from (1)]
∴140° + m∠Q = 180°
∴m∠Q = 180° – 140°
∴m∠Q = 40°
The measures of the remaining angles are m∠R = 70° and m∠Q = 40°.

Solution 3:

In ΔPST, seg PS ≅ seg PT. ….(Given)
∴ By Isosceles Triangle Theorem,
∠PTS ≅ ∠PST ….(1)
The sum of the measures of the angles of a triangle is 180°.
∴m∠P + m∠PTS + m∠PST = 180° ….(2)
∴m∠P + 2m∠PTS = 180° ….. [From (1) and (2)] …..(3)
Seg PS ≅ seg PT and seg SQ ≅ seg TR …(Given)
∴ PS = PT and SQ = TR
∴ PS + SQ = PT + TR
∴ PQ = PR ….(P-S-Q and P-T-R)
∴ seg PQ ≅ seg PR.
In ΔPQR, seg PQ ≅ seg PR ….(Proved)

∴ ∠PRQ ≅ ∠PQR ….(4)
m∠P + m∠PQR + m∠PRQ = 180° …..(5)
∴m∠P + 2m∠PQR = 180° ……[From (4) and (5)]….(6)
From (3) and (6), ∠PTS = ∠PRQ
∴ seg ST ∥ seg QR ….(corresponding angles test for parallel lines)
i.e. Side ST ∥ Side QR.

Solution 4:

Proof with justification:
In ΔPOR and ΔSOQ,
seg OP ≅ seg OS and seg OR ≅ seg OQ …..(Given)
∠POR ≅ ∠SOQ ….(Vertically opposite angles)
∴ ΔPOR ≅ ΔSOQ ….(SAS test)
∴ seg PR ≅ seg SQ ….(c.s.c.t.)
Also, ∠P ≅∠S and ∠R ≅∠Q …..(c.a.c.t.)
For seg PR and seg SQ, PQ is the transversal.
∴ For seg PR and seg SQ to be parallel, the alternate angles, ∠ P and ∠Q, should be congruent.
But these angles are not congruent.
∴ seg PR and seg SQ are not parallel.
Also, taking RS as the transversal, we can show that seg PR and seg SQ are not parallel.

Solution 5:

∠ABP ≅ ∠CBQ …..(Given)
∴ ∠ABP = ∠CBQ
Adding ∠PBC to both the sides
∠ABP + ∠PBC = ∠CBQ + ∠PBC
∠ABC ≅ ∠PBQ ….(1)
seg AB = seg PB and seg BC ≅ seg BQ …..(Given)
∴ΔABC ≅ ΔPBQ …..(SAS test)
∴ seg AC ≅ seg PQ ….(c.s.c.t.)

Solution 6:

Solution with justification:
In Δ ABC, seg AB ≅ seg AC ……(Given)
∴ by Isosceles Triangle Theorem,
∠ABC ≅ ∠ACB.
i.e. ∠ABC = ∠ACB …(1)
The sum of the measures of the angles of a triangle is 180°

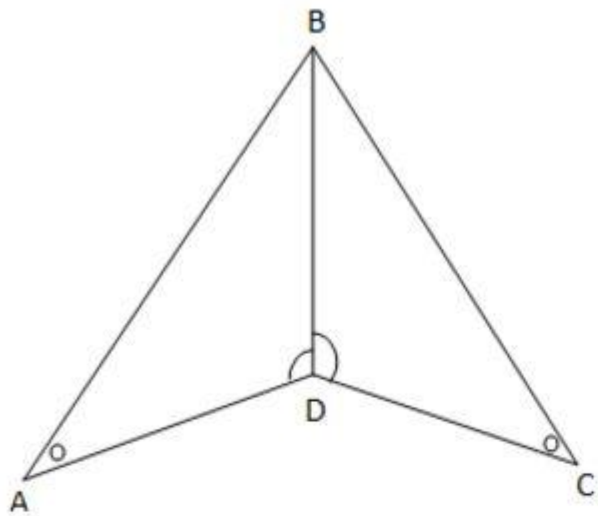
$\therefore m\angle A + m\angle ABC + m\angle ACB = 180°$
$\therefore m\angle A + m\angle ABC + m\angle ABC = 180°$ …..[From (1)]
$\therefore m\angle A + 2m\angle ABC = 180°$
$\therefore 40° + 2m\angle ABC = 180°$ …[Given: $m\angle A = 40°$]
$\therefore 2m\angle ABC = 180° – 40° = 140°$
$\therefore m\angle ABC = 70°$
$\therefore m\angle ABC = m\angle ACB = 70°$ …(2)
In ΔABP, seg AB ≅ seg PB …(Given)
$\therefore \angle P \cong \angle PAB$ i.e. $\angle P = \angle PAB$ …(3)
By Remote Interior Angle Theorem,
$\angle ABC = \angle P + \angle PAB$
From (2) and (3), $70° = m\angle P + m\angle P$
$\therefore m\angle P = m\angle PAB = 35°$ ….(4)
Similarly, we can prove that
$m\angle Q = m\angle QAC = 35°$ ….(5)
In ΔAPQ, $m\angle PAQ + m\angle P + m\angle Q = 180°$
$\therefore m\angle PAQ + 35° + 35° = 180°$ ….[From (4) and (5)]
$\therefore m\angle PAQ = 180° – 70°$
$\therefore m\angle PAQ = 110°$ …..(6)
So, $m\angle P = m\angle Q = m\angle PAB = m\angle QAC = 35°$
PC = PB + BC and QB = QC + BC …..(7)
But PB = QC ….(Given) ….(8)
∴ PC = QB ….[From (7) and (8)]
In ΔPAC and ΔQAB,
PC = QB …..(Proved)
AC = AB ….(Given)
$\angle ACB = \angle ABC$ …..[From (2)]
i.e. $\angle ACP = \angle ABQ$
$\therefore \Delta PAC \cong \Delta QAB$ …..(SAS test)
Similarly, we can prove that
$\Delta PAB \cong \Delta QAC$.
Hence, $m\angle PAQ = 110°$; Also $\Delta PAC \cong \Delta QAB$ and $\Delta PAB \cong \Delta QAC$ are the congruent triangles.

