Chapter – 5

Motion of System of Particles & Rigid Bodies

Multiple Choice Questions

Question 1.
The center of mass of a system of particles does not depend upon,
(a) position of particles
(b) relative distance between particles
(c) masses of particles
(d) force acting on particle

Answer:
(d) force acting on particle

Question 2.
A couple produces, [AIPMT 1997, AIEEE 2004]
(a) pure rotation
(b) pure translation
(c) rotation and translation
(d) no motion [AIPMT 1997]

Answer:
(a) pure rotation

Question 3.
A particle is moving with a constant velocity along a line parallel to positive X – axis. The magnitude of its angular momentum with respect to the origin is –
(a) zero
(b) increasing with x
(c) decreasing with x
(d) remaining constant [IIT 2002]

Answer:
(d) remaining constant

Question 4.
A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force 30 N?
(a) 0.25 rad s^{-2}
(b) 25 rad s^{-2}
(c) 5 m s^{-2}
(d) 25 m s^{-2}
[NEET 2017]

Answer:
(b) 25 rad s^{-2}

Question 5.
A closed cylindrical container is partially filled with water. As the container rotates in a horizontal plane about a perpendicular bisector, its moment of inertia,
(a) increases
(b) decreases
(c) remains constant
(d) depends on direction of rotation. [IIT 1998]

Answer:
(a) increases

Question 6.
A rigid body rotates with an angular momentum L. If its kinetic energy is halved, the angular momentum becomes,
(a) L
(b) L / 2
(c) 2 L
(d) L / 2 [AFMC 1998, AIPMT 2015]

Answer:
(d) L / 2

Question 7.
A particle undergoes uniform circular motion. The angular momentum of the particle remain conserved about –
(a) the center point of the circle.
(b) the point on the circumference of the circle
(c) any point inside the circle.
(d) any point outside the circle. [IIT 2003]

Answer:
(a) the center point of the circle.

Question 8.
When a mass is rotating in a plane about a fixed point, its angular momentum is directed along –
(a) a line perpendicular to the plane of rotation
(b) the line making an angle of 45° to the plane of rotation
(c) the radius
(d) tangent to the path [AIPMT 2012]

Answer:
(a) a line perpendicular to the plane of rotation

Question 9.
Two discs of same moment of inertia rotating about their regular axis passing through center and perpendicular to the plane of disc with angular velocities ω_1 and ω_1. They are brought in to contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is-
(a) $\frac{1}{4}$ $I(\omega_1 - \omega_2)\omega^2$
(b) $I(\omega_1 - \omega_2)\omega_2$
(c) $\frac{1}{8}$ $I(\omega_1 - \omega_2)\omega^2$
(d) $\frac{1}{2}$ $I(\omega_1 - \omega_2)\omega^2$

Answer:
(a) $\frac{1}{4}$ $I(\omega_1 - \omega_2)\omega^2$

Question 10.
A disc of moment of inertia I_a is rotating in a horizontal plane about its symmetry axis with a constant angular speed to. Another disc initially at rest of moment of inertia I_b is dropped coaxially on to the rotating disc. Then, both the discs rotate with same constant angular speed. The loss of kinetic energy due to friction in this process is-

(a) $\frac{1}{2}\frac{I_b^2}{(I_a+I_b)}\omega^2$

(b) $\frac{I_b^2}{(I_a+I_b)}\omega^2$

(c) $\frac{(I_b-I_a)^2}{(I_a+I_b)}\omega^2$

(d) $\frac{1}{2}\frac{I_b I_b}{(I_a+I_b)}\omega^2$

[AIPMT 2001]

Answer:

(d) $\frac{1}{2}\frac{I_b I_b}{(I_a+I_b)}\omega^2$

Question 11.
The ratio of the acceleration for a solid sphere (mass m and radius R) rolling down an incline of angle 0 without slipping and slipping down the incline without rolling is –
(a) 5 : 7
(b) 2 : 3
(c) 2 : 5
(d) 7 : 5
[AIPMT 2014]

Answer:
(a) 5 : 7

Question 12.
From a disc of radius R a mass M, a circular hole of diameter R, whose rim

passes through the center is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis passing through it?
(a) $15MR^2/32$
(b) $13MR^2/32$
(c) $11MR^2/32$
(d) $9MR^2/32$ [NEET 2016]

Answer:
(b) $13MR^2/32$

Question 13.
The speed of a solid sphere after rolling down from rest without sliding on an inclined plane of vertical height h is,
(a) $\sqrt{\frac{4}{3}gh}$
(b) $\sqrt{\frac{10}{7}gh}$
(c) $\sqrt{2gh}$
(d) $\sqrt{\frac{1}{2}gh}$

Answer:
(a) $\sqrt{\frac{4}{3}gh}$

Question 14.
The speed of the center of a wheel rolling on a horizontal surface is vQ. A point on the rim in level with the center will be moving at a speed of speed of,
(a) zero
(b) v_0
(c) $\sqrt{2}v_0$
(d) $2\ v_0$
[PMT 1992, PMT 2003, IIT 2004]

