DPP – 11
CLASS –10th
TOPIC – HCF & LCM

Q.1 Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together? (excluding the one at start)

Q.2 The traffic lights at three different road crossings change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20:00 hrs, when will they again change simultaneously?

Q.3 A merchant has 120 ltrs of and 180 ltrs of two kinds of oil. He wants the sell oil by filling the two kinds of oil in tins of equal volumes. What is the greatest of such a tin.

Q.4 Find the least number of soldiers in a regiment such that they stand in rows of 15, 20, 25 and form a perfect square.

Q.5 Find the least number of square tiles by which the floor of a room of dimensions 16.58 m and 8.32 m can be covered completely.

Q.6 A wine seller had three types of wine. 403 liters of 1st kind, 434 liters of 2nd kind and 465 liters of 3rd kind. Find the least possible number of casks of equal size in which different types of wine can be filled without mixing.

Q.7 The sum of two numbers is 588 and their HCF is 49. How many such pairs of numbers can be formed ?

Q.8 The product of two numbers is 2028 and their H.C.F. is 13. Find the number of such pairs

Q.9 Lenin is preparing dinner plates. He has 12 pieces of chicken and 16 rolls. If he wants to make all the plates identical without any food left over, what is the greatest number of plates Lenin can prepare?

Q.10 The drama club meets in the school auditorium every 2 days, and the choir meets there every 5 days. If the groups are both meeting in the auditorium today, then how many days from now will they next have to share the auditorium ?

DPP – 11

CLASS –10th

TOPIC – HCF & LCM

Sol.1 For example, let the two bells toll after every 3 secs and 4 secs respectively.

Then the first bell tolls after every 3, 6, 9, 12 seconds…

Like this, the second bell tolls after every 4, 8, 12 seconds…

So, if the two bell toll together now, again they will toll together after 12 seconds. This 12 seconds is nothing but the L.C.M of 3 seconds and 4 seconds

The same thing happened in our problem. To find the time, when they will all toll together, we have to find the L.C.M of (2, 4, 8, 6, 10, 12).

L.C.M of (2, 4, 8, 6, 10, 12) is 120 seconds = 2 minutes.

So, after every two minutes, all the bell will toll together.

For example, in 10 minutes, they toll together :

10/2 = 5 times

That is, after 2,4,6,8,10 minutes. It does not include the one at the start

Similarly, in 30 minutes, they toll together :

30/2 = 15 times

(excluding one at the start).

Sol.2 For example, let the two signals change after every 3 secs and 4 secs respectively.

Then the first signal changes after 3, 6, 9, 12 seconds…

Like this, the second signal changes after 4, 8, 12 seconds…

So, if the two signals change simultaneously now, again they will change simultaneously after 12 seconds. This 12 seconds is nothing but the L.C.M of 3 seconds and 4 seconds

The same thing happened in our problem. To find the time, when they will all change simultaneously, we have to find the L.C.M of (48, 72, 108).

L.C.M of (48, 72, 108) is 432 seconds = 7 min 12 sec

So, after every 7 min 12 sec, all the signals will change simultaneously.

At 8:20:00 hrs, if all the three signals change simultaneously, again they will change simultaneously after 7 min 12 sec. That is at 8:27:12 hrs.

Hence, three signals will change simultaneously at 8:27:12 seconds.

Sol.3 The given two quantities 120 and 180 can be divided by 10, 20,… exactly. That is, both the kinds of oils can be sold in tins of equal volume of 10, 20,… ltrs.

But, the target of the question is, the volume of oil filled in tins must be greatest.

So, we have to find the largest number which exactly divides 120 and 180.That is nothing but the H.C.F of (120, 180)

H.C.F of (120, 180) = 60

The 1st kind 120 ltrs is sold in 2 tins of of volume 60 ltrs in each tin.

The 2nd kind 180 ltrs is sold in 3 tins of volume 60 ltrs in each tin.

Hence, the greatest volume of each tin is 60 ltrs.

Sol.4 To answer this question, we have to find the least number which is exactly divisible by the given numbers 15, 20 and 25.That is nothing but the L.C.M of (15, 20, 25)

L.C.M of (15, 20, 25) = 300

So, we need 300 soldiers such that they stand in rows of 15, 20, 25.

