3

Averages

THEORY

The average of a number is a measure of the central tendency of a set of numbers. In other words, it is an estimate of where the center point of a set of numbers lies.

The basic formula for the average of n numbers x_1, x_2, x_3, … x_n is

$A_n = (x_1 + x_2 + x_3 + \ldots + x_n)/n$ = (Total of set of n numbers)/n

This also means $A_n \times n$ = total of the set of numbers.

The average is always calculated for a set of numbers.

Concept of Weighted Average: When we have two or more groups whose individual averages are known, then to find the combined average of all the elements of all the groups we use weighted average. Thus, if we have k groups with averages A_1, A_2 ... A_k and having n_1, n_2 ... n_k elements then the weighted average is given by the formula:

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \ldots + n_k A_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

Another Meaning of Average The average [also known as *arithmetic mean* (AM)] of a set of numbers can also be defined as the number by which we can replace each and every number of the set without changing the total of the set of numbers.

Properties of Average (AM) The properties of averages [arithmetic mean] can be elucidated by the following examples:

Example 1: The average of 4 numbers 12, 13, 17 and 18 is:

Solution: Required average = (12 + 13 + 17 + 18)/4 = 60/4 = 15

This means that if each of the 4 numbers of the set were replaced by 15 each, there would be no change in the total.

This is an important way to look at averages. In fact, whenever you come across any situation where the average of a group of 'n' numbers is given, you should visualise that there are 'n' numbers, each of whose value is the average of the group. This view is a very important way to visualise averages.

This can be visualised as

$$\begin{aligned} 12 &\rightarrow +3 \rightarrow 15 \\ 13 &\rightarrow +2 \rightarrow 15 \\ 17 &\rightarrow -2 \rightarrow 15 \\ 18 &\rightarrow -3 \rightarrow 15 \\ \hline 60 &\rightarrow +0 \rightarrow 60 \end{aligned}$$

Example 2: In Example 1, visualise addition of a fifth number, which increases the average by 1.

$$\begin{aligned} 15 + 1 &= 16 \\ 15 + 1 &= 16 \\ 15 + 1 &= 16 \\ 15 + 1 &= 16 \end{aligned}$$

The +1 appearing 4 times is due to the fifth number, which is able to maintain the average of 16 first and then 'give one' to each of the first 4.

Hence, the fifth number in this case is 20.

Example 3: The average always lies above the lowest number of the set and below the highest number of the set.

Example 4: The net deficit due to the numbers below the average always equals the net surplus due to the numbers above the average.

Example 5: *Ages and averages:* If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$.

Also, n years ago their average age would have been $(x - n)$. This happens due to the fact that for a group of people, 1 year is added to each person's age every year.

Example 6: A man travels at 60 kmph on the journey from A to B and returns at 100 kmph. Find his average speed for the journey.

Solution:

Average speed = (total distance)/(total time)

If we assume distance between 2 points to be d
Then

Average speed = $2d/[(d/60) + (d/100)] = (2 \times 60 \times 100)/(60 + 100) = (2 \times 60 \times 100)/160 = 75$

Average speed = $(2S_1 \cdot S_2)/(S_1 + S_2)$
[S_1 and S_2 are speeds]

of going and coming back, respectively.

Short Cut The average speed will always come out by the following process:

The ratio of speeds is $60:100 = 3:5$ (say $r_1 : r_2$)

Then, divide the difference of speeds (40 in this case) by $r_1 + r_2$ (3 + 5 = 8, in this case) to get one part. (40/8 = 5, in this case)

The required answer will be three parts away (i.e. r_1 parts away) from the lower speed.

Check out how this works with the following speeds:

$S_1 = 20$ and $S_2 = 40$

Step 1: Ratio of speeds = $20:40 = 1:2$

Step 2: Divide difference of 20 into 3 parts $(r_1 + r_2) \rightarrow = 20/3 = 6.66$

Required average speed = $20 + 1 \times 6.66$

Note: This process is essentially based on alligations and we shall see it again in the next chapter.

Exercise for Self-practice

Find the average speed for the above problem if

(1) $S_1 = 20$ $S_2 = 200$
(2) $S_1 = 60$ $S_2 = 120$
(3) $S_1 = 100$ $S_2 = 50$
(4) $S_1 = 60$ $S_2 = 180$

Space for Notes

WORKED-OUT PROBLEMS

Problem 3.1 The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?

Solution *Normal process:*

Runs in 26th inning = Runs total after 26 innings – Runs total after 25 innings

$$= 26 \times 58 - 25 \times 56$$

For mental calculation use:

$$(56 + 2) \times 26 - 56 \times 25$$

$$= 2 \times 26 + (56 \times 26 - 56 \times 25)$$

$$= 52 + 56 = 108$$

Short Cut Since the average increases by 2 runs per innings it is equivalent to 2 runs being added to each score in the first 25 innings. Now, since these runs can only be added by the runs scored in the 26th inning, the score in the 26th inning must be $25 \times 2 = 50$ runs higher than the average after 26 innings (i.e. new average = 58).

Hence, runs scored in 26th inning = New Average + Old innings × Change in average

$$= 58 + 25 \times 2 = 108$$

Visualise this as

Average in first 25 innings	Average after 26 innings
56	58
56	58
56	58
…	…
…	…
25 times…	26 times…

Difference in total is two, 25 times and 58 once, that is, $58 + 25 \times 2$.

Problem 3.2 The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher. If the initial average is 14 years, find the age of the class teacher.

Solution *Normal process:*

Age of teacher = Total age of (students + teacher)

– Total age of students

$$= 31 \times 14 - 30 \times 13.5$$

$$= 434 - 405$$

$$= 29 \text{ years}$$

Short Cut The teacher after fulfilling the average of 14 (for the group to which he belonged) is also able to give 0.5 years to the age of each of the 30 students. Hence, he has $30 \times 0.5 \rightarrow 15$ years to give over and above maintaining his own average age of 14 years.

$$\text{Age of teacher} = 14 + 30 \times 0.5 = 29 \text{ years}$$

(*Note:* This problem should be viewed as change of average from 13.5 to 14 when teacher is included.)

Problem 3.3 The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?

Solution *Normal process:*

Let initial average be x.

Then the initial total is $20x$

New average will be $(x - 4)$ and the new total will be $20(x - 4) = 20x - 80$.

The reduction of 80 is created by the replacement.

Hence, the new student has 80 marks less than the student he replaces. Hence, he must have scored 10 marks.

Short Cut The replacement has the effect of reducing the average marks for each of the 20 students by 4. Hence, the replacement must be $20 \times 4 = 80$ marks below the original.

Hence, answer = 10 marks.

Problem 3.4 The average marks of 3 students A, B and C is 48 marks. Another student D joins the group and the new average becomes 44 marks. If another student E, who has 3 marks more than D, joins the group, the average of the 4 students B, C, D and E becomes 43 marks. Find how many marks A got in the exam.

Solution Solve while reading. The first sentence gives you a total of 144 for A, B and C's marks. *Second sentence*: When D joins the group, the total becomes $44 \times 4 = 176$. Hence D must get 32 marks.

Alternatively, you can reach this point by considering the first 2 statements together as:

D's joining the group reduces the average from 48 to 44 marks (i.e., 4 marks).

This means that to maintain the average of 44 marks, D has to take 4 marks from A, 4 from B and 4 from $C \rightarrow A$ total of 12 marks. Hence, he must have got 32 marks.

From here:

The first part of the third sentence gives us information about E getting 3 marks more than 32 $\rightarrow$ Hence, E gets 35 marks.

Now, it is further stated that when A is replaced by E, the average marks of the students reduces by 1 to 43.

Mathematically this can be shown as

$A + B + C + D = 44 \times 4 = 176$ while, $B + C + D + E$

$$= 43 \times 4 = 172$$

Subtracting the two equations, we get $A - E = 4$ marks.

Hence, A would have got 39 marks.

Alternatively, you can think of this as:

The replacement of A with E results in the reduction of 1 mark from each of the 4 people who belong to the group. Hence, the difference is 4 marks. Hence, A would get 4 marks more than E i.e., A gets 39 marks.

Problem 3.5 The mean temperature of Monday to Wednesday was 27 °C and of Tuesday to Thursday was 24 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?

Solution From the first sentence, we get that the total from Monday to Wednesday was 81 while from Tuesday to Thursday was 72. The difference is arising out of the replacement of Monday by Thursday.

This can be mathematically written as

$$\text{Mon} + \text{Tue} + \text{Wed} = 81 \quad (1)$$

$$\text{Tue} + \text{Wed} + \text{Thu} = 72 \quad (2)$$

Hence, $$\text{Mon} - \text{Thu} = 9$$

We have two unknown variables in the above equation. To solve for 2 unknowns, we need a new equation. Looking back at the problem we get the equation:

$$\text{Thu} = (2/3) \times \text{Mon}$$

Solving the two equations we get: Thursday = 18 °C.

However, in the exam, you should avoid using equation-solving as much as possible. You should, ideally, be able to reach half way through the solution during the first reading of the question, and then meet the gap through the use of options.

The answer to this problem should be got by the time you finish reading the question for the first time.

Thus suppose we have the equations:

$M - T = 9$ and $T = 2M/3$ or $T/M = 2/3$ and have the options for T as

(a) 12 (b) 15

(c) 18 (d) 27

To check which of these options is the appropriate value, we need to check one by one.

Option (a) gives $T = 12$, then we have $M = 21$. But $12/21 \neq 2/3$. Hence, this is not the correct option.

Option (b) gives $T = 15$, then $M = 24$. But again $15/24 \neq 2/3$. Hence, this is not the correct option.

Option (c) gives $T = 18$, then $M = 27$. Now $18/27 = 2/3$. Hence, this is the correct option.

So we no longer need to check for option (d).

However, if we had checked for option (d) then $T = 27$, so $M = 36$. But again $27/36 \neq 2/3$. Hence, this is not the correct option.

In the above, we used 'solving-while-reading' and 'option-based' approaches.

These two approaches are very important and by combining the two, you can reach amazing speeds in solving the question.

You are advised to practice both these approaches while solving questions, which will surely improve your efficiency and speed. You will see that, with practice, you will be able to arrive at the solution to most of the LOD I problems (given later in this chapter) even as you finish reading the questions. And since it is the LOD I level problems that appear in most examinations (like IIFT, SNAP, NMAT, CLAT, CET, BANK PO, SSC, BBA/BMS entrance etc) you will gain a significant advantage in solving these problems.

On LOD II, LOD III and CAT type problems, you will find that using solving-while-reading and option-based approaches together would take you through anywhere between 30 and 70% of the question by the time you finish reading the question for the first time.

This will give you a tremendous time advantage over the other students appearing in the examination.

Problem 3.6 A person covers half his journey by train at 60 kmph, the remainder half by bus at 30 kmph and the rest by cycle at 10 kmph. Find his average speed during the entire journey.

Solution Recognise that the journey by bus and that by cycle are of equal distance. Hence, we can use the short cut illustrated earlier to solve this part of the problem.

Using the process explained above, we get average speed of the second half of the journey as

$$10 + 1 \times 5 = 15 \text{ kmph}$$

Then we employ the same technique for the first part and get

$$15 + 1 \times 9 = 24 \text{ kmph (Answer)}$$

Problem 3.7 A school has only 3 classes that contain 10, 20 and 30 students, respectively. The pass percentage of these classes are 20%, 30% and 40% respectively. Find the pass percentage of the entire school.

Solution

Using weighted average: $\dfrac{10 \times 0.2 + 20 \times 0.3 + 30 \times 0.4}{10 + 20 + 30}$

$= \dfrac{20}{60} = 33.33\%$

Alternatively, we can also use solving-while-reading as

Recognise that the pass percentage would be given by

$$\frac{\text{Passed students}}{\text{Total students}}$$

As soon as you get into the second line of the question get back to the first sentence and get the total number of passed students = 2 + 6 + 12 and you are through with the problem.