Answer:
(c) $\sqrt{2}v_0$

Question 15.
A round object of mass m and radius r rolls down without slipping along an inclined plane. The fractional force,
(a) dissipates kinetic energy as heat.
(b) decreases the rotational motion.
(c) decreases the rotational and transnational motion,
(d) converts transnational energy into rotational energy [PMT 2005]

Answer:
(d) converts transnational energy into rotational energy

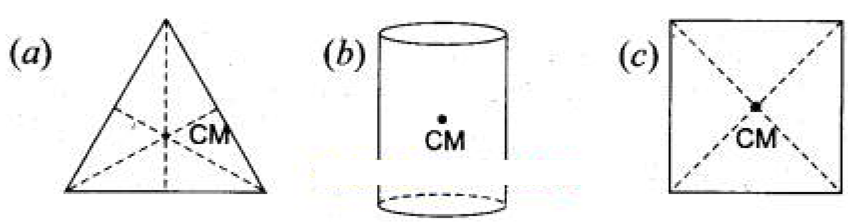
Short Answer Questions

Question 1.
Define center of mass.

Answer:
The center of mass of a body is defined as a point where the entire mass of the body appears to be concentrated.

Question 2.
Find out the center of mass for the given geometrical structures.
(a) Equilateral triangle
(b) Cylinder
(c) Square

Answer:



(a) For equilateral triangle, center of mass lies at its centro-id.
(b) For cylinder, center of mass lies at its geometrical center.
(c) For square, center of mass lies at the point where the diagonals meet.

Question 3.
Define torque and mention its unit.

Answer:
Torque is defined as the moment of the external applied force about a point or axis of rotation. The expression for torque is,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Question 4.
What are the conditions in which force cannot produce torque?

Answer:
The forces intersect (or) passing through the axis of rotation cannot produce torque as the perpendicular distance between the forces is 0 i.e. r = 0.

$$\therefore \vec{\tau} = \vec{r} \times \vec{F} = 0$$

Question 5.
Give any two examples of torque in day – to – day life.

Answer:

- The opening and closing of a door about the hinges.
- Turning of a nut using a wrench.

Question 6.
What is the relation between torque and angular momentum?

Answer:
We have the expression for magnitude of angular momentum of a rigid body as, $L = I\,\omega$. The expression for magnitude of torque on a rigid body is, $\tau = I\,\alpha$. We can further write the expression for torque as,

$$\tau = I \frac{d\omega}{dt} \quad (\because \alpha = \frac{d\omega}{dt})$$

Where, ω is angular velocity and α is angular acceleration. We can also write equation,

$$\tau = \frac{d(I\omega)}{dt}$$
$$\tau = \frac{dL}{dt}$$

Question 7.
What is equilibrium?

Answer:
A rigid body is said to be in mechanical equilibrium where both its linear momentum and angular momentum remain constant.

Question 8.
How do you distinguish between stable and unstable equilibrium?

Answer:

Stable Kquilibrium:

- The body tries to come back to equilibrium if slightly disturbed and released.
- The center of mass of the body shifts slightly higher if disturbed from equilibrium.
- Potential energy of the body is minimum and it increases if disturbed.

Unstable Equilibrium:

- The body cannot come back to equilibrium if slightly disturbed and released.
- The center of mass of the body shifts slightly lower if disturbed from equilibrium.
- Potential energy of the body is not minimum and it decreases if disturbed.

Question 9.
Define couple.

Answer:
A pair of forces which are equal in magnitude but opposite in direction and separated by a perpendicular distance so that their lines of action do not coincide that causes a turning effect is called a couple.

Question 10.
State principle of moments.

Answer:
Principle of moment states that when an object is in equilibrium the sum of the anticlockwise moments about a point is equal to the sum of the clockwise moments.

Question 11.
Define center of gravity.

Answer:
The center of gravity of a body is the point at which the entire weight of the body acts, irrespective of the position and orientation of the body.

Question 12.
Mention any two physical significance of moment of inertia

Answer:
Moment of inertia for point mass,

$I = m_i r_i^2$

Moment of inertia for bulk object,

$I = \sum m_i r_i^2$

Question 13.
What is radius of gyration?

Answer:
The radius of gyration of an object is the perpendicular distance from the axis of rotation to an equivalent point mass, which would have the same mass as well as the same moment of inertia of the object.

Question 14.
State conservation of angular momentum.

Answer:
The law of conservation of angular momentum states that when no external torque acts on the body the net angular momentum of a rotating rigid body remains constant.

Question 15.
What are the rotational equivalents for the physical quantities, (i) mass and (ii) force?

Answer:
The rotational equivalents for (i) mass and (ii) force are moment of inertia and torque respectively.

Question 16.
What is the condition for pure rolling?

Answer:
In pure rolling, there is no relative motion of the point of contact with the surface when the rolling object speeds up or shows down. It must accelerate or decelerate respectively.

