Similarity (As a Size Transformation) 14

STUDY NOTES

- Any two figures are said to be similar if they have exactly the same shape but not necessarily the same size.
- Size transformation is the process in which a figure is enlarged or reduced by a scale factor k such that the resulting figure is similar to the given figure. The given figure is called an object or the pre-image and the resulting figure is called its image.
- Let k be the scale factor of a size transformation, then
  (i) $k > 1$ implies enlargement
  (ii) $k = 1$ implies identity transformation.
  (iii) $k < 1$ implies reduction
- Each side of the resulting figure = $k \times$ corresponding side of the given figure.
- Area of the resulting figure = $k^2 \times$ area of the given figure.
- Volume of the resulting figure = $k^3 \times$ volume of the given figure.
- Let the model of a plane figure be drawn to the scale 1 : p. Then, scale factor $k = \frac{1}{p}$.
  (i) Length of the model = $k \times$ length of actual figure.
  (ii) Area of the model = $k^2 \times$ area of the actual figure.
  (iii) Volume of the model = $k^3 \times$ volume of the actual figure.
- Let the map of a plane figure be drawn to the scale 1 : p. Then, scale factor $k = \frac{1}{p}$.
  (i) Length of the map = $k \times$ actual length
  (ii) Area of the map = $k^2 \times$ actual area.

QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

1. Figures which have exactly the same shape, but not necessarily the same ____________, are said to be similar.
(a) angle (b) side (c) size (d) volume

2. All regular polygons having the same number of _____________ are similar.
(a) sides (b) medians (c) diagonals (d) altitudes

3. Two circles are always :
(a) congruent (b) similar (c) enlarged (d) concentric

4. In size transformation, the given figure is called an object and the resulting figure is called its :
(a) pre-image (b) image (c) post-image (d) enlarge object

5. Let k be the scale factor of a given size transformation. Then $k < 1$ as the transformation is a :
(a) enlargement (b) identify transformation (c) reduction (d) preserved

6. Each side of the resulting figure = ___________ times the corresponding side of the given figure.
(a) k^2 (b) k (c) k^3 (d) $2k$

7. The transformation is an ______________, if $k = 1$ where k is the scale factor of a given size transformation.

(a) identify transformation (b) reduction (c) enlargement (d) none

8. In case of solids, we have volume of the resulting figure = ____________ × (volume of the given figure), where k is the scale factor.

(a) k (b) k^2 (c) k^3 (d) $3k$

9. If scale factor, $k = \frac{1}{p}$, then area of the model = __________ × (area of the actual figure).

(a) k^2 (b) k (c) k^2 (d) $\frac{1}{p}$

10. Let the map of a plane figure be drawn to the scale 1 : p. Then scale factor k = ________, length in the map = k × (Actual length).

(a) $\frac{p}{1}$ (b) $\frac{1}{p}$ (c) $\frac{1}{k}$ (d) k

Answers :

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (c) **6.** (b) **7.** (a) **8.** (c) **9.** (a) **10.** (b)

B. Short Answer Type Questions [3 Marks]

1. ΔABC with sides AB = 12 cm, BC = 8 cm and AC = 14 cm is enlarged to ΔA′B′C′ such that the smallest side of ΔA′B′C′ is 12 cm. Find the scale factor and use it to find the length of the other sides of the image ΔA′B′C′.

Sol. Seale factor $k = \frac{B'C'}{BC} = \frac{12}{8} = \frac{3}{2}$

So, $A'B' = k \times AB = \frac{3}{2} \times 12 = 18$ cm

$A'C' = k \times AC = \frac{3}{2} \times 14 = 21$ cm.

2. ΔABC is reduced by a scale factor 0.72. If the area of ΔABC is 62.5 cm^2, find the area of the image.

Sol. Area of image = k^2 × area of original figure

$= (0.72)^2 \times 62.5 \text{ cm}^2 = 32.4 \text{ cm}^2$.

3. A rectangle having an area of 60 cm^2 is transformed under enlargement about a point in space. If the area of its image is 135 cm^2, find the scale factor of the enlargement.

Sol. Here, area of image = k^2 × area of original rectangle

$\Rightarrow 135 = k^2 \times 60$

$\Rightarrow k^2 = \frac{135}{60} = \frac{9}{4} \Rightarrow k = \frac{3}{2} = 1.5$

4. In the map of a rectangular plot of land the length = 2.5 cm and breadth = 1.4 cm. If the scale is 1 : 10000, then find the area of the plot in m^2.