Space for Rough Work

LEVEL OF DIFFICULTY (I)

1. The average of the first fifteen natural numbers is
   (a) 8.5 (b) 7.5
   (c) 6.5 (d) 8
2. The average of the first ten whole numbers is
   (a) 4.5 (b) 5
   (c) 5.5 (d) 4
3. The average of the first ten even numbers is
   (a) 18 (b) 22
   (c) 9 (d) 11
4. The average of the first ten odd numbers is
   (a) 11 (b) 10
   (c) 17 (d) 9
5. The average of the first ten prime numbers is
   (a) 15.5 (b) 12.5
   (c) 10 (d) 12.9
6. The average of the first ten composite numbers is
   (a) 12.9 (b) 11
   (c) 11.2 (d) 10
7. The average of the first ten prime numbers, which are odd, is
   (a) 12.9 (b) 13.8
   (c) 17 (d) 15.8
8. The average age of 13 boys and the principal is 10 years. When the principal's age is excluded, the average age decreases by 2 year. What is the age of the principal?
   (a) 34 (b) 36
   (c) 38 (d) 40
9. The average weight of 3 boys *A*, *B* and *C* is 74 kg. Another boy *D* joins the group and the average now becomes 70 kg. If another boy *E*, whose weight is 3 kg more than that of *D*, replaces *A* then the average weight of *B*, *C*, *D* and *E* becomes 75 kg. The weight of *A* is
   (a) 40 kg (b) 42 kg
   (c) 49 kg (d) 41 kg
10. The mean temperature of Monday to Wednesday was 35 °C and of Tuesday to Thursday was 30 °C. If the temperature on Thursday was ½ that of Monday, the temperature on Thursday was
    (a) 30 °C (b) 15 °C
    (c) 20 °C (d) 25 °C
11. Five years ago, the average age of *A*, *B* and *C* was 25 years and that of *B* and *C* 10 years ago was 20 years. *A*'s present age is
    (a) 30 years (b) 35 years
    (c) 40 years (d) 48 years
12. Ganguly has a certain average for 4 innings. In the 5^{th} inning, he scores 40 runs thereby increasing his average by 4 runs. His new average is
    (a) 20 (b) 24
    (c) 28 (d) 32
13. The average of the first six multiples of 5 is
    (a) 18.50 (b) 21
    (c) 28 (d) 17.50
14. There are three fractions *A*, *B* and *C*. If *A* = 1/5 and *B* = 1/8 and the average of *A*, *B* and *C* is 1/10. What is the value of *C*?
    (a) –1/20 (b) – 1/60
    (c) –1/30 (d) – 1/40
15. The marks obtained by Alan in Mathematics, English and Biology are respectively 90 out of 100, 70 out of 150 and 150 out of 200. Find his average score in percent.
    (a) 87.83 (b) 68.88
    (c) 76.33 (d) 77.33
16. The average monthly expenditure of a family was 2250 for the first 3 months, ₹2150 for the next three months and ₹ 5750 for the next three months. Find the average income of the family for the 9 months, if they save ₹500 per month.
    (a) ₹3866.66 (b) ₹3883.33
    (c) ₹3666.66 (d) ₹3222.66
17. The average age of a family of 5 members is 20 years. If the age of the youngest member be 5 years, what was the average age of the family at the birth of the youngest member?
    (a) 15.25 (b) 18.75
    (c) 21.25 (d) 12.50
18. The average age of 5 persons in a group is increased by 10 years when two men aged 30 years and 40 years are substituted by two women. Find the average age (in years) of the two women.
    (a) 60 (b) 65
    (c) 51 (d) 62
19. The average temperature for Wednesday, Thursday and Friday was 20 °C. The average for Thursday, Friday and Saturday was 21 °C. If the temperature on Saturday was 22 °C, what was the temperature on Wednesday?
    (a) 19 °C (b) 24 °C
    (c) 18 °C (d) 21 °C
20. The speed of the train in going from Kanpur to Lucknow is 60 km/hr while when coming back from Lucknow to Kanpur, its speed is 40 km/hr. Find the average speed (in km/hr) during the whole journey.
    (a) 45 (b) 48
    (c) 50 (d) 46

21. The average weight of a class of 19 students is 20 kg. If the weight of the teacher be included, the average rises by 1 kg. What is the weight of the teacher?
(a) 40 kg (b) 50 kg
(c) 45 kg (d) 55 kg

22. The average of 3 numbers is 20 and that of the first two is 25. Find the third number.
(a) 15 (b) 10
(c) 20 (d) 12

23. The average weight of 29 men in a ship is increased by 5 kg when one of the men, who weighs 120 kg, is replaced by a new man. Find the weight of the new man (In kg)
(a) 265 (b) 205
(c) 245 (d) 240

24. The age of A and B is in the ratio 1: 3. After 10 years, the ratio of their ages will become 1:2. Find the average of their ages after 20 years.
(a) 22 (b) 40
(c) 37 (d) 30

25. Find the average of the first 100 natural numbers.
(a) 50.50 (b) 52.50
(c) 51.50 (d) 49

26. Find the average of all prime numbers between 20 and 50.
(a) 35.8 (b) 34.65
(c) 35.85 (d) 31.8

27. If we take four numbers, the average of the first three is 20 and that of the last three is 25. If the last number is 30, the first number is
(a) 20 (b) 21
(c) 23 (d) 15

28. The average of 15 results is 40 and that of 25 more results is 48. For all the results taken together, the average is
(a) 45 (b) 42
(c) 46 (d) 44

29. The average of 7 consecutive numbers is 21. The highest of these numbers will be
(a) 20 (b) 23
(c) 24 (d) 22

30. The average age of 8 students is 11 years. If 2 more students of age 15 and 17 years join, their average will become
(a) 13 years (b) 12 years
(c) 14 years (d) 15 years

31. The average of 9 numbers is 14. If each number is increased by 4, the new average will be
(a) 16 (b) 15
(c) 18 (d) 17

32. The average of 11 consecutive numbers is n. If the next two numbers are also included, the average will.
(a) increase by 1 (b) remain the same
(c) increase by 1.4 (d) increase by 2

33. The average of 40 numbers is 45. If two numbers, namely, 65 and 25 are discarded, the average of the remaining numbers is
(a) 35 (b) 45
(c) 40 (d) 43

34. The average of 15 numbers is 18. If each number is multiplied by 9, then the average of the new set of numbers is
(a) 162 (b) 152
(c) 144 (d) 164

35. In a family of 5 males and a few ladies, the average monthly consumption of grain per head is 9 kg. If the average monthly consumption per head be 12 kg in the case of males and 8 kg in the case of females, find the number of females in the family.
(a) 18 (b) 12
(c) 9 (d) 15

36. Average marks obtained by a student in 3 papers is 63 and in the fourth paper he obtains 67 marks. Find his new average.
(a) 54 (b) 62
(c) 64 (d) 65

37. The average earning of Srikanth for the initial three months of the calendar year 2012 is ₹2100. If his average earning (in ₹) for the second and third month is ₹2250 find his earning in the first month?
(a) 1950 (b) 1500
(c) 1700 (d) 1800

38. In a hotel where rooms are numbered from 201 to 230, each room gives an earning of ₹2000 for the first fifteen days of a month and for the latter half, ₹1000 per room. Find the average earning per room per day (in ₹) over the month. (Assume 30 day month)
(a) 1450 (b) 1500
(c) 1750 (d) 1666.66

39. The average weight of 10 men is decreased by 2 kg when, one of them weighing 140 kg is replaced by another person. Find the weight of the new person.
(a) 142 kg (b) 130 kg
(c) 138 kg (d) 120 kg

40. The average age of a group of men is increased by 6 years when a person aged 26 years is replaced by a new person of aged 56 years. How many men are there in the group?
(a) 3 (b) 4
(c) 5 (d) 6

41. The average score of a cricketer in three matches is 33 runs and in two other matches, it is 23 runs. Find the average in all the five matches.
(a) 31 (b) 26
(c) 29 (d) 28

42. The average of 15 papers is 50. The average of the first 8 papers is 48 and of the last eight papers is 54. Find the marks obtained in the 8th paper.

(a) 66 (b) 64
(c) 58 (d) 56

43. The average age of the Indian cricket team playing the Coimbatore test is 28. The average age of 5 of the players is 26 and that of another set of 5 players, totally different from the first five, is 29. If it is the captain who was not included in either of these two groups, then find the age of the captain.
(a) 35 (b) 30
(c) 33 (d) 28

44. Siddhartha has earned an average of 3200 dollars for the first eleven months of the year. If he justifies his staying on in the US on the basis of his ability to earn at least 4000 dollars per month for the entire year, how much should he earn (in dollars) in the last month to achieve his required average for the whole year?
(a) 11,800 (b) 12,800
(c) 10,800 (d) 13,800

45. A bus goes to Ranchi from Patna at the rate of 80 km per hour. Another bus leaves Ranchi for Patna at the same time as the first bus at the rate of 90 km per hour. Find the average speed for the journeys of the two buses combined if it is known that the distance from Ranchi to Patna is 720 kilometres.
(a) 84.705 kmph (b) 84 kmph
(c) 81.63 kmph (d) 82.82 kmph

46. A train travels 12 km in the first quarter of an hour, 15 km in the second quarter and 30 km in the third quarter. Find the average speed of the train per hour over the entire journey.
(a) 76 km/h (b) 72 km/h
(c) 77 km/h (d) 73 km/h

47. The average weight of 6 men is 58.5 kg. If it is known that Ram and Tram weigh 65 kg each, find the average weight of the others.
(a) 55 kg (b) 54.25 kg
(c) 54 kg (d) 55.25 kg

48. The average score of a class of 30 students is 56. What will be the average score of the rest of the students if the average score of 10 of the students is 59.
(a) 52.5 (b) 54.5
(c) 52 (d) 53

49. The average age of 60 students of IIM, Bangalore of the 2005 batch is 23 years. What will be the new average if we include the 40 faculty members whose average age is 35 years?
(a) 27 years (b) 26.5 years
(c) 27.8 years (d) 28 years

50. Out of three numbers, the first is twice the second and three times the third. The average of the three numbers is 132. The smallest number is
(a) 36 (b) 72
(c) 42 (d) 48

51. The sum of three numbers is 147. If the ratio between the first and second is 2 : 3 and that between the second and the third is 5 : 8, then the second number is
(a) 30 (b) 45
(c) 72 (d) 48

52. The average height of 40 girls out of a class of 50 is 150 cm and that of the remaining girls is 155 cm. The average height of the whole class is
(a) 151 cm (b) 152 cm
(c) 156 cm (d) 153 cm

53. The average weight of 6 persons is increased by 1.5 kg when one of them, whose weight is 60 kg is replaced by a new man. The weight of the new man is
(a) 71 kg (b) 72 kg
(c) 68 kg (d) 69 kg

54. The average age of three boys is 24 years. If their ages are in the ratio 2:5:5, the age of the youngest boy is
(a) 16 years (b) 11 years
(c) 21 years (d) 12 years

55. The average age of P, Q, R and S four years ago was 46 years. By including M, the present average age of all the five is 51 years. The present age of M is
(a) 52 years (b) 49 years
(c) 55 years (d) 58 years

56. The average salary of 30 workers in an office is ₹ 1800 per month. If the manager's salary is added, the average salary becomes ₹ 1900 per month. What is the manager's annual salary?
(a) ₹ 48000 (b) ₹ 58,800
(c) ₹ 46,800 (d) None of these

57. If p, q, r, s and t are five consecutive even numbers, then their average is
(a) $(p + q+r+s+t)$ (b) $(p\ q\ r\ s\ t)/5$
(c) $5(p + q + r + s + t)$ (d) None of these