Question 17.
What is the difference between sliding and slipping?

Answer:

Sliding:

- Velocity of center of mass is greater than Rω i.e. $V_{CM} > R\omega$.
- Velocity of transnational motion is greater than velocity of rotational motion.
- Resultant velocity acts in the forward direction.

Slipping:

- Velocity of center of mass is lesser than Rω. i.e. $V_{CM} < R\omega$
- Velocity of translation motion is lesser than velocity of rotational motion.
- Resultant velocity acts in the backward direction.

Long Answer Questions

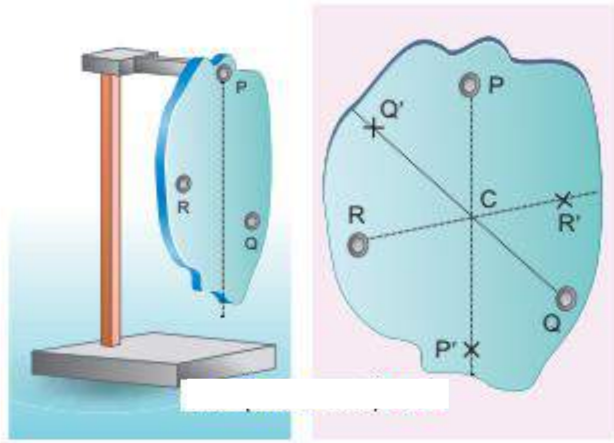
Question 1.
Explain the types of equilibrium with suitable examples.

Answer:

- Transnational motion – A book resting on a table.
- Rotational equilibrium – A body moves in a circular path with constant velocity.
- Static equilibrium – A wall – hanging, hanging on the wall.
- Dynamic equilibrium – A ball decends down in a fluid with its terminal velocity.
- Stable equilibrium – A table on the floor
- Unstable equilibrium – A pencil standing on its tip.
- Neutral equilibrium – A dice rolling on a game board.

Question 2.
Explain the method to find the center of gravity of a irregularly shaped lamina.

Answer:
There is also another way to determine the center of gravity of an irregular lamina. If we suspend the lamina from different points like P, Q, R as shown in figure, the vertical lines I PP', QQ', RR' all pass through the center of gravity. Here, reaction force acting at the point of suspension and the gravitational force acting at the center of gravity cancel each other and the torques caused by them also cancel each other.

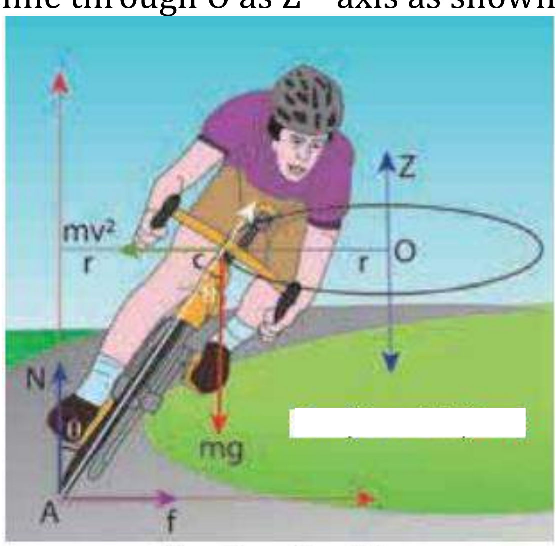


Determination of center of gravity of plane lamina by suspending

Determination of center of gravity of plane lamina by suspending

Question 3.
Explain why a cyclist bends while negotiating a curve road? Arrive at the expression for angle of bending for a given velocity.

Answer:
Let us consider a cyclist negotiating a circular level road (not banked) of radius r with a speed v. The cycle and the cyclist are considered as one system with mass m. The center gravity of the system is C and it goes in a circle of radius r with center at O. Let us choose the line OC as X – axis and the vertical line through O as Z – axis as shown in Figure.

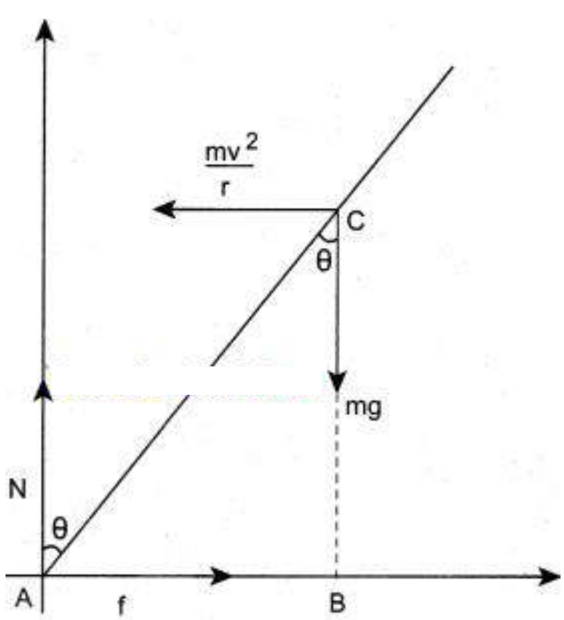


Bending of cyclist

The system as a frame is rotating about Z – axis. The system is at rest in this rotating frame. To solve problems in rotating frame of reference, we have to apply a centrifugal force (pseudo force) on the system which will be $\frac{mv^2}{r}$ This force will act through the center of gravity. The forces acting on the system are,

- gravitational force (mg)
- normal force (N)
- frictional force (f)
- centrifugal force ($\frac{mv^2}{r}$).