Exercise – 3.2

Solution 1:

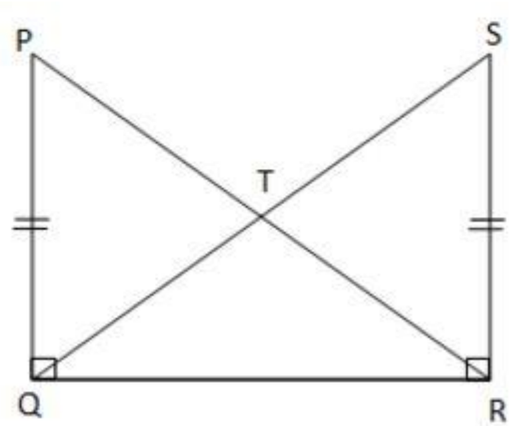
In ΔACO and ΔDBO,
$\angle ACO \cong \angle DBO$ ….(Given)
seg CO ≅ seg BO …(Given)
$\angle AOC \cong \angle DOB$ …..(Vertically opposite angles)
$\therefore \Delta ACO \cong \Delta DBO$ ….(ASA test)
The remaining congruent parts:
$\angle A \cong \angle D$, seg AC ≅ seg DB, and seg AO ≅ seg DO

Solution 2(i):



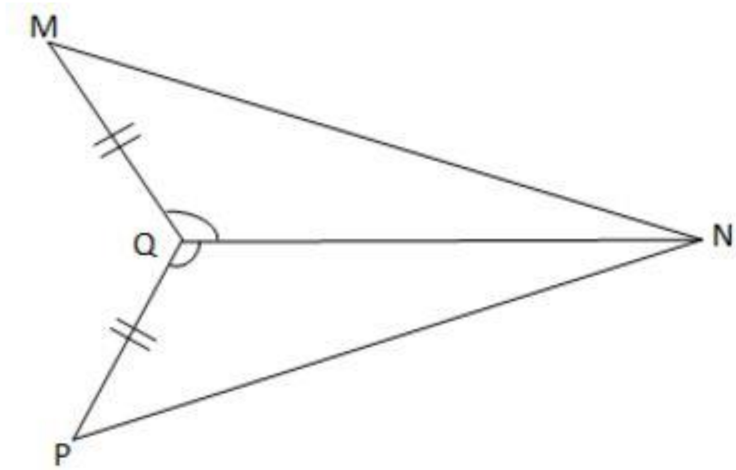
ΔBDA ≅ ΔBDC
1. Test of congruence: SAA test
2. Correspondence: BDA ↔ BDC
3. Congruent Parts:
∠ABD ≅ ∠CBD
seg AB ≅ seg CB
seg AD ≅ seg CD

Solution 2(ii):



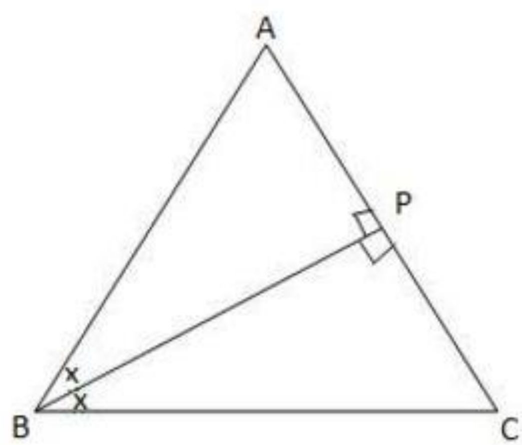
ΔPQR ≅ ΔSRQ
1. Test of congruence: SAS test
2. Correspondence: PQR ↔ SRQ
3. Congruent Parts:
seg PR ≅ seg SQ
∠QPR ≅ ∠RSQ
∠PRQ ≅ ∠SQR

