CHAPTER 15

Wave Particle Duality and Atomic Physics

LEVEL 1

Q. 1: A beam of red light (λ_r = 800 nm) is made up of a stream of photons. In the diagram shown the size of dots represent the photon energy and the spacing represents the spatial distribution of photons.

Copy the diagram in your notebook; and just below it draw a similar diagram representing a beam of blue light (λ_b = 400 nm) having same intensity.

Q. 2: The brightness of an incandescent light bulb is controlled by a regulator. The regulator is a variable resistor connected in series with the bulb. What happens to the colour of the light given off by the bulb as the resistance of the regulator is increased? What happens to the average energy of photons given out by the bulb? Does it increase?

Q. 3: Find the change in energy of a photon of blue light (λ = 400 Å) when the light enters into a medium of refractive index $\mu = \frac{4}{3}$.

Q. 4: A 0.6×10^{-3} W laser is aimed at the Moon. The wavelength of emitted light is 600 nm and the laser beam spreads out of the source at a divergence angle of $\theta = 0.5 \times 10^{-3}$ rad. The Earth-Moon distance is nearly 4×10^8 m. Calculate the maximum number of photons arriving per second per square meter on the Moon? Neglect any absorption by the atmosphere.

Q. 5: A laser beam of wavelength $\lambda = 5 \times 10^{-7}$ m strikes normally a blackened plate and produces a force of 10^{-5} N. Mass of the plate is 10 g and its specific heat capacity is 400 J kg^{-1} K^{-1}. At what rate will the temperature of the plate rise? Assume no heat loss to the surrounding.

Q. 6: When photons of wavelength λ_1 = 2920 Å strike the surface of metal A, the ejected photoelectron have maximum kinetic energy of k_1 eV and the smallest de-Broglie wavelength of λ. When photons of wavelength λ_2 = 2640 Å strike the surface of metal B the ejected photoelectrons have kinetic energy ranging from zero to $k_2 = (k_1 - 1.5)$ eV. The smallest de-Broglie wavelength of electrons emitted from metal B is 2λ. Find

(a) Work functions of metal A and B.

(b) k_1

Take hc = 12410 eV Å

Q. 7: Work function of metal X is equal to 3.5 eV and work function of material Y is equal to ionization energy of He^+ ion in its first excited state. Light of same wavelength is incident on both X and Y. The maximum kinetic energy of photoelectrons emitted from X is twice that of photoelectrons emitted from Y. Find the wavelength of incident light. hc = 12400 eV Å.

Q. 8: A particle of mass m is allowed to fall freely under gravity on an elastic horizontal surface. The quantum effect become important if the smallest de-Broglie wavelength of the particle is of the same order as the height from which it was dropped. Write the mechanical energy of the particle if quantum effects become important.

Q. 9: A microscope can make you "see" something as small as the wavelength of wave used to make the observation. Calculate the minimum energy of an electron needed in an electron microscope to see a hydrogen atom.

Q. 10: Electrons originally at rest are accelerated through a potential difference of V volts. The applied potential difference is measured using a voltmeter having a least count of ΔV. The electrons in the beam have a de-Broglie wavelength of $\lambda \pm \Delta\lambda$ [$\Delta\lambda << \lambda$]. Find $\left|\frac{\Delta\lambda}{\lambda}\right|$.

Q. 11: Calculate the ratio of de-Broglie wavelength of molecules of hydrogen and oxygen kept in two separate jars at 27°C and 127°C respectively.

Q. 12: Calculate de-Broglie wavelength of hydrogen molecules (H_2) present in a container at 400 K. Assume that molecules travel at rms speed.

Boltzmann constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Mass of H_2 molecule = 2.0 amu

Q. 13: In a nuclear physics experiment, electrons are to be accelerated so as to have de-Broglie wavelength of the order of the diameter of a heavy nucleus ($\sim 10^{-14}$ m). Determine the momentum of the electron needed to make the wavelength of electrons equal to diameter. Use the classical formula for momentum, $p = m_0 v$ to determine the speed of electrons. Is this speed permissible? Now use the relativistic mass of the electrons in the expression of momentum to find the speed of electrons. Relativistic mass is given by $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$.
Where m_0 is rest mass and v is speed of the particle.

Q. 14: A parallel beam of monochromatic light of frequency v is incident on a surface. Intensity of the beam is I and area of the surface is A. Find the force exerted by the light beam on the surface for following cases–

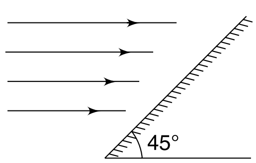
(i) the surface is perfectly absorbing and the light beam is incident normally on it.

(ii) the surface is perfectly reflecting and the light beam is incident normally.

(iii) the surface is perfectly absorbing and the light beam is incident at an angle of incidence θ.

(iv) the surface is perfectly reflecting and the light beam is incident at an angle of incidence θ.

Q. 15: A horizontal beam of light in incident on a plane mirror inclined at 45° to the horizontal. The percentage of light energy reflected from the mirror is 80%. Find the direction in which the mirror will experience force due to the incident light.



Q. 16: Intensity of sunlight on the surface of the earth is $I = 1400$ W/m^2 (neglecting atmospheric absorption).

(a) Find the Wattage of the Sun.

(b) Assuming that light emitted from the sun is monochromatic having wavelength $\lambda = 6000$ Å, estimate the number of photons emitted from the sun in one second.

(c) According to mass energy equivalence principle, estimate the decrease in mass of the sun in one second.

Given: $h = 6.64 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s

Q. 17: A perfectly absorbing solid sphere with a known fixed density, hovers stationary above the sun. This is because the gravitational attraction of the sun is balanced by the pressure due to the sun's light. Assume the sun is far enough away so that it closely approximates a point source of light. Find the radius of the sphere and prove that it is independent of the distance of the sphere from the Sun.

Q. 18: Prove that Bohr's condition for quantization of angular momentum in hydrogen atom can be obtained by requiring an integer number of standing waves around an electron orbit. Use de-Broglie wavelength as the wavelength of wave associated with electron.

Q. 19: The circumference of circular orbit of electron in a He^+ ion is five times the de-Broglie wavelength associated with the electron. Find the radius of the orbit.

Q. 20: A gas of hydrogen like atoms can absorb radiations having photons of energy 68 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal or smaller then the absorbed photon.

(a) Find the initial state of gas atoms.

(b) Identify the gas.

(c) Find the minimum wavelength of emitted radiations.

(d) Find the ionisation energy of the atom.

Q. 21: A particle of mass m moves in a circular orbit of radius r. It is under the influence of a force field where its potential energy is given as $U = ar^2$ where a is a positive constant. Assume that Bohr's model of quantization of angular momentum is applicable and calculate the radius of n^{th} allowed orbit.

Q. 22: Hydrogen like atoms (atomic number = Z) in a sample are in excited state with principal quantum number n. The emission spectrum of the sample has 15 different lines. The second most energetic photon emitted by the sample has energy of 27.2 eV. Find Z.

Q. 23: A μ – meson (charge = $-e$, mass = 208 m_e) moves in a circular orbit around a heavy nucleus having charge $+3e$. Find the quantum state n for which the radius of the orbit is same as that of first Bohr orbit for hydrogen atom.

Q. 24: A hydrogen like atom (atomic number = Z) is in higher excited state of quantum number n. This excited atom can make a transition to first excited state by successively emitting two photons of energy 22.94 eV and 5.15 eV. The atom from the same state n can make transition to second excited state by successively emitting two photons of energies 2.4 eV and 8.7 eV. Find values of n and Z.

Q. 25: A system of positron ($_{+1}e^0$) and electron ($_{-1}e^0$) is called Positronium atom. The radii of Positronium atom is expanded but energy levels are reduced by a factor of x compared to H – atom.

(a) Find x

(b) Is it true to say that the entire spectrum of Positronium will be shifted towards longer wavelength compared to H – atom.

Q. 26: A negatively charged muon (mass 207 times the mass of an electron) is captured in a Bohr's orbit of high principal quantum number (n). The atom thus formed is called mesic atom. The muon in high energy state cascades down to lower orbits by emitting photons.

(a) The emitted photons are X rays. Why?

(b) Find the atomic number (Z) of a mu-mesic atom in which the orbit with $n = 1$ will just touch the nuclear surface. Assume that the nucleus has equal number of protons and neutrons.

Q. 27: Consider a gas of hydrogen atom filling a cubical box of side length 1 m. Assume that all hydrogen atoms are in their ninth excited state and they fill up the space like footballs filling up a room. Estimate the number of hydrogen atoms in the room.

Q. 28: Hydrogen atoms in ground state are bombarded by electrons accelerated through a potential difference of V volts. For what range of values of V, will we get only one spectral line in the spectrum of hydrogen atoms excited by impacts of electrons? Take ground state energy of hydrogen atom to be – 13.6 eV.

Q. 29: A glass tube has Be^{+++} ions excited to their second excited state. The radiation coming out of this tube is incident on a metal plate having work function $\phi = 5$ eV. Find the minimum de-Broglie wavelength of emitted photoelectrons.

Q. 30: An electron in an excited hydrogen atom makes transition from a state of quantum number n to a state of quantum number (n – 1).

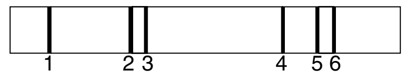
(a) Show that the frequency of emitted radiation is intermediate between the frequencies of orbital revolution in initial and final states.

(b) What is the relationship between frequency of emitted photon and the orbital frequency of the electron when n is large?

Q. 31: Hydrogen atoms in very high quantum state have been observed in radio astronomy. The wavelength of radiation emitted when hydrogen atom makes transition from $n = 110$ state to $n = 109$ state is $\lambda = 5.8$ cm. Imagine a He atom in which one of the electrons has been excited to $n = 110$ state. What will be wavelength of emitted radiation if the electron makes transition to $n = 109$?

Q. 32: A moving hydrogen atom absorbs a photon and comes to rest. What is the maximum possible wavelength of photon? [$hc = 12420$ eV Å]

Q. 33: A tube contains a sample of hydrogen atoms which are all in their third excited state. The atoms de-excite and a spectrum of the radiation emitted is obtained. The spectrum is shown in the given figure.



(a) Which of the lines (1, 2, 3, 4, 5 or 6) represent a transition from quantum state $n = 3$ to $n = 2$.

(b) Which of the lines represent the one with second smallest wavelength?

Q. 34 The biding energy of an electron in the ground state of He atom is 25 eV. Find the energy required to remove both the electrons.

Q. 35: Which of the followings can excite a hydrogen atom in ground state?

(i) A photon having 11 eV energy

(ii) A neutron having 11 eV kinetic energy

(iii) An electron having 11 eV kinetic energy

Give reasons for your answer.

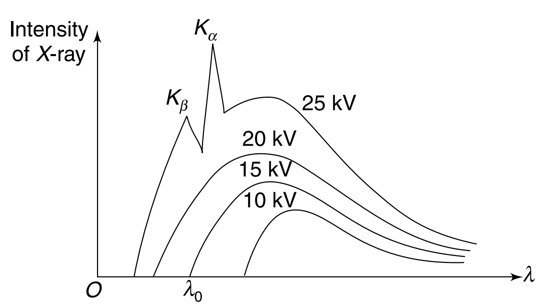
Q. 36: Absorption spectrum is obtained for a sample of gas having atomic hydrogen in ground state. Which lines of the spectrum will be in the range 94 nm to 122 nm.

Take Rydberg's constant $R = 1.1 \times 10^7$ m^{-1}

Q. 37: Radiation coming out of a sample of hydrogen gas excited to first excited state is used for illuminating certain metal plate. When the radiation from some unknown hydrogen like gas excited to the same level is used to expose the same plate, it is found that the de–Broglie wavelength of the fastest photoelectron has decreased 2.3 times. It is given that the energy corresponding to the longest wavelength of the Lyman series of the unknown gas is 3 times the ionization energy of hydrogen (13.6 eV). Find the work function (W) of photoelectric plate in eV. (Take $(2.3)^2 = 5.25$.

Q. 38: An X ray tube with Copper target is found to emit characteristic X rays other than only due to Copper. The k_α lie of Copper has a wavelength of 1.5405 Å. The other K_α line observed is having a wavelength of 1.6578 Å. Identify the impurity.

Q. 39: X-ray spectrum obtained from a metallic target has been shown in the figure. The spectrum has been obtained for four different accelerating voltages in the X-ray tube: 25 kV, 20 kV, 15 kV and 10 kV.



(a) Find the value of λ_0 shown in the graph.

(b) What can you say about the energy of a K-shell electron in the target atom?