Sol. Here $k = \frac{1}{10000}$

Area, on the map = k^2 × actual area

$\Rightarrow 2.5 \times 1.4 \text{ cm}^2 = \left(\frac{1}{10000}\right)^2 \times \text{Actual area}$

$\Rightarrow$ Actual area $= 3.5 \times (10000)^2 \text{ cm}^2$

$= \frac{3.5 \times (10000)^2}{10000} \text{ m}^2 = 35000 \text{ m}^2$

5. The surface area of a solid is 5 m^2, while the surface area of its model is 20 cm^2. Find.

(i) the scale factor

(ii) the volume of the solid if the volume of the model is 100 cm^3.

Sol. (i) Area of model = k^2 × area of solid

$\Rightarrow 20 \text{ cm}^2 = k^2 \times 5 \times 10000 \text{ cm}^2$

$$\Rightarrow k^2 = \frac{20}{5\times10000} = \frac{1}{2500} \Rightarrow k = \frac{1}{50}$$

(ii) Volume of model = k^3 × volume of solid

$$\Rightarrow 100 \text{ cm}^3 = \frac{1}{125000} \times \text{volume of solid}$$

$$\Rightarrow \text{volume of solid} = \frac{125000 \times 100}{1000000} \text{ m}^3 = 12.5 \text{ m}^3.$$

6. Two bottles of sauce of circular cross-section are completely similar in every respect. One is 24 cm high and the other is 32 cm high.

(i) Calculate the external diameter of the smaller bottle, given that the corresponding diameter for the other bottle is 8 cm.

(ii) The smaller bottle can hold 270 cm^3 of sauce. How much sauce can the bigger bottle hold?

Sol. (i) $\frac{H_1}{H_2} = \frac{D_1}{D_2}$

$$\Rightarrow \frac{24}{32} = \frac{D_1}{8} \Rightarrow D_1 = 6 \text{ cm}.$$

(ii) $\frac{{D_1}^3}{{D_2}^3} = \frac{270}{V_1}$

$$\Rightarrow V_1 = \frac{8^3}{6^3} \times 270 = 640 \text{ cm}^3.$$

7. The model of a building is constructed with scale factor 1 : 30.

(i) If the height of the model is 80 cm, find the actual height of the building in metre.

(ii) If the actual volume of the tank on the top of the building is 27 m^3, find the volume of the tank on the top of the model.

Sol. Here $k = \frac{1}{30}$

(i) Height of mode = k × Actual height

$$\Rightarrow 80 \text{ cm} = \frac{\text{Actual height}}{30}$$

$\Rightarrow$ Actual height = 2400 cm = 24 m.

(ii) Volume of model = k^3 × volume of tank

$$= \left(\frac{1}{30}\right)^3 \times 27 \text{ m}^3$$

$$= \frac{27\times1000000}{27000} \text{ cm}^3 = 1000 \text{ cm}^3.$$

8. Two similar jugs have heights of 4 cm and 6 cm respectively. If the capacity of the smaller jug is 48 cm^3, find the capacity of the larger jug.

Sol. Here, $\frac{{H_1}^3}{{H_2}^3} = \frac{V_1}{V_2}$

$$\Rightarrow \frac{4^3}{6^3} = \frac{48}{V_2} \Rightarrow V_2 = \frac{6^3 \times 48}{4^3}$$

$\Rightarrow V_2 = 162$ cm^3.

9. Two similar cylindrical tins have base radii of 6 cm and 8 cm respectively. Find the capacity of the smaller tin, if the capacity of the larger is 256 cm^3.

Sol. Here, $\frac{{R_1}^3}{{R_2}^3} = \frac{V_1}{V_2}$

$$\Rightarrow \frac{6^3}{8^3} = \frac{V_1}{256} \Rightarrow V_1 = \frac{256 \times 6^3}{8^3} = 108 \text{ cm}^3.$$

C. Long Answer Type Questions [4 Marks]

1. The model of a ship is made to a scale 1 : 200.

(i) The length of the model is 4 m. Calculate the length of the ship.

(ii) The area of the deck of the ship is 1,60,000 m^2. Find the area of the deck of the model.

(iii) The volume of the model is 200 litres. Calculate the volume of the ship in m^3.

Sol. Here, $k = \frac{1}{200}$

(i) Length of the model = k × length of the ship

$\Rightarrow 4 \text{ m} = \frac{1}{200} \times$ length of the ship

$\Rightarrow$ Length of the ship = 800 m

(ii) Area of the deck of the model = k^2 × actual area of the deck

$= \left(\frac{1}{200}\right)^2 \times 1{,}60{,}000 \text{ m}^2 = 4 \text{ m}^2.$

(iii) Volume of the model = k^3 × actual volume of the ship

$\frac{200}{1000} \text{ m}^3 = \left(\frac{1}{200}\right)^3 \times$ volume of the ship

$\Rightarrow$ Volume of the ship $= \frac{200 \times (200)^3}{1000} \text{ m}^3 = 16{,}00{,}000 \text{ m}^3.$

2. On a map drawn to a scale of 1 : 2,50,000 a triangular plot of land has the following measurements:
AB = 3 cm, BC = 4 cm, $\angle ABC = 90°$
Calculate : (i) the actual length of AB in km (ii) the area of the plot in km^2.