Solution 2(iii):



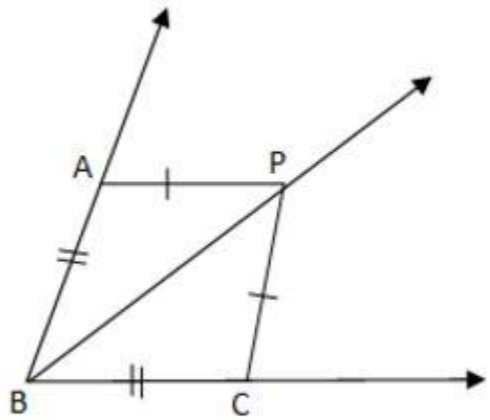
ΔMQN ≅ ΔPQN
1. Test of congruence: SAS test
2. Correspondence: MQN ↔ PQN
3. Congruent Parts:
seg MN ≅ seg PN
∠QMN ≅ ∠QPN
∠QNM ≅ ∠QNP

Solution 2(iv):



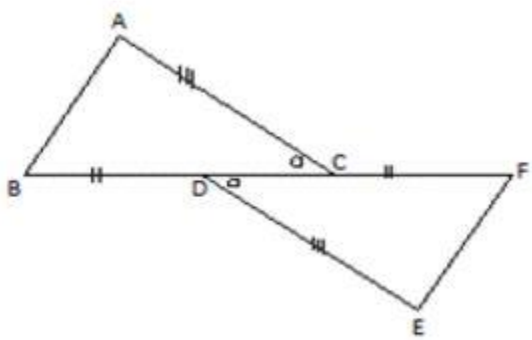
ΔBPA ≅ ΔBPC
1. Test of congruence: ASA test
2. Correspondence: BPA ↔ BPC
3. Congruent Parts:
∠BAP ≅ ∠BCP
seg BA ≅ seg BC
seg PA ≅ seg PC

Solution 2(v):



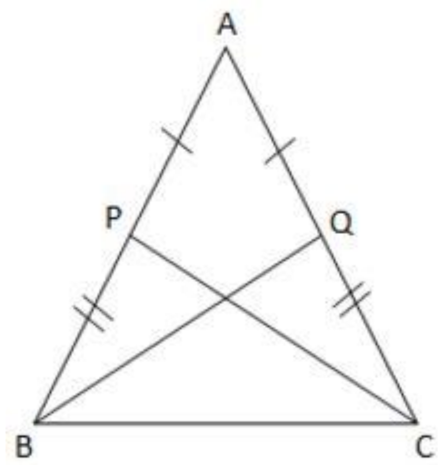
Δ ABP ≅ ΔCBP
1. Test of congruence: SSS test
2. Correspondence: ABP ↔ CBP
3. Congruent Parts:
∠PBA ≅ ∠PBC
∠BAP ≅ ∠BCP
∠BPA ≅ ∠BPC

Solution 2(vi):



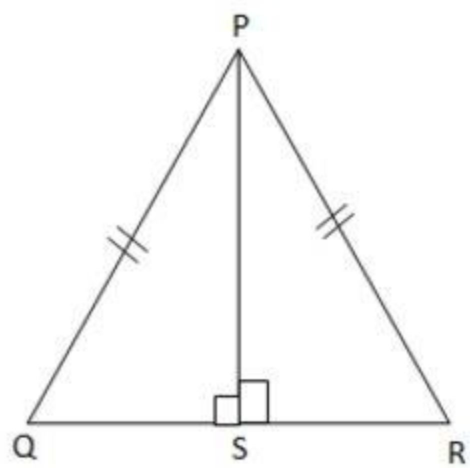
BD + DC = FC + DC
∴BC = FD
Δ ABC ≅ ΔEFD
1. Test of congruence: SAS test
2. Correspondence: ABC ↔ EFD
3. Congruent Parts:
seg AB ≅ seg EF
∠ABC ≅ ∠EFD
∠BAC ≅ ∠FED

Solution 2(vii):



AP = AQ
PB = QC
∴ AP + PB = AQ + QC
∴AB = AC …..(A-P-B; A-Q-C)
∆ ABQ ≅ ∆ACP
1. Test of congruence: SAS test
2. Correspondence: ABQ ↔ ACP
3. Congruent Parts:
seg BQ ≅ seg CP
∠ABQ ≅ ∠ACP
∠AQB ≅ ∠APC