Q. 40: When the voltage applied to an X-ray tube increases from 10 to 20 kV the wavelength difference between the k_α line and the short wave cut-off of continuous X-ray spectrum increases by a factor of 3.0. Identify the target element.

Take $\frac{4}{3R} \simeq 1200$ Å where R is Rydberg's contant and $hc = 12.4$ keV Å

Q. 41: An X-ray tube has nickel as target. The wavelength difference between the k_α X-ray and the cut-off wavelength of continuous X-ray spectrum is 84 pm. Find the accelerating voltage applied in the X-ray tube.

[Take $hc = 12400$ eV Å]

Q. 42: How many elements have k_α lines between 241 pm and 180 pm?

Take Rydberg's constant $R = 1.09 \times 10^7$ m^{-1}

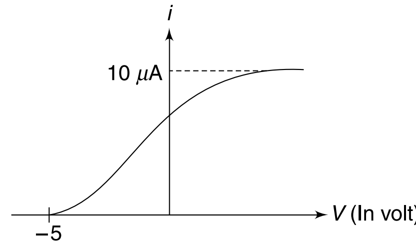
Q. 43: In an electron capture process, a nucleus captures an electron from K shell of the atom. The electron is having a binding energy of B_0. Followed by this capture of electron, several photons are emitted due to electronic transitions. What will be Sum of energy of all such photons emitted?

LEVEL 2

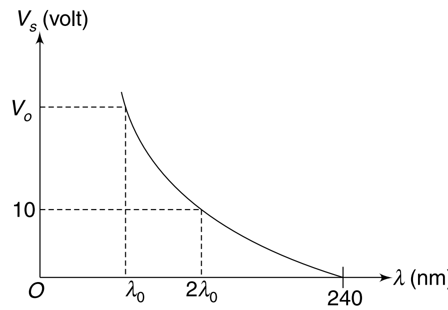
Q. 44: A normal human eye can detect yellow light if more than 10 photons enter into it per second. A star is generating as much power as the Sun and is emitting predominantly yellow light ($\lambda = 6000$ Å). How far is the star if our eye is barely able to see it? It is given that intensity of solar light on surface of the earth is $I = 1400$ Wm^{-2} and the distance of the Sun from the Earth is $r = 1.5 \times 10^{11}$ m. The diameter of pupil of our eye is $d = 6$ mm.

Q. 45: A microwave oven operates at 2.5 GHz. Assume that 5% of microwave photons are absorbed by 200 ml of water kept inside the oven. The time needed to warm the water from 20°C to 70°C is 2 minute. Specific heat capacity of water is 4.2 Jg^{-1} °C^{-1}. Neglect any heat loss by water to the surrounding. Calculate the number of photons emitted per second per kilowatt of power consumed by the microwave. Efficiency of the oven to convent electrical energy into microwave energy is 70%.

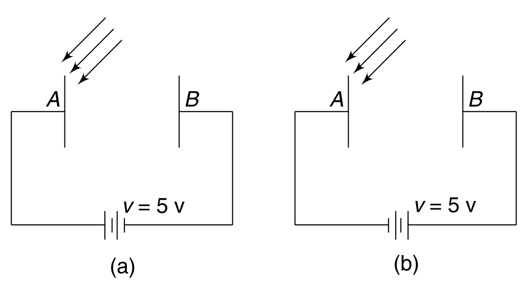
Q. 46: In a photoelectric experiment a metal plate of work function $\phi = 2.0$ eV is irradiated with beam of monochromatic light. The photoelectric current (i) versus the applied potential difference (V) graph is as shown in the figure. It is known that the efficiency of photoemission is 10^{-3}%. Calculate the power of light incident on the metal plate.



Q. 47: In a photoelectric experiment light of different wavelengths are used on a metal surface. For each wavelength the stopping potential difference is recorded. The given graph shows the variation of stopping potential difference (V_s) versus the wavelength (λ) of light used. Find the value of V_0 shown in the graph. Given $h = 4 \times 10^{-15}$ eV s and $c = 3 \times 10^8$ ms^{-1}.



Q. 48: In a photoelectric experiment, a monochromatic light is incident on the metal plate A. It was observed that with $V = 5$ volt, the maximum kinetic energy of photoelectrons striking plate B was 1 eV. The polarity of the applied potential difference was as shown in figure (a). With polarity of the applied potential difference reversed (as shown in figure (b)) and frequency of incident light doubled, it was observed that in saturation state, the kinetic energy of electrons striking plate B ranged between 5 eV to 20 eV. Find the work function of metal used in plate A.



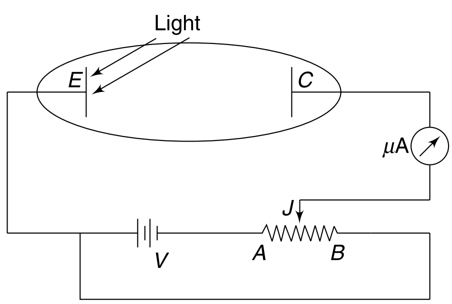
Q. 49: In a photoelectric experimental set-up ultraviolet light of wavelength 350 nm is incident on emitter plate (E). The work function of the emitter plate is $\phi = 2.2$ eV.

AB is a uniform wire resistor having length $L = 100$ cm. Resistance of the wire AB is 10 Ω and the emf of the battery is $V = 10$ volt. The sliding contact J can be moved along the wire AB. When the sliding contact is placed at end B of the wire resistor the micro-ammeter shows a reading of $i = 6\ \mu A$. For answering the following questions assume that photoelectric current is very small compared to the current through the cell.

(a) Find the reading of the ammeter when the slider is moved to end A of the wire.

(b) The slider is moved gradually away from A. Plot the variation of ammeter reading versus distance of the slider measured from A.

(c) Find the maximum speed with which an electron will hit the collector plate (C) if slider J is placed at a distance of 95 cm from A.



Q. 50: A point source is emitting 0.2 W of ultraviolet radiation at a wavelength of $\lambda = 2537$ Å. This source is placed at a distance of 1.0 m from the cathode of a photoelectric cell. The cathode is made of potassium (Work function = 2.22 eV) and has a surface area of 4 cm^2.

(a) According to classical theory, what time of exposure to the radiation shall be required for a potassium atom to accumulate sufficient energy to eject a photoelectron. Assume that radius of each potassium atom is 2 Å and it absorbs all energy incident on it.

(b) Photon flux is defined as number of light photons reaching the cathode in unit time. Calculate the photon flux.

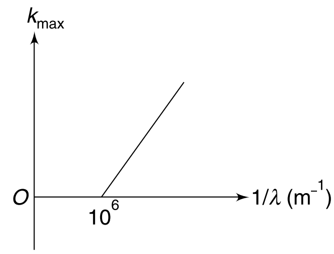
(c) Photo efficiency is defined as probability of a photon being successful in knocking out an electron from the metal surface. Calculate the saturation photocurrent in the cell assuming a photo efficiency of 0.1.

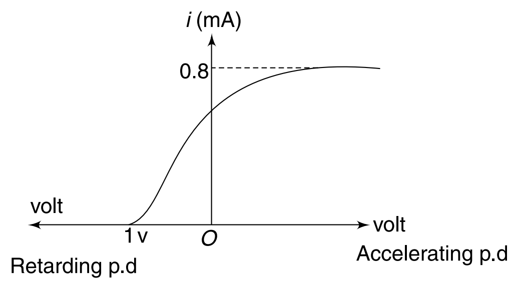
(d) Find the cut – off potential difference for the cell.

Q. 51: In photoelectric experiment set up, the maximum kinetic energy (k_{max}) of emitted photoelectrons was measured for different wavelength (λ) of light used. The graph of k_{max} vs $\dfrac{1}{\lambda}$ was obtained as shown in first figure. In the same setup, keeping the wavelength of incident light fixed at λ, the applied potential difference was varied and the photoelectric current was recorded. The result has been shown in graph in second figure.

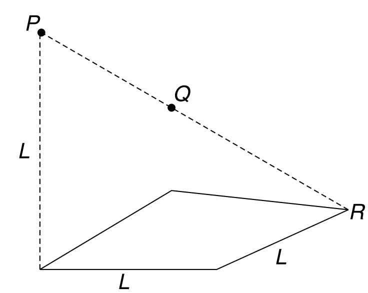
(a) Find λ is Å

(b) Taking the photo efficiency to be 2% (i.e. percentage of incident photons which produce photoelectrons) find the power of light incident on the emitter plate in the experiment. [Take hc = 12400 eV Å]



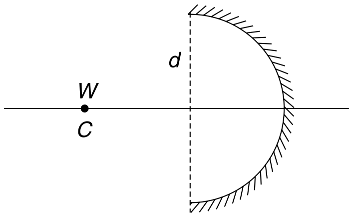


Q. 52: A square plate of side length L absorbs all radiation incident on it. A point source of light is placed at a point P which is directly above a corner of the plate at a height L. The incident light on the plate produces a force of magnitude F_0 on the plate. Calculate the magnitude of force on the plate if the source is moved to a point Q (Q is midpoint of the line PR).

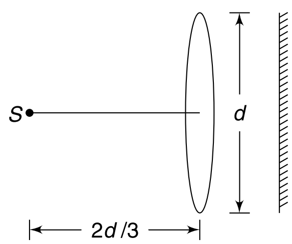


Q. 53: A light photon of wavelength λ_0 is absorbed by an atom of mass m at rest in ground state. The atom is free to move. The atom after absorbing the incident photon, emits another photon in direction opposite to that of the incident photon and returns back to its ground state. In the process the atom acquires a kinetic energy k. Find λ_0 in terms of k and m.

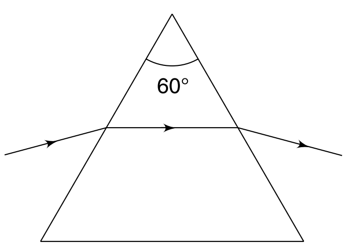
Q. 54: A spherical concave mirror has aperture diameter d and radius of curvature R. A point source of light is placed at its centre of curvature. Source emits power W and the mirror surface is completely reflecting. Find force on the mirror due to light incident on it.



Q. 55: A point source (S) of light having power 500 W is kept at the focus of a lens of aperture diameter d. The focal length of the lens is $\frac{2d}{3}$. Assume that 40% of the incident light energy is transmitted through the lens and the complete transmitted light is incident normally on a perfectly reflecting surface placed behind the lens. Calculate the force on the reflecting surface.



Q. 56: An equilateral glass prism kept on a table has refractive index of $\mu = \sqrt{2}$. It is illuminated by a narrow laser beam having power P_0 and wavelength λ. The path of the laser beam inside the prism is parallel to the base of the prism. Calculate change in weight of the prism due to the incident laser beam.



Q. 57: A proton X is projected in a region of uniform electric field E with speed u. Another proton Y is projected simultaneously with same speed u in a separate region having a uniform magnetic field 'B'. It was found that both X and Y had same de-Broglie wavelength after time T. Specific charge on a proton is σ (C/kg).

(a) Find the angle α at which the proton X was projected with respect to the direction of electric field.

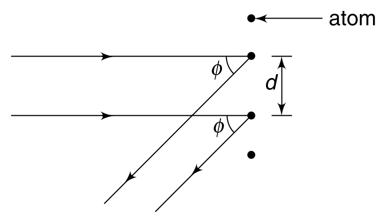
(b) Find the displacement of the proton X in time T.

Q. 58: It is possible for a photon to materialize into an electron and a position. The process is called pair production.

(a) Using conservation of energy and momentum prove that pair production cannot occur in empty space.

(b) Argue qualitatively that such pair production is possible near a nucleus.

Q. 59: The figure shows the reflection of a plane wave by two neighbouring atoms on the surface of a crystal. The depicted waves are actually de-Broglie wave of electrons accelerated from rest by a potential difference of V volt. Neglect relativistic effects and take mass and charge of an electron to be m and e respectively. Calculate ϕ for the case of a constructive interference.



Q. 60: A particle of mass m is trapped between two perfectly rigid parallel walls. The particle bounces back and forth between the walls without losing any energy. From a wave point of view, the particle trapped between the walls is like a standing wave in a stretched string between the walls. Distance between the two walls is L.

(a) Calculate the energy difference between third energy state and the ground state (lowest energy state) of the particle.

(b) Calculate the ground state energy if the mass of the particle is $m = 1$ mg and separation between the walls is $L = 1$ cm.

Q. 61: A moving H-atom makes a head on perfectly inelastic collision with a stationary Li^{++} ion. Just before collision, both - H and Li^{++} are in their first excited state. Immediately after collision H was found to be in ground state and Li^{++} was in its second excited state. Find the kinetic energy of H – atom before collision. No photon is emitted in the process. Assume that mass of Li^{++} is nearly 7 times the mass of H.

