RBT-2 for (JEE-Advanced)

PHYSICS, CHEMISTRY & MATHEMATICS

PAPER - 2

Time Allotted: 3 Hours **Maximum Marks: 186**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & B** in the OMR.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Part.

(i) **PART–A** (**01–08**) contains **(**8) Multiple Choice Questions which have **One or More Correct** answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
***Negative Marks*: –2 In all other cases.**
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in –**2** marks, as a wrong option is also darkened.

(ii) **Part-A (09-12) –** This section contains Two (02) List-Match Sets, each List-Match set has Two (02) Multiple Choice Questions. Each List-Match set has two lists: List-I and List-II. FOUR options are given in each Multiple Choice Question based On List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question. Each question carries **+3 Marks** for correct combination chosen and **-1 marks** for wrong options chosen.

(iii) **Part-B (01-06)** contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :________________________________

Batch :__________________ **Date of Examination :**__________________

Enrolment Number :________________________________

SECTION – I : PHYSICS

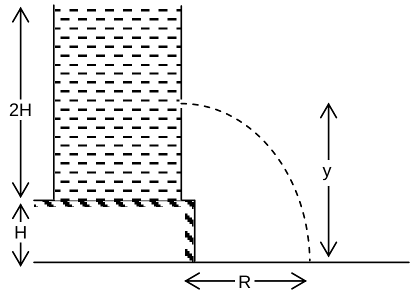
(PART – A)

(One or More Than One Options Correct Type)

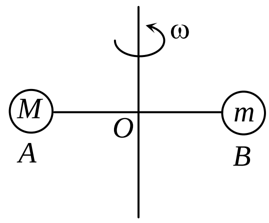
This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. An artificial satellite is in a circular orbit around the earth. The universal gravitational constant starts decreasing at time t = 0 at a constant rate with respect to time t. Then the satellite has its
(A) path gradually spiralling out away from the centre of the earth.
(B) path gradually spiralling in towards the centre of the earth.
(C) angular momentum about the centre of the earth remains constant.
(D) potential energy increases.

2. A tank is filled upto a height 2H with a liquid and is placed on a platform of height H from the ground. The distance y from the ground where a small hole is made in the tank, to get the maximum horizontal range R is
(A) 2H
(B) 3 H/2
(C) 5 H/4
(D) H



3. Two balls of mass *M* = 9 g and *m* = 3 g are attached by massless threads *AO* and *OB*. The length *AB* is 1 m. They are set in rotational motion in a horizontal plane about a vertical axis at *O* with constant angular velocity ω. The ratio of length *AO* and *OB* $\left(\frac{AO}{OB}\right)$ for which the tension in threads are same will be

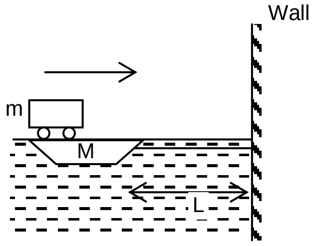


(A) $\frac{1}{3}$ (B) 3 (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

4. A particle moves along positive branch of the curve $y = \frac{x}{2}$ where $x = \frac{t^3}{3}$, x and y are measured in metres and t in seconds, then:
(A) The velocity of particle at t = 1 s is $\hat{i} + \frac{1}{2}\hat{j}$
(B) The velocity of particle at t = 1 s is $\frac{1}{2}\hat{i} + \hat{j}$
(C) The acceleration of particle at t = 1 s is $2\hat{i} + \hat{j}$
(D) The acceleration of particle at t = 2 s is $\hat{i} + 2\hat{j}$

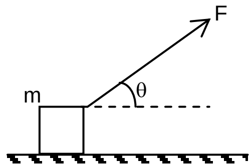
Space For Rough Work

5. A car of mass m is initially at rest on the boat of mass M tied to the wall of dock through a massless inextensible string. The car accelerates from rest to velocity v_0 in times t_0. At $t = t_0$ the car applies brake and comes to rest relative to the boat in negligible time. Neglect friction between the boat and water; the time 't' at which boat will strike the wall is



