

CHAPTER

8

Circle

Section-A

JEE Advanced/ IIT-JEE

A

Fill in the Blanks

- If A and B are points in the plane such that $PA/PB = k$ (constant) for all P on a given circle, then the value of k cannot be equal to (1982 - 2 Marks)
- The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are and (1983 - 2 Marks)
- The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is (1984 - 2 Marks)
- Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point $(4, 5)$ with a pair of radii form a quadrilateral of area (1985 - 2 Marks)
- From the origin chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is (1985 - 2 Marks)
- The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is (1986 - 2 Marks)
- From the point $A(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that $AM = 2AB$. The equation of the locus of M is (1986 - 2 Marks)
- The area of the triangle formed by the tangents from the point $(4, 3)$ to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is (1987 - 2 Marks)
- If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that common chord is of maximum length and has a slope equal to $3/4$, then the coordinates of the centre of C_2 are (1988 - 2 Marks)
- The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is, (1989 - 2 Marks)
- If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of $\lambda =$ (1991 - 2 Marks)
- The equation of the locus of the mid-points of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $2\pi/3$ at its centre is (1993 - 2 Marks)
- The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB . Equation of the circle with AB as a diameter is (1996 - 1 Mark)
- For each natural number k , let C_k denote the circle with radius k centimetres and centre at the origin. On the circle C_k , α -particle moves k centimetres in the counter-clockwise direction. After completing its motion on C_k , the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at $(1, 0)$. If the particle crosses the positive direction of the x-axis for the first time on the circle C_n then $n =$ (1997 - 2 Marks)
- The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to circle $x^2 + y^2 = 1$ pass through the point (1997 - 2 Marks)

B

True / False

- No tangent can be drawn from the point $(5/2, 1)$ to the circumcircle of the triangle with vertices $(1, \sqrt{3})$, $(1, -\sqrt{3})$, $(3, -\sqrt{3})$. (1985 - 1 Mark)
- The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. (1989 - 1 Mark)

C

MCQs with One Correct Answer

- A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinate axes. The one vertex of the square is (1980)
 - $(1 + \sqrt{2}, -2)$
 - $(1 - \sqrt{2}, -2)$
 - $(1, -2 + \sqrt{2})$
 - none of these
- Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point $(1, 1)$ is (1980)
 - $x^2 + y^2 - 6x + 4 = 0$
 - $x^2 + y^2 - 3x + 1 = 0$
 - $x^2 + y^2 - 4y + 2 = 0$
 - none of these

3. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is (1983 - 1 Mark)
- (a) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (b) $\left(-\frac{16}{7}, \frac{53}{10}\right)$
- (c) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (d) none of these
4. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is (1983 - 1 Mark)
- (a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$
- (b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
- (c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
- (d) none of these
5. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is (1984 - 2 Marks)
- (a) $x + y = 2$ (b) $x^2 + y^2 = 1$
- (c) $x^2 + y^2 = 2$ (d) $x + y = 1$
6. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is (1988 - 2 Marks)
- (a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
- (b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
- (c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
- (d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$
7. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then (1989 - 2 Marks)
- (a) $2 < r < 8$ (b) $r < 2$ (c) $r = 2$ (d) $r > 2$
8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. Then the equation of this circle is (1989 - 2 Marks)
- (a) $x^2 + y^2 + 2x - 2y = 62$
- (b) $x^2 + y^2 + 2x - 2y = 47$
- (c) $x^2 + y^2 - 2x + 2y = 47$
- (d) $x^2 + y^2 - 2x + 2y = 62$
9. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is (1992 - 2 Marks)
- (a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{2}\right)$
10. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation: (1993 - 1 Marks)
- (a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$
- (c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$
11. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if (1994)
- (a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \leq r \leq 8$
12. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9 \sin^2 \alpha + 13 \cos^2 \alpha = 0$ is 2α . The equation of the locus of the point P is (1996 - 1 Mark)
- (a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$
- (c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$
13. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then (1999 - 2 Marks)
- (a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$
14. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) and (-4, 3) respectively, then $\angle QPR$ is equal to (2000S)
- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$
15. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$, $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is (2000S)
- (a) 2 or $-\frac{3}{2}$ (b) -2 or $-\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$
16. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is (2001S)
- (a) a parabola (b) a circle
- (c) an ellipse (d) a pair of straight lines
17. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals (2001S)
- (a) $\sqrt{PQ \cdot RS}$ (b) $(PQ + RS)/2$
- (c) $2PQ \cdot RS / (PQ + RS)$ (d) $\sqrt{(PQ^2 + RS^2)} / 2$
18. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets a straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is (2002S)
- (a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$
19. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is (2003S)
- (a) (4, 7) (b) (7, 4) (c) (9, 4) (d) (4, 9)
20. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is (2004S)
- (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2
21. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is (2005S)
- (a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \leq 0\}$
- (b) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \leq 0\}$
- (c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \leq 0\}$
- (d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \leq 0\}$