Q. 62: In a Lithium atom there is one electron in second orbit. The interaction of this electron with the two inner electrons can be accounted for by assuming that this electron (the electron in second orbit) sees a nuclear attraction of Z' protons (where $Z' < Z = 3$). Experimentally, it is known that 5.39 eV of energy is needed to remove the outermost electron from the atom.

(a) Find Z'

(b) Find the longest wavelength of photon that can be absorbed by the electron in $n = 2$ state.

Q. 63: In a hypothetical hydrogen-like atom the wavelength in $A°$ for the spectral lines for transitions from $\boldsymbol{n = P}$ to $\boldsymbol{n = 1}$ are given by $\lambda = \dfrac{\mathbf{1500}\boldsymbol{p}^2}{\boldsymbol{p}^2 - \mathbf{1}}$ where $\boldsymbol{p}$ is an integer larger than 1. Find the energy of three lowest states for this atom. What is the ionization potential of this atom?

Q. 64: Assume that structure of hydrogen atom is governed by classical mechanics. An electron is circulating around a proton and it is radiating energy at a rate given by $\dfrac{dE}{dt} = \dfrac{e^2 a^2}{6\pi c^3 \epsilon_0}$ where a is the acceleration of the electron. Assume that speed of the electron is v ($<< c$).

(a) Estimate the fraction of kinetic energy lost by the electron per revolution in terms of v. Make suitable assumptions.

(b) Is the energy loss per revolution large? Is it safe to assume that orbit is circular during a small time interval?

Q. 65: In a hypothetical hydrogen atom the electrostatic potential energy of interaction of proton and electron is given by $U = U_0 \ell n\left(\dfrac{r}{r_0}\right)$ where U_0 and r_0 are constants and r is radius of circular orbit of electron. For such hydrogen atom the energy difference between n^{th} and m^{th} state is represented by ΔE_{nm}. Calculate the ratio $\Delta E_{12} : \Delta E_{24}$ Assume Bohr's assumption of angular momentum quantization to hold.

Q. 66: When radiations of wavelength λ_1, λ_2 ($= 0.6\ \lambda_1$), λ_3 ($= 4\lambda_2$) and λ_4 are incident on a metal plate, photoelectrons with maximum kinetic energy of -5.2 eV are emitted for λ_1, 12 eV are emitted for λ_2, 0.95 eV are emitted for λ_4 and no electron is emitted for λ_3. It is known that a hydrogen like atom (atomic number Z) in a higher excited state of quantum number n can make a transition to first excited state by successively emitting two photons of wavelength λ_1 and λ_2 respectively. Alternatively, the atom from same excited state can make a transition to the second excited state by successively emitting two photons of wavelength λ_3 and λ_4 respectively.

(a) Find the work function of the metal.

(b) Find the values of n and Z for the atom.

Q. 67: An electron approaches a fixed proton from a large distance with a kinetic energy of 12.2 eV. The electron gets captured by the proton to form a hydrogen atom in an excited state and a photon of wavelength $\lambda_1 = 796$ Å is emitted. Later, the hydrogen atom de-excites by emitting a single photon of wavelength λ_2. Find λ_2. [Take $h = 6.62 \times 10^{-34}$ Js, $c = 3 \times 10^8$ m/s]

Q. 68: A free hydrogen atom in its ground state is at rest. A neutron having kinetic energy k_0 collides head one with the atom. Assume that mass of both neutron and the atom is same.

(a) Find minimum value of k_0 so that this collision can be inelastic.

(b) If $k_0 = 25$ eV, find the kinetic energy of neutron after collision if its excites the hydrogen atom to its second excited state.

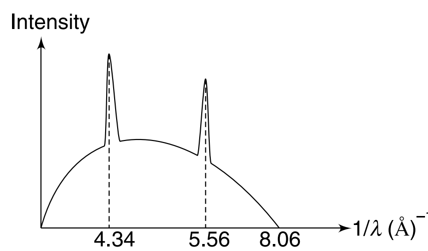
Take ionization energy of hydrogen atom in ground state to be 13.6 eV.

Q. 69: The X-ray spectrum of a metallic target has been shown in figure

(a) What is the accelerating potential difference for bombarding electrons?

(b) Two characteristic X-rays have been shown in the figure one of them is k_α X-ray and the other one is k_β X-ray. What is wavelength of k_α X-ray?

(c) Find the atomic number of the target atom.



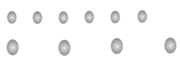
Q. 70: In a X-ray tube, after an electron has been removed from an inner shell of a target atom, an electron from outer shell falls into the vacancy and the excess energy is usually released in form of a photon. Many a times an atom chooses to release this excess energy by ejecting another electron. The ejected electron is called Auger electron. In one such event, the bombarding electron removed an electron from K shell of a target atom. An electron from L shell falls to occupy the vacant K shell position, and the excess energy is used by the atom to eject an Auger electron from L shell. What will be kinetic energy of such an Auger electron if the ionization energies for K and L shell of the atom is E_k and E_L respectively.

LEVEL 3

Q. 71: A hydrogen atom in ground state is moving with a kinetic energy of 30 eV. It collides with a deuterium atom in ground state at rest. The hydrogen atom is scattered at right angle to its original line of motion. Assume that energy of n^{th} state in both the atoms is given by $E_n = -\dfrac{13.6}{n^2}$ eV and the mass of deuterium is twice that of hydrogen. Write the maximum and minimum possible kinetic energy of deuterium after collision.

Q. 72: A hydrogen atom at rest de-excites from $n = 2$ state to $n = 1$ state. Calculate the percentage error in calculation of momentum of photon if recoil of atom is not taken into account. The energy equivalent of rest mass of hydrogen atom is 940 MeV.

ANSWERS

1. 

2. It becomes reddish. Average photon energy decreases.

3. No change.

4. $5.7 \times 10^4\ m^{-2}s^{-1}$

5. $750\ Ks^{-1}$

6. (a) $\phi_A = 2.25$ eV, $\phi_B = 4.2$ eV
(b) 2 eV

7. 523 Å

8. $E = \left(\dfrac{mg^2h^2}{2}\right)^{1/3}$

9. 0.15 keV

10. $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\Delta V}{2V}$

11. $\dfrac{8}{\sqrt{3}}$

12. 1.02 Å

13. No

14. (i) $\dfrac{IA}{c}$ (ii) $\dfrac{2IA}{c}$
(iii) $\dfrac{IA\cos\theta}{c}$ (iv) $\dfrac{2IA\cos^2\theta}{c}$

15. $\theta = \tan^{-1}(0.8)$ with horizontal

16. (a) 3.96×10^{26} W (b) 1.19×10^{45}
(c) 4.4×10^9 kg/s.

17. $R = \dfrac{3P_{sun}}{16\pi cdGM_{sun}}$

19. 6.613 Å

20. (a) 2 (b) $Z = 6$
(c) $\lambda_{min} = 28.5\ Å$ (d) 489.6 eV

21. $r = \left[\dfrac{n^2h^2}{8am\pi^2}\right]^{1/4}$

22. $Z = 3$

23. $n \simeq 25$

24. $n = 7$; $Z = 3$

25. (a) 2 (b) yes

26. (b) $z \simeq 48$

27. 8×10^{23}

28. 10.2 volt < V ≤ 12.09 volt.

29. 0.9 Å

30. (b) The two are same

31. 5.8 cm

32. 1218 Å

33. (a) 2 (b) 5

34. 79.4 eV

35. An electron having 11 eV kinetic energy.

36. 94.7 nm, 96.9 nm, 102.2 nm, 121 nm

37. 3 eV

38. $_{28}$Ni

39. (a) 0.08 nm
(b) Energy of an electron in K-shell lies between −25 keV and −20 keV.

40. Cu ($Z = 29$)

41. 14.8 kV

42. 4 elements ($Z = 24, 25, 26, 27$)

43. B_0

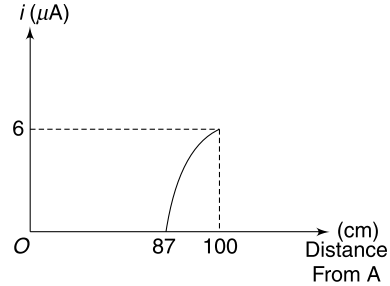
44. 1.64×10^{19} m

45. 4.22×10^{26}

46. 7 W

47. 25 Volt

48. $\phi = 3$ eV

49. (a) Zero
(b)


(c) 0.8 eV

50. (a) 178 s (b) $8.12 \times 10^{12}\ s^{-1}$
(c) 65 nA (d) 2.68 V

51. (a) 5536 Å (b) 0.089 W

52. $4F_0$

53. $\lambda_0 = \dfrac{2h}{\sqrt{2mk} + \dfrac{k}{c}}$

54. $\dfrac{Wd^2}{8R^2c}$

55. 1.33×10^{-7} N

56. $\dfrac{2P_0}{c}\cos 15°$

57. (a) $\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{\sigma ET}{2u}\right)$ (b) $\sqrt{u^2 - \dfrac{\sigma^2E^2T^2}{4}}\cdot T$

59. $\phi = \sin^{-1}\left(\dfrac{nh}{d\sqrt{2\text{ meV}}}\right)$

60. (a) $\dfrac{h^2}{mL^2}$ (b) 5×10^{-54} J

61. 7.77 keV

62. (a) $Z' = 1.26$ (b) $\lambda_{max} = 415$ nm

63. -8.28 eV, -2.07 eV, -0.92 eV. Ionization Potential $= 8.28$ eV

64. (a) $\dfrac{8\pi}{3}\left(\dfrac{v}{c}\right)^3$ (b) No, Yes

65. 1

66. (a) 5 eV (b) $n = 6$, $Z = 3$

67. 1217 Å

68. (a) 20.4 eV (b) 8.72 eV or 4.19 eV

69. (a) 10^5 volt (b) $\lambda_{k\alpha} = 0.23$ Å (c) $Z = 74$

70. $E_k - 2E_L$

71. 20 eV, 13.2 eV

72. $5.4 \times 10^{-7}\%$

SOLUTIONS

2. The heat dissipated in the filament will decrease and the temperature of the bulb will fall. More light of longer wavelength will be emitted (Wien's law). A photon of longer wavelength has lesser energy.

4. Number of photons emitted (per second) from the source is

$$N = \frac{\text{Power}}{\dfrac{hc}{\lambda}} = \frac{0.6 \times 10^{-3} \times 600 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$= 1.8 \times 10^{15} \text{ per second}$$

Beam diameter $d = r\theta = 4 \times 10^8 \times 0.5 \times 10^{-3}$

$$= 2 \times 10^5 \text{ m}$$

Area illuminated on the moon, $A = \pi\left(\dfrac{d^2}{4}\right)$

$$= 3.14 \times \frac{4 \times 10^{10}}{4} = 3.14 \times 10^{10} \text{ m}^2$$

$\therefore$ Number of photons incident per unit area per second

$$= \frac{1.8 \times 10^{15}}{3.14 \times 10^{10}} = 5.7 \times 10^4 \text{ m}^{-2}\text{s}^{-1}$$

5. Force = (Change in momentum of one photon) × (number of photons striking per second)

$\Rightarrow$ $$10^{-5} = \left(\frac{h}{\lambda}\right)(n) = \frac{hc}{\lambda}\frac{n}{c} = \frac{E}{c}$$

Where E = energy incident per second.

$\therefore$ $$E = 10^{-5} \times 3 \times 10^8 = 3 \times 10^3 \text{ Js}^{-1}$$

Now $$ms\frac{d\theta}{dt} = 3 \times 10^3$$

$\Rightarrow$ $$\frac{d\theta}{dt} = \frac{3 \times 10^3}{10 \times 10^{-3} \times 400} = 750 \text{ Ks}^{-1}$$

6. Energy of photons incident on A is

$$E_1 = \frac{hc}{\lambda_1} = \frac{12410}{2920} = 4.25 \text{ eV}$$

$$\therefore \quad k_1 = 4.25 - \phi_1 \qquad \text{...(i)}$$

Energy of photons incident on B is

$$E_2 = \frac{hc}{\lambda_2} = \frac{12410}{2640} = 4.70 \text{ eV}$$

$$\therefore \quad k_2 = 4.70 - \phi_2$$

$$\therefore \quad k_1 - 1.5 = 4.70 - \phi_2 \qquad \text{...(ii)}$$

Combining (i) and (ii) gives

$$\phi_2 - \phi_1 = 1.95 \text{ eV} \qquad \text{... (iii)}$$

The smallest de-Broglie wavelength corresponds to the electron with maximum kinetic energy

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mk_1}}$$

And $$2\lambda = \frac{h}{\sqrt{2mk_2}}$$

Taking ratio $$2 = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k_1}{k_1 - 1.5}}$$

$$\Rightarrow \quad 4 = \frac{k_1}{k_1 - 1.5} \quad \Rightarrow \quad k_1 = 2 \text{ eV}$$

Put this in (i) to get $\phi_1 = 2.25$ eV

Substituting in (iii) gives

$$\phi_2 = 4.2 \text{ eV.}$$

7. $$\phi_x = 3.5 \text{ eV}$$

$$\therefore \quad E - 3.5 = k_x$$

Where E = energy of incident photon

k_x = maximum KE of emitted photoelectrons.