58. The average of first five multiples of 7 is
(a) 21 (b) 28
(c) 14 (d) 18

59. The average weight of a class of 30 students is 50 kg. If the weight of the teacher be included, the average weight increases by 500 gm. The weight of the teacher is
(a) 50.5 kg (b) 65.5 kg
(c) 62.5 kg (d) 60.5 kg

60. In a management entrance test, a student scores 3 marks for every correct answer and loses 1 mark for every wrong answer. A student attempts all the 100 questions and scores 160 marks. The number of questions he answered correctly was
(a) 62 (b) 64
(c) 65 (d) 68

61. The average age of five children is 7 years, which is increased by 5 years when the age of the father is included. Find the age of the father.
(a) 32 (b) 37
(c) 39 (d) 35

62. The average weight of a class of 20 students is 50 kg. If, however, the weight of the teacher is included, the average becomes 51 kg. The weight of the teacher is
(a) 69 kg (b) 72 kg
(c) 70 kg (d) 71 kg

63. Ram bought 2 toys for ₹ 5.50 each, 3 toys for ₹ 3.66 each and 6 toys for ₹ 1.833 each. The average price per toy is (in ₹)
(a) 3 (b) 10
(c) 5 (d) 9

64. 40 oranges and 65 apples were purchased for ₹ 480. If the price per apple was ₹ 4, then the average price of oranges was (in ₹)
(a) 5.5 (b) 6.5
(c) 6 (d) 7

65. The average income of Aditya and Vikas is ₹ 4,000 and that of Sudhir and Raunak is ₹ 2500. What is the average income of Aditya, Vikas, Sudhir and Raunak (in ₹)?
(a) 3150 (b) 3250
(c) 3350 (d) 3550

66. A batsman made an average of 55 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after the fifth inning?
(a) 54 (b) 64
(c) 44 (d) 49

67. The average weight of 50 teachers of a school is 70 kg. If, however, the weight of the principal be included, the average decreases by 0.5 kg. What is the weight of the principal?
(a) 44.5 kg (b) 45 kg
(c) 43.5 kg (d) None of these

68. The average temperature of 4th, 5th and 6th December was 25.6 °C. The average temperature of the first two days was 27 °C. The temperature on the 6th of December was:
(a) 22.2 °C (b) 22.8 °C
(c) 26.4 °C (d) None of these

69. The average age of Gita and Sita is 30 years. Their average age 4 years hence will be
(a) 30 years (b) 26 years
(c) 28 years (d) 34 years

70. Three years ago, the average age of a family of 6 members was 18 years. A baby having been born, the average of the family is the same today. What is the age of the baby?
(a) 1 year (b) 2 years
(c) 3 years (d) 0 years

71. Ramu's average daily expenditure is ₹ 21 during July, ₹24 during August and ₹11 during September. His approximate daily expenditure for the 3 months is
(a) ₹18 (b) ₹18.75
(c) ₹17 (d) ₹18.25

72. A ship sails out to a mark at the rate of 25 km per hour and sails back at the rate of 30 km/h. What is its average rate of sailing?
(a) 27.27 km (b) 23.24 km
(c) 25.85 km (d) 28.45 km

73. The average temperature on Monday, Tuesday and Wednesday was 52 °C and on Tuesday, Wednesday and Thursday it was 50 °C. If on Thursday it was exactly 49 °C, then on Monday, the temperature was
(a) 55°C (b) 56 °C
(c) 53 °C (d) 51°C

74. The average of 15 results is 20 out of which the first 5 results are having an average of 10. The average of the rest 10 results is
(a) 50 (b) 40
(c) 20 (d) 25

75. A man had ten children. When their average age was 15 years a child aged 6 years died. The average age of the remaining 9 children is
(a) 16 years (b) 13 years
(c) 17 years (d) 15 years

76. The average income of Hari and Prasad is ₹200. The average income of Rahul and Ravi is ₹ 250. The average income of Hari, Prasad, Rahul and Ravi is
(a) ₹275 (b) ₹ 225
(c) ₹450 (d) ₹250

77. The average weight of 40 students is 40 kg. If the teacher of weight 122 kg is also included, then the average weight will become:
(a) 41 kg (b) 42 kg
(c) 40 kg (d) 45 kg

78. The average of *a*, *b* and c is 25. *a* is as much more than the average as *b* is less than the average. Find the value of *c*.
(a) 45 (b) 25
(c) 35 (d) 15

79. Find the average of four numbers $2\frac{3}{4}, 4\frac{4}{5} \cdot 5\frac{1}{5}, 3\frac{3}{4}$
(a) 4.125 (b) 3.20
(c) 1.60 (d) None of these.

80. The average salary per head of all the workers in a company is ₹9000. The average salary of 15 officers is ₹5000 and the average salary per head of the rest is ₹10,000. Find the total number of workers in the company.
(a) 75 (b) 80
(c) 50 (d) 40

81. The average age of 10 men is increased by 3 years when one of them, whose age is 54 years is replaced by a woman. What is the age of the woman?
    (a) 68 years (b) 82 years
    (c) 72 years (d) 84 years
82. The average monthly expenditure of Aman was ₹ 100 during the first 3 months, ₹ 200 during the next 4 months and ₹ 400 during the subsequent five months of the year. If the total saving during the year was ₹3000, find Aman's average monthly income (to the closest rupee)
    (a) 508 (b) 515
    (c) 1033 (d) 425
83. Ram bought 2 articles for ₹5.50 each, and 3 articles for ₹3.50 each, and 3 articles for ₹5.50 and 5 articles for ₹ 1.50 each. The average price for one article is
    (a) ₹ 3 (b) ₹ 8.50
    (c) ₹3.50 (d) None of these.
84. In a bag, there are 150 coins of ₹1, 50 p and 25 p denominations. If the total value of coins is ₹150, then find how many rupees can be constituted by 50 p coins.
    (a) 16 (b) 20
    (c) 28 (d) None of these
85. What is the average of the first seven natural number multiples of 11?
86. The age of two friends Aman and Baman is in the ratio 1:2. After 5 years, the ratio of their ages will become 3:5. Find the average of their ages after 10 years.
87. If we take four numbers, the average of the first three is 20 and that of the last three is 10. If the first number is 20, the last number is:
88. Sachin has a batting average of 40 runs per innings in 15 ODIs. It was found that in the 2 ODI match series against South Africa (which were part of his first 15 ODIs), his average score was 40 and not 80. His correct average is:
89. The average of temperatures at noontime from Monday to Friday is 40; the lowest one is 35. What is the possible maximum temperature at noontime on any of these days?
90. The average age of a family of 3 members is 30 years. If the age of the youngest member is 5 years, then what was the average age of the family immediately prior to the birth of the youngest member?
91. The average of 50 numbers is zero. Of them, at the most, how many may be greater than zero?
92. The average of 40 numbers is 25. If two numbers namely 40 and 50 are discarded, the average of the remaining numbers is:
93. The average of ten numbers is 60. The average of the first five numbers is 50 and that of last four numbers is 30. Then the 6^{th} number is:
94. The average of 10 integers is found to be 10. But after the calculation, it was detected that, by mistake, the integer 20 was copied as 30, while calculating the average. After the due correction is made, the new average will be:

Space for Rough Work

LEVEL OF DIFFICULTY (II)

1. A bus travels with a speed of 10 km/h in the first 15 minutes, goes 5 km in the next 15 minutes, 15 km in the next 15, 10 km in the next 15. What is the average speed of the bus in kilometre per hour for the journey described?
   (a) 45 kmph (b) 32.50 kmph
   (c) 50.50 kmph (d) 40 kmph
2. With an average speed of 25 km/h, a train reaches its destination in time. If it goes with an average speed of 20 km/h, it is late by 1 hour. The length of the total journey is
   (a) 90 km (b) 100 km
   (c) 120 km (d) 80 km
3. In the month of January of a certain year, the average daily expenditure of an organisation was ₹60. For the first 15 days of the month, the average daily expenditure was ₹ 80 and for the last 17 days, ₹50. Find the amount spent by the organisation on the 15th of the month.
   (a) 190 (b) 160
   (c) 180 (d) 130
4. One-fifth of a certain journey is covered at the rate of 20 km/h, one-fourth at the rate of 50 km/h and the rest at 55 km/h. Find the average speed for the whole journey.
   (a) 53 km/h (b) 40 km/h
   (c) 35 km/h (d) 38 km/h
5. A batsman makes a score of 40 runs in the 5^{th} inning and thus increases his average by 4. Find the possible value of the new average.
   (a) 28 (b) 24
   (c) 12 (d) 20
6. Aman can type a sheet in 10 minutes, Baman in 20 minutes and Chaman in 30 minutes. The average number of sheets typed per hour per typist for all three typists is
   (a) 11/3 (b) 30/7
   (c) 55/9 (d) 32/11
7. Find the average increase rate (per annum) if increase in the population in the first year is 10% and that in the second year is 20%.
   (a) 11 (b) 16
   (c) 20 (d) 18
8. The average income of a person for the first 5 days of a month is ₹20, for the next 10 days it is ₹24, for the next 10 days it is ₹ 30 and for the remaining days of the month it is ₹ 10. Find the average income (in ₹) per day.
   (a) 30 (b) 35
   (c) 25 (d) Cannot be determined
9. In hotel Clarks, the rooms are numbered from 101 to 150 on the first floor, 201 to 240 on the second floor and 316 to 355 on the third floor. In the month of May 2018, the room occupancy was 50% on the first floor, 50% on the second floor and 30% on the third floor. If it is also known that the room charges are ₹ 2000, ₹1000 and ₹1500 on each of the floors, then find the average income per room (in ₹) for the month of May 2017.
   (a) 676.92 (b) 880.18
   (c) 783.3 (d) 650.7
10. A salesman gets a bonus according to the following structure: If he sells articles worth ₹x then he gets a bonus of ₹(x/10 – 1000). In the month of January, his sales value was ₹10000, in February it was ₹12000, from March to November it was ₹30000 for every month and in December it was ₹12000. Apart from this, he also receives a basic salary of ₹3000 per month from his employer. Find his average income per month (in ₹) during the year.
    (a) 4533 (b) 4517
    (c) 4532 (d) 4668
11. The average of 61 results is 43. If the average of the first 48 results is 36 and that of the last 12 is 49. Find the 49th result.
    (a) 302 (b) 307
    (c) 304 (d) 328
12. A man covers half of his journey by train at 50 km/hr, half of the remainder by bus at 25 km/h and the rest by cycle at 5 km/h. Find his average speed during the entire journey.
    (a) 100/7 kmph (b) 25/3 kmph
    (c) 14 kmph (d) 18 kmph
13. In 2010, Sachin Tendulkar, the Indian cricketer, scored 900 runs for his county at an average of 30, in 2011, he scored 950 runs at an average of 30.65; in 2012, 1300 runs at an average of 32.50 and in 2013, 1100 runs at an average of 35.49. What was his county average for the four years?
    (a) 34.23 (b) 32.19
    (c) 33.88 (d) 30.98
14. The average weight of 10 men is decreased by 5 kg when, one of them weighing 100 kg is replaced by another person. This new person is again replaced by another person, whose weight is 10 kg lower than the person he replaced. What is the overall change in the average due to this dual change?