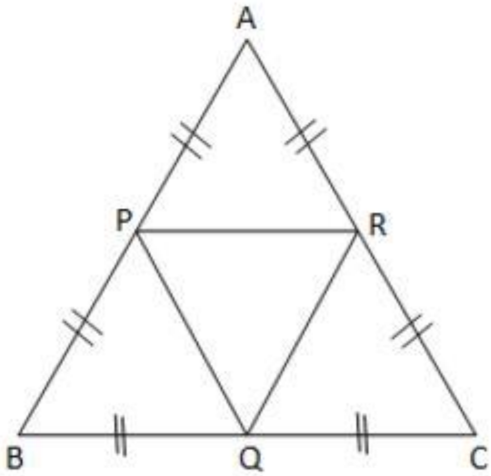
Solution 2(viii):



∆ PSQ ≅ ∆PSR
1. Test of congruence: Hypotenuse-Side theorem

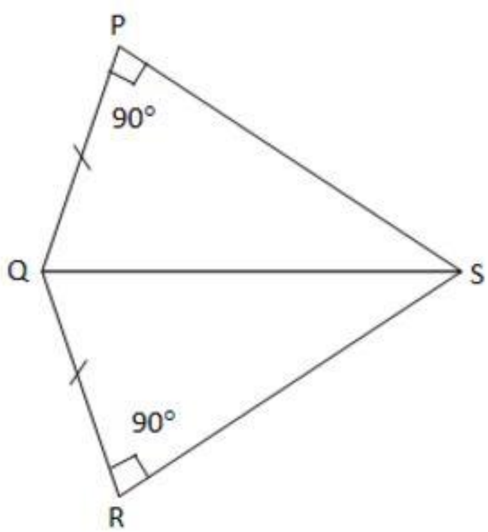
2. Correspondence: PSQ ↔ PSR
3. Congruent Parts:
seg QS ≅ seg RS
∠QPS ≅ ∠RPS
∠PQS ≅ ∠PRS

Solution 2(ix):



Δ APR ≅ ΔBPQ ≅ ΔCQR ≅ ΔQPR
1. Test of congruence: SSS test
2. Correspondence:
APR ↔ BPQ ↔ CQR ↔ QPR or any correspondence
3. Congruent Parts:
All the angles of ΔAPR, ΔBPQ, ΔCQR and ΔQPR are 60° each.

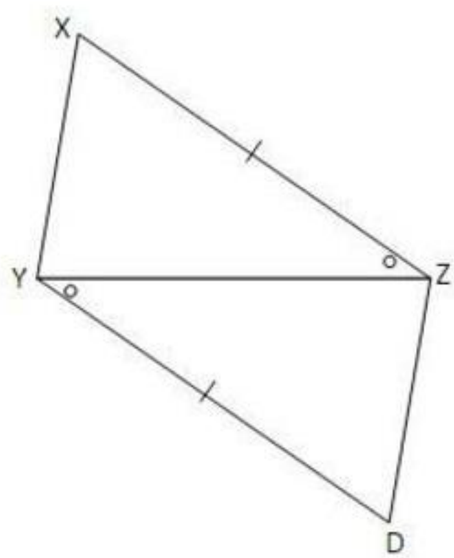
Solution 2(x):



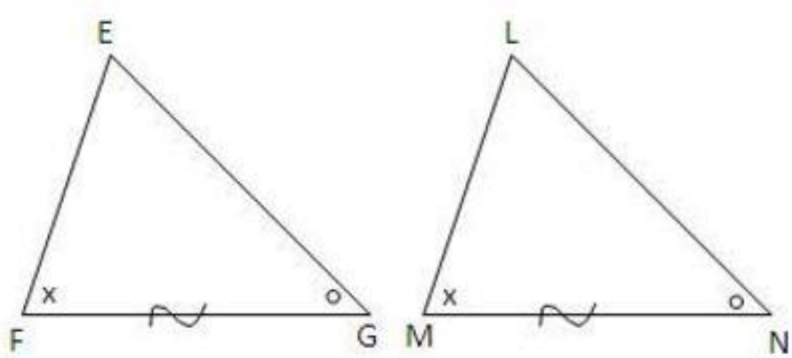
Δ PQS ≅ ΔRQS
1. Test of congruence: Hypotenuse – side theorem
2. Correspondence: PQS ↔ RQS
3. Congruent Parts:
seg PS ≅ seg RS
∠PQS ≅ ∠RQS
∠PSQ ≅ ∠RSQ

Solution 3:

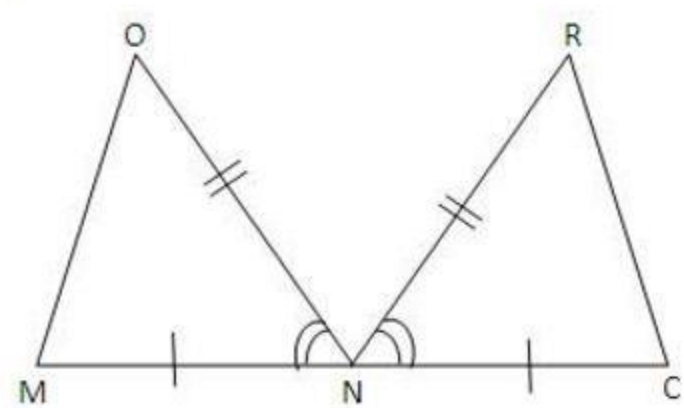
i.