Also $$\phi_Y = \frac{13.6}{(2)^2}(2)^2 = 13.6 \text{ eV}$$

$$\therefore \quad E - 13.6 = k_y$$

Given $$k_x = 2k_y$$

$$\Rightarrow \quad E - 3.5 = 2E - 27.2$$

$$\Rightarrow \quad E = 23.7 \text{ eV}$$

$$\Rightarrow \quad \frac{hc}{\lambda} = 23.7 \text{ eV}$$

$$\therefore \quad \lambda = \frac{12400 \text{ eVÅ}}{23.7 \text{ eV}} = 523 \text{ Å}$$

8. Mechanical energy $$E = mgH = \frac{p^2}{2m}$$

p = momentum at the lowest point (i.e., maximum momentum of the particle)

$$\therefore \quad mgH = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} \qquad \left[\because \ p = \frac{h}{\lambda}\right]$$

When quantum effects are considerable $\lambda \simeq H$

$$\therefore \quad mgH = \frac{h^2}{2mH^2}$$

$$\therefore \quad H = \left(\frac{h^2}{2m^2g}\right)^{1/3}$$

$$\therefore \quad E = mgH = \left(\frac{mg^2h^2}{2}\right)^{1/3}$$

9. Diameter of H – atom $\simeq$ 1.0 Å

De-Broglie wavelength of electron having kinetic energy k is

$$\lambda = \frac{h}{\sqrt{2mk}}$$

One can "see" hydrogen atom if

$$\lambda = 1\text{Å}$$

$$\Rightarrow \quad \frac{h}{\sqrt{2mk}} = 10^{-10} \quad \Rightarrow \quad k = \frac{(6.63 \times 10^{-34})^2}{10^{-20} \times 2 \times 9.1 \times 10^{-31}}$$

$$k = 2.4 \times 10^{-17}\text{ J} = \frac{2.4 \times 10^{-17}}{1.6 \times 10^{-19}}\text{ eV}$$

$$= 1.5 \times 10^2\text{ eV} \simeq 0.15\text{ keV}$$

10. KE of an electron accelerated through a potential difference of V is

$$k = \text{eV}$$

$$\Rightarrow \quad \frac{p^2}{2m} = \text{eV} \quad \Rightarrow \quad p = \sqrt{2\text{ meV}}$$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2\text{ meV}}}$$

$$\Rightarrow \quad \frac{\Delta\lambda}{\lambda} = -\frac{1}{2}\frac{\Delta V}{V}$$

11.
$$V_{\text{rms}} = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \quad \frac{V_H}{V_0} = \sqrt{\frac{T_H}{M_H}\cdot\frac{M_0}{T_0}} = \sqrt{\frac{300}{2}\cdot\frac{32}{400}} = \sqrt{12}$$

$$\frac{\lambda_H}{\lambda_0} = \frac{h}{m_HV_H}\cdot\frac{m_0V_0}{h} = \frac{m_0}{m_H}\frac{V_0}{V_H}$$

$$= \frac{32}{2}\cdot\frac{1}{\sqrt{12}} = \frac{8}{\sqrt{3}}$$

12. Mean kinetic energy of one atom $= \frac{3}{2}kT$

$$p = \sqrt{2m\left(\frac{3}{2}kT\right)}$$

$$= \sqrt{3 \times 2 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 400}$$

$$= 6.50 \times 10^{-24}\text{ kg m/s}$$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6.50 \times 10^{-24}}$$

$$= 1.02 \times 10^{-10}\text{ m} = 1.02\text{ Å}$$

13.
$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}\ \text{m}}\ \text{J-s} = 6.63 \times 10^{-20}\ \text{kg ms}^{-1}$$
$$m_0 v = p$$
$$v = \frac{6.63 \times 10^{-20}\ \text{J-s}}{9.1 \times 10^{-31}\ \text{kg}} = 7.3 \times 10^{10}\ \text{m/s}$$

This speed is not possible as it is larger than speed of light.

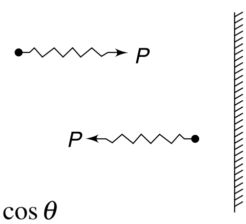
$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$
$$\therefore \quad \frac{m_0}{\sqrt{1 - v^2/c^2}} \cdot v = p$$
$$\frac{9.1 \times 10^{-31} v}{\sqrt{1 - \frac{v^2}{(3 \times 10^8)^2}}} = 6.63 \times 10^{-20}$$
$$\Rightarrow \quad (1.37 \times 10^{-12})^2 v^2 = 1 - 1.11 \times 10^{-17} v^2$$
$$\Rightarrow \quad [1.11 \times 10^{-17} + 1.88 \times 10^{-24}]\ v^2 = 1$$
$$\Rightarrow \quad v^2 \simeq \frac{10^{17}}{1.11} = 0.9 \times 10^{17}$$
$$v = 3 \times 10^8\ \text{m/s}$$

14. (i) Incident power $= IA$

Momentum incident per unit time $= \frac{IA}{c}$ $\quad \left[\because \ p = \frac{E}{c}\right]$

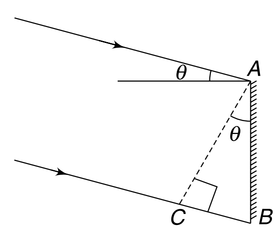
$\therefore$ Force $\quad F = \frac{IA}{c}$

(ii) Momentum transferred to the wall per unit time



$$= \frac{2IA}{c}$$
$$\therefore \quad F = \frac{2IA}{c}$$

(iii) Energy incident on the surface AB in unit time = Energy incident on AC in unit time $= IA\cos\theta$ [where $A\cos\theta$ = area of surface AC]



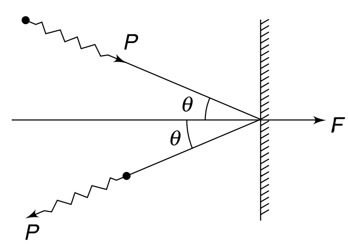
Momentum transferred per unit time $= \frac{IA\cos\theta}{c}$

$\therefore \quad F = \frac{IA\cos\theta}{c}$ [Force is along the direction of the beam]

(iv) Incident power $= IA\cos\theta$

Momentum incident per unit time $= \frac{IA\cos\theta}{c}$

Momentum transferred per unit time to the surface



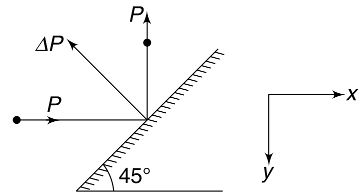
$$= 2\left(\frac{IA\cos\theta}{c}\right)\cos\theta$$

Since change in momentum of each photon is $2p\cos\theta$ if p is momentum of each photon.

$$\therefore \quad F = \frac{2IA}{c}\cos^2\theta$$

Force is normal to the surface, i.e., along the direction of change in momentum.

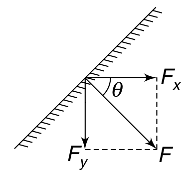
15. If n is number of photons incident per second on the mirror, $0.8n$ is the number that suffers reflection. $0.2n$ is absorbed.



The reflected photons suffer change in momentum in a direction perpendicular to the mirror.

If each incident photon has momentum = p, then change in momentum of each incident photon is $\Delta p = \sqrt{2}p$ (normal to mirror)

And change in momentum of each absorbed photon is = p (in horizontal direction)



$$\therefore \quad F_x = 0.8n(\sqrt{2}p)\cos 45° + 0.2 \cdot n \cdot p = np$$

$$F_y = 0.8n(\sqrt{2}p)\sin 45° = 0.8np$$

Alternate:

All photons lose their entire momentum in x direction.

$$\therefore \quad F_x = np$$

$0.8n$ photons acquired a momentum p in upward direction

$$\therefore \quad F_y = 0.8np$$

Resultant makes an angle θ with horizontal where

$$\tan\theta = \frac{F_y}{F_x} = 0.8.$$

16. (a) Power of the Sun

$$p = 4\pi r^2 \cdot I$$

$$p = 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 1400$$

$$= 3.96 \times 10^{26} \text{ W}$$

(b) Energy of one photon

$$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}}$$

$$= 3.32 \times 10^{-19} \text{ J}$$

Number of photons emitted per second

$$n = \frac{3.96 \times 10^{26}}{3.32 \times 10^{-19}} = 1.19 \times 10^{45} \text{ photons per second}$$

(c) $1 \text{ kg} = 1 \times (3 \times 10^8)^2 \text{ J/c}^2$

$= 9 \times 10^{16} \text{ J/c}^2$

$\therefore$ Decrease in mass of the sun in one second

$$= \frac{3.96 \times 10^{26}}{9 \times 10^{26}} = 4.4 \times 10^9 \text{ kg}$$

17. The radiation pressure is give by $P = \frac{I}{c}$.

If r is distance of the sphere from the centre of the Sun and R is radius of the sphere itself then-

$$\frac{GM_{sun}m}{r^2} = \frac{I}{c}\pi R^2$$

But $m = \frac{4\pi R^3}{3}d$ and $I = \frac{P_{sun}}{4\pi R^2}$

$$\therefore \quad \frac{GM_{sun}\frac{4\pi R^3}{3}d}{r^2} = \frac{\frac{P_{sun}}{4\pi r^2}}{c}\pi R^2$$

$$\therefore \quad R = \frac{3P_{sun}}{16\pi c d G M_{sun}}$$

This is independent of distance from the sun.

18. $$2\pi r = n\lambda$$

$\Rightarrow$ $$2\pi r = n\frac{h}{mv}$$

$\Rightarrow$ $$mvr = \frac{nh}{2\pi}$$

19. As per question $n = 5$; $Z = 2$

$\therefore$ $$r = 0.529\frac{n^2}{Z}\ \text{Å}$$

$$= 0.529 \times \frac{25}{2} = 6.613\ \text{Å}$$

20. (a) Since 3 radiations are emitted, the final state is $n = 3$ and initial state must be is $n = 2$; because the emitted radiations have equal or smaller wavelength than of absorbed photon.

(b) $13.6Z^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 68 \quad \Rightarrow Z = 6$

(c) $\frac{1}{\lambda_{min}} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \quad \Rightarrow \lambda_{min} = 28.5 Å$

(d) $E = 13.6Z^2 = 13.6 \times 6^2 = 489.6$ eV

21. $$U = ar^2$$

$\Rightarrow$ $$F = -\frac{dU}{dr} = -2ar$$

Negative sign indicates that force is towards the centre of the circle. It must be centripetal force.

$\therefore$ $$\frac{mV^2}{r} = 2ar$$

$\Rightarrow$ $$v = \sqrt{\frac{2a}{m}}\cdot r \qquad \ldots(i)$$

Given: $$mvr = n\frac{h}{2\pi}$$

$\Rightarrow$ $$m\left(\sqrt{\frac{2a}{m}}\cdot r\right)r = \frac{nh}{2\pi}$$

$\Rightarrow$ $$r = \left[\frac{n^2h^2}{8am\pi^2}\right]^{1/4}$$

22. Number of lines in the spectrum = ${}^nC_2$

$\Rightarrow$ $$\frac{n(n-1)}{2} = 15 \quad \Rightarrow \quad n = 6$$

The second most energetic photon will be emitted due to transition $n = 6 \rightarrow n = 2$ [The most energetic photon will result from $n = 6 \rightarrow n = 1$ transition.