Sol. (i) Here, $k = \frac{1}{2{,}50{,}000}$

Length of AB on map = k × Actual length of AB.

$\Rightarrow 3 \text{ cm} = \frac{1}{2{,}50{,}000} \times$ Actual length of AB

$\Rightarrow$ Actual length of AB $= \frac{3 \times 2{,}50{,}000}{100000}$ km = 7.5 km.

(ii) Area on the map = k^2 × actual area

$= \frac{4 \times 3}{2} \text{ cm}^2 = \left(\frac{1}{2{,}50{,}000}\right)^2 \times$ Actual area

$\Rightarrow$ Actual area $= \frac{6 \times (2{,}50{,}000)^2}{(100000)^2} \text{ km}^2 = 37.5 \text{ km}^2.$

3. On a map drawn to a scale of 1 : 25,000, a rectangular plot of land ABCD has the following measurements.
AB = 12 cm, BC = 16 cm.

(i) The diagonal distance of the plot in km.

(ii) The area of the plot in km^2.

Sol. Here, $k = \frac{1}{25{,}000}$

Diagonal on the map = $\sqrt{12^2 + 16^2} = \sqrt{400}$ cm = 20 cm

(i) Diagonal distance on the map = k × actual diagonal

$\Rightarrow 20 \text{ cm} = \frac{1}{25{,}000} \times$ actual diagonal

$\Rightarrow$ Actual diagonal $= \frac{20 \times 25{,}000}{1{,}00{,}000}$ km = 5 km.

(ii) Area on the map = k^2 × area of the plot

$$\Rightarrow 12 \times 16 \text{ cm}^2 = \left(\frac{1}{25,000}\right)^2 \times \text{area of the plot}$$

$$\Rightarrow \text{Area of the plot} = \frac{12 \times 16 \times (25000)^2}{(1,00,000)^2} \text{ km}^2 = 12 \text{ km}^2.$$

4. The scale of a model ship is 1 : 300.

(i) If the length of the model is 250 cm, find the actual length in m.

(ii) If the deck area of the model is 1 m^2, find the deck area of the ship and the cost of painting it at ₹10 per m^2.

(iii) If the volume of the ship is 10,80,00,000 m^3, find the volume of the model.

Sol. Here, $k = \frac{1}{300}$

(i) Length of model = k × Actual length

$$\Rightarrow 250 \text{ cm} = \frac{1}{300} \times \text{actual length}$$

$$\Rightarrow \text{Actual length} = \frac{250 \times 300}{100} \text{ m} = 750 \text{ m}$$

(ii) Area on the model = k^2 × actual area

$$\Rightarrow 1 \text{ m}^2 = \left(\frac{1}{300}\right)^2 \times \text{actual area}$$

$$\Rightarrow \text{Actual area} = 300 \times 300 \text{ m}^2 = 90,000 \text{ m}^2.$$

Cost of painting = ₹10 × 90,000 = ₹9,00,000.

(iii) Volume of the model = k^3 × volume of the ship

$$= \left(\frac{1}{300}\right)^3 \times 10,80,00,000 \text{ m}^3 = 4 \text{ m}^3.$$

5. The dimensions of the model of a multistoreyed building are 1 m × 60 cm × 1.25 m. If the model is drawn to a scale 1 : 60, find the actual dimensions of the model in metres. Also find

(i) the floor area of a room of the building, whose area in the model is 250 cm^2.

(ii) the volume of the room in the model whose actual volume is 648 m^3.

Sol. Here, $k = \frac{1}{60}$

Dimensions in model = k × actual dimensions.

So, dimensions of the building are (1 m × 60) × (60 cm × 60) (1.25 m × 60), i.e., 60 m × 36 m × 75 m

(i) Area in the model = k^2 × Actual area

$$\Rightarrow 250 \text{ cm}^2 = \left(\frac{1}{60}\right)^2 \times \text{Actual area}$$

$$\Rightarrow \text{Actual Area} = \frac{250 \times (60)^2}{10,000} \text{ m}^2 = 90 \text{ m}^2$$

(ii) Volume of model = k^3 × actual volume

$$= \left(\frac{1}{60}\right)^3 \times 648 \text{ m}^3 = \frac{648 \times 10,00,000}{(60)^3} \text{ cm}^3 = 3,000 \text{ cm}^3.$$

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