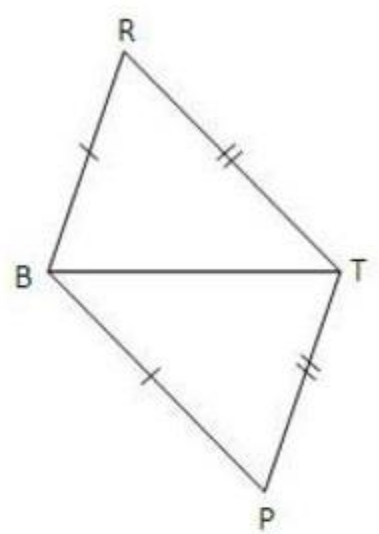
1. ΔXYZ and ΔDYZ are drawn as above.
2. Congruent parts seg XZ ≅ seg DY and
∠XZY ≅ ∠ZYD are shown in the figure by identical marks.
3. Test of congruence: SAS test
4. Congruent triangle: Δ XYZ ≅ ΔDZY.
Correspondence: XYZ ↔ DZY
ii.



1. ∆EFG and ∆LMN are drawn as above.
2. Congruent parts seg FG ≅ seg MN,
∠G ≅ ∠N and ∠F ≅ ∠M are shown in the figure by identical marks.
3. Test of congruence: ASA test
4. Congruent triangle: ∆ EFG ≅ ∆LMN.
Correspondence: EFG ↔ LMN
iii.

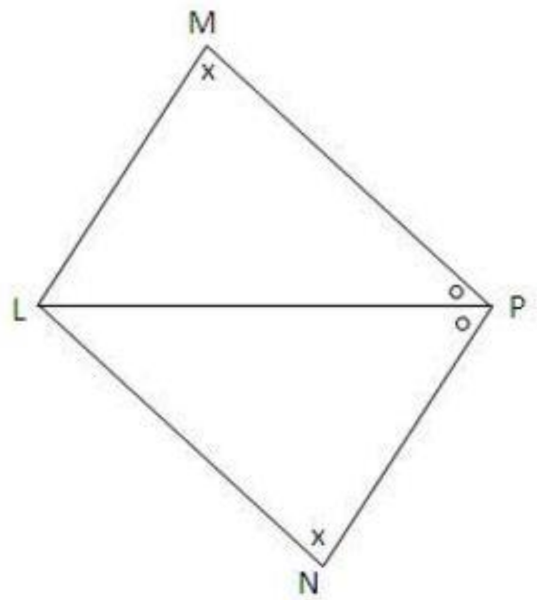


1. ∆MNO and ∆CNR are drawn as above.
2. Congruent parts seg MN ≅ seg CN,
seg NO ≅ seg NR and ∠MNO ≅ ∠CNR are shown in the figure by identical marks.
3. Test of congruence: SAS test
4. Congruent triangle: ∆MNO ≅ ∆CNR.
Correspondence: MNO ↔ CNR
iv.



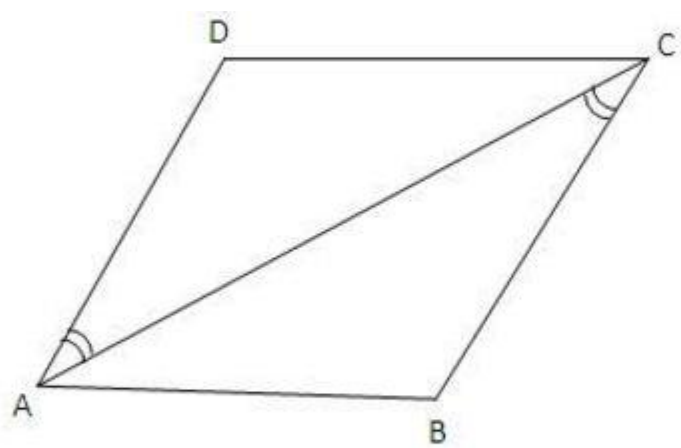
1. ∆BTR and ∆BTP are drawn as above.
1. Congruent parts seg BR ≅ seg BP,
Seg RT ≅ seg PT are shown in the figure by identical marks.
2. Test of congruence: SSS test
3. Congruent triangle: ∆BTR ≅ ∆BTP.