$\therefore$ $$13.6\left[\frac{1}{2^2} - \frac{1}{6^2}\right]Z^2 = 27.2$$

$\Rightarrow$ $$Z^2 = 2 \times \frac{36}{8}$$

$\Rightarrow$ $$Z = 3$$

23. $$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

and $$mvr = \frac{nh}{2\pi}$$

Solving the above two equations we get

$$r = \frac{n^2 h^2 \varepsilon_0}{Z \cdot \pi m e^2}$$

Put $Z = 3$; $m = 208 m_e$

$$r_\mu = \frac{n^2 h^2 \varepsilon_0}{624 \pi m_e e^2}$$

According to the question

$$\frac{n^2 h^2 \varepsilon_0}{624 \pi m_e e^2} = \frac{h^2 \varepsilon_0}{\pi m_e e^2}$$

$$\Rightarrow \quad n^2 = 624$$

$$\Rightarrow \quad n \simeq 25.$$

24. $$13.6 \left[\frac{1}{2^2} - \frac{1}{n^2}\right] Z^2 = 22.94 + 5.15 = 28.09$$

$$\Rightarrow \quad \left(\frac{1}{4} - \frac{1}{n^2}\right) Z^2 = 2.065 \quad \ldots\text{(i)}$$

$$\Rightarrow \quad E_n - E_2 = 28.09 \text{ eV}$$

$$E_n - E_3 = 2.4 + 8.7 = 11.1 \text{ eV}$$

$$\Rightarrow \quad E_3 - E_2 = 16.99 \text{ eV}$$

$$\Rightarrow \quad 13.6\, Z^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 16.99$$

$$\Rightarrow \quad Z^2 \simeq 9$$

$$Z = 3$$

Put in (i) to get $n = 7$

25. (a) Reduced mass $\mu = \dfrac{m \cdot m}{m + m} = \dfrac{m}{2}$

$\therefore$ Energy values are $\frac{1}{2}$ compared to hydrogen.

(b) Transition between any two energy states in Positronium will result in emission of photon having energy half that of corresponding photon in hydrogen.

$\therefore$ Each line in spectrum will have twice the wavelength of the corresponding lines in H – atom.

26. (a) For hydrogen like atom the energy in n^{th} state is $E_n = -13.6 \dfrac{\mu Z^2}{n^2}$

For mesic atom $\mu \simeq 200 m_e$

The transition energies are enhanced by a factor of 200, so that emitted radiation falls in X – ray region instead of UV, IR or visible part of electromagnetic spectrum.

(b) Nuclear radius $$R = R_0 A^{1/3}$$

$$= (1.2 \times 10^{-15})\,(2Z)^{1/3}$$

Atomic radius of first orbit is

$$r_1 = \frac{0.529 \times 10^{-10}}{200 \times Z}$$

Since $$r_1 = R$$

$$\therefore \quad \frac{0.529 \times 10^{-10}}{200 Z} = (1.2 \times 10^{-15})(2Z)^{1/3}$$

$$\Rightarrow \quad Z^{4/3} = \frac{0.529 \times 10^5}{1.2 \times (2)^{1/3} \times 200}$$

$$\Rightarrow \quad Z^4 = \left(\frac{0.529 \times 10^3}{1.2 \times 2}\right)^3 \times \frac{1}{2} = 5.35 \times 10^6$$

$$\Rightarrow \quad Z^2 = 2.31 \times 10^3$$

$$\Rightarrow \quad Z \simeq 48$$

27. Radius of each atom

$$r = r_0 \cdot n^2 = 0.53\,\text{Å} \times (10)^2$$

$$= 53\ \text{Å}$$

No. of atoms in one line

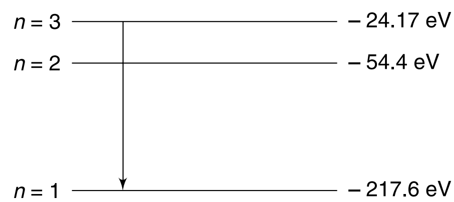
$$N = \frac{1\ \text{m}}{2 \times 53\ \text{Å}} = \frac{10^{10}}{106} = 9.4 \times 10^7$$

$\therefore$ No. of atoms in the box

$$= N^3 = (9.4 \times 10^7)^3 = 8 \times 10^{23}$$

29. Energy levels for Be^{+++} ($Z = 4$) are as shown.

Transition ($3 \rightarrow 1$) releases maximum energy photons



$$E = -24.17 - (-217.6) = 193.4\ \text{eV}$$

When these photons hit the metal plate, electrons are emitted with maximum KE given by

$$K_{\max} = E - \phi = 193.4 - 5 = 188.4\ \text{eV}$$

$$= 188.4 \times 1.6 \times 10^{-19}\ \text{J} = 3.01 \times 10^{-17}\ \text{J}$$

De-Broglie wavelength of such electrons will be minimum

$$\lambda_{\min} = \frac{h}{\sqrt{2mK_{\max}}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.01 \times 10^{-17}}}$$

$$= 0.90\ \text{Å}$$

30. (a) Radius of orbit for n^{th} state and speed of electron in the orbit is given by

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

$$v = \frac{Ze^2}{2\varepsilon_0 hn}$$

$\therefore$ Frequency of orbital revolution of electron in n^{th} state is

$$f_n = \frac{v}{2\pi r} = \frac{me^4 Z^2}{4\varepsilon_0^2 h^3 n^3} \qquad \text{...(i)}$$

Frequency of emitted radiation can be obtained as

$$hf_0 = E_n - E_{n-1}$$

$$hf_0 = \frac{me^4}{8\varepsilon_0^2 h^2} Z^2 \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$f_0 = \frac{me^4}{8\varepsilon_0^2 h^3} Z^2 \left[\frac{2n-1}{(n-1)^2 n^2}\right] \qquad \text{...(ii)}$$

You can show (or else verify) that

$$\frac{1}{n^3} < \frac{2n-1}{(n-1)^2 n^2} < \frac{1}{(n-1)^3}$$

Hence, from (i) and (ii)

$$f_n < f_0 < f_{n-1}$$

(b) When $n >> 1$

$$f_0 \simeq \frac{me^4 Z^2}{8\varepsilon_0^2 h^3}\frac{2}{n^3} = \frac{me^4 Z^2}{4\varepsilon_0^2 h^3 n^3}$$

Therefore, for large quantum number the frequency of emitted radiation is equal to the orbital frequency of the electron. It means for large quantum numbers the results predicted by quantum mechanics are nearly same as that given by classical mechanics.

31. The outer electron will effectively see attraction of one positive charge. The electron present in first orbit will screen the attraction of two protons. Since outer electron is in an orbit of large radius it will see one positive charge at nucleus.

32. The hydrogen atom is having momentum equal to that of the photon and the two momenta are in opposite directions.

The kinetic energy of hydrogen atom will be negligible compared to the energy of photon. The entire photon energy is absorbed as excitation energy by the hydrogen atom. Hence minimum energy of photon must be 10.2 eV.

$$\lambda_{max} = \frac{hc}{10.2 \text{ eV}} = \frac{1240 \text{ eVÅ}}{10.2 \text{ eV}} = 1218 \text{ Å}.$$

33. There are 6 lines in spectrum. 4, 5 and 6 represent transitions: $2 \to 1$, $3 \to 1$, $4 \to 1$ respectively. The wavelength difference between 4 and 5 will be higher than the difference between 5 and 6.

Line 2 and 3 are the first two lines of Balmer series and line 1 is the first line of Paschen series. The gap between 3 and 4 is large which means energy difference of photon corresponding to 3 and 4 is large. In fact, among all emitted photons, the energy difference between these two photons is largest.

34. When one e^- is removed the remaining atom is hydrogen like atom with $Z = 2$.

Binding energy of such atom will be

$$= 13.6 \times 2^2 = 54.4 \text{ eV}$$

$\therefore$ Total energy needed $= 54.4 + 25 = 79.4$ eV

36. Absorption lines will correspond to Lyman series in the emission spectrum.

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \qquad n = 2, 3, 4, 5...$$

For $n = 2$; $\lambda = \frac{4}{3R} = 1.21 \times 10^{-7}\text{ m} = 121\text{ nm}$

Similarly, for $n = 3$; $\lambda = 102.2$ nm

for $n = 4$; $\lambda = 96.9$ nm

for $n = 5$; $\lambda = 94.7$ nm

for $n = 6$; $\lambda = 93.5$ nm

$\therefore$ Required answers are 94.7 nm, 96.9 nm, 102.2 nm and 121 nm.

37. We have,

$$10.2 = W + K_1 \qquad ...(i)$$

and

$$10.2\, Z^2 = W + K_2 \qquad ...(ii)$$

Also

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = 2.3 \quad \Rightarrow \quad K_2 = 5.25\, K_1 \qquad ...(iii)$$

Also $10.2\, Z^2$ = energy corresponding to longest wavelength of the Lyman series $= 3 \times 13.6$

$\Rightarrow$ $Z = 2$.

$\therefore$ From equations (i), (ii) and (iii)

$$W = 3 \text{ eV}.$$

38. $$\frac{1}{\lambda_{k\alpha}} = \frac{3}{4} R(Z-1)^2$$

$$\therefore \quad \frac{\lambda}{\lambda_{cu}} = \frac{(29-1)^2}{(Z-1)^2}$$

$$\therefore \quad (Z-1)^2 = 28^2 \times \frac{1.5404}{1.6578}$$

$$\therefore \quad Z-1 = 27$$

$$\therefore \quad Z = 28$$

40. $$\lambda_k = \frac{4}{3R(Z-1)^2} = \frac{1200}{(Z-1)^2} \text{ Å}$$

$$\lambda_c = \frac{12.4}{V} \quad \text{where } V \text{ is in } kV$$

$$\therefore \quad \frac{1200}{(Z-1)^2} - \frac{12.4}{V} = \Delta\lambda$$

$$\Rightarrow \quad \frac{1200}{(Z-1)^2} - \frac{12.4}{10} = \Delta\lambda \qquad \text{...(i)}$$

$$\text{and} \quad \frac{1200}{(Z-1)^2} - \frac{12.4}{20} = 3\Delta\lambda \qquad \text{...(ii)}$$

Eliminating $\Delta\lambda$ between (i) and (ii) we get

$$Z = 28.8 \simeq 29$$

41. For nickel $Z = 28$

$$\lambda_k = \frac{4}{3R(Z-1)^2} = \frac{4}{3 \times 1.09 \times 10^7 \times (28-1)^2} \text{ m}$$

$$= \frac{4000}{3 \times 1.09 \times 27^2} \text{ Å}$$

$$= \frac{1223}{27^2} \text{ Å} = 1.68 \text{ Å}$$

$$\lambda_k - \lambda_c = 0.84$$

$$\Rightarrow \quad \lambda_c = 0.84 \text{ Å}$$

$$\Rightarrow \quad \frac{hc}{eV} = 0.84 \text{ Å}$$

$$\Rightarrow \quad \frac{12400 \text{ eV Å}}{V} = 0.84 \text{ Å}$$

$$\Rightarrow \quad V = 14762 \text{ volt} = 14.8 \text{ kV.}$$

42. $$\lambda_k = \frac{4}{3R(Z-1)^2} = \frac{4}{3 \times 1.09 \times 10^7 (Z-1)^2} \text{ m.}$$

$$= \frac{4000}{3 \times 1.09} \frac{1}{(Z-1)^2} \text{ Å}$$

$$= \frac{1223}{(Z-1)^2} \text{ Å}$$

$$\text{For} \quad \lambda_k = 2.41 \text{ Å}$$

$$(Z-1)^2 = \frac{1223}{2.41} = 507$$

$$\Rightarrow \quad Z - 1 = 22.53 \quad \Rightarrow \quad Z = 23.53$$

Similarly for $\lambda_k = 1.80$ Å

$$(Z-1)^2 = \frac{1223}{1.8} = 679.4$$

$$\Rightarrow \quad Z = 27.06$$

$\therefore$ The elements have $Z = 24, 25, 26, 27$.

44. Power of the sun $P = I.4\pi r^2$

The star has same power but it is at a distance x from us.