As the system is in equilibrium in the rotational frame of reference, the net external force and net external torque must be zero. Let us consider all torques about the point A in Figure.
For rotational equilibrium,
$\tau_{net} = 0$
The torque due to the gravitational force about point A is (mg AB) which causes a clockwise turn that is taken as negative. The torque due to the centripetal force is I BC which causes an ($\frac{mv^2}{r}$ BC) Which causes an anticlockwise turn that is taken as positive.



Force diagrams for the cyclist in turns

$$-mg\ AB + \frac{mv^2}{r} BC = 0$$

$$mg\ AB = \frac{mv^2}{r} BC$$

From ΔABC

$AB = AC \sin\theta$ and $BC = AC \cos\theta$

$$mg\ AC \sin\theta = \frac{mv^2}{r} AC \cos\theta$$

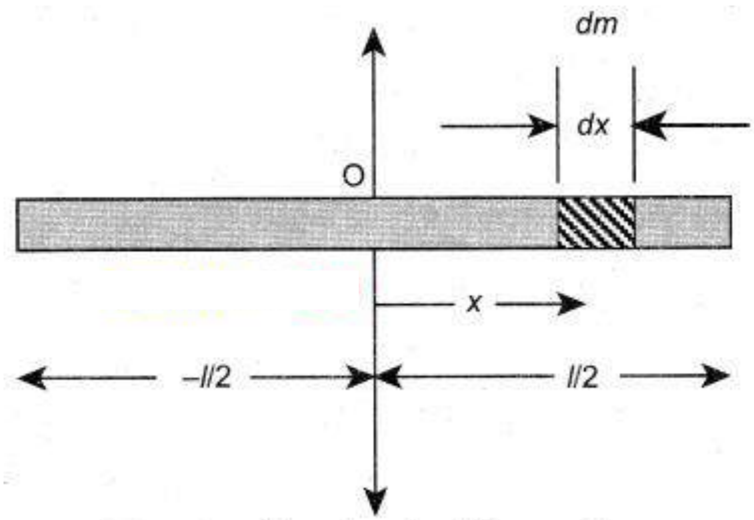
$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

While negotiating a circular level road of radius r at velocity v, a cyclist has to bend by an angle 0 from vertical given by the above expression to stay in equilibrium (i.e. to avoid a fall).

Question 4.
Derive the expression for moment of inertia of a rod about its center and perpendicular to the rod.

Answer:
Let us consider a uniform rod of mass (M) and length (l) as shown in figure. Let us find an expression for moment of inertia of this rod about an axis that passes through the center of mass and perpendicular to the rod. First an origin is to be fixed for the coordinate system so that it coincides with the center of mass, which is also the geometric center of the rod. The rod is now along the x axis. We take an infinitesimally small mass (dm) at a distance (x) from the origin. The moment of inertia (dI) of this mass (dm) about the axis is, dI = (dm) x^2



Moment of inertia of uniform rod

As the mass is uniformly distributed, the mass per unit length (λ) of the rod is, $\lambda = \frac{M}{l}$

The (dm) mass of the infinitesimally small length as, $dm = \lambda\, dx = \frac{M}{l}\, dx$

The moment of inertia (I) of the entire rod can be found by integrating dI,

$$I = \int dI = \int (dm)x^2 = \int \left(\frac{M}{l} dx\right) x^2$$

$$I = \frac{M}{l} \int x^2 dx$$

As the mass is distributed on either side of the origin, the limits for integration are taken from to – l/2 to l/2.

$$I = \frac{M}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$I = \frac{M}{l}\left[\frac{l^3}{24} - \left(-\frac{l^3}{24}\right)\right] = \frac{M}{l}\left[\frac{l^3}{24} + \frac{l^3}{24}\right]$$

$$I = \frac{M}{l}\left[2\left(\frac{l^3}{24}\right)\right]$$

$$I = \frac{1}{12} ml^2$$

Question 5.
Derive the expression for moment of inertia of a uniform ring about an axis passing through the center and perpendicular to the plane.

Answer:
Let us consider a uniform ring of mass M and radius R. To find the moment of inertia of the ring about an axis passing through its center and perpendicular to the plane, let us take an infinitesimally small mass (dm) of length (dx) of the ring. This (dm) is located at a distance R, which is the radius of the ring from the axis as shown in figure.