Circle

22. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B . The equation of the circumcircle of the triangle PAB is (2009)
- (a) $x^2 + y^2 + 4x - 6y + 19 = 0$
 (b) $x^2 + y^2 - 4x - 10y + 19 = 0$
 (c) $x^2 + y^2 - 2x + 6y - 29 = 0$
 (d) $x^2 + y^2 - 6x - 4y + 19 = 0$
23. The circle passing through the point $(-1, 0)$ and touching the y -axis at $(0, 2)$ also passes through the point. (2011)
- (a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) $(-4, 0)$
24. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is (2012)
- (a) $20(x^2 + y^2) - 36x + 45y = 0$
 (b) $20(x^2 + y^2) + 36x - 45y = 0$
 (c) $36(x^2 + y^2) - 20x + 45y = 0$
 (d) $36(x^2 + y^2) + 20x - 45y = 0$

D MCQs with One or More than One Correct

1. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are (1988 - 2 Marks)
- (a) $x = 0$ (b) $y = 0$
 (c) $(h^2 - r^2)x - 2rhy = 0$ (d) $(h^2 - r^2)x + 2rhy = 0$
2. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is (1998 - 2 Marks)
- (a) 0 (b) 1 (c) 3 (d) 4
3. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then (1998 - 2 Marks)
- (a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$
 (c) $x_1 x_2 x_3 x_4 = c^4$ (d) $y_1 y_2 y_3 y_4 = c^4$
4. Circle(s) touching x -axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y -axis is (are) (JEE Adv. 2013)
- (a) $x^2 + y^2 - 6x + 8y + 9 = 0$ (b) $x^2 + y^2 - 6x + 7y + 9 = 0$
 (c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x - 7y + 9 = 0$
5. A circle S passes through the point $(0, 1)$ and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then (JEE Adv. 2014)
- (a) radius of S is 8 (b) radius of S is 7
 (c) centre of S is $(-7, 1)$ (d) centre of S is $(-8, 1)$
6. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q . The normal to the circle at P intersects a line drawn through Q parallel to RS at point E . Then the locus of E passes through the point(s) (JEE Adv. 2016)
- (a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
 (c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

E Subjective Problems

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point $(5, 5)$. (1978)
2. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points $B(1, 7)$ and $D(4, -2)$ on the circle meet at the point C . Find the area of the quadrilateral $ABCD$. (1981 - 4 Marks)
3. Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$. (1982 - 3 Marks)
4. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. (1983 - 5 Marks)
5. The abscissa of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter. (1984 - 4 Marks)
6. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines. (1986 - 5 Marks)
7. Let a given line L_1 intersects the x and y axes at P and Q , respectively. Let another line L_2 , perpendicular to L_1 , cut the x and y axes at R and S , respectively. Show that the locus of the point of intersection of the lines PS and QR is a circle passing through the origin. (1987 - 3 Marks)
8. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find k . (1987 - 4 Marks)
9. If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0$, $i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$. (1989 - 2 Marks)
10. A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle. (1990 - 5 Marks)
11. Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. (1991 - 4 Marks)
12. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, ($a \neq 0, b \neq 0$). Find the condition on a and b if two chords, each bisected by the x -axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$. (1992 - 6 Marks)