$\therefore$ Intensity of star light at earth is

$$I_S = \frac{P}{4\pi x^2}$$

$\therefore$ Energy from star that enters an eye in one second is

$$E = I_S \cdot \pi\left(\frac{d}{2}\right)^2 = \frac{P}{16}\frac{d^2}{x^2}$$

This must be at least equal to 10 times the energy of a photon of yellow light

$$\frac{Pd^2}{16x^2} = 10\frac{hc}{\lambda}$$

$$\Rightarrow \quad x^2 = \frac{Pd^2\lambda}{160\,hc} = \frac{I.4\pi r^2 d^2 \lambda}{160\,hc}$$

$$x^2 = \frac{1.4 \times 10^3 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times (6 \times 10^{-3})^2 \times (6 \times 10^{-7})}{160 \times 6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$= 2.68 \times 10^{38}$$

$$\Rightarrow \quad x = 1.64 \times 10^{19} \text{ m}$$

45. Energy needed to warm the water

$$E_0 = ms\Delta\theta = (200 \text{ g})(4.2 \text{ Jg}^{-1}{}^\circ\text{C}^{-1})(50^\circ\text{C})$$

$$= 4.2 \times 10^4 \text{ J}$$

Energy of one photon, $E = hv = 6.63 \times 10^{-34} \times 2.5 \times 10^9 = 1.66 \times 10^{-24}$ J

Number of microwave photons absorbed by water $= \dfrac{4.2 \times 10^4}{1.66 \times 10^{-24}} = 2.53 \times 10^{28}$

Number of photons emitted by the oven in 2 min is $= \dfrac{2.53 \times 10^{28}}{0.05} = 5.06 \times 10^{29}$

Number of photons emitted per second $= \dfrac{5.06 \times 10^{29}}{2 \times 60} = 4.22 \times 10^{27}$

Photon energy emitted per second by oven $= \dfrac{1}{0.05} \times \dfrac{4.2 \times 10^4 \text{ J}}{120 \text{ s}} = 7000$ W

Electrical energy consumed per second $= \dfrac{7000}{0.7} = 10000$ W $= 10$ kW

Number of photons emitted per kW of power consumed $= \dfrac{4.22 \times 10^{27}}{10} = 4.22 \times 10^{26}$

46. Stopping potential difference = 5 volt

$\therefore$ Energy of incident photon $h\upsilon = 5 + \phi = 7$ eV

Saturation current is $i_s = 10\,\mu$A

$\therefore$ Number of electrons emitted per second is

$$n_e = \frac{i_s}{e} = \frac{10 \times 10^{-6}}{1.6 \times 10^{-19}} = \frac{1}{1.6} \times 10^{14}$$

No of photons incident per second

$$n_p = n_e \times \frac{100}{10^{-3}} = 10^5 n_e = \frac{10^{19}}{1.6}$$

$\therefore$ Power incident $= (7\,\text{eV})\, n_p$

$$= (7 \times 1.6 \times 10^{-19}\text{ J}) \times \left(\frac{10^{19}}{1.6}\text{ s}^{-1}\right)$$

$$= 7\text{ J s}^{-1} = 7\text{ W}$$

47. Threshold wavelength $\lambda_{th} = 240\,\text{nm}$

$\therefore$ Work function, $\phi = \frac{hc}{\lambda_{th}}$

$$= \frac{(4 \times 10^{-15}\,\text{eVs})(3 \times 10^8\,\text{m s}^{-1})}{240 \times 10^{-9}} = 5\text{ eV}$$

With light of wavelength $2\lambda_0$, the maximum kinetic energy of emitted electrons is 10 eV.

$\therefore$ Energy of photon of wavelength $2\lambda_0$ is $= 10 + 5 = 15$ eV

$\therefore$ Energy of photon of wavelength λ_0 is $= 15 \times 2 = 30$ eV

With light of wavelength λ_0, the maximum kinetic energy of emitted electrons is $= 30 - 5 = 25$ eV

$\therefore$ $V_0 = 25$ V

48. In the experiment performed in figure (a), the applied potential difference is retarding.

$\therefore$ Electrons lose 5 eV kinetic energy in travelling from A to B.

Hence K_{max} for electrons emitted from plate A must be 6 eV. An electron emitted with KE of 6 eV will reach plate B with a KE of 1 eV

$$h\upsilon = \phi + 6\text{ eV} \quad \text{...(i)}$$

In experiment in figure (b), the applied potential difference is accelerating. Electrons gain a KE of 5 eV in travelling from plate A to B.

$\therefore$ Maximum KE of an electrons emitted from A is 15 eV.

$$2h\upsilon = \phi + 15\text{ eV} \quad \text{...(ii)}$$

Solving (i) & (ii)

$$\phi = 3\text{ eV}$$

49. When slider is at A, the applied potential difference across the photoelectric cell is a retarding potential difference of 10 V.

Energy of incident photon

$$E = \frac{hc}{\lambda} = \frac{1240\text{ eVnm}}{350\text{ nm}} = 3.5\text{ eV}$$

Maximum KE of emitted photoelectrons is

$$\text{KE}_{max} = \frac{hc}{\lambda} - \phi = 3.5 - 2.2 = 1.3\text{ eV}$$

$\therefore$ Stopping potential difference $V_S = 1.3$ V

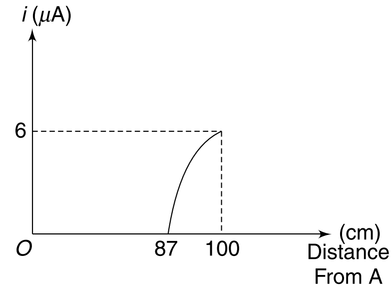
(a) The applied potential difference is

$$V(= 10\ V) > V_S\ (= 1.3\text{ V})$$

Hence there is no photocurrent and reading of Ammeter is zero.

(b) When slider is at a distance of 13 cm from B, the applied retarding potential difference is $V_B - V_J = 1.3$ V

When slider moves from A through 87 cm, there is no photocurrent because applied retarding potential difference is higher than V_S.



After 87 cm, the photocurrent increases as applied potential difference becomes less than V_S.

(c) Applied retarding potential difference = 0.5 V

$\therefore$ Maximum KE of electrons hitting plate C

$$= K_{max} - 0.5 \text{ eV}$$
$$= (1.3 - 0.5)\text{eV} = 0.8 \text{ eV}$$

50. (a) Energy required to eject a photoelectron is $\phi = 2.2$ eV

$$= 2.22 \times 1.6 \times 10^{-19} \text{ J} = 3.552 \times 10^{-19} \text{ J}$$

According to classical theory, energy flow is a continuous process and a photo electron will be ejected if a potassium atom receives this amount of energy over a length of time. The potassium atom is at a distance of 1.0 m from the source.

Hence, intensity of ultraviolet radiation on the potassium surface is

$$I = \frac{W}{4\pi r^2} = \frac{0.2}{4\pi(1)^2} = \frac{0.2}{4\pi} \text{ J/m}^2\text{/s}$$

Cross sectional area of one K atom is

$$A = \pi(2 \times 10^{-10}\text{m})^2 = 4\pi \times 10^{-20} \text{ m}^2$$

Hence, the exposure time is given as

$$t = \frac{3.552 \times 10^{-19} \text{ J}}{\frac{0.2}{4\pi} \text{Jm}^{-2}\text{s}^{-1} \times 4\pi \times 10^{-20}\text{m}^2} = 178 \text{ s}$$

(b) Energy of photon

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2537 \times 10^{-10}}$$
$$= 7.84 \times 10^{-19} \text{ J} = 4.9 \text{ eV}$$

Let N be the number of photons reaching the cathode per second. Then intensity at cathode is $I' = NE$

But energy falling per second on cathode is $\frac{0.2}{4\pi} \times 4 \times 10^{-4}$ J

$\therefore$
$$N = \frac{0.2 \times 4 \times 10^{-4}}{4\pi \times 7.84 \times 10^{-19}} = 8.12 \times 10^{12} \text{ photons/s}$$

(c) 10 % of photons are able to eject electrons. Hence, current is

$$i = 0.1 \times N \times 1.6 \times 10^{-19} = 65 \text{ nA}$$

(d) Cut off potential

$$V_0 = \frac{h\nu - \phi}{e} = \left(\frac{4.90 - 2.22}{e}\right) \text{eV} = 2.68 \text{ volts}$$

51. (a) Photoelectric equation is $k_{max} = \frac{hc}{\lambda} - \phi$

Graph between k_{max} and $\frac{1}{\lambda}$ is a straight line. $k_{max} = 0$ for wavelength given by-

$$\frac{1}{\lambda_0} = \frac{\phi}{hc}$$

$$\phi = \frac{hc}{\lambda_0} = (12400 \text{ eVÅ}) (10^6 \times 10^{-10} \text{ Å}^{-1})$$

$\left[\text{From graph } \frac{1}{\lambda_0} = 10^6\text{m}^{-1} = 10^6 \times 10^{-10} \text{ Å}^{-1}\right]$

$\therefore$
$$\phi = 1.24 \text{ eV}$$

When light of wavelength λ is used the stopping potential is $V_S = 1$ volt

$\therefore \qquad k_{max} = 1 \text{ eV}$

$\therefore$ Energy of incident photon

$$E = \frac{hc}{\lambda} = \phi + k_{max} = 2.24 \text{ eV}$$

$$\therefore \qquad \lambda = \frac{hc}{2.24 \text{ eV}} = \frac{12400 \text{ eVÅ}}{2.24 \text{ eV}} = 5536 \text{ Å}$$

(b) Saturation current $i_s = 0.8 \times 10^{-3}$ A

Number of photoelectrons emitted is

$$n_e = \frac{i_s}{e} = \frac{0.8 \times 10^{-3}}{1.6 \times 10^{-19}} \text{ s}^{-1} = 5 \times 10^{15} \text{ s}^{-1}$$

Since out of 100 photons, only 2 emit electrons; number of photons incident will be-

$$N_P = 50 \times n_e = 2.5 \times 10^{17} \text{ s}^{-1}$$

Incident power $\qquad P = EN_P = (2.24 \times 1.6 \times 10^{-19})(2.5 \times 10^{17})$ J/s

$$= 8.96 \times 10^{-2} = 0.089 \text{ watt.}$$

52. Point Q is the centre of a cube of side length L, one of the faces of which is the plate. One sixth of the total radiation strikes the plate.

Point P is centre of a cube of side length 2 L and the given plate is $\frac{1}{4}$ of one face of cube. Hence radiation striking the plate is $\frac{1}{24}$ of the total radiation emitted.

$\therefore \qquad F = 4F_0$

53. Momentum conservation gives

$$\frac{h}{\lambda_1} = -\frac{h}{\lambda_2} + \sqrt{2mk} \qquad \text{...(i)}$$

Energy conservation given

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + k$$

$$\Rightarrow \qquad \frac{h}{\lambda_1} = \frac{h}{\lambda_2} + \frac{k}{c} \qquad \text{...(ii)}$$

Add (i) and (ii)

$$\frac{2h}{\lambda_1} = \sqrt{2mk} + \frac{k}{c}$$

$$\Rightarrow \qquad \lambda_1 = \frac{2h}{\sqrt{2mk} + \frac{k}{c}}$$

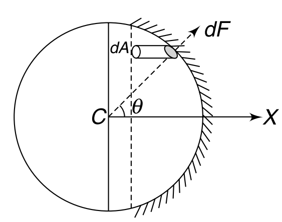
You must be able to argue why the wavelength of emitted photon will be different from the incident one?

54. Intensity of light on the mirror surface $I = \frac{W}{4\pi R^2}$

Consider a small patch of area dS on the surface of the mirror. Energy incident per unit time on this area is

$$dE = IdS = \frac{WdS}{4\pi R^2}$$

Momentum incident on area $\qquad dS = \frac{dE}{c} = \frac{WdS}{4\pi R^2 c}$



Light is reflected back towards the centre of the sphere, hence change in momentum per unit time for area dS is

$$= \frac{2WdS}{4\pi R^2 c}$$

This is equal to force on dS.

$$dF = \frac{2\,WdS}{4\,\pi R^2 c}$$

By symmetry the resultant force is along X direction

$$\therefore \qquad dF_X = dF\cos\theta = \frac{2W(dS\cos\theta)}{4\,\pi R^2 c}$$

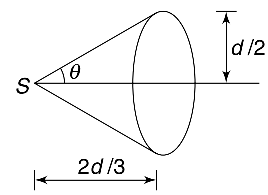
Projection of dS on vertical plane (in figure) is $dA = dS\cos\theta$

$$\therefore \qquad F_X = \frac{2W}{4\,\pi R^2 c}\int dS\cos\theta = \frac{2W}{4\,\pi R^2 c}\int dA$$

$$= \frac{2W}{4\,\pi R^2 c}\,\pi\left(\frac{d}{2}\right)^2 = \frac{Wd^2}{8R^2 c}$$

55.

$$\tan\theta = \frac{d}{2}\times\frac{3}{2d} = \frac{3}{4}$$



$$\therefore \qquad \cos\theta = \frac{4}{5}$$

$\therefore$ Solid angle subtended by the lens at the source S is given by

$$\Omega = 2\pi(1-\cos\theta) = 2\pi\left(1-\frac{4}{5}\right) = \frac{2\pi}{5}\ sr.$$

$\therefore$ Power incident on lens

$$P_0 = \frac{500}{4\pi}\Omega = 50 \text{ watt}$$

Transmitted power (P) = $0.4 \times 50 = 20$ W

The transmitted beam strikes the reflecting surface normally. Momentum incident per unit time on the reflecting surface = $\frac{P}{c}$

$$\therefore \qquad \text{Force} = \frac{2P}{c} = \frac{2\times 20}{3\times 10^8} = 1.33\times 10^{-7}\text{ N}$$

56. The prism is set for minimum deviation condition

$$r_1 = r_2 = A \quad\Rightarrow\quad r_1 = r_2 = 30^\circ$$

Angle of incidence (i) = Angle of emergence (e)
Snell's law gives-

$$1\cdot\sin i = \sqrt{2}\sin 30^\circ \quad\Rightarrow\quad i = 45^\circ$$

$\therefore$ Deviation

$$\delta = i + e - A$$

$$= 45^\circ + 45^\circ - 60^\circ = 30^\circ$$

The incident photons have momentum directed at 15° to the horizontal (up with horizontal) and the emergent photons have their momentum making 15° to the horizontal (down).