4. Correspondence: BTR ↔ BTP
v. In ΔLMP and ΔLPN,
∠LMP ≅ ∠LNP, ∠LPM ≅ ∠NPL.
Ans. (1) and (2)



1. ΔLMP and ΔLPN are drawn as above.
2. Congruent parts ∠LMP ≅ ∠LNP and ∠LPM ≅ ∠NPL are shown in the figure by identical marks.
3. Test of congruence: SAA test
4. Congruent triangle: ΔLMP ≅ ΔLNP.
Correspondence: LMP ↔ LNP

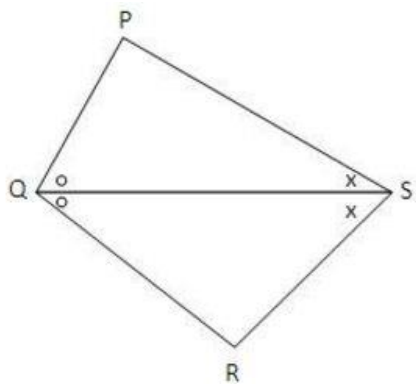
Solution 4:



Missing information required and sufficient to prove
ΔADC ≅ ΔCBA:
SAS test
seg AD ≅ seg CB

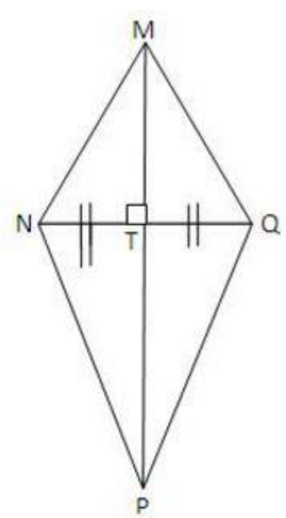
SAA test
∠D ≅ ∠B
ASA test
∠ACD ≅ ∠CAB

Solution 5:



Seg QS is the bisector of ∠PQR ….(Given)
∴∠PQS ≅ ∠RQS …(1)
Seg QS is the bisector of ∠PSR ….(Given)
∴∠PSQ ≅ ∠RSQ …(2)
In ∆PQS and ∆RQS,
∠PQS ≅ ∠RQS …[From (1)]
seg QS ≅ seg QS …(Common side)
∠PSQ ≅ ∠RSQ …..[From (2)]
∴ ∆PQS ≅ ∆RSQ ….(ASA test)
∴∠P ≅ ∠R ….(c.a.c.t.)

Solution 6:

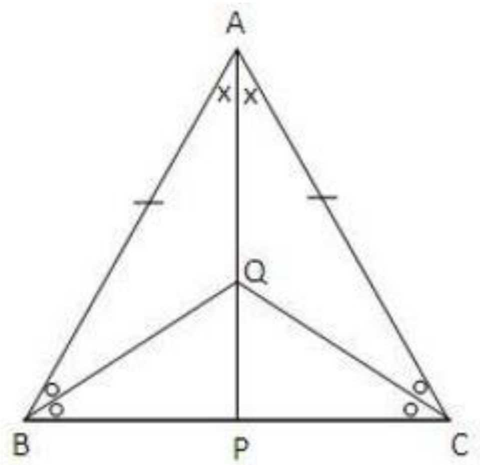


Diagonal MP is the perpendicular bisector of diagonal NQ.
∴ seg NT ≅ seg QT and
$m\angle NTM = m\angle QTM = m\angle PTN = m\angle PTQ = 90°$ …..(1)
Now, in ΔMNT and ΔMQT,
seg NT ≅ seg QT …[From (1)]
∠MTN ≅ ∠MTQ …[From (1)]
seg MT ≅ seg MT …(Common side)
∴ ΔMNT ≅ΔMQT …(SAS test)
∴ seg MN ≅ seg MQ …(c.s.c.t.)
In ΔPNT and ΔPQT,
seg NT ≅ seg QT ….[From (1)]
∠PTN ≅ ∠PTQ ….[From (1)]
seg PT ≅ seg PT …(Common side)
∴ ΔPNT ≅ΔPQT …(SAS test)
∴ seg NP ≅ seg QP …(c.s.c.t.)
In ΔNMP and ΔQMP,
seg MN ≅ seg MQ
seg NP ≅ seg QP
seg MP ≅ seg MP …(Common side)
∴ ΔPNT ≅ΔPQT …(SSS test)