The moment of inertia (dl) of this small mass (dm) is,
$dI = (dm)R^2$

The length of the ring is its circumference ($2\pi R$). As the mass is uniformly distributed, the mass per unit length (λ) is,

$$\lambda = \frac{mass}{lengh} = \frac{M}{2\pi R}$$

The mass (dm) of the infinitesimally small length is,

$$dm = \lambda\ dx = \frac{M}{2\pi R}\ dx$$

Now, the moment of inertia (I) of the entire ring is,

$$I = \int dI = \int (dm)R^2 = \int \left(\frac{M}{2\pi R} dx \right) R^2$$

$$I = \frac{MR}{2\pi} \int dx$$

To cover the entire length of the ring, the limits of integration are taken from 0 to 2πR.

$$I = \frac{MR}{2\pi} \int_0^{2\pi R} dx$$

$$I = \frac{MR}{2\pi} [x]_0^{2\pi R} = \frac{MR}{2\pi} [2\pi R - 0]$$

$$I = MR^2$$

Question 6.
Derive the expression for moment of inertia of a uniform disc about an axis passing through the center and perpendicular to the plane.

Answer:
Consider a disc of mass M and radius R. This disc is made up of many infinitesimally small rings as shown in figure. Consider one such ring of mass (dm) and thickness (dr) and radius (r). The moment of inertia (dl) of this small ring is,
$dI = (dm)R^2$

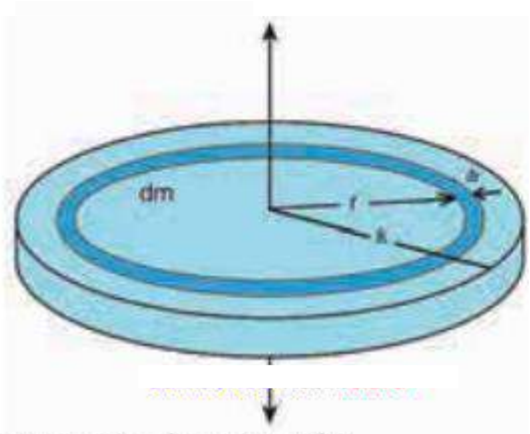
As the mass is uniformly distributed, the mass per unit area (σ) is $\sigma = \frac{mass}{area} = \frac{M}{\pi R^2}$

The mass of the infinitesimally small ring is,

$$dm = \sigma\, 2\pi r\, dr = \frac{M}{\pi R^2}\, 2\pi r\, dr$$

where, the term (2πr dr) is the area of this elemental ring (2πr is the length and dr is the thickness), $dm = \frac{2M}{R^2} r\, dr$

$$dI = \frac{2M}{R^2} r^3 \, dr$$



Moment of inertia of a uniform disc

$$I = \int dI$$

$$I = \int_0^R \frac{2M}{R^2} r^3 dr = \frac{2M}{R^2} \int_0^R r^3 dr$$

$$I = \frac{2M}{R^2} \left[\frac{r^4}{4} \right]_0^R = \frac{2M}{R^2} \left[\frac{R^4}{4} - 0 \right]$$

$$I = \frac{1}{2} MR^2$$

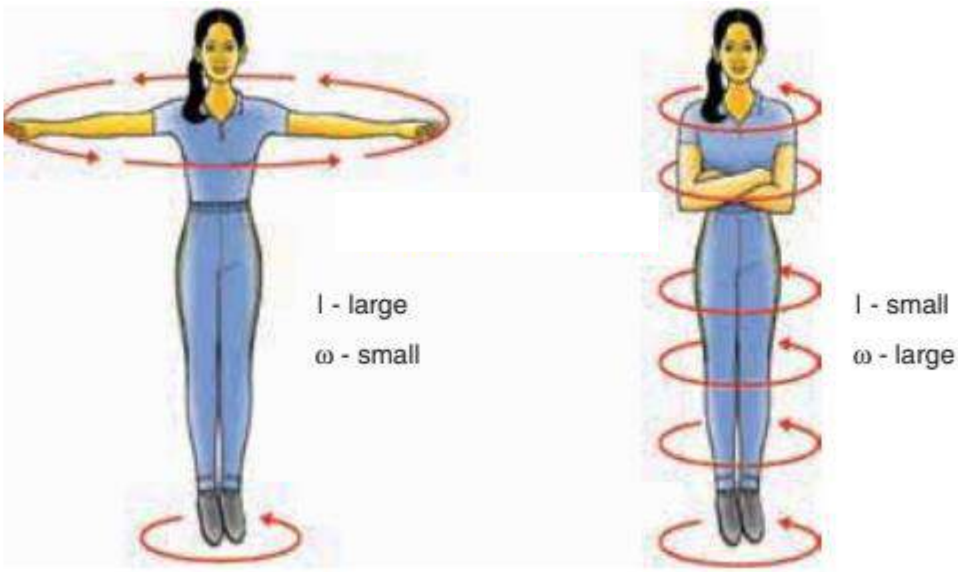
Question 7.
Discuss conservation of angular momentum with example.