$\therefore$ Change in momentum of one photon is

$$\Delta P_{\text{one}} = 2P\cos 15^\circ \text{ (in vertical direction)}$$

$$= \frac{2h}{\lambda}\cos 15^\circ$$

Number of photons incident per second is

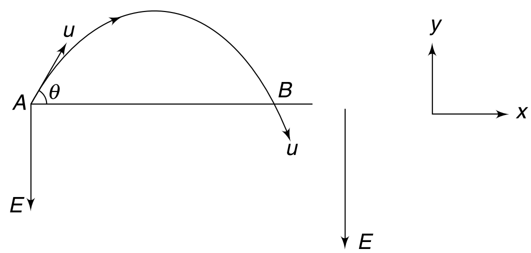
$$n = \frac{P_0}{\frac{hc}{\lambda}} = \frac{P_0\lambda}{hc}$$

$\therefore$ Vertically downward force on the prism applied by the incident beam is

$$F = n\Delta P_{\text{one}} = \frac{P_0\lambda}{hc}\frac{2h}{\lambda}\cos 15°$$

$$= \frac{2P_0}{c}\cos 15°$$

57. Speed of 'Y' does not change. de – Broglie wavelength of 'X' will becomes equal to that of 'Y' when speed of 'X' again become equal to its initial speed 'u'. This is possible in a situation shown in figure. Proton starts from A with speed 'u' and reaches point B with same speed 'u'. [compare this to a projectile]



(a) T = time of flight from A to B

$$T = \frac{2u_y}{a} \qquad \left[a = \frac{eE}{m}\right]$$

$\therefore$ $$u_y = \frac{eET}{2m}$$

$\Rightarrow$ $$u\sin\theta = \frac{eET}{2m}$$

$$\theta = \sin^{-1}\left(\frac{eET}{2mu}\right)$$

Desired angle $$\alpha = \frac{\pi}{2} + \theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{eET}{2mu}\right)$$

(b) $$u_x = \sqrt{u^2 - u_y^2} = \frac{\pi}{2} + \sin^{-1}\left(\frac{\sigma ET}{2u}\right)$$

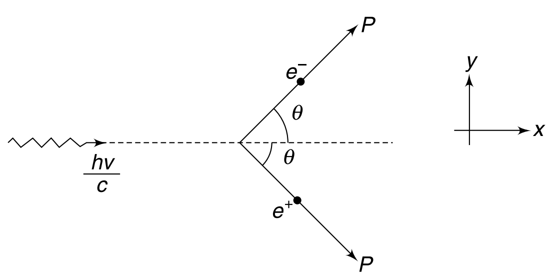
$$AB = u_x \cdot T$$

$$= \sqrt{u^2 - u_y^2}\cdot T$$

$$= \sqrt{u^2 - \frac{e^2E^2T^2}{4m^2}}\cdot T$$

$$= \sqrt{u^2 - \frac{\sigma^2E^2T^2}{4}}\cdot T$$

58. (a) To conserve momentum, the two particles (electron and positron) will move with equal momentum (P) making equal angle with the original line of motion of the photon.



For conservation of momentum along X

$$2P\cos\theta = \frac{hv}{c}$$

$\Rightarrow$ $$hv = 2Pc\cos\theta$$

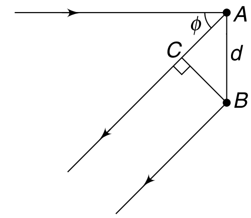
$\Rightarrow$ $$hv = 2(mV)\,c\cos\theta = 2mc^2\left(\frac{V}{c}\right)\cos\theta$$

Since $\frac{V}{c} < 1$ and $\cos\theta \le 1$

$\therefore$ $$hv < 2mc^2$$

But energy conservation requires that hv must be equal to rest mass energy ($2mc^2$) of the particle plus their kinetic energies. Hence it is impossible to conserve both energy and momentum unless some other particle is also involved.

(b) In presence of a nucleus the process is possible. The massive nucleus can carry a lot of momentum despite having negligible kinetic energy. Now the pair particles are required to carry less momentum and it is possible to conserve momentum and energy.



59. Path difference between the wave reflected at A and B is

$$\Delta x \simeq AC = d\sin\phi$$

For constructive interference $\Delta x = n\lambda \qquad n = 1, 2, 3, 4 \ldots$

$$\Rightarrow \qquad d\sin\phi = n\lambda$$

But
$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2\text{meV}}}$$

$$\therefore \qquad d\sin\phi = \frac{nh}{\sqrt{2\text{meV}}}$$

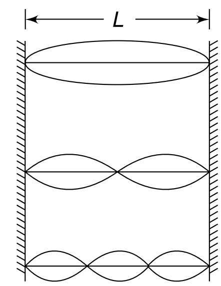
$$\therefore \qquad \phi = \sin^{-1}\left(\frac{nh}{d\cdot\sqrt{2\text{meV}}}\right)$$

60. The permitted wavelengths are

$2L, L, \frac{2L}{3} \ldots$

i.e.,
$$\lambda = \frac{2L}{n} \quad \text{where} \quad n = 1, 2, 3, 4 \ldots$$

de-Broglie wavelength will be given by-



$$\lambda_d = \frac{h}{P} = \frac{h}{\sqrt{2mK_n}} \qquad [K_n \text{ is KE for } n^{\text{th}} \text{ mode}]$$

$$\therefore \qquad \frac{h}{\sqrt{2mK_n}} = \frac{2L}{n}$$

$$\therefore \qquad K_n = \frac{n^2h^2}{8\text{ mL}^2}$$

Since the particle has only KE, hence energy of the particle is

$$E_n = \frac{n^2h^2}{8\text{ mL}^2};\ n = 1, 2, 3, 4$$

(a) $E_3 - E_1 = (3^2 - 1^2)\dfrac{h^2}{8\text{ mL}^2} = \dfrac{h^2}{\text{mL}^2}$

(b) $E_1 = \dfrac{h^2}{8\text{ mL}^2} = \dfrac{(6.6\times10^{-34})^2}{8\times10^{-6}\times(10^{-2})^2} = 5\times10^{-54}\text{ J}$

Note: Quantization of energy is conspicuous only when m and L both are very small.

61. Energy absorbed by Li^{++} to move from $n = 2$ to $n = 3$ state is

$$\Delta E = E_3 - E_2 = 13.6\,(3)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= \frac{13.6\times9\times5}{36} = 17\text{ eV}$$

Energy released by H atom in transition $n = 2$ to $n = 1$ is

$$\Delta E' = E_2 - E_1 = 10.2\text{ eV}$$

Thus $\Delta E - \Delta E' = 6.8$ eV must come from loss in KE in collision.

Momentum conservation warrants that momentum of H and Li^{++} after collision = momentum of hydrogen before collision (say = P).

$$\text{Loss in KE} = \frac{P^2}{2m_{\text{H}}} - \frac{P^2}{2(m_{\text{H}} + m_{\text{Li}})}$$

$$\Rightarrow \qquad 6.8 \text{ eV} = \frac{P^2}{2m_H}\left[1 - \frac{m_H}{m_H + m_{Li}}\right]$$

$$\Rightarrow \qquad 6.8 \text{ eV} = K_H\left[\frac{m_{Li}}{m_H + m_{Li}}\right]$$

$$\Rightarrow \qquad K_H = 6.8 \times \frac{8m_H}{7m_H} = 6.8 \times \frac{8}{7} = 7.77 \text{ keV}$$

62. (a) $E'_\infty - E'_2 = 5.39$ eV

$$\Rightarrow \qquad 0 + 13.6\,\frac{(Z')^2}{2^2} = 5.36$$

$$\Rightarrow \qquad Z' = 1.26$$

It is logical to expect $1 < Z' < 3$ since we do not expect the screening by two electrons to be perfect.

(c) By absorbing longest wavelength photon the electron will transit from $n = 2$ to $n = 3$

$$\therefore \qquad \frac{hc}{\lambda} = E'_3 - E'_2$$

$$= Z'^2 \times 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= (1.26)^2 \times 13.6 \times \frac{5}{36}$$

$$= 2.99 \text{ eV}$$

$$\therefore \qquad \lambda = \frac{hc}{2.99 \text{ eV}} = \frac{12420\,\text{eVÅ}}{2.99\,\text{eV}} = 4154 \text{ Å} \simeq 415 \text{ nm}$$

63. As we know energy of a photon is given by $E = \dfrac{hc}{\lambda}$

From the given condition $\lambda = \dfrac{1500p^2}{p^2 - 1}$

Hence, $E = \dfrac{hc}{\lambda} = \dfrac{hc}{1500}\left(1 - \dfrac{1}{p^2}\right) \times 10^{10}$ J; converted into eV it becomes

$$= \frac{hc}{(1500)(1.6 \times 10^{-19})}\left(1 - \frac{1}{p^2}\right) \times 10^{10} \text{ eV}$$

$$= 8.28\left(1 - \frac{1}{p^2}\right) \text{ eV}$$

Hence energy of n^{th} state is given by $E_n = -\dfrac{8.28}{n^2}$

i.e., ionization energy $= 8.28$ eV

Hence, ionization potential $= 8.28$ V

The energy of three lowest states are obtained by putting $n = 1, 2, 3$ in the above expression. Values are -8.28 eV, -2.07 eV, -0.92 eV.

$n = 3$ _____ -0.92 eV
$n = 2$ _____ -2.07 eV
$n = 1$ _____ -8.28 eV

64. (a) We assume that orbit is circular (though the electron will spiral down into the nucleus). If orbital radius at an instant is r then equating the electrostatic force to the centripetal force we get

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \Rightarrow a = \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 mr} \qquad \text{...(i)}$$

And $$v = \sqrt{\frac{e^2}{4\pi\in_0 r}}$$

Kinetic energy

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{1}{4\pi\in_0}\frac{e^2}{r} = \frac{e^2}{8\pi\in_0 r} \quad \text{...(ii)}$$

Time period of circular motion is

$$T = \frac{2\pi r}{v}$$

Energy loss in one revolution is $\Delta E \simeq \left(\frac{dE}{dt}\right)T$

$$= \frac{e^2a^2}{6\pi\in_0 c^3}\cdot\frac{2\pi r}{v} = \frac{e^2 r}{3\in_0 c^3 v}\left(\frac{v^2}{r}\right)^2 \quad \left[\because \ a = \frac{v^2}{r}\right]$$

$$= \frac{e^2v^3}{3\varepsilon_0 c^3 r}$$

$$\frac{\Delta E}{E} = \frac{\Delta E}{\frac{1}{2}mv^2} = \frac{e^2v^3}{3\in_0 c^3 r}\times\frac{8\pi\in_0 r}{e^2} \quad \text{[using (ii)]}$$

$$= \frac{8\pi}{3}\left(\frac{v}{c}\right)^3$$

(b) Since $v < < c$

$\therefore \quad \frac{\Delta E}{E} << 1$

This means that fraction of energy lost per revolution is small. And because of this reason the radius of the circular path is changing slowly. We can assume the path to be circular for a small interval of time.