13. Consider a family of circles passing through two fixed points $A(3, 7)$ and $B(6, 5)$. Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point. (1993 - 5 Marks)

14. Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y = -4$ and $x^2 + y^2 - 12x - 8y = -36$ touch each other. Also find equations common tangents touching the circles in the distinct points. (1993 - 5 Marks)

15. Find the intervals of values of a for which the line $y + x = 0$

bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$

to the circle $2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$.

(1996 - 5 Marks)

16. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d , prove that the area of the circle is

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta \cos(\beta - \alpha)}$$

(1996 - 5 Marks)

17. Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C . (A rational point is a point both of whose coordinates are rational numbers.)

(1997 - 5 Marks)

18. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1 . From a point P on C_2 , tangents PA and PB are drawn to C_1 . Prove that the centroid of the triangle PAB lies on C_1 . (1998 - 8 Marks)

19. Let T_1, T_2 be two tangents drawn from $(-2, 0)$ onto the circle $C: x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. (1999 - 10 Marks)

20. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA . (2001 - 5 Marks)

21. Let C_1 and C_2 be two circles with C_2 lying inside C_1 . A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C . (2001 - 5 Marks)

22. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point $P(6, 8)$ to the circle and the chord of contact is maximum.

(2003 - 2 Marks)

23. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at $(1, -1)$ and cutting orthogonally the circle having line segment joining $(0, 3)$ and $(-2, -1)$ as diameter.

(2004 - 4 Marks)

24. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. (2005 - 2 Marks)

G

Comprehension Based Questions

PASSAGE-1

$ABCD$ is a square of side length 2 units. C_1 is the circle touching all the sides of the square $ABCD$ and C_2 is the circumcircle of square $ABCD$. L is a fixed line in the same plane and R is a fixed point.

1. If P is any point of C_1 and Q is another point on C_2 , then

$$\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2} \text{ is equal to } (2006 - 5M, -2)$$

- (a) 0.75 (b) 1.25 (c) 1 (d) 0.5

2. If a circle is such that it touches the line L and the circle C_1 externally, such that both the circles are on the same side of the line, then the locus of centre of the circle is

(2006 - 5M, -2)

- (a) ellipse (b) hyperbola
(c) parabola (d) pair of straight line

3. A line L' through A is drawn parallel to BD . Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts L' at T_2 and T_3 and AC at T_1 , then area of $\Delta T_1 T_2 T_3$ is

(2006 - 5M, -2)

- (a) $\frac{1}{2}$ sq. units (b) $\frac{2}{3}$ sq. units
(c) 1 sq. units (d) 2 sq. units

PASSAGE-2

A circle C of radius 1 is inscribed in an equilateral triangle PQR . The points of contact of C with the sides PQ, QR, RP are D, E, F , respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$

and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin

and the centre of C are on the same side of the line PQ .

4. The equation of circle C is (2008)

- (a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$
(b) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$
(c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$
(d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

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5. Points E and F are given by (2008)

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

6. Equations of the sides QR, RP are (2008)

(a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$

(b) $y = \frac{1}{\sqrt{3}}x, y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$

(d) $y = \sqrt{3}x, y = 0$

PASSAGE-3

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L , perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$. (2012)

7. A possible equation of L is

(a) $x - \sqrt{3}y = 1$ (b) $x + \sqrt{3}y = 1$

(c) $x - \sqrt{3}y = -1$ (d) $x + \sqrt{3}y = 5$

8. A common tangent of the two circles is

(a) $x = 4$ (b) $y = 2$

(c) $x + \sqrt{3}y = 4$ (d) $x + 2\sqrt{2}y = 6$

H Assertion & Reason Type Questions

1. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

STATEMENT-1 : The tangents are mutually perpendicular.