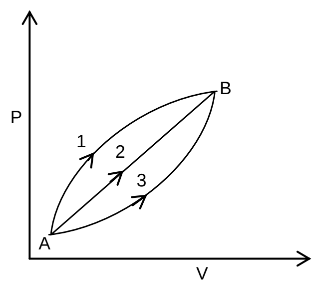
(A) $t_0 + (L/v_0)$
(B) $t_0 + \frac{L(M+m)}{mv_0}$
(C) $\frac{LM}{mv_0}$
(D) none of these

6. A force F acts on a smooth block of mass m placed on a horizontal floor at an angle θ with horizontal. On the block



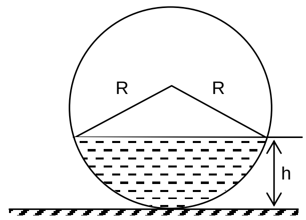
(A) the net force $= F\cos\theta$ if $F\sin\theta < mg$
(B) acceleration $= \frac{F\cos\theta}{m}$ when $F > mg\ \mathrm{cosec}\theta$
(C) acceleration $= \frac{F}{m}$ if $F\sin\theta > mg$
(D) $N = mg - F\sin\theta$ if $F\sin\theta < mg$

7. In figure, a certain mass of gas traces three paths 1,2,3 from state A to state B. If work done by the gas along three paths are W_1, W_2, W_3 respectively, then



(A) $W_1 < W_2 < W_3$
(B) $W_1 = W_2 = W_3$
(C) $W_1 > W_2 > W_3$
(D) cannot say

8. A liquid is filled in a spherical container of radius R till a height h. At this position the liquid surface at the edges is also horizontal. The contact angle is



(A) 0
(B) $\cos^{-1}\left(\frac{R-h}{R}\right)$
(C) $\cos^{-1}\left(\frac{h-R}{R}\right)$
(D) $\sin^{-1}\left(\frac{R-h}{R}\right)$

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

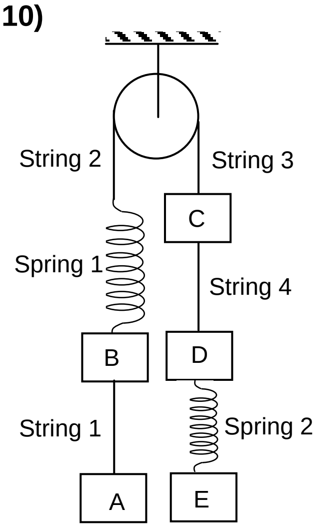
List Match Set (09-10)

The system shown below is initially in equilibrium

$m_A = m_B = 3kg$

$m_C = m_D = m_E = 2kg$

Take g = 10 m/s^2



List–I gives the four strings while List–II the value of the tension in the strings.

List – I		**List – II**	
(I)	String 1	(P)	10 N
(II)	String 2	(Q)	20 N
(III)	String 3	(R)	30 N
(IV)	String 4	(S)	40 N
		(T)	60 N
		(U)	0 N

9. If the spring 1 is cut, match the correct option for the tension in the strings just after the cutting.

(A) I $\rightarrow$ R, II $\rightarrow$ T, III $\rightarrow$ T, IV $\rightarrow$ P
(B) I $\rightarrow$ U, II $\rightarrow$ T, III $\rightarrow$ T, IV $\rightarrow$ P
(C) I $\rightarrow$ U, II $\rightarrow$ U, III $\rightarrow$ U, IV $\rightarrow$ P
(D) I $\rightarrow$ R, II $\rightarrow$ S, III $\rightarrow$ S, IV $\rightarrow$ P

10. If the spring 2 is cut, match the correct option for the tension in the strings just after the cutting.

(A) I $\rightarrow$ R, II $\rightarrow$ T, III $\rightarrow$ T, IV $\rightarrow$ R
(B) I $\rightarrow$ R, II $\rightarrow$ U, III $\rightarrow$ U, IV $\rightarrow$ R
(C) I $\rightarrow$ U, II $\rightarrow$ R, III $\rightarrow$ R, IV $\rightarrow$ P
(D) I $\rightarrow$ P, II $\rightarrow$ Q, III $\rightarrow$ Q, IV $\rightarrow$ R

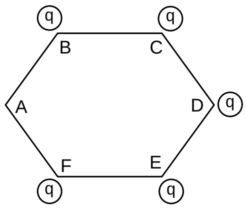
Space For Rough Work

List Match Set (11-12)

If some charges are place at the corners of a regular hexagon and E is the electric field due to one charge at the centre.