Solution 7:

i. In ΔABC and ΔCDE,
$m\angle B = m\angle D = 90°$
hypotenuse AC ≅ hypotenuse CE ….(Given)
seg BC ≅ seg ED ….(Given)
∴ ΔABC ≅ ΔCDE ….(Hypotenuse- side theorem)
ii. As ΔABC ≅ ΔCDE
∴∠BAC ≅ ∠DCE ….(c.a.c.t.) …(1)
iii. In ΔABC, $m\angle BAC + m\angle ACB = 90°$ … (Acute angles of a right angled triangle)…(2)
From (1) and (2),
$m\angle DCE + m\angle ACB = 90°$ ….(3)
$m\angle ACB + m\angle ACE + m\angle DCE = 180°$
∴ $m\angle ACB + m\angle DCE + m\angle ACE = 180°$
∴ $90° + m\angle ACE = 180°$ ….[From (3)]
∴ $m\angle ACE = 90°$

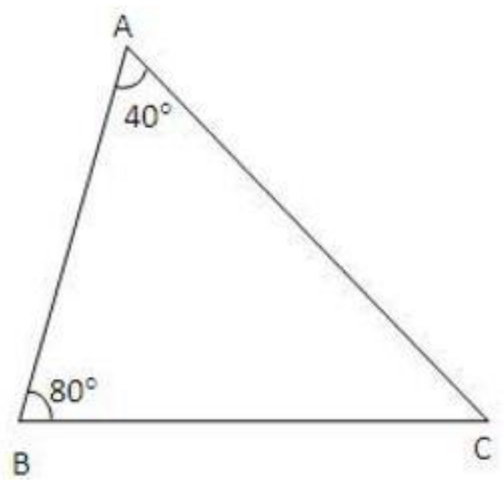
Solution 8:



i. The pairs of congruent triangles:
a. ∆ABP and ∆ACP
b. ∆ABQ and ∆ACQ
c. ∆BQP and ∆CQP
a.
In ∆ABP and ∆ACP,
seg AB ≅ seg AC …(Given)
∠BAP ≅ ∠CAP ….(AP is the bisector of ∠BAC)
seg AP ≅ seg AP …(Common side)
∴ ∆ABP ≅ ∆ACP ….(SAS test)
∴ seg BP ≅ seg CP …(c.s.c.t.)….(1)
b.
In ∆ABQ and ∆ACQ,
seg AB ≅ seg AC …(Given)
∠BAQ ≅ ∠CAQ ….(AP, i.e. AQ is the bisector of ∠BAC)
seg AQ ≅ seg AQ …(Common side)
∴ ∆ABQ ≅ ∆ACQ ….(SAS test)
∴ seg BQ ≅ seg CQ …(c.s.c.t.)….(2)
c.
In ∆BQP and ∆CQP,
seg BP ≅ seg CP …[From (1)]
seg BQ ≅ seg CQ …[From (2)]
seg QP ≅ seg QP …(Common side)
∴ ∆BQP ≅ ∆CQP …(SSS test)
ii ∆BQC is an isosceles triangle. ….[From (2)]
seg AB ≅ seg AC …(Given)
∆ABC is an isosceles triangle.

Exercise – 3.3

Solution 1:



m∠A + m∠B + m∠C = 180° …(Angles of a triangles)
∴40° + 80° + m∠C = 180°
∴m∠C = 180° – 120° ∴m∠C = 60°
The descending order of the measures of the angles is ∠B > ∠C > ∠A
∴ side AC > side AB > side BC ….(Sides opposite to the angles)
The shortest side is side BC and the longest side is side AC.

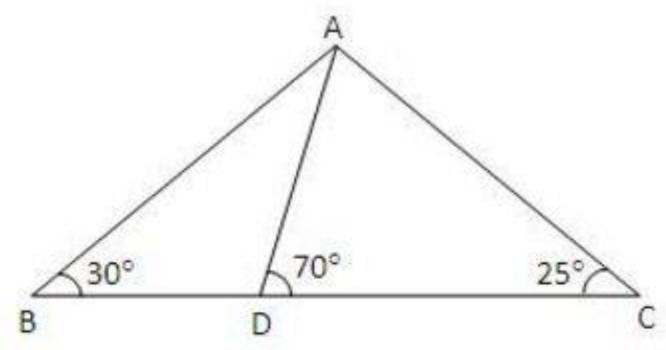
Solution 2:

AB = 5 cm, BC = 8 cm, AC = 10 cm.
∴ AC > BC > AB
∴∠B > ∠A > ∠C …..(Inequality property of a triangle)
The smallest angle is ∠C and the greatest angle is ∠B.
Their descending order is ∠B > ∠A > ∠C.