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$. (2007 -3 marks)

- (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
- (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (c) Statement-1 is True, Statement-2 is False
- (d) Statement-1 is False, Statement-2 is True.

2. Consider $L_1 : 2x + 3y + p - 3 = 0$

$$L_2 : 2x + 3y + p + 3 = 0$$

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$

STATEMENT - 1 : If line L_1 is a chord of circle C , then line L_2 is not always a diameter of circle C

and

STATEMENT - 2 : If line L_1 is a diameter of circle C , then line L_2 is not a chord of circle C . (2008)

- (a) Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1
- (b) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1
- (c) Statement - 1 is True, Statement - 2 is False
- (d) Statement - 1 is False, Statement - 2 is True

I Integer Value Correct Type

1. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C , then the radius of the circle C is (2009)

2. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts.

If $S = \left\{ \left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right) \right\}$ then the number of points (s) in S lying inside the smaller part is (2011)

Section-B

JEE Main / AIEEE

- If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is [2002]
 - $2 \pm \sqrt{2}$
 - $-2 \pm \sqrt{2}$
 - $-1 \pm \sqrt{2}$
 - none of these
- The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is [2002]
 - $4 \leq x^2 + y^2 \leq 64$
 - $x^2 + y^2 \leq 25$
 - $x^2 + y^2 \geq 25$
 - $3 \leq x^2 + y^2 \leq 9$
- The centre of the circle passing through $(0, 0)$ and $(1, 0)$ and touching the circle $x^2 + y^2 = 9$ is [2002]
 - $\left(\frac{1}{2}, \frac{1}{2}\right)$
 - $\left(\frac{1}{2}, -\sqrt{2}\right)$
 - $\left(\frac{3}{2}, \frac{1}{2}\right)$
 - $\left(\frac{1}{2}, \frac{3}{2}\right)$
- The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is [2002]
 - $x^2 + y^2 = 9a^2$
 - $x^2 + y^2 = 16a^2$
 - $x^2 + y^2 = 4a^2$
 - $x^2 + y^2 = a^2$
- If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct point, then [2003]
 - $r > 2$
 - $2 < r < 8$
 - $r < 2$
 - $r = 2$
- The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is [2003]
 - $x^2 + y^2 - 2x + 2y = 62$
 - $x^2 + y^2 + 2x - 2y = 62$
 - $x^2 + y^2 + 2x - 2y = 47$
 - $x^2 + y^2 - 2x + 2y = 47$
- If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is [2004]
 - $2ax - 2by - (a^2 + b^2 + 4) = 0$
 - $2ax + 2by - (a^2 + b^2 + 4) = 0$
 - $2ax - 2by + (a^2 + b^2 + 4) = 0$
 - $2ax + 2by + (a^2 + b^2 + 4) = 0$
- A variable circle passes through the fixed point $A(p, q)$ and touches x -axis. The locus of the other end of the diameter through A is [2004]
 - $(y-q)^2 = 4px$
 - $(x-q)^2 = 4py$
 - $(y-p)^2 = 4qx$
 - $(x-p)^2 = 4qy$
- If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π , then the equation of the circle is [2004]
 - $x^2 + y^2 + 2x - 2y - 23 = 0$
 - $x^2 + y^2 - 2x - 2y - 23 = 0$
 - $x^2 + y^2 + 2x + 2y - 23 = 0$
 - $x^2 + y^2 - 2x + 2y - 23 = 0$
- Intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB . Equation of the circle on AB as a diameter is [2004]
 - $x^2 + y^2 + x - y = 0$
 - $x^2 + y^2 - x + y = 0$
 - $x^2 + y^2 + x + y = 0$
 - $x^2 + y^2 - x - y = 0$
- If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for [2005]
 - exactly one value of a
 - no value of a
 - infinitely many values of a
 - exactly two values of a
- A circle touches the x -axis and also touches the circle with centre at $(0, 3)$ and radius 2. The locus of the centre of the circle is [2005]
 - an ellipse
 - a circle
 - a hyperbola
 - a parabola
- If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is [2005]
 - $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$
 - $2ax + 2by - (a^2 - b^2 + p^2) = 0$