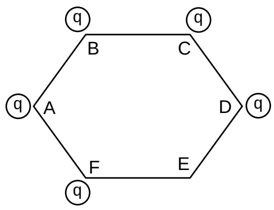
List – I		List – II	
(I)	If charge at B is removed, then electric field at centre become	(P)	Zero
(II)	If charge at C is removed, then electric field at centre become.	(Q)	2E
(III)	If charge at D is removed, then electric field at centre become.	(R)	$\frac{2E}{\sqrt{3}}$
(IV)	If charge at B and C both are removed, then electric field at centre become	(S)	$\sqrt{3}$ E
		(T)	$\frac{E}{2}$
		(U)	E

11. If five charges are kept at five vertices of regular hexagon as shown, then



(A) I $\rightarrow$ S, II $\rightarrow$ U, III $\rightarrow$ Q, IV $\rightarrow$ P
(B) I $\rightarrow$ S, II $\rightarrow$ U, III $\rightarrow$ P, IV $\rightarrow$ Q
(C) I $\rightarrow$ S, II $\rightarrow$ Q, III $\rightarrow$ P, IV $\rightarrow$ U
(D) I $\rightarrow$ Q, II $\rightarrow$ U, III $\rightarrow$ P, IV $\rightarrow$ S

12. If five charges are kept at five vertices of regular hexagon as shown, then



(A) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ U
(B) I $\rightarrow$ P, II $\rightarrow$ U, III $\rightarrow$ R, IV $\rightarrow$ S
(C) I $\rightarrow$ P, II $\rightarrow$ U, III $\rightarrow$ S, IV $\rightarrow$ U
(D) I $\rightarrow$ Q, II $\rightarrow$ U, III $\rightarrow$ S, IV $\rightarrow$ R

(PART – B)

(Numerical Type)

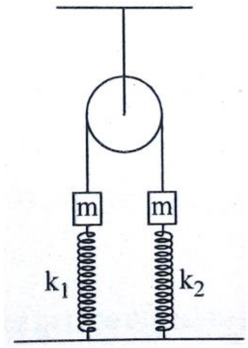
Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30).

1. At a distance of 20 m from a point source, the loudness is 30 dB. After a minimum distance of $k(300\sqrt{10})$ metre the sound is inaudible. Value of 'k' is

Space For Rough Work

2. A proton is fired from very far away towards a nucleus with charge = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = (5/3) \times 10^{-27}$ kg ; $h/e = 4.2 \times 10^{-15}$ J.s./C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm = 10^{-15} m)

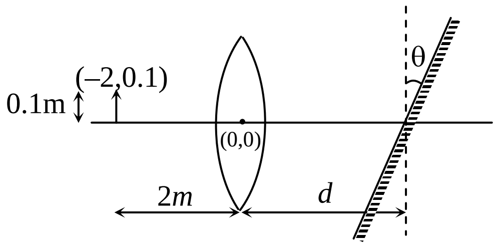
3. On the diagram, two blocks of equal mass are connected by an ideal string. The values of m = 1 kg and k_2 = 100 N/m are given. Initially, both springs are relaxed. Then the left block is slowly pulled down a distance 0.1 m and released. The maximum possible value of k_1 (in N/m) for which both blocks will have same magnitude of acceleration just after releasing, is k. Both spring have natural length of 2m. Then find the value of $\frac{k}{100}$.



4. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube is raised by 2 cm. The size of the cube is 20 k cm, then the value of k is

5. Two vibrating strings of the same material but of lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and other with frequency f_2. Find the ration $\frac{f_1}{f_2}$?