Solution 3:

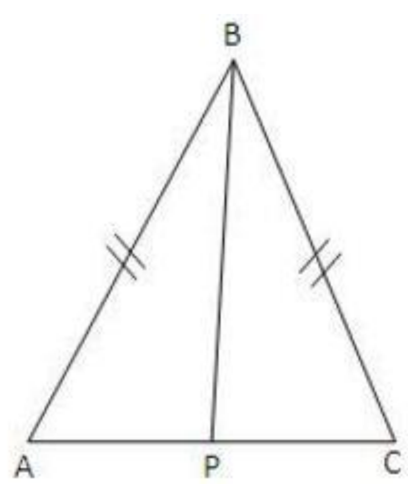
In ∆ABC, AB = 4 cm, AC = 6 cm.
∴ AC > AB.
Angle opposite to the greater side is greater.
∴a > b
b is the exterior angle of ∆ACD.
∴ By exterior angle theorem b > d ….(2)
From (1) and (2) a > b > d ….(3)
Now, c is the exterior angle of ∆ABC.
∴ c > a ….(By exterior angle theorem) …(4)
From (3) and (4) c > a > b > d.

Solution 4:



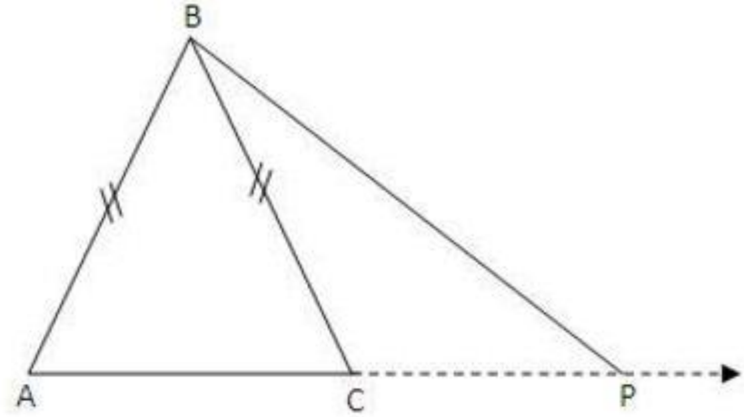
In ∆ABC, m∠B = 30°, m∠C = 25°,
∴ ∠B > ∠C
∴side AC > side AB
…..(In a triangle, the side opposite to the greater angle is greater) ….(1)
m∠ADB + m∠ADC = 180° …(Angles in a linear pair)
∴m∠ADB + 70° = 180°
∴m∠ADB = 110°
In ∆ABD, m∠ADB = 110° and m∠B = 30°
∴ ∠ADB > ∠B
∴ side AB > side AD …(2)
From (1) and (2),
side AC > side AB > side AD.

Solution 5:



(i) In ∆ABC, side AB ≅ side BC and A-P-C. …(Given)
∴ ∠A ≅ ∠C …(Isosceles Triangle Theorem) ..(1)
∠BPC > A …(Exterior Angle Theorem) …(2)
From (1) and (2), ∠BPC > ∠C
∴ BC > BP

…(Side opposite to greater angle)
i.e. BP < BC …(3)
AB ≅ BC ….(Given)…(4)
From (3) and (4),
BP
∴ BP< congruent sides.
(ii) In ∆ABC, side AB ≅ side BC and A-C-P.
….(Given)
∴ ∠A ≅ ∠BCA ….(Isosceles Triangle Theorem) …(1)



∠BCA > ∠P …(Exterior Angle Theorem)…(2)
From (1) and (2), ∠A > ∠P
∴ BP > BA …(Side opposite to greater angle) ..(3)
side AB ≅ side BC …(Given) …(4)
From (3) and (4),
BP > BA and BP > BC.
∴ BP > congruent sides.

Solution 6:

∠APC > ∠B …(Exterior Angle Theorem) …(1)
∠B ≅ ∠ACB …(Isosceles Triangle Theorem) …(2)
From (1) and (2), ∠APC > ∠ACP.
∴ AC > AP …(Side opposite to greater angle) … (3)
AB = AC ….(Given) …(4)
From (3) and (4), AB > AP
i.e. AP < AB …(5)
∠ACB > ∠AQC …(Exterior Angle Theorem) ..(6)
∴ ∠ABC > ∠AQC …[From (2) and (6)]
i.e. ∠ABQ > ∠AQC
∴ in ∆ABQ, AQ > AB
…(Side opposite to greater angle)
i.e. AB < AQ …(7)
From (5) and (7), AP < AQ.

Solution 7:

Let the length of side PR be x cm.
The sum of the lengths of any two sides of a triangle is greater than the third side.
∴ PQ + QR > PR
∴ 4 + 6 > PR
∴ 10 > PR
∴ 10 > x …(1)
The difference between the lengths of any two sides of a triangle is less than the length of the third side.
∴ QR – PQ < PR
∴ 6 – 4 < PR
∴ 2 < PR i.e. PR > 2
∴ x > 2 …(2)
From (1) and (2), 10 > x > 2
The length of side PR is greater than 2 cm but less than 10 cm.