Answer:
When no external torque acts on the body, the net angular momentum of a rotating rigid body remains constant. This is known as law of conservation of angular momentum.

$$\tau = \frac{dL}{dt}$$

If $\tau = 0$ then, L = constant.
As the angular momentum is $L = I\omega$, the conservation of angular momentum could further be written for initial and final situations as,

$I_i\omega_i = I_i\omega_i$ (or) $I\omega$ = constant
The above equations say that if I increases ω will decrease and vice – versa to keep the angular momentum constant.



Conservation of angular momentum for ice dancer

There are several situations where the principle of conservation of angular momentum is applicable. One striking example is an ice dancer as shown in Figure A. The dancer spins slowly when the hands are stretched out and spins faster when the hands are brought close to the body.

Stretching of hands away from body increases moment of inertia, thus the angular velocity decreases resulting in slower spin. When the hands are brought close to the body, the moment of inertia decreases, and thus the angular velocity increases resulting in faster spin. A diver while in air as in Figure B curls the body close to decrease the moment of inertia, which in turn helps to increase the number of somersaults in air.

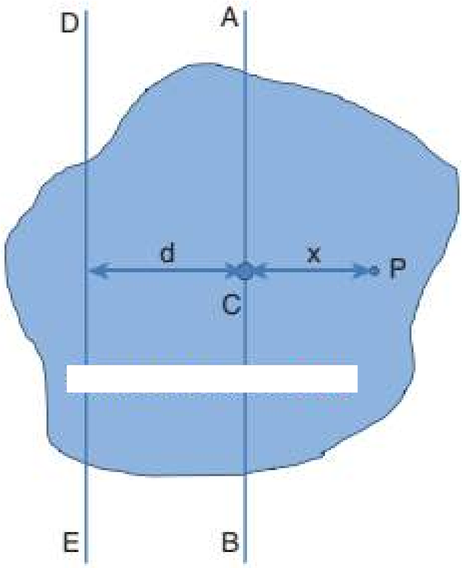
Conservation of angular momentum for a diver

Question 8.
State and prove parallel axis theorem.

Answer:

Parallel axis theorem:
Parallel axis theorem states that the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two axes.

If I_C is the moment of inertia of the body of mass M about an axis passing through the center of mass, then the moment of inertia I about a parallel axis at a distance d from it is – given by the relation,
$I = I_C + M d^2$
Let us consider a rigid body as shown in figure. Its moment of inertia about an axis AB passing through the center of mass is I_C. DE is another axis parallel to AB at a perpendicular distance d from AB. The moment of inertia of the body about DE is I. We attempt to get an expression for I in terms of I_C. For this, let us consider a point mass m on the body at position x from its center of mass.



Parallel axis theorem

The moment of inertia of the point mass about the axis DE is, $m\,(x + d)^2$. The moment of inertia I of the whole body about DE is the summation of the above expression.

$I = \sum m\,(x + d)^2$
This equation could further be written as,
$I = \sum m(x^2 + d^2 + 2xd)$
$1 = \sum (mx^2 + md^2 + 2\,dmx)$
$l = \sum mx^2 + md^2 + 2d \sum mx$
Here, $\sum mx^2$ is the moment of inertia of the body about the center of mass.

Hence, $I_C = \sum mx^2$
The term, $\sum mx = 0$ because, x can take positive and negative values with respect to the axis AB. The summation $(\sum mx)$ will be zero.
Thus, $I = I_C + \sum m\,d^2 = I_C + (\sum m)\,d^2$
Here, $\sum m$ is the entire mass M of the object $(\sum m = M)$.
$I = I_C + Md^2$

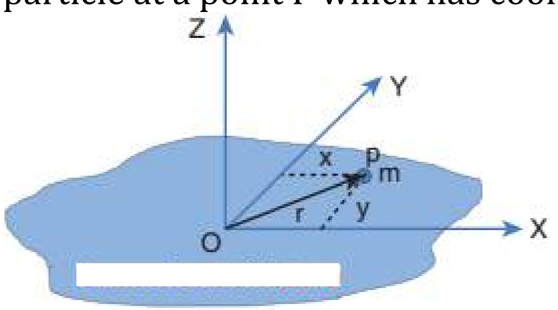
Question 9.
State and prove perpendicular axis theorem.

Answer:
Perpendicular axis theorem:
This perpendicular axis theorem holds good only for plane laminar objects.

The theorem states that the moment of inertia of a plane laminar body about an axis perpendicular to its plane is equal to the sum of moments of inertia about two perpendicular axes lying in the plane of the body such that all the three axes are mutually perpendicular and have a common point.

Let the X and Y – axes lie in the plane and Z – axis perpendicular to the plane of the laminar object. If the moments of inertia of the body about X and Y-axes are I_X and I_Y respectively – and I_Z is the moment of inertia about Z-axis, then the perpendicular axis theorem could be expressed as,
$I_Z = I_X + I_Y$

To prove this theorem, let us consider a plane laminar object of negligible thickness on which lies the origin (O). The X and Y – axes lie on the plane and Z – axis is perpendicular to it as shown in figure. The lamina is considered to be made up of a large number of particles of mass m. Let us choose one such particle at a point P which has coordinates (x, y) at a distance r from O.