(a) 5 kg (b) 6 kg
(c) 12 kg (d) 15 kg

15. Find the average weight of four packets, if it is known that the weight of the first packet is 20 kg and the weights of the second, third and fourth packets' each is defined by $f(x) = x^2 - \frac{2}{5} \times (x^2)$ where $x = 10$.
(a) 50 kg (b) 90 kg
(c) 70 kg (d) 40 kg

16. There are five boxes in a box hold. The weight of the first box is 40 kg and the weight of the second box is 50% higher than the weight of the third box, whose weight is 25% higher than the first boxes' weight. The fourth box at 150 kg is 50% heavier than the fifth box. Find the difference in the average weight of the four heaviest boxes and the four lightest boxes.
(a) 21.5 kg (b) 25 kg
(c) 27.5 kg (d) 22.5 kg

17. For Question 16, find the difference in the average weight of the heaviest three and the lightest three.
(a) 66.66 kg (b) 25 kg
(c) 50 kg (d) 53.33 kg

18. 20 persons went to a hotel for a combined dinner party. 15 of them spent ₹ 90 each on their dinner and the rest spent ₹ 30 more than the average expenditure of all the 20. What was the total money spent (in ₹) by them?
(a) 1700 (b) 2000
(c) 2200 (d) None of these

19. There were 30 students in a hostel. Due to the admission of 20 new students, the expenses of the mess increase by ₹1600 per day while the average expenditure per head diminished by ₹8. What was the original expenditure of the mess?
(a) 3000 (b) 1600
(c) 2000 (d) 1200

20. The average price of 3 precious diamond studded platinum thrones is ₹ 97610498312. If their prices are in the ratio 4:7:9. The price of the cheapest is
(a) 5, 65, 66, 298.972 (b) 5, 85, 66, 29, 8987.2
(c) 58, 56, 62, 889.72 (d) None of these

21. The average weight of 23boxes is 3kg. If the weight of the container (in which the boxes are kept) is included, the calculated average weight per box increases by 1 kg. What is the weight of the container?
(a) 26 kg (b) 4 kg
(c) 5 kg (d) None of these

22. A man covers 1/4th of his journey by cycle at 40 km/h, the ½ of the remaining by car at 20 km/h, and the rest by walking at 10 km/h. Find his average speed during the whole journey.
(a) 16 kmph (b) 15 kmph
(c) 18 kmph (d) 17 kmph

23. The average age of a group of 15 persons is 25 years and 5 months. Two persons, each 40 years old, left the group. What will be the average age of the remaining persons in the group?
(a) 23.17 years (b) 24.25 years
(c) 25.35 years (d) 25 years

24. The average salary of the entire staff in an department is ₹2000 per month. The average salary of officers is ₹ 3000 and that of non-officers is ₹ 1500. If the number of officers is 10, then find the number of non-officers in the office?
(a) 20 (b) 25
(c) 15 (d) 10

25. $\sum_{r=1}^{n}(n+1)r$, where $r = n$.
(a) $\frac{[(n-1)n(n+1)]}{2}$ (b) $\frac{[n(n+1)^2]}{2}$
(c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n^2}{2}$

26. The average of 'n' numbers is z. If the number x is replaced by the number x^1, then the average becomes z^1. Find the relation between n, z, z^1, x and x^1.
(a) $\left[\frac{z^1-2}{x^1-x}=\frac{1}{n}\right]$ (b) $\frac{x^1-x}{z^1}=\frac{1}{n}$
(c) $\left[\frac{z-z^1}{x-x^1}=\frac{1}{n}\right]$ (d) $\left[\frac{x-x^1}{z-z^1}=\frac{1}{n}\right]$

27. A person travels three equal distances at a speed of x km/h, y km/h and z km/h respectively. What will be the average speed during the whole journey?
(a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/xyz$
(c) $3xyz/(xy + yz + xz)$ (d) None of these

Directions for questions 28 to 30: Read the following passage and answer the questions that follow.

Aman, Binod, Charan, Dharam and Ehsaan are the members of the same family. Each and everyone loves one another very much. Their birthdays are in different months and on different dates. *Aman* remembers that his birthday is between 25th and 30th, of *Binod* it is between 20th and 25th, of *Charan* it is between 10th and 20th, of *Dharam* it is between 5th and 10th and of *Ehsaan* it is between 1st to 5th of the month. The sum of the date of birth is defined as the addition of the date and the month, for example 12th January will be written as 12/1 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

28. What may be the maximum average of their sum of the dates of birth?
(a) 24.6 (b) 15.2
(c) 28 (d) 32

29. What may be the minimum average of their sum of the dates of births?

(a) 24.6 (b) 15.2
(c) 28 (d) 32

30. If it is known that the dates of birth of three of them are even numbers then find maximum average of their sum of the dates of birth.
(a) 24.6 (b) 15.2
(c) 27.6 (d) 28

31. If the dates of birth, of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.
(a) 27.2 (b) 26.4
(c) 28 (d) None of these

32. The average age of a group of persons going for a movie is 20 years. 10 new persons with an average age of 10 years join the group on the spot due to which the average of the group becomes 18 years. Find the number of persons initially going for the movie.
(a) 20 (b) 40
(c) 50 (d) 30

33. An engineering college has only four batches that contain 20, 40, 60 and 80 students respectively. The pass percentage of these classes, are 10%, 20%, 30% and 40% respectively. Find the pass percentage of the entire college.
(a) 50% (b) 70%
(c) 30% (d) 60%

34. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ at $x = 1$. $f(x)$ is equal to $x^3 + 12$, $g(x) = 15x^3 - 10$, $h(x) = \log 10x^2$ and $d(x) = x^2$
(a) 5 (b) 10
(c) 4 (d) 7

35. In question 34 find the average of $f(x) - g(x)$, $g(x) - h(x)$, $h(x) - d(x)$, $d(x) - f(x)$ at x = 1
(a) 0 (b) –2
(c) 5 (d) None of these.

36. The average salary of employees in TCP is ₹20,000, the average salary of managers being ₹40,000 and the management trainees being ₹5000. The total number of workers could be
(a) 350 (b) 300
(c) 100 (d) 500

Directions for questions 37 to 40: Read the following and answer the questions that follows.

During the final match of ICC championship 2000, India playing against Australia scored in the following manner:

Partnership	*Runs scored*
1st wicket	62
2nd wicket	48
3rd wicket	32
4th wicket	62
5th wicket	26
6th wicket	13

37. Find the average runs scored by the first four batsmen.
(a) 83.5 (b) 60.5
(c) 66.8 (d) Cannot be determined

38. The maximum average runs scored by the first five batsmen could be
(a) 48.6 (b) 36.8
(c) 46 (d) Cannot be determined.

39. The minimum average runs scored by the last five batsmen to get out could be
(a) 13.6 (b) 24.4
(c) 36.8 (d) 0

40. If the fifth down batsman gets out for a duck, then find the average runs scored by the first six batsmen.
(a) 27.1 (b) 33.3
(c) 28.5 (d) Cannot be determined

41. The weight of a metal piece as calculated by the average of 7 different experiments is 53.735 gm. The average of the first three experiments is 54.005 gm, of the fourth is 0.004 gm greater than the fifth, while the average of the sixth and seventh experiment was 0.010 gm less than the average of the first three. Find the weight of the body obtained by the fourth experiment.
(a) 49.353 gm (b) 51.712 gm
(c) 53.072 gm (d) 54.512 gm

42. Sumer's average expenditure for the first 4 months of the year was ₹251.25. For the next 5 months the average monthly expenditure was ₹26.27 more than what it was during the first 4 months. If the person spent ₹760 in all during the remaining 3 months of the year, find what percentage of his annual income of ₹3000 he saved in the year.
(a) 13.667% (b) –5.0866%
(c) 12.333% (d) None of these

43. A certain number of tankers were required to transport 60 lakh liters of oil from the IOCL factory in Mathura. However, it was found that since each tanker could take 50,000 liters of oil less, another 4 tankers were needed. How many tankers were initially, planned to be used?
(a) 10 (b) 15
(c) 20 (d) 25

44. One collective farm got an average harvest of 21 tons of wheat per hectare and another collective farm that had 12 acres of land less given to wheat, got 25 tons from a hectare. As a result, the second farm harvested 300 tons of wheat more than the first. How many tons of wheat did each farm harvest?
(a) 3150, 3450 (b) 3250, 3550
(c) 2150, 2450 (d) None of these

45. If the product of n positive integers is n^n, then what is the minimum value of their average for $n = 6$?

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46. The average of 7 consecutive numbers is P. If the next three numbers are also added, the average increases by **(IIFT 2013)**

Directions for question number 47-48: In 2001 there were 6 members in Binod's family and their average age was 28 years. He got married between 2001 and 2004 and in 2004 there was an addition of a child in his family. In 2006, the average age of his family is 32 years.

47. What is the present age (in 2006) of Binod's wife (in years) is:
48. If Binod's age is greater than his wife and in 2001 his age was a prime number then what is the minimum possible value of Binod's present age? (in 2006) (in years)
49. If the average of 21, 23, x, 24, 27 lies in between 25, 28 (including both). Find the number of possible integral values of x.
50. Rozer a great tennis player has 10 boxes with him, which have an average of 25 tennis balls per box. If each box has at least 8 balls and no two boxes have an equal number of balls, then what is the maximum possible number of balls in any box?
51. If the average of five different numbers is 6, and if all the numbers are positive integers then what is the largest possible value of the average of the 2 biggest numbers?

Directions for question number 52–53: In CMS, Delhi students of two different sections appeared for a test. The average score of students of Class $9^{th}$is 80 and the average score of class 10^{th} is 73. Average scores of girls and boys of class 9^{th} are 70 and 85, respectively and that of class 10^{th} are 70 and 74, respectively. The number of boys of class 10^{th} is 3 times the number of girls of class 9^{th}

52. What is the ratio of girls and boys of class 9^{th}?
53. What is the average score of all the students of class 9^{th} and 10^{th} together?
54. There are four numbers w, x, y, z. If the sum of all possible distinct groups, each having two numbers from amongst w, x, y, z is 1440, then what is the average of these four numbers?

Space for Rough Work

LEVEL OF DIFFICULTY (III)

Directions for Questions 1 to 8: Read the following:

There are 3 classes having 20, 25 and 30 students respectively having average marks in an examination as 20, 25 and 30, respectively. If the three classes are represented by A, B and C and you have the following information about the three classes, answer the questions that follow:

$A \rightarrow$ Highest score 22, Lowest score 18

$B \rightarrow$ Highest score 31, Lowest score 23

$C \rightarrow$ Highest score 33, Lowest score 26

If five students are transferred from A to B.

1. What will happen to the average score of B?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
2. What will happen to the average score of A?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a transfer of 5 students from A to C

3. What will happen to the average score of C?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
4. What will happen to the average score of A?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a transfer of 5 students from B to C (Questions 5–6)

5. What will happen to the average score of C?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
6. Which of these can be said about the average score of B?
   (a) Increases if C decreases
   (b) Decreases if C increases
   (c) Increases if C decreases
   (d) Decreases if C decreases
7. In a transfer of 5 students from A to B, the maximum possible average achievable for group B is
   (a) 25 (b) 24.5
   (c) 25.5 (d) 24
8. For the above case, the maximum possible average achieved for group A will be
   (a) 20.66 (b) 21.5
   (c) 20.75 (d) 20.5
9. What will be the minimum possible average of Group A if 5 students are transferred from A to B?
   (a) 19.55 (b) 21.5
   (c) 19.33 (d) 20.5
10. If 5 students are transferred from B to A, what will be the minimum possible average of A?
    (a) 20.69 (b) 21
    (c) 20.75 (d) 20.6
11. For question 10, what will be the maximum average of A?
    (a) 23.2 (b) 22.2
    (c) 18.75 (d) 19

Directions for Questions 12 to 17: Read the following and answer the questions that follow.