Circle

- (c) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$
 (d) $2ax + 2by - (a^2 + b^2 + p^2) = 0$
14. If the pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then [2005]
- (a) $3a^2 - 10ab + 3b^2 = 0$ (b) $3a^2 - 2ab + 3b^2 = 0$
 (c) $3a^2 + 10ab + 3b^2 = 0$ (d) $3a^2 + 2ab + 3b^2 = 0$
15. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is [2006]
- (a) $x^2 + y^2 + 2x - 2y - 47 = 0$
 (b) $x^2 + y^2 + 2x - 2y - 62 = 0$
 (c) $x^2 + y^2 - 2x + 2y - 62 = 0$
 (d) $x^2 + y^2 - 2x + 2y - 47 = 0$
16. Let C be the circle with centre $(0, 0)$ and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is
- (a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$ [2006]
 (c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$
17. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x -axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval [2007]
- (a) $-\frac{1}{2} \leq k \leq \frac{1}{2}$ (b) $k \leq \frac{1}{2}$
 (c) $0 \leq k \leq \frac{1}{2}$ (d) $k \geq \frac{1}{2}$
18. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is [2008]
- (a) $(3, -4)$ (b) $(-3, 4)$ (c) $(-3, -4)$ (d) $(3, 4)$
19. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is
- (a) $(x-2)y'^2 = 25 - (y-2)^2$
 (b) $(y-2)y'^2 = 25 - (y-2)^2$
 (c) $(y-2)^2 y'^2 = 25 - (y-2)^2$
 (d) $(x-2)^2 y'^2 = 25 - (y-2)^2$
20. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and $(1, 1)$ for: [2009]
- (a) all except one value of p
 (b) all except two values of p
 (c) exactly one value of p
 (d) all values of p
21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if [2010]
- (a) $-35 < m < 15$ (b) $15 < m < 65$
 (c) $35 < m < 85$ (d) $-85 < m < -35$
22. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ ($c > 0$) touch each other if [2011]
- (a) $|a| = c$ (b) $a = 2c$
 (c) $|a| = 2c$ (d) $2|a| = c$
23. The length of the diameter of the circle which touches the x -axis at the point $(1, 0)$ and passes through the point $(2, 3)$ is: [2012]
- (a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{3}$
24. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point [JEE M 2013]
- (a) $(-5, 2)$ (b) $(2, -5)$ (c) $(5, -2)$ (d) $(-2, 5)$
25. Let C be the circle with centre at $(1, 1)$ and radius = 1. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to [JEE M 2014]
- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$
26. Locus of the image of the point $(2, 3)$ in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in \mathbf{R}$, is a : [JEE M 2015]
- (a) circle of radius $\sqrt{2}$.
 (b) circle of radius $\sqrt{3}$.
 (c) straight line parallel to x -axis
 (d) straight line parallel to y -axis
27. The number of common tangents to the circles $x^2 + y^2 - 4x - 6x - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is : [JEE M 2015]
- (a) 3 (b) 4 (c) 1 (d) 2

28. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: [JEE M 2016]
- (a) a hyperbola
 - (b) a parabola
 - (c) a circle
 - (d) an ellipse which is not a circle
29. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is: [JEE M 2016]
- (a) 5
 - (b) 10
 - (c) $5\sqrt{2}$
 - (d) $5\sqrt{3}$