6. A convex lens of focal length 1.5m is placed in a system of coordinate axis such that its optical centre is at origin and principal axis coinciding with the x-axis. An object and a plane mirror are arranged on the principal axis as shown in figure. Find the value of d (in m) so that y-coordinate of final image (after refraction and reflection) is 0.3m. (Take $\tan\theta = 0.3$)



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SECTION – II : CHEMISTRY

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Compound/s which contain 3c – $2e^-$ bond.
(A) $(BeH_2)_n$
(B) $Al_2(CH_3)_6$
(C) B_2H_6
(D) Al_2Cl_6

2. ψ(x, y, z) describe the wave function of a particle the probability of finding the particle between x and x + dx, y and y + dy, z and z + dz can be expressed as
(A) $\psi^*(x, y, z)\ \psi(x, y, z)$
(B) $|\psi(x, y, z)|^2$ dxdydz
(C) $\psi^*(x, y, z)\ \psi(x, y, z)$ dxdydz
(D) $\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz\, \psi^*(x, y, z)\ \psi(x, y, z)$

3. In water, the enthalpy of a protein in its folded state (H_f) is lower than that in its unfolded state(H_{uf}). The entropies of the folded and unfolded states are S_f and S_{uf} respectively. The condition(s) under which this protein spontaneously folds in water at a temperature T is(are)
(A) $S_{uf} < S_f$
(B) $S_{uf} = 0$
(C) $S_{uf} = S_f$
(D) $(S_f – S_{uf}) > H_f – H_{uf}/T$

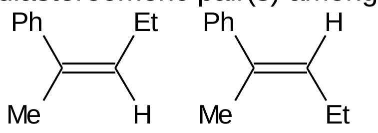
4. The transition metal complex(es) with zero magnetic moment, zero dipole moment and CFSE of $-2.4\Delta_0$ is(are)
(A) $[Mn(CO)_5\ (CH_3)]$
(B) [Trans-Ni(ethylenediamine)$_2Cl_2$]
(C) [trans-$Co(CN)_4(H_2O)_2]^-$
(D) [Trans-$Fe(CN)_4Cl_2]^{4-}$

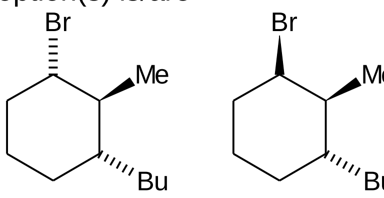
5. The organometallic reagent(s) among the following is/are
(A) lithium divnylcuprate
(B) lilthium disopropylamide
(C) potassium tert-butoxide
(D) isopropyl magnesiumiodide

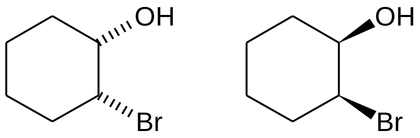
6. The correct statement(s) among the following is/are
(A) secondary structure of a polypeptide describes the number and type of amino acid residues
(B) uracil is a pyrimidine nucleobase
(C) natural fatty acids have odd number of carbon atoms
(D) reaction of D-glucose with $Ca(OH)_2$ gives a product mixture containing(D)-fructose, D-mannose and (D)-glucose

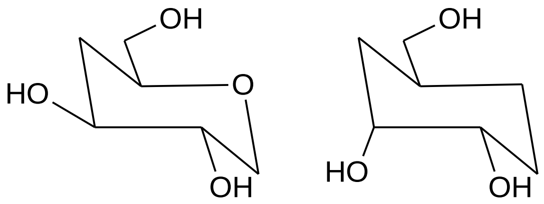
Space For Rough Work

7. The diastereomeric pair(s) among the following option(s) is/are

(A) 

(B) 

(C) 

(D) 

8. With reference to the variation of molar conductivity ($\wedge_m$) with concentration for a strong electrolyte in aq. solution, the incorrect statement is

(A) the asymmetric effect contributes to decrease $\wedge_m$ whereas the electrophoretic effect contributes to increase $\wedge_m$.

(B) the asymmetric effect contributes to increase $\wedge_m$ whereas the electrophoretic effect contributes to decrease $\wedge_m$.

(C) both asymmetry effect and electrophoretic effect contribute to decrease $\wedge_m$.