If 5 people are transferred from A to B and another independent set of 5 people are transferred back from B to A, then after this operation (Assume that the set transferred from B to A contains none from the set of students that came to B from A)

12. What will happen to B's average?
    (a) Increase if A's average decreases
    (b) Decrease always
    (c) Cannot be said
    (d) Decrease if A's average decreases
13. What can be said about A's average?
    (a) Will decrease
    (b) Will always increase if B's average changes
    (c) May increase or decrease
    (d) Will increase only if B's average decreases
14. At the end of the 2 steps mentioned above (in the *direction*) what could be the maximum value of the average of class B?
    (a) 25.4 (b) 25
    (c) 24.8 (d) 24.6
15. For question 14, what could be the minimum value of the average of class B?
    (a) 22.4 (b) 24.2
    (c) 25 (d) 23
16. What could be the maximum possible average achieved by class A at the end of the operation?
    (a) 25.2 (b) 26
    (c) 23.25 (d) 23.75
17. What could be the minimum possible average of class A at the end of the operation?
    (a) 21.4 (b) 19.2
    (c) 28.5 (d) 20.25

Directions for Questions 18 to 23: Read the following and answer the questions that follow.

If 5 people are transferred from C to B, further, 5 more people are transferred from B to A, then 5 are transferred

from *A* to *B* and finally, 5 more are transferred from *B* to *C*.

18. What is the maximum possible average achieved by class *C*?
    (a) 30.833 (b) 30
    (c) 29.66 (d) 30.66
19. What is the maximum possible average of class *B*?
    (a) 26 (b) 27
    (c) 25 (d) 28
20. What is the maximum possible average value attained by class *A*?
    (a) 22.75 (b) 23.75
    (c) 23.5 (d) 24
21. The minimum possible value of the average of group *C* is
    (a) 26.3 (b) 27.5
    (c) 29.6 (d) 28
22. The minimum possible average of group *B* after this set of operation is
    (a) 21.6 (b) 22.2
    (c) 21.8 (d) 21.4
23. The minimum possible average of group *A* after the set of 3 operation is
    (a) 20 (b) 20.3
    (c) 20.4 (d) 19.8
24. Which of these will definitely not constitute an operation for getting the minimum possible average value for group *A*?
    (a) Transfer of five 31s from *B* to *A*
    (b) Transfer of five 26s from *C* to *B*
    (c) Transfer of five 22s from *A* to *B*
    (d) Transfer of five 33s from *C* to *B*
25. For getting the lowest possible value of *C*'s average, the sequence of operations could be
    (a) Transfer five 33s from *C* to *B*, five 23s from *B* to *A*, five 18s from *A* to *B*, five 18s from *B* to *C*
    (b) Transfer five 33s from *C* to *B*, 31s from *B* to *A*,
    (c) Both a and b
    (d) None of the above
26. If we set the highest possible average of class *C* as the primary objective and want to achieve the highest possible value for class *B* as the secondary objective, what is the maximum value of class *B*'s average that is attainable?
    (a) 27 (b) 26
    (c) 25 (d) 24
27. For Question 26, if the secondary objective is changed to achieving the minimum possible average value of class *B*'s average, the lowest value of class *B*'s average that could be attained is
    (a) 22.6 (b) 23
    (c) 22.2 (d) 22
28. For question 27, what can be said about class *A*'s average?
    (a) Will be determined automatically at 22.25
    (b) Will have a maximum possible value of 22.25
    (c) Will have a minimum possible value of 22.25
    (d) Will be determined automatically at 22.5
29. A team of miners planned to mine 1800 tons of ore during a certain number of days. Due to technical difficulties in one-third of the planned number of days, the team was able to achieve an output of 20 tons of ore less than the planned output. To make up for this, the team overachieved for the rest of the days by 20 tons. The end result was that the team completed the task one day ahead of time. How many tons of ore did the team initially plan to ore per day?
    (a) 50 tons (b) 100 tons
    (c) 150 tons (d) 200 tons
30. According to a plan, a team of woodcutters decided to harvest 216 m^3 of wheat in several days. In the first three days, the team fulfilled the daily assignment, and then it harvested 8 m^3 of wheat over and above the plan everyday. Therefore, a day before the planned date, they had already harvested 232 m^3 of wheat. How many cubic metres of wheat a day did the team have to cut according to the plan?
    (a) 12 (b) 13
    (c) 24 (d) 25
31. On an average, two litres of milk and one litre of water are needed to be mixed to make 1 kg of sudha shrikhand of type *A*, and 3 litres of milk and 2 litres of water are needed to be mixed to make 1 kg of sudha shrikhand of type *B*. How many kilograms of each type of shrikhand was manufactured if it is known that 130 litres of milk and 80 litres of water were used?
    (a) 20 of type *A* and 30 of type *B*
    (b) 30 of type *A* and 20 of type *B*
    (c) 15 of type *A* and 30 of type *B*
    (d) 30 of type *A* and 15 of type *B*
32. There are 500 seats in Minerva Cinema, Mumbai, placed in similar rows. After the reconstruction of the hall, the total number of seats became 10% less. The number of rows was reduced by 5 but each row contained 5 seats more than before. How many rows and how many seats in a row were there initially in the hall?
    (a) 20 rows and 25 seats
    (b) 20 rows and 20 seats
    (c) 10 rows and 50 seats
    (d) 50 rows and 10 seats

33. One fashion house has to make 810 dresses and another one 900 dresses during the same period of time. In the first house, the order was ready 3 days ahead of time and in the second house, 6 days ahead of time. How many dresses did each fashion house make a day if the second house made 21 dresses more a day than the first?
    (a) 54 and 75 (b) 24 and 48
    (c) 44 and 68 (d) 04 and 25
34. A shop sold 64 kettles of two different capacities. The smaller kettle cost a rupee less than the larger one. The shop made 100 rupees from the sale of large kettles and 36 rupees from the sale of small ones. How many kettles of either capacity did the shop sell and what was the price of each kettle?
    (a) 20 kettles for 2.5 rupees each and 14 kettles for 1.5 rupees each
    (b) 40 kettles for 4.5 rupees each and 24 kettles for 2.5 rupees each
    (c) 40 kettles for 2.5 rupees each and 24 kettles for 1.5 rupees each
    (d) either a or b
35. An enterprise got a bonus and decided to share it in equal parts between the exemplary workers. It turned out, however, that there were 3 more exemplary workers than it had been assumed. In that case, each of them would have got 4 rupees less. The administration had found the possibility to increase the total sum of the bonus by 90 rupees and as a result each exemplary worker got 25 rupees. How many people got the bonus?
    (a) 9 (b) 18
    (c) 8 (d) 16

Directions for Questions 36 to 39: Read the following and answer the questions that follows.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

36. If the minister for economic affairs decided to reverse the process of calculation of average income and average expenditure, what will happen to the estimated savings of a person living on Hoola Boola Moola island?
    (a) It will increase
    (b) It will decrease
    (c) It will remain constant
    (d) Will depend on the value
37. If it is known that Mr. Magoo Hoola Boola estimates his savings at 10 Moolahs and if it is further known that his actual expenditure is 288 Moolahs in an year (Moolahs, for those who are not aware, is the official currency of Hoola Boola Moola), then what will happen to his estimated savings if he suddenly calculates on the basis of a 12 month calendar year?
    (a) Will increase by 5 (b) Will increase by 15
    (c) Will increase by 10 (d) Will triple
38. Mr. Boogie Woogie comes back from the USA to Hoola Boola Moola and convinces his community comprising 546 families to start calculating the average income and average expenditure on the basis of 12 months per calendar year. Now if it is known that the average estimated income on the island is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).
    (a) 251.60 Moolahs (b) 565.5 Moolahs
    (c) 625.5 Moolahs (d) Cannot be determined
39. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income on the basis of 12 months per calendar year. Now if it is known that the average estimated income in his community is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).
    (a) 251.60 Moolahs (b) 282.75 Moolahs
    (c) 312.75 Moolahs (d) Cannot be determined

Directions for Questions 40 to 44: Read the following and answer the questions that follows.

The Indian cricket team has to score 360 runs on the last day of a test match in 90 overs, to win the test match. This is the target set by the opposing captain Brian Lara after he declared his innings closed at the overnight score of 411 for 7.

The Indian team coach has the following information about the batting rates (in terms of runs per over) of the different batsmen:

Assume that the run rate of a partnership is the weighted average of the individual batting rates of the batsmen involved in the partnership (on the basis of the ratio of the strike each batsman gets, i.e., the run rate of a partnership is defined as the weighted average of the run rates of the two batsmen involved weighted by the ratio of the number of balls faced by each batsman).

Since decimal fractions of runs are not possible for any batsman, assume that the estimated runs scored by a

batsman in an inning (on the basis of his run rate and the number of overs faced by him) is rounded off to the next higher integer immediately above the estimated value of the runs scored during the innings.

For example, if a batsman scores at an average of 3 runs per over for 2.1666 overs, then he will be estimated to have scored $2.1666 \times 3 = 6.5$ runs in his innings, but since this is not possible, the actual number of runs scored by the batsman will be taken as 7 (the next higher integer above 6.5).

Runs scored per over in different batting styles

Name of Batsman	*Defensive*	*Normal*	*Aggressive*
Das	3	4	5
Dasgupta	2	3	4
Dravid	2	3	4
Tendulkar	4	6	8
Laxman	4	5	6
Sehwag	4	5	6
Ganguly	3	4	5
Kumble	2	3	4
Harbhajan	3	4	5
Srinath	3	4	5
Yohannan	2	3	4

Also, this rounding off can take place only once for one innings of a batsman.

Assume no extras unless otherwise stated.

Assume that the strike is equally shared unless otherwise stated.

40. If the first wicket pair of Das and Dasgupta bats for 22 overs and during this partnership Das has started batting normally and turned aggressive after 15 overs while Dasgupta started off defensively but shifted gears to bat normally after batting for 20 overs, find the expected score after 22 overs.
    (a) 65 (b) 71
    (c) 82 (d) 58
41. Of the first-wicket partnership between Das and Dasgupta as per the previous question, the ratio of the number of runs scored by Das to those scored by Dasgupta is:
    (a) 46:25 (b) 96:46
    (c) 41:32 (d) Cannot be determined
42. The latest time by which Tendulkar can come to bat and still win the game, assuming that the run rate at the time of his walking the wicket is into 2.5 runs per over, is (assuming he shares strike equally with his partner and that he gets the maximum possible support at the other end from his batting partner and both play till the last ball).
    (a) After 50 overs (b) After 55 overs
    (c) After 60 overs (d) Cannot be determined
43. For question 42, where Tendulkar batted aggressively and assuming that it is the Tendulkar–Laxman pair that wins the game for India (after Tendulkar walks into bat with the current run rate at 2.5 per over, and at the latest possible time for him to win the game with maximum possible support from the opposite end), what will be Tendulkar's score for the innings (assume equal strike)?
    (a) 105 (b) 120
    (c) 135 (d) None of these
44. For questions 42 and 43, if it was Laxman who batted with Tendulkar for his entire innings, then how many runs would Laxman score in the innings?
    (a) 105 (b) 75
    (c) 90 (d) Cannot be determined

Directions for Questions 45 to 49: Read the following and answer the questions that follow (with reference to the data provided in the table for questions 40 to 44).