But, it has to form a perfect square (as per the question)

To form a perfect square, we have to multiply 300 by some number such that it has to be a perfect square.

To make 300 as perfect square, we have to multiply 300 by 3.

Then, it is 900 which is a perfect square.

Hence, the least number of soldiers required is 900.

Sol.5 We require the least number of square tiles. So, each tile must be of maximum dimension.

To get the maximum dimension of the tile, we have to find the largest number which exactly divides 16.58 and 8.32. That is nothing but the H.C.F of (16.58, 8.32).

To convert meters into centimeters, we have to multiply by 100.

$16.58 \cdot 100 = 1658$ cm

$8.32 \cdot 100 = 832$ cm

H.C.F of (1658, 832) = 2

Hence the side of the square tile is 2 cm

Required no. of tiles :

= (Area of the floor) / (Area of a square tile)

= $(1658 \cdot 832) / (2 \cdot 2)$

= 344,864

Hence, the least number of square tiles required is 344,864.

Sol.6 For the least possible number of casks of equal size, the size of each cask must be of the greatest volume.

To get the greatest volume of each cask, we have to find the largest number which exactly divides 403, 434 and 465. That is nothing but the H.C.F of (403, 434, 465)

The H.C.F of (403, 434, 465) = 31 liters

Each cask must be of the volume 31 liters.

Req. No. of casks is

= (403/31) + (434/31) + (465/31)

= 13 + 14 + 15

= 42

Hence, the least possible number of casks of equal size required is 42.

Sol.7 Because the H.C.F is 49, the two numbers can be assumed as 49x and 49y

Their sum is 588. So, we have

$49x + 49y = 588$

Divide each side 49.

$x + y = 12$

We have to find the values of "x" and "y" such that their sum is 12.

The possibles pairs of values of (x, y) are

(1, 11), (2, 10), (3, 9), (4, 8), (5, 7), (6, 6)

Here, we have to check an important thing. That is, in the above pairs of values of (x, y), which are all co-primes ?

[Co-primes = Two integers are said to be co-primes or relatively prime if they have no common positive factor other than 1 or, equivalently, if their greatest common divisor is 1]

Therefore in the above pairs, (1, 11) and (5, 7) are the co-primes.

Hence, the number of pairs is 2.

Sol.8 Since the H.C.F is 13, the two numbers could be 13x and 13y

Their product is 2028.

So, we have

$(13x) \cdot (13y) = 2028$

$169xy = 2028$

Divide each side by 169.

$xy = 12$

We have to find the values of "x" and "y" such that their product is 12.

The possibles pairs of values of (x, y) are

(1, 12), (2, 6), (3, 4)

Here, we have to check an important thing. That is, in the above pairs of values of (x, y), which are all co-primes?

[Co-primes = Two integers are said to be co-primes or relatively prime if they have no common positive factor other than 1 or, equivalently, if their greatest common divisor is 1]

Therefore in the above pairs, (1, 12) and (3, 4) are the co-primes.

Hence, the number of pairs is 2

Sol.9 To make all the plates identical and find the greatest number of plates, we have to find the greatest number which can divide 12 and 16 exactly.

That is nothing but H.C.F of 12 and 16.

H.C.F of (12, 16) = 4

That is, 12 pieces of chicken would be served in 4 plates at the rate of 3 pieces per plate.

And 16 rolls would be served in 4 plates at the rate of 4 rolls per plate.

In this way, each of the 4 plates would have 3 pieces of chicken and 4 rolls. And all the 4 plates would be identical.

Hence, the greatest number of plates Lenin can prepare is 4

Sol.10 If the drama club meets today, again they will meet after 2, 4, 6, 8, 10, 12…. days.

Like this, if the choir meets today, again they will meet after 5, 10, 15, 20 …. days.

From the explanation above, If both drama club and choir meet in the auditorium today, again, they will meet after 10 days.

And also, 10 is the L.C.M of (2, 5).

Hence, both the groups will share the auditorium after ten days.