(D) both asymmetry effect and electrophoretic effect contribute to increase $\wedge_m$.

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Match the lists and answer the question.

List– I		**List – II**	
(I)	One mole of nitrogen gas contained in a cylinder at 300 K is allowed to expand isothermally against an external pressure of 5 atm	(P)	$w > 0$
(II)	3 moles of helium gas at 1 atm are compressed reversibly and isothermally at 400 K to 5 atm pressure	(Q)	$\Delta U = 0$
(III)	An ideal gas is compressed reversibly and adiabatically from a volume of 1 dm^3 to a volume of 0.25 dm^3	(R)	$\Delta H = 0$
(IV)	One mole of liquid benzene is converted reversibly into vapour at its boiling point by heating it	(S)	$\Delta H > \Delta U$

9. Which combination is correct according to the above list?

(A) I $\rightarrow$ P
(B) II $\rightarrow$ S
(C) III $\rightarrow$ PS
(D) IV $\rightarrow$ PR

Space For Rough Work

10. Which combination is correct according to the above list?
(A) I $\rightarrow$ QR (B) II $\rightarrow$ QS (C) III $\rightarrow$ Q (D) IV $\rightarrow$ PR

List Match Set (11-12)

Match the lists and answer the question.
When we titrate sodium carbonate solution(in beaker) with hydrochloric acid

List– I		List – II	
(I)	At the start of titration	(P)	Buffer solution of HCO_3^- & CO_3^{2-}
(II)	Before the first equivalent point	(Q)	Buffer solution of H_2CO_3 & HCO_3^-
(III)	At the first equivalent point	(R)	Amphiprotic anion pH = $1/2\left(pK_{a_1} + pK_{a_2}\right)$
(IV)	Between the first and second equivalent points	(S)	Hydrolysis of CO_3^{2-}

11. Which combination is correct according to the above list?
(A) I $\rightarrow$ S (B) II $\rightarrow$ R (C) III $\rightarrow$ Q (D) IV $\rightarrow$ P

12. Which combination is correct according to the above list?
(A) I $\rightarrow$ P (B) II $\rightarrow$ R (C) III $\rightarrow$ S (D) IV $\rightarrow$ Q

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. The enthalpy of formation for CH_4(g), C(g) and H(g) are -75, 717 and 218 kJ mol^{-1} respectively the enthalpy of the C – H bond in kJ mol^{-1} is

2. The number of P – H bonds in Hypophosphoric acid is _______

3. Number of vertices in an icosahedral doso borane is

4. The number of gauche butane-interaction(s) in cis-1, 2-dimethyl cyclohexane the following compound is ______

5. 5.3 g of benzaldehyde was reacted with excess of acetophenone to produce 5.2 g of enone product as per the reaction shown below the yield of the reaction is _______%
(round off to the nearest integer)

6. Assume that the reaction of MeMgBr with ethyacetate proceeds with 100% conversion to give tert butanol. The volume of 0.2 M solution of MeMgBr required to convert 10 mL of a 0.025 M solution of ethylacetate to tert butanol is _______mL
(Round off to one decimal place)

Space For Rough Work

SECTION – III : MATHEMATICS

(PART – A)

(One or More Than One Options Correct Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Consider the function $f(x) = xe^x + (xe^x)^{-1}$ where $x \in R - \{0\}$ then which of following statement(s) is CORRECT?
(A) $f(x)$ attains it's local maxima at $x = x_0$ where $x_0 \in (0,1)$
(B) $f(x)$ attains local minima at $x = -1$
(C) $f(x)$ attains local minima at $x = x_0$ where $x_0 \in (1,\infty)$
(D) $f(x)$ attains it's local minima at $x = x_0$ where $x_0 \in (0,1)$

2. $f : R \to R$ is one – one, onto and differentiable function and graph of $y = f(x)$ is symmetrical about the point (4, 0), then
(A) $f^{-1}(2010) + f^{-1}(-2010) = 8$
(B) $\int_{-2010}^{2018} f(x)\,dx = 0$
(C) If $f'(-100) > 0$, then roots of $x^2 - f'(10)x - f'(10) = 