If Sachin Tendulkar walks into bat after the fall of the fifth wicket and has to share partnerships with Ganguly, Kumble, Harbhajan, Srinath and Yohannan, who have batted normally, defensively, defensively, defensively and defensively, respectively while Tendulkar has batted normally, aggressively, aggressively, aggressively and aggressively, respectively in each of the five partnerships that lasted for 12, 10, 8, 5 and 10 overs, respectively, sharing strike equally with Ganguly and keeping two-thirds of the strike in his other four partnerships, then answer the following questions:

45. How many runs did Sachin score during his innings?
    (a) 128 (b) 212
    (c) 176 (d) None of these
46. The highest partnership that Tendulkar shared in was worth
    (a) 60 (b) 61
    (c) 62 (d) 58
47. The above partnership was shared with:
    (a) Ganguly (b) Yohannan
    (c) Kumble (d) All three
48. If India proceeded to win the match based on the runs scored by these last five partnerships (assuming the last wicket pair remained unbeaten), what could be the maximum score at which Tendulkar could have come into bat?
    (a) 103 for 5 (b) 97 for 5
    (c) 100 for 5 (d) 104 for 5
49. For Question 48, what could be the minimum score at which Tendulkar could have come to bat?
    (a) 103 for 5 (b) 97 for 5
    (c) 104 for 5 (d) 98 for 5

ANSWER KEY

Level of Difficulty (I)

1. (d)	2. (a)	3. (d)	4. (b)
5. (d)	6. (c)	7. (d)	8. (b)
9. (d)	10. (b)	11. (a)	12. (b)
13. (d)	14. (d)	15. (b)	16. (b)
17. (b)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (a)	24. (b)
25. (a)	26. (c)	27. (d)	28. (a)
29. (c)	30. (b)	31. (c)	32. (a)
33. (b)	34. (a)	35. (d)	36. (c)
37. (d)	38. (b)	39. (d)	40. (c)
41. (c)	42. (a)	43. (c)	44. (b)
45. (a)	46. (d)	47. (d)	48. (b)
49. (c)	50. (b)	51. (b)	52. (a)
53. (d)	54. (d)	55. (c)	56. (b)
57. (d)	58. (a)	59. (b)	60. (c)
61. (b)	62. (d)	63. (a)	64. (a)
65. (b)	66. (c)	67. (a)	68. (b)
69. (d)	70. (d)	71. (b)	72. (a)
73. (a)	74. (d)	75. (a)	76. (b)
77. (b)	78. (b)	79. (a)	80. (a)
81. (d)	82. (a)	83. (c)	84. (d)
85. 44	86. 25	87. -10	88. 34.66
89. 60	90. 37.50	91. 49	92. 39.56
93. 230	94. 9		

Level of Difficulty (II)

1. (b)	2. (b)	3. (a)	4. (b)
5. (b)	6. (a)	7. (b)	8. (d)
9. (a)	10. (a)	11. (b)	12. (a)
13. (b)	14. (b)	15. (a)	16. (c)
17. (d)	18. (b)	19. (a)	20. (b)
21. (d)	22. (a)	23. (a)	24. (a)
25. (b)	26. (c)	27. (c)	28. (c)
29. (b)	30. (d)	31. (a)	32. (b)
33. (c)	34. (a)	35. (a)	36. (a)
37. (d)	38. (a)	39. (d)	40. (d)
41. (c)	42. (b)	43. (c)	44. (a)
45. 6	46. 1.5	47. 56	48. 58
49. 16	50. 142	51. 12	52. 2
53. 76	54. 120		

Level of Difficulty (III)

1. (b)	2. (d)	3. (b)	4. (d)
5. (d)	6. (b)	7. (b)	8. (a)
9. (c)	10. (d)	11. (b)	12. (b)
13. (b)	14. (c)	15. (a)	16. (c)
17. (d)	18. (a)	19. (b)	20. (b)
21. (b)	22. (b)	23. (a)	24. (c)
25. (a)	26. (d)	27. (c)	28. (a)
29. (b)	30. (c)	31. (a)	32. (a)
33. (a)	34. (c)	35. (b)	36. (a)
37. (b)	38. (d)	39. (b)	40. (b)
41. (b)	42. (c)	43. (b)	44. (d)
45. (b)	46. (b)	47. (b)	48. (d)
49. (b)			

Solutions and Shortcuts

Level of Difficulty (I)

1. Required average = (1 + 2 + 3 + … + 15)/15 = 120/15 = 8. Alternately you could use the formula for sum of the first n natural numbers as $n(n + 1)/2$ with n as 15. Then average = Sum/15 = (15 × 16/2)/15 = 8
2. Required average = (0 + 1 + 2 + … + 9)/10 = 45/10 = 4.5
3. Required average = (2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20)/10 = 110/10 = 11. Alternately you could use the formula for sum of the first n even natural numbers as $n(n + 1)$ with n as 10. Then average = Sum/10 = 10 × 11/10 = 11.
4. The sum of the first n odd numbers = n^2. In this case $n = 10 \rightarrow$ Sum = $10^2 = 100$. Required average = 100/10 = 10.
5. Required average = (2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29)/10 = 129/10 = 12.9.
6. Required average = (4 + 6 + 8 + 9 + 10 + 12 + 14 + 15 + 16 + 18)/10 = 112/10 = 11.2.
7. Required average = (3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 + 31)/10 = 158/10 = 15.8.
8. $P = 14 \times 10 - 13 \times 8 = 140 - 104 = 36$
9. D's weight = 4 × 70 – 3 × 74 = 280 – 222 = 58. E's weight = 58 + 3 = 61.
   Now, we know that $A + B + C + D = 4 \times 70 = 280$ and $B + C + D + E = 75 \times 4 = 300$. Hence, A's weight is 20 kg less than E's weight. A = 61 - 20 = 41 kg.
10. Monday + Tuesday + Wednesday = 3 × 35 = 105;
    Tuesday + Wednesday + Thursday = 3 × 30 = 90.
    Thus, Monday – Thursday = 15 and
    Thursday = Monday/2 → Monday = 30 and Thursday = 15
11. Total present age of A, B and C = 25 × 3 + 15 = 75 + 15 = 90.
    Total present age of B and C = 20 × 2 + 20 = 60.
    A's age = 90 – 60 = 30.
12. $4x + 40 = 5(x + 4) \rightarrow x = 20$ (average after 4 innings). Hence, new average = 20 + 4 = 24.
13. (5+10+15+20+25+30)/6 = 105/6 = 17.5
14. $1/5 + 1/8 + C = 3 \times 1/10 \rightarrow C = -1/40$.
15. His total score is 90 + 70 + 150 = 310 out of 450. This works out to a percentage score of = 68.88%
16. Average income over 9 months = [3 × (2250 + 500) + 3 × (2150 + 500) + 3 × (5750+500)]/9 = 3883.33
17. Total age (at present) = 5 × 20 = 100 years. Total age of the family excluding the youngest member (for

the remaining 5 people) = 100 –5 = 95. Average age of the other 4 people in the family = 95/4 years.

5 years ago their average age = 95/4 – 5 = 75/4 = 18.75 years.

18. If the average age of 5 persons has gone up by 10 years it means the total age has gone up by 50 years. Thus the total age of the two women would be: 30 + 40 + 50 = 120. Hence, their average age = 60 years.
19. $W + T + F = 60$; $T + F + S = 63 \rightarrow S - W = 3$. Hence temperature on Wednesday = 22 – 3 = 19.
20. Average speed = $\frac{2 \times 60 \times 40}{60 + 40} = 48$ km/hr.
21. Teacher's weight = 21 × 20 – 20 × 19 = 420– 380 = 40.
22. 3 × 20 – 2 × 25 = 60 – 50 = 10.
23. The weight of the new man would be 29 × 5 kgs more than the weight of the man he replaces. New man's weight = 120 + 29 × 5 = 265 kgs.
24. Let their current ages be x and $3x$. Then their ages after 10 years would be $x + 10$ and $3x + 10$. Now it is given that $(x + 10)/(3x + 10) = 1/2 \rightarrow x = 10$ and hence their current ages are 10 years and 30 years, respectively. So their current average age is 20 years. After 20 years their average age would be 20+20 = 40 years.
25. The average would be given by the average of the first and last numbers (since the series 1, 2, 3, 4…100 is an Arithmetic Progression).

    Hence, the average = (1 + 100)/2 = 50.50
26. We need the average of the numbers: 23, 29, 31, 37, 41, 43 and 47

    Average = Total/number of numbers → 251/7 = 35.85
27. Let the numbers be a, b, c and d, respectively. $a + b + c = 20 \times 3 = 60$ and $b + c + d = 25 \times 3 = 75$.

    Also, since d-a = 15, we have 30- a = 15 or a = 15.
28. Required average = (15 × 40 + 25× 48)/40 = 1800/40 = 45.
29. The numbers would form an *AP* with common difference 1 and the middle term (also the 4th term) as 21. Thus, the numbers would be 18, 19, 20, 21, 22, 23 and 24. The highest of these numbers would be 24.
30. Required average = (8 × 11 + 15 + 17)/10 = 120/10 = 12.
31. The new average would also go up by 4. Hence, 14 + 4 = 18.
32. If the numbers are $a + 1$, $a + 2$, $a + 3$, $a + 4$,$a + 5$….a+11 the average would be $a + 6$. If we take 13 numbers as:

    $a + 1$, $a + 2$, $a + 3$, $a + 4$, $a + 5$, $a + 6$…. and $a + 13$ their average would be a+7. Hence, the average increases by 1. You can also experimentally verify this by taking any 11 consecutive numbers and finding their average, then adding the next two numbers and finding the average of the 13 numbers.
33. Total of 38 numbers = 40 × 45 – 65 – 25 = 1710. Average of 38 numbers = 1710/38 = 45.
34. When we multiply each number by 9, the average would also get multiplied by 9. Hence, the new average = 18 × 9 = 162.
35. Let the number of ladies be n. Then we have $5 \times 12 + n \times 8 = (5 + n) \times (9) \rightarrow 60 + 8n = 45 + 9n \rightarrow n = 15$.
36. (3 × 63 + 67)/4 = 256/4 = 64.
37. 2100 × 3 – 2250 × 2 = 1800.
38. (2000 × 15 + 1000 × 15)/30 = (2000 + 1000)/2 = 1500.
39. The decrease in weight would be 20 kgs (10 people's average weight drops by 2 kgs). Hence, the new person's weight = 140 - 20 = 120.
40. When a person aged 26 years, is replaced by a person aged 56 years, the total age of the group goes up by 30 years. Since this leads to an increase in the average by 6 years, it means that there are 30/6 = 5 persons in the group.
41. (33 × 3 + 23 × 2)/5 = 145/5 = 29.
42. Let the number of marks in the 8th paper be M. Then the total of the first eight papers = 8 × 48 while the total of the last 8 (i.e., 8th to 15th papers) would be 8 × 54.

    Total of 1st 8 + total of 8th to 15th = total of all 15 + marks in the 8th paper →

    $8 \times 48 + 8 \times 54 = 15 \times 50 + M$

    $816 = 750 + M \rightarrow M = 66$

    (Note: We write this equation since marks in the eighth paper is counted in both the first 8 and the last 8)
43. Let the captain's age by C. Then: $11 \times 28 = 26 \times 5 + 29 \times 5 + C \rightarrow 308 = 130 + 145 + C \rightarrow C = 33$.
44. His earning in the 12th month should be: 4000 × 12 – 3200 × 11 = 48000 – 35200 = 12800.
45. Total distance divided by total time = 1440/17 = 84.705.
46. In three quarters of an hour the train has traveled 57 km. Thus, in a full hour the train would have traveled 1/3rd more (as it gets 1/3rd time more). Thus, the speed of the train = 57+ 1 × 57/3 = 57 + 19 = 76.
47. Total weight of all 6 = 58.5 × 6. Total weight of Ram and Tram = 65 × 2 = 130. Average weight of the 4 people excluding Ram and Tram = (58.5 × 6 – 130)/4 = 55.25 kg.
48. $10 \times 59 + 20 \times A = 30 \times 56 \rightarrow A = (1680 - 590)/20 = 1090/20 = 54.5$
49. (60 × 23 + 40 × 35)/100 = 2780/100 = 27.8
50. If we take the first number as 6n, the second number would be $3n$ and the third would be $2n$. Sum of the three numbers = $6n + 3n + 2n = 11n = 132 \times 3 \rightarrow n = 36$. The smallest number would be $2n = 72$.