65. $$U = U_0 \ell nr - U_0 \ell n r_0$$

$\Rightarrow$ $$\frac{dU}{dr} = \frac{U_0}{r}$$

$\therefore$ Force between the particles is

$$F = -\frac{dU}{dr} = -\frac{U_0}{r}$$

Negative sign indicates attraction

$\therefore$ $$\frac{mv^2}{r} = \frac{U_0}{r} \quad \text{...(i)}$$

And $$mvr = \frac{nh}{2\pi} \quad \text{...(ii)}$$

From (i) and (ii) $$r = \frac{nh}{2\pi\sqrt{mU_0}}$$

Kinetic energy of electron $$K = \frac{1}{2}mv^2 = \frac{U_0}{2} \quad \text{[using (i)]}$$

Energy of electron in nth orbit is

$$E_n = \text{KE} + \text{PE} = \frac{U_0}{2} + U_0 \ell n\left(\frac{r}{r_0}\right)$$

$$= \frac{U_0}{2} + U_0 \ell n\left[\frac{nh}{2\pi\sqrt{mU_0}}\cdot\frac{1}{r_0}\right]$$

$$= \frac{U_0}{2}\left[1 + \ell n\left(\frac{n^2 h^2}{4\pi^2 m U_0 r_0^2}\right)\right]$$

$$\Delta E_{nm} = E_n - E_m$$

$$= \frac{U_0}{2}\ell n\left(\frac{n^2 h^2}{4\pi^2 m U_0 r_0^2}\right) - \frac{U_0}{2}\ell n\left(\frac{m^2 h^2}{4\pi^2 m U_0 r_0^2}\right)$$

$$= \ell n\left(\frac{n}{m}\right)^2 = 2\ell n\left(\frac{n}{m}\right)$$

$$\therefore \quad \frac{\Delta E_{12}}{\Delta E_{24}} = \frac{\ell n\left(\frac{1}{2}\right)}{\ell n\left(\frac{2}{4}\right)} = 1$$

66. Let E_1, E_2, E_3 and E_4 be energy of photons of wavelength λ_1, λ_2, λ_3 and λ_4 respectively.

$$E_1 = \phi + 5.2 \text{ eV} \quad \text{...(i)}$$

$$E_2 = \phi + 12 \text{ eV} \quad \text{...(ii)}$$

$$E_4 = \phi + 0.95 \text{ eV} \quad \text{...(iii)}$$

But $\lambda_2 = 0.6\,\lambda_1$

$\Rightarrow$ $E_1 = 0.6\,E_2$ ...(iv)

From (i) and (ii)

$$0.6 = \frac{\phi + 5.2}{\phi + 12}$$

Solving we get: $\phi = 5$ eV

$\therefore$ $E_1 = 10.2$ eV; $E_2 = 17$ eV; $E_4 = 5.95$ eV

And $\because$ $\lambda_3 = 4\lambda_2$

$\therefore$ $E_2 = 4E_3 \Rightarrow E_3 = \frac{17}{4} = 4.25$ eV

For atom

$$-13.6Z^2\left[\frac{1}{n^2} - \frac{1}{4}\right] = 10.2 + 17$$

$$\Rightarrow \quad Z^2\left[\frac{1}{4} - \frac{1}{n^2}\right] = 2 \quad \text{...(A)}$$

Also
$$-13.6Z^2\left[\frac{1}{n^2} - \frac{1}{9}\right] = 4.25 + 5.95$$

$$\Rightarrow \quad Z^2\left[\frac{1}{9} - \frac{1}{n^2}\right] = 10.2 \quad \text{...(B)}$$

Solving (A) and (B)

$$Z = 3 \text{ and } n = 6$$

67. Energy conservation gives

Energy of H atom in nth state + $\frac{hc}{\lambda_1}$ = 12.2 eV

$$\Rightarrow \quad -\frac{13.6}{n^2}\text{ eV} + \frac{hc}{\lambda_1} = 12.2 \text{ eV}$$

$$\Rightarrow \quad -\frac{13.6}{n^2} + \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{796 \times 10^{-10} \times 1.6 \times 10^{-19}} = 12.2$$

$$\Rightarrow \qquad \frac{13.6}{n^2} = 15.6 - 12.2 = 3.4 \text{ eV}$$

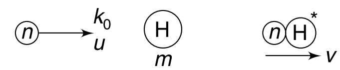
$$\therefore \qquad n = 2$$

Hydrogen atom formed is in 1st excited state.

$$\therefore \qquad \frac{hc}{\lambda_2} = 10.2 \text{ eV}$$

$$\lambda_2 = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{10.2 \times 1.6 \times 10^{-19}} = 1217 \text{ Å}$$

68. (a) For collision to be inelastic, the atom must get excited. Minimum excitation energy for hydrogen atom is



$$\Delta E = -\frac{13.6}{4} - (-13.6) = 10.2 \text{ eV}$$

In a collision, maximum loss of KE (i.e. maximum energy available for excitation of the atom) will happen when both the particles move together with same speed after collision.

$$2mv = mu$$

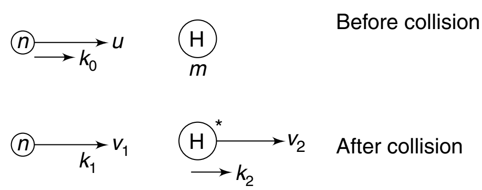
$$\Rightarrow \qquad v = \frac{u}{2}$$

$$\text{Loss in KE} = \frac{1}{2}mu^2 - \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{1}{4}mu^2 = \frac{k_0}{2}$$

$$\Rightarrow \qquad \frac{k_0}{2} = 10.2$$

$$\Rightarrow \qquad k_0 = 20.4 \text{ eV} = \text{minimum } k_0 \text{ for excitation of hydrogen atom.}$$

(b) Excitation energy



$$\Delta E = -\frac{13.6}{9} - (-13.6) = 12.09 \text{ eV}$$

Momentum conservation gives-

$$mv_1 + mv_2 = mu$$

$$\Rightarrow \qquad \sqrt{2mk_1} + \sqrt{2mk_2} = \sqrt{2mk_0}$$

$$\Rightarrow \qquad k_1 + k_2 + 2\sqrt{k_1 k_2} = k_0 = 25 \qquad \text{...(i)}$$

Energy conservation gives-

$$k_1 + k_2 + \Delta E = k_0$$

$$\Rightarrow \qquad k_1 + k_2 = 25 - 12.09 = 12.91 \text{ eV} \qquad \text{...(ii)}$$

From (i) and (ii)

$$2\sqrt{k_1 k_2} = 12.09$$

$$(k_1 - k_2)^2 = (k_1 + k_2)^2 - 4k_1k_2$$

$$= (12.91)^2 - (12.09)^2 = 20.5$$

$$\therefore \qquad k_1 - k_2 = \pm 4.53 \text{ eV} \qquad \text{...(iii)}$$

Solving (ii) and (iii)

$$k_1 = 8.72 \text{ eV}; \ k_2 = 4.19 \text{ eV}$$

Or, $\qquad k_1 = 4.19 \text{ eV}; \ k_2 = 8.72 \text{ eV}$

69. (a) From figure

$$\frac{1}{\lambda_{\min}} = 8.06 \ (\text{Å})^{-1}$$

If the bombarding electrons have KE of E (in eV units) then

$$\frac{12420}{\lambda_{\min}} = E \quad [\text{Where } \lambda_{\min} \text{ is in Å}]$$

$$\therefore \qquad E = 12420 \times 8.06 \simeq 10^5 \text{ eV}$$

$\therefore$ Accelerating potential difference is 10^5 volt.

(b) k_a will have larger wavelength $\left(\text{and hence smaller } \frac{1}{\lambda}\right)$

$$\therefore \qquad \frac{1}{\lambda_{k\alpha}} = 4.34\ (\text{Å})^{-1} \quad \Rightarrow \quad \lambda_{k\alpha} = 0.23\,\text{A}^\circ$$

(c) We know

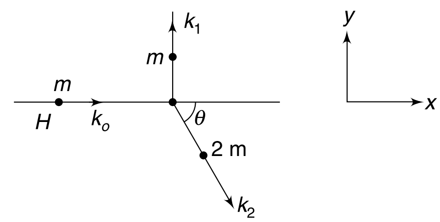
$$\frac{1}{\lambda_{k\alpha}} = (Z-1)^2 R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \qquad 4.34 \times 10^{10} \text{ m} = (Z-1)^2 \times 1.09 \times 10^7 \times \frac{3}{4}$$

$$\therefore \qquad (Z-1) = 73$$

$$Z = 74$$

71. Let the initial line of motion of hydrogen atom be along x direction and its kinetic energy be $k_0 = 30$ eV
After collision the kinetic energy of hydrogen and deuterium be k_1 and k_2 respectively.
For conservation of momentum along x and y we must have



$$\sqrt{2mk_0} = \sqrt{2(2m)k_2} \cos\theta \qquad \text{...(i)}$$

$$\sqrt{2mk_1} = \sqrt{2(2m)k_2} \sin\theta \qquad \text{...(ii)}$$

Squaring and adding the above two equations gives

$$k_0 + k_1 = 2k_2$$

$$\therefore \qquad 2k_2 - k_1 = 30 \text{ eV} \qquad \text{...(iii)}$$

The sum $k_1 + k_2$ will be equal to 30 eV minus the excitation energy of atoms. Let's check all possible cases.

Case-1: None of the atoms is excited.

$$k_1 + k_2 = 30 \qquad \text{...(iv)}$$

Solving (iii) and (iv)

Gives $\qquad k_2 = 20$ eV and $k_1 = 10$ eV

Case-2: One of the atoms is excited to $n = 2$ state.

$$k_1 + k_2 = 30 - 10.2 = 19.8 \text{ eV} \qquad \text{...(v)}$$

Solving (iii) and (v) gives

$$k_2 = 16.6 \text{ eV and } k_1 = 3.2 \text{ eV}$$

Case-3: One of the atoms is excited to $n = 3$ state.

$$k_1 + k_2 = 30 - 12.09 = 17.91 \text{ eV} \qquad \text{...(vi)}$$

Solving (iii) and (vi) gives

$$k_2 = 15.97 \text{ eV and } k_1 = 1.94 \text{ eV}$$

Case-4: Now consider the case when one atom is excited to $n = \infty$ state

$$k_1 + k_2 = 30 - 13.6 = 16.4 \text{ eV} \qquad \text{... (vii)}$$

Solving (iii) and (vii) gives

$$k_2 = 15.47 \text{ eV}, k_1 = 0.93$$

Values of kinetic energies are positive. It means one atom can get excited to all possible states. There can be infinite cases between case 3 and 4.

Case-5: Both atoms are excited to $n = 2$ state.

$$k_1 + k_2 = 30 - 2 \times 10.2 = 9.6 \text{ eV} \qquad \text{... (viii)}$$

Solving (iii) and (viii) gives

$$k_2 = 13.2 \text{ eV and } k_1 = -3.6 \text{ eV}$$

Negative KE is meaningless. Therefore, excitation of both atoms is not possible

$$\therefore \qquad (k_2)_{max} = 20 \text{ eV}$$

$$\text{And} \qquad (k_2)_{min} = 13.2 \text{ eV}$$

72. The initial momentum of the atom is zero. Thus if the emitted photon carries momentum P, the atom must recoil with equal momentum.

For energy to remain conserved we must have

$$KE_{atom} + E_{Photon} = \Delta E \qquad [\Delta E = 10.2 \text{ eV}]$$

$$\therefore \qquad \frac{P^2}{2m} + P \cdot c = \Delta E \quad \Rightarrow \quad P^2 + (2mc)\,P - 2m\,\Delta E = 0$$

$$\Rightarrow \qquad P = \frac{-2mc \pm (4m^2c^2 + 8m\Delta E)^{1/2}}{2}$$

$$= mc\left[-1 + \left(1 + \frac{2\,\Delta E}{mc^2}\right)^{1/2}\right]$$

We have retained + sign only because magnitude of momentum cannot be negative.

$$\text{Now} \qquad \frac{2\Delta E}{mc^2} = \frac{20.2\,\text{eV}}{940\,\text{MeV}} = 2.2 \times 10^{-8}$$

$$\therefore \qquad \frac{\Delta E}{mc^2} << 1$$

$$\therefore \qquad P \simeq mc\left[-1 + 1 + \frac{\Delta E}{mc^2} - \frac{1}{8}\left(\frac{2\Delta E}{mc^2}\right)^2\right]$$

We have neglected all other higher order terms.

$$\therefore \qquad P = mc\left[\frac{\Delta E}{mc^2} - \frac{1}{8}\left(\frac{2\Delta E}{mc^2}\right)^2\right]$$

$$P = \frac{\Delta E}{c}\left[1 - \frac{\Delta E}{2mc^2}\right]$$

$\frac{\Delta E}{c} = P_0$ = momentum of photon if recoil of atom is not taken into account.

$$\therefore \qquad P = P_0\left[1 - \frac{\Delta E}{2mc^2}\right]$$

$$\therefore \qquad \frac{P - P_0}{P_0} = -\frac{\Delta E}{2mc^2}$$

$$\text{Percentage error} = \frac{\Delta E}{2mc^2} \times 100$$

$$= \frac{10.2 \text{ eV}}{2 \times 940 \times 10^6 \text{ eV}} \times 100$$

$$= 5.4 \times 10^{-7}\%$$