Perpendicular axis theorem

The moment of inertia of the particle about Z – axis is, mr^2.
The summation of the above expression gives the moment of inertia of the entire lamina about Z – axis as, $I_Z = \sum mr^2$
Here, $r^2 = x^2 + y^2$
Then, $I_Z = \sum m\,(x^2 + y^2)$
$I_Z = \sum m\,x^2 + \sum m\,y^2$

In the above expression, the term $\sum m\,x^2$ is the moment of inertia of the body about the Y-axis and similarly the term $\sum m\,y^2$is the moment of inertia about X- axis. Thus,
$I_X = \sum m\,y^2$ and $I_Y = \sum m\,x^2$

Substituting in the equation for Iz gives,

$I_Z = I_X + I_Y$

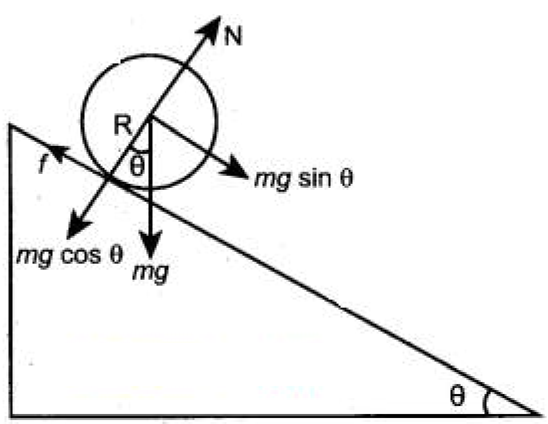
Thus, the perpendicular axis theorem is proved.

Question 10.

Discuss rolling on inclined plane and arrive at the expression for the acceleration.

Answer:

Let us assume a round object of mass m and radius R is rolling down an inclined plane without slipping as shown in figure. There are two forces acting on the object along the inclined plane. One is the component of gravitational force (mg sin θ) and the other is the static frictional force (f). The other component of gravitation force (mg cos θ) is cancelled by the normal force (N) exerted by the plane. As the motion is happening along the incline, we shall write the equation for motion from the free body diagram (FBP) of the object.



Rolling on inclined plane

For transnational motion, mg sin θ is the supporting force and f is the opposing force, mg sin θ f = ma

For rotational motion, let us take the torque with respect to the center of the object. Then mg sin 0 cannot cause torque as it passes through it but the frictional force f can set torque of Rf = Iα

By using the relation, a = rα, and moment of inertia $I = mK^2$ we get,

$Rf = mK^2 \frac{a}{R}$; $f = ma\left(\frac{K^2}{R^2}\right)$

Now equation becomes,

$$mg \sin\theta - ma\left(\frac{K^2}{R^2}\right) = ma$$

$$mg \sin\theta = ma + ma\left(\frac{K^2}{R^2}\right)$$

$$a\left(1 + \frac{K^2}{R^2}\right) = g \sin\theta$$

After rewriting it for acceleration, we get,

$$a = \frac{g \sin\theta}{\left(1 + \frac{K^2}{R^2}\right)}$$

We can also find the expression for final velocity of the rolling object by using third equation of motion for the inclined plane.
$v^2 = u^2 + 2as$. If the body starts rolling from rest, $u = 0$. When h is the vertical height of the incline, the length of the incline s is, $s = \frac{h}{sin\theta}$

$$v^2 = 2\frac{g \sin\theta}{\left(1+\frac{K^2}{R^2}\right)}\left(\frac{h}{\sin\theta}\right) = \frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}$$ By taking square root,

$$v = \sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}$$

The time taken for rolling down the incline could also be written from first equation of motion as, $v = u + at$. For the object which starts rolling from rest, $u = 0$. Then,

$$t = \frac{v}{a}$$

$$t=\left(\sqrt{\frac{2gh}{\left(1+\frac{K^2}{R^2}\right)}}\right)\left(\frac{\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}\right)$$

$$t=\sqrt{\frac{2h\left(1+\frac{K^2}{R^2}\right)}{g\sin^2\theta}}$$

The equation suggests that for a given incline, the object with the least value of radius of gyration K will reach the bottom of the incline first.

Numerical Problems

Question 1.
A uniform disc of mass 100 g has a diameter of 10 cm. Calculate the total energy of the disc when rolling along a horizontal table with a velocity of 20 cm s^{-2}.

Answer:
Given,
Mass of the disc = 100 g = 100 x 10^{-3} kg = 1/10kg
Velocity of disc = 20 cm s^{-1} = 20 x 10^{-2} ms^{-1} = 0.2 ms^{-1}

$$r = 5\text{ cm} = 5\times10^{-2}\text{ m},\quad \omega = \frac{v}{r} = \frac{20\times10^{-2}}{5\times10^{-2}} = 4$$

$$\text{Energy} = \frac{1}{2}mV^2+\frac{1}{2}I\omega^2 = \frac{1}{2}(mV^2+I\omega^2)\text{, where } I=\frac{1}{2}mr^2$$

$$= \frac{1}{2}\left[\frac{1}{10}\times0.2\times0.2+\frac{1}{2}\times\frac{1}{10}\times25\times\frac{1}{10^4}\times16\right]$$

$$= \frac{1}{2}\left[\frac{4}{1000}+\frac{2}{1000}\right]=\frac{1}{2}\left[\frac{6}{1000}\right]$$

$$\text{Energy} = 3\times10^{-3}\text{ J}$$

Question 2.
A particle of mass 5 units is moving with a uniform speed of v = $3\sqrt{2}$ units in the XOY plane along the line y = x + 4. Find the magnitude of angular momentum.

Answer:
Given,
Mass = 5 units
Speed = v = $3\sqrt{2}$ units
Y = X + 4

Angular momentum = L = $m(\bar{r} \times \bar{v})$

$= m(x\hat{i} + y\hat{j}) x (v\hat{i} + v\hat{j}) = m[xv\hat{k} - vy\hat{k}] = m[xv\hat{k} - v(x + 4)\hat{k}]$

$L = -mv\hat{k} = -4 \times 5 \times 3\sqrt{2}\hat{k} = -60\sqrt{2}\hat{k}$

$L = 60\sqrt{2}$ units.

Question 3.
A fly wheel rotates with a uniform angular acceleration. If its angular velocity increases from 20π rad/s to 40π rad/s in 10 seconds, find the number of rotations in that period.

Answer:
Given,
Initial angular velocity $\omega_0 = 20\pi$ rad/s
Final angular velocity $\omega = 40\pi$ rad/s
Time t = 10 s

Solution:

Angular acceleration $\alpha = \frac{\omega - \omega_0}{t} = \frac{40\pi - 20\pi}{10}$

$\alpha = 2\pi$ rad/s^2
According to equation of motion for rotational motion

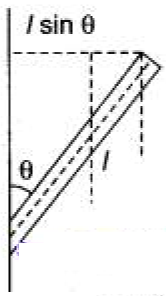
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 20\pi \times 10 + \frac{1}{2} 2\pi \times 100 = 300\pi \text{ rad}$$

The number of rotations = n = $\frac{\theta}{2\pi}$

n = $\frac{300\pi}{2\pi}$ = 150 rotations.

Question 4.
A uniform rod of mass m and length / makes a constant angle 0 with an axis of rotation which passes through one end of the rod. Find the moment of inertia about this gravity is.

Answer:
Moment of inertia of the rod about the axis which is passing through its center of gravity is



$$= I_o = \frac{Ml^2}{12} = Ml^2 \sin^2 \theta/12$$

Moment of inertia of a uniform rod of mass m and length l about one axis which passes through one end of the rod

$$= I_o + M\frac{l^2 \sin^2 \theta}{4}$$

$$= \frac{Ml^2 \sin^2 \theta}{12} + \frac{Ml^2 \sin^2 \theta}{4}$$

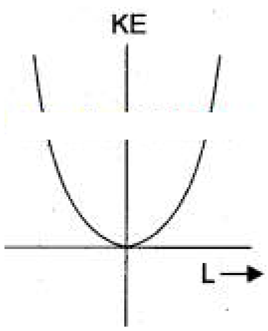
$$= \frac{1}{3} Ml^2 \sin^2 \theta$$

Question 5.
Two particles P and Q of mass 1 kg and 3 kg respectively start moving towards each other from rest under mutual attraction. What is the velocity of their center of mass?

Answer:
Given,

Mass of particle P = 1 kg Mass of particle Q = 3 kg

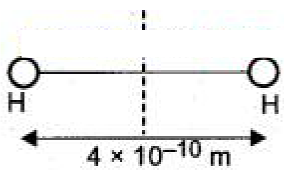
Solution:
Particles P and Q forms a system. Here no external force is acting on the system,



We know that M = d/dt (V_{CM}) = f
It means that, C.M. of an isolated system remains at rest when no external force is acting and internal forces do not change its center of mass.

Question 6.
Find the moment of inertia of a hydrogen molecule about an axis passing through its center of mass and perpendicular to the inter-atomic axis. Given: mass of hydrogen atom 1.7 x 10^{27}kg and inter atomic distance is equal to 4 x 10^{-10}m.

Answer:
Given,
Inter-atomic distance : 4 x 10^{-10} m
Mass of H_2 atom : 1.7 x 10^{-27} kg
Moment of inertia of H_2 =



$$= 2M\left(\frac{R}{2}\right)^2 = \frac{1}{2}MR^2$$

$$= \frac{1}{2}\times 1.7\times 10^{-27}\times 16\times 10^{-20}$$

$$= 1.86 \times 10^{-46}\ \text{kg m}^2$$