51. The ratio between the first, second and third would be: 10:15:24. Since their total is 147, the numbers would be 30, 45 and 72 respectively. The second number is 45.
52. $(40 \times 150 + 10 \times 155)/50 = 151$. (Note: this question can also be solved using the alligation method explained in the next chapter.
53. The total weight of the six people goes up by 9 kgs (when the average for 6 persons goes up by 1.5 kg). Thus, the new person must be 9 kgs more than the person who he replaces. Hence, the new person's weight = 60 +9 = 69
54. Total age = $3 \times 24 = 72$. Individual ages being in the ratio 2:5:5 their ages would be 12, 30 and 30 years respectively. The youngest boy would be 12 years.
55. $50 \times 4 + M = 51 \times 5 \rightarrow M = 55$.
56. $(30 \times 1800 + M) = 31 \times 1900 \rightarrow M = 4900$. Hence, the salary is4900 per month which also means ₹ 58,800 per year.
57. Five consecutive even numbers would always be in an Arithmetic progression and their average would be the middle number. The average would be 'r' in this case.
58. The average of 7, 14, 21, 28 and 35 would be 21.
59. $30 \times 50 + T = 31 \times 50.5 \rightarrow T = 1565.5 - 1500 = 65.5$ kgs.
60. If the number of questions correct is N, then the number of wrong answers is $100 - N$. Using this we get: $N \times 3 - (100 - N) \times 1 = 160 \rightarrow 4N = 260 \rightarrow N = 65$.
61. Required age of the father will be given by the equation: $6 \times 12 = 5 \times 7 + F \rightarrow F = 37$.
62. Teacher's weight = $21 \times 51 - 20 \times 50 = 1071 - 1000 = 71$.
63. Required average = $(2 \times 5.5 + 3 \times 3.666 + 6 \times 1.8333)/11 = (11 + 11 + 11)/11 = 3$.
64. $40 \times m + 65 \times 4 = 480 \rightarrow P = (480 - 260)/40 = 5.5$.
65. Required average = $(2 \times 4000 + 2 \times 2500)/4 = 13000/4 = 3250$.
66. Required average = Total runs/ total innings = $(55 \times 4 + 0)/5 = 220/5 = 44$.
67. Principal's weight = $51 \times 69.5 - 50 \times 70 = 3544.5 - 3500 = 44.5$
68. Temperature on 6th December = $25.6 \times 3 - 27 \times 2 = 76.8 - 54 = 22.8$
69. Average age 4 years hence would be 4 years more than the current average age. Hence, 30 + 4 = 34.
70. Total age 3 years ago for 6 people = $18 \times 6 = 108$. Today, the family's total age = $18 \times 7 = 126$. The age of the 6 older people would be $108 + 3 \times 6 = 126$. Hence, the baby's age is 0 years.
71. Required average = $(21 \times 31 + 24 \times 31 + 11 \times 30)/92 = (651 + 744 + 330)/92 = 1725/92 = 18.75$
72. Assume a distance of 150 km. In such a case, the Required average = Total distance/Total time = $(150 + 150)/(6 + 5) = 300/11 = 27.27$
73. (Mon + Tue + wed) = $52 \times 3 = 156$. (Tue + Wed + Thu) = $50 \times 3 = 150$.
    Mon – Thu = 156 – 150 = 6. Since Thursday's temperature is given as 49, Monday's temperature would be 49 + 6 = 55.
74. Required average = $(15 \times 20 - 5 \times 10)/10 = 250/10 = 25$.
75. Total age of 10 children = $15 \times 10 = 150$ years. When the 6 year old child dies, the total age of the remaining 9 children would be 150 – 6 = 144. Required average = 144/9 = 16 years
76. Required average = $(2 \times 200 + 2 \times 250)/4 = 900/4 = 225$.
77. Average weight including Teacher's weight = $(40 \times 40 + 122)/41 = (1600+ 122)/41 = 1722/41 = 42$kgs.
78. The statement 'a' is as much more than the average as *'b'* is less than the average signifies that the numbers a, b, c form an Arithmetic Progression with c as the middle term. c's value would then be equal to the average of the three numbers. This average is given as 25. Hence, the correct answer is $c = 25$.
79. The sum of the given 4 numbers is 16.50. The required average = 16.50/4 = 4.125. Option (a) is correct.
80. Let the number of non officer workers in the company be W. Then we will have the following equation: $(15 \times 5000 + W \times 10000) = (15 + W) \times 9000 \rightarrow W = 60$. Thus, the total number of workers in the company would be 60 + 15 = 75.
81. The woman's age would be $10 \times 3 = 30$ years more than the age of the man she replaces. Age of the woman = $54 + 3 \times 10 = 84$ years.
82. Required average income = (Total expenditure + total savings]/12
    = $[(100 \times 3 + 200 \times 4 + 400 \times 5) + 3000]/12 = 6100/12 \approx 508$.
83. Required average = $(2 \times 5.5 + 3 \times 3.5 + 3 \times 5.5 + 5 \times 1.5)/13 = 45.5/13 = 3.5$.
84. For 150 coins to be of a value of ₹ 150, using only 25 paise, 50 paise and 1 Rupee coins, we cannot have any coins lower than the value of ₹1. Thus, the number of 50 paise coins would be 0. Option (d) is correct.
85. Since you can see that this is an Arithmetic Progression, the average of the six numbers is simply the average of the first and the last numbers in the series i.e., the average of 11 and 77 (which is 44).
86. Let the present ages of Aman and Baman be x and $2x$ respectively.

$$\frac{x+5}{2x+5} = \frac{3}{5}$$

$5x + 25 = 6x + 15$

$x = 10$. Hence their present ages are 10 and 20. In 10 years time their ages would be 20 and 30 years respectively. The average age would be 25 years.

87. Let the numbers be a, b, c, d

$a + b + c = 3 \times 20$ .....(1)

$b + c + d = 3 \times 10$ .....(2)

Equation (1) – Equation (2)

$a - d = 30$

$d = a - 30$

$d = 20 - 30 = -10$.

88. Correct average

$$= \frac{15 \times 40 - 2(80 - 40)}{15} = \frac{520}{15} = 34.66$$

89. M + T + W + Th + Fr = 40 × 5 = 200

If four of these were equal to the lowest possible, i.e. 35 each, the maximum possible temperature for the fifth day is 200 – 35× 4 = 60.

90. Present sum of ages of the family = 3 × 30 = 90

Sum of ages (5 years ago) = 90 – 3 × 5 = 75.

Required average = 75/2 = 37.50 years

91. If the Average of 50 numbers is 0, then at most 49 of them can be greater than 0 and 50th number can be such that its' negative value equals to the positive value of the first 49 numbers.

92. Required average = $\frac{40.25 - 50 - 40}{23} = \frac{910}{23} = 39.56.$

93. 6th number = (10 × 60 – 5 × 50 – 4 × 30) = 230

94. New average = $\frac{10 \times 10 - 30 + 20}{10} = 9.$

Level of Difficulty (II)

1. Find the total distance covered in each segment of 15 minutes. You will get total distance = 32.50 kilometres in 60 minutes.
2. The train needs to travel 60 minutes extra @ 20 kmph. Hence, it is behind by 20 kms. The rate of losing distance is 5 kmph. Hence, the train must have travelled for 20/5 = 4 hours@ 25 kmph → 100 km.

   Alternatively, you can also see that 20% drop in speed results in 25% increase in time. Hence, total time required is 4 hours @ 25 kmph → 100 kilometres.

   Alternatively, solve through options.
3. Standard question requiring good calculation speed. Obviously, the 15th day is being double counted. Calculations can be reduced by thinking as:

   Surplus in first 15 days – Deficit in last 17 days = 15 × 20 – 17 × 10 = 300 – 170 → Net surplus of 130. This means that the sum is advancing by 130 due to the double counting of the 15th day. This can only mean that the 15th day's expenditure is 60 + 130 = 190.

   (Lengthy calculations would have yielded the following calculations: 80 × 15 + 50 × 17 – 60 × 31 = 190).
4. Assume that the distance is 100 km. Hence, 20 km is covered @ 20 kmph, 25 @ 50 kmph and 55 km @ 55 kmph.

   Then average speed is total distance/total time = 100/(1+0.5+1) = 40 kmph.
5. 4x + 40 = (x + 4) × 5 (where x is the average of first four innings.)

   On solving we get, x = 20. x is the old average here. Hence, the new average = x + 4 = 20 + 4 = 24.
6. In one hour the total number of sheets typed will be: 60/10 + 60/20 + 60/30 = 11

   Hence the number of sheets/hour per typist is 11/3.
7. 100 → 110 → 132. Hence, 32/2 = 16.
8. You do not know the number of days in the month. Hence, the question cannot be answered.
9. The number of rooms is 50 + 40 + 40 = 130 on the three floors respectively.

   Total revenues are: 25 × 2000 + 20 × 1000 + 12 × 1500 = 88000. Hence the required average = 88000/130 = 676.92
10. Replace x with the sales value to calculate the bonus in a month.

    Bonus = 0 in January, 200 in February, 2000 each from March to November and 200 in December. Hence, his Total bonus = 0 + 200 + 2000 × 9 + 200 = 18400. Salary for the year = 3000 × 12. Total annual income = 36000+18400 = 54400. Hence, the average monthly income = 4533.33. Option (a) is closest and hence is the correct answer.
11. 61 × 43 = 48 × 36 + x + 12 × 49 → x = 307.
12. Use the same process as Q. No. 4 above. Let the journey be 100 km. The average speed is given by: Total Distance/Total time = 100/(1+1+5) = 100/7 kmph.
13. Find out the number of innings in each year. Then the answer will be given by:

$$\frac{\text{Total runs in 4 years}}{\text{Total innings in 4 years}} = (4250/132 = 32.19)$$

14. The weight of the second man is 50 kg and that of the third is 40 kg. Hence, net result is a drop of 60 for 10 people. Hence, 6 kg is the drop in the average.
15. Put $x = 10$ to get the weight of the last three packets. These packets would weigh 60 kgs each. Thus the total of the four packets would be 60 × 3 + 20 = 200. Their average weight 200/4 = 50 kg.
16. The weight of the boxes are 1st box → 40, 3rd box → 50 kg, 2nd box → 75 kg, 4th box → 150 and 5th box → 100 kg. Hence difference between the heaviest 4 and the lightest 4 is 110 kg. Hence, difference in the averages is 27.5 kg.

17. Difference between heaviest three and lightest three totals is: (150 +100) – (40 + 50) = 160
Difference in average weights is 160/3 = 53.33 kg.

18. Assume x is the average expenditure of 20 people. Then, $20x = 15 \times 90 + 5(x + 30)$. On solving we get $x = 100$
Total expenditure = 20 × 100 = ₹ 2000.

19. $30 \times A + 1600 = 50 \times (A - 8) \rightarrow 20 \times A = 2000 \rightarrow A = 100$. Total expenditure original = 100 × 30 = ₹ 3000.

20. The total price of the three stones would be 97610498312 × 3 = 292831494936. Since, this price is divided into the three stones in the ratio of 4: 7: 9, the price of the cheapest one would be = (4 × 2928314936/20) = 58566298987.2.

21. The average weight per box is asked. Hence, the container does not have to be counted as the 5th item. Also, since the average for 23 boxes goes up by 1 kg, the total weight must have gone up by 23 kgs. That weight is the actual weight of the container. Hence, option (d) is correct.

22. Solve through the same process as the Q. No. 4 of this chapter. Assume the distance to be 160. Then the journey would get broken up into: 40 kms @ 40kmph, 60 kms @20kmph and 60 kms @10kmph. The average speed = Total distance/ total time = 160/ (1 + 3 + 6) = 16 kmph.

23. (15 × 305 – 2 × 480)/13 = 278.07 months or 23.17 years.

24. Use alligation to solve.

non officer — officer

2000

1500 — 3000

3000 – 2000 = 1000 | 3000 – 2000 = 1000

1000:500 = 2:1

Non-officers = 20.

25. Solve through options by assuming the value of n and checking the value of the summation – and experimentally verifying it with the given options. At n = 1, we get the value of the summation as 2. Only option (b) gives the same summation for n = 1. Hence, the other options get rejected and option (b) stands as the correct answer.

26. $nz - x + x^1 = nz^1 \rightarrow$ Simplify to get Option (c) correct.

27. Let the equal distances be 'd' each. Then using the formula for the average speed as: Total distance/ total time we get: Average speed = $3d/(d/x + d/y + d/z) = 3xyz/(yx + yz + xz)$.

28–30. You have to take between 25th and 30th to mean that both these dates are also included.

28. The maximum average will occur when the maximum possible values are used. Thus:
Aman should have been born on 30th, *Binod* on 25th, *Charan* on 20th, *Dharam* on 10th and *Ehsaan* on 5th. Further, the months of births in random order will have to be between August to December to maximize the average.
Hence the maximum total will be 30 + 25 + 20 + 10 + 5 + 12 + 11 + 10 + 9 + 8 = 140. Hence, the maximum average is 28.

29. The minimum average will be when we have 1 + 5 + 10 + 20 + 25 + 1 + 2 + 3 + 4 + 5 = 76. Hence, average is = 76/5 = 15.2.

30. This does not change anything. Hence the answer is the same as Q. 28.

31. The prime dates must be 29th, 23rd, 19th and 5th. This represents a reduction in the totals from 30+25+20+5 to 29+23+19+5 – a drop of 4. Hence, the maximum possible average will reduce by 4/5 = 0.8. Hence, the answer will be 27.2.

32.

18

20 — 10

18 – 10 = 8 | 20 – 18 = 2

8:2
4:1

Number of person initially going to movie = 40

33. The number of pass candidates are = 2 + 8 + 18 + 32 = 60 out of a total of 200. Hence, 30%.

34. Put $x = 1$ in the given equations and find the average of the resultant values. You will get the respective values of f(x) = 13; g(x) = 5; h(x) = 1 and d(x) = 1. Hence, the required average = 20/4 = 5.

35. The values are: f(x) – g(x) = 8; g(x) – h(x) = 4; h(x) – d(x) = 0 and d(x) – f(x) = –12. The average of these four values = $\frac{8+4+0-12}{4} = 0$.

36. By alligation the ratio is 4: 3.

Management trainee — Manager

20,000

5000 — 40000

40000 – 20000 | 20000 – 5000 = 15000

20000:15000
4:3

Hence, only 350 (multiple of 4+ 3 = 7) is a possible value for the number of people.

37-40.

37. You don't know who got out when. Hence, cannot be determined.

38. Since possibilities are asked about, you will have to consider all possibilities. Assume, the sixth and seventh batsmen have scored zero. Only then will the possibility of the first 5 batsmen scoring the highest possible average arise. In this case the maximum possible average for the first 5 batsmen could be 243/5 = 48.6

39. Again it is possible that only the first batsman has scored runs. Hence, the minimum average would be 0.
40. We cannot find out the number of runs scored by the 7th batsman. Hence the answer is (d).
41. You can take 53 as the base to reduce your calculations. Otherwise the question will become highly calculation intensive. Let the fifth experiments measurement be 'x' above 53. Then you get: $0.735 \times 7 = 1.005 \times 3 + (x + 0.004) + x + 0.995 \times 2 \rightarrow 5.145 = 3.015 + 2x + 0.004 + 1.99$. On solving this you get $x = 0.068$. Hence, the weight of the fifth body is 53.068 and the weight of the fourth body is 53.072. Hence, option (c) is correct.
42. $251.25 \times 4 + 277.52 \times 5 + 760 = 3152.6$
    Required percentage
    $$= \frac{3000 - 3152.6}{3000} = -5.087\%$$
43. Solve using options. 20 is the only possible value.
44. Check through options to solve. Option (a) is correct since if the first farm harvested 3150 tons of wheat, with an average harvest of 21 tons per hectare – the number of hectares would be 3150/21 = 150. The second farm would then harvest 12 hectares less (as given in the question) - thus would harvest 138 hectares with an average output of 25 tons per hectare. The total harvest would be in this case 138 × 3450 – and is 300 tons more than the first farm as required. Hence, this answer is correct.
45. Let the numbers be $n_1, n_2, n_3, n_4, \ldots\ldots n_n$.
    $(n_1+n_2+n_3+n_4, \ldots\ldots+n_n)/n \geq (n_1, n_2, n_3, n_4, \ldots\ldots n_n)^{(1/n)}$
    $(n_1+n_2+n_3+n_4, \ldots\ldots+n_n)/n \geq n$
    For $n = 6$, the minimum value of $(n_1+ n_2+ n_3+ n_4, \ldots\ldots+n_n) = 6$
    Therefore, the minimum value of the average of these numbers = 6
46. If the numbers are 1, 2, 3, 4, 5, 6, 7 and we add 8, 9,10.
    Initial average = (1 + 2 + 3 + 4 + 5 + 6 + 7)/7 = 4
    Final average = (1 + 2 + 3 + 4 + -----10)/10 = 5.5
    Therefore, the average increases by 1.5.
47. If present age of Binod's wife is x years.
    Then according to the question:
    $$\frac{33 \times 6 + x + 2}{8} = 32$$
    $x + 2 + 198 = 256$
    $x + 2 = 58$
    $x = 56$
    $x = 56$ years
48. Binod's wife's age in 2001 was 51 years. Therefore Binod's age in 2001 was greater than 51 and minimum possible prime number above 51 is 53. Therefore minimum possible present age of Binod = 53 + 5 = 58 years.
49. $$25 \leq \frac{21 + 23 + x + 24 + 27}{5} \leq 28$$
    $125 \leq 95 + x \leq 140$
    $30 \leq x \leq 45$
    Number of possible values of $x = 16$.
50. Total number of balls in all 10 boxes = 10 × 25 = 250.
    Minimum possible number of balls in 9 of these 10 boxes = 8 + 9 + 10 +11 +12 + 13 + 14 + 15 + 16 = 108.
    Maximum possible number of balls in any box = 250 – 108 = 142.
51. Sum of these five numbers = 6 × 5 = 30.
    Maximum average of the largest two numbers would occur when the three smaller numbers are as small as possible (i.e., when they are 1, 2 and 3, respectively) → Max. Average = (30 – 1 – 2 – 3)/2 = 24/2 = 12.
52. Let there be B number of boys, G number of girls in class 9th.
    $$\frac{B}{G} = \frac{80 - 70}{85 - 80} = \frac{10}{5} = 2:1$$
53. Let the number of girls and boys in class 10TH are g and b respectively.
    $$\frac{b}{g} = \frac{73 - 70}{74 - 73} = 3:1, b = 3g$$
    Number of boys of class 10th is three times the number of girls in 9th
    b = 3G
    let B = 2k, G = k, b = 3k, g = k
    Average score of all students of class 9th and 10th
    $$\frac{2k.85 + k.70 + 3k.74 + k.70}{2k + k + 3k + k} = \frac{532}{7} = 76$$
54. Total possible groups = (w, x), (w, y), (w, z), (x, y), (x, z), (y, z)
    According to the question = $w + x + w + y + w + z + x + y + x + z + y + z = 1440$
    $3(w + x + y + z) = 1440$. Hence, $(w + x + y + z) = 480$.
    Required average = 480/4 = 120.

Level of Difficulty (III)

1. Definitely decrease, since the highest marks in Class *A* is less than the lowest marks in Class *B*.
2. Cannot say since there is no indication of the values of the numbers which are transferred.
3. It will definitely decrease since the highest possible transfer is lower than the lowest value in *C*.

4. The effect on *A* will depend on the profile of the people who are transferred. Hence, anything can happen.
5. Cannot say since there is a possibility that the numbers transferred are such that the average can either increase, decrease or remain constant.
6. If *C* increases, then the average of *C* goes up from 30. For this to happen it is definite that the average of *B* should drop.
7. The maximum possible average for *B* will occur if all the 5 transferees from *A* have 22 marks.
8. The average of Group *A* after the transfer in Q. 7 above is:

   (400 – 18*5)/15 = 310/15 = 20.66
9. (400 – 22*5)/15 = 19.33
10. 400 + 23*5 = 515. Average = 515/25 = 20.6
11. 400 + 31*5 = 555. Average = 555/25 = 22.2
12. Will always decrease since the net value transferred from *B* to *A* will be higher than the net value transferred from *A* to *B*.
13. Since the lowest score in Class *B* is 23 which is more than the highest score of any student in Class *A*. Hence, *A*'s average will always increase.
14. The maximum possible value for *B* will happen when the *A* to *B* transfer has the maximum possible value and the reverse transfer has the minimum possible value.
15. For the minimum possible value of *B* we will need the *A* to *B* transfer to be the lowest possible value while the *B* to *A* transfer must have the highest possible value. Thus, *A* to *B* transfer → 18* 5 while *B* to *A* transfer will be 31*5. Hence answer is 22.4.
16. The maximum value for *A* will happen in the case of Q. 15. Then the increment for group *A* is:

   31*5 – 18*5 = 5*(31 – 18) = 65.

   Thus maximum possible value is 465/20 = 23.25.
17. Minimum possible average will happen for the transfer we saw in Q. 14. Thus the answer will be 405/20 = 20.25.
18. The maximum possible value for *C* will be achieved when the transfer from *C* is of five 26's and the transfer back from *B* is of five 31's. Hence, difference is totals will be +25. Hence, max. average = (900 + 25)/30 = 30.833.

   [Note here that 900 has come by 30*30]
19. For the maximum possible value of Class *B* the following set of operations will have to hold:

   Five 33's are transferred from *C* to *B*, whatever goes from *B* to *A* comes back from *A* to *B*, then five 23's are transferred from *B* to *C*. This leaves us with:

   Increase of 50 marks → average increases by 2 to 27.
20. A will attain maximum value if five 33's come to *A* from *C* through *B* and five 18's leave *A*. In such a case the net result is going to be a change of +75. Thus the average will go up by 75/20 = 3.75 to 23.75.

21–23. Will be solved by the same pattern as the above questions.

   Note: For question 22, you need to realise that there are only a maximum of six 31's in group B.

24. Option (c) is correct, since you need to transfer out whatever you got into *A*, in order to keep the value of *A*'s average at the minimum.

25–28. Will be solved by the same pattern as above questions.

29–35. These are standard questions using the concept of averages. Hence, analyse each and every sentence by itself and link the interpretations. If you are getting stuck, the only reason is that you have not used the information in the questions fully.

36. Monthly estimates of income is reduced as the denominator is increased from 12 to 14 at the same time the monthly estimate of expenditure is increased as the denominator is reduced from 12 to 9. Hence, the savings will be underestimated.

37–39. Use the averages formulae and common sense to answer.

40–49. The questions are commonsensical with a lot of calculations and assumptions involved. You have to solve these using all the information provided.

40. Das's score = 15*2 + 7*2.5 = 47.5 → 48.

   Dasgupta's score = 20*1 + 2*1.5 = 23.
41. From the above the answer is 48:23 = 96:46.

42–44. By maximum possible support from the other end, you have to assume that he has Laxman or Sehwag batting aggressively for the entire tenure at the crease. Strike has to be shared equally.

42. Through options, After 60 overs, score would be 150. Then Tendulkar can score @ 4 runs per over (sharing the strike and batting aggressively) and get maximum support @ 3 runs per over. Thus in 30 overs left the target will be achieved.
43. Tendulkar's score for the innings will be 30*4 = 120.
44. We do not know when Laxman would have come into bat. Hence this cannot be determined.

45–49. Build in each of the conditions in the problem to form a table like:

Partnership	Partner	Overs faced	Tendulkar's score	Partner's score
6th wicket	Ganguly	12	6 overs × 6	6 overs × 4
7th wicket	and so on			
8th wicket				
9th wicket				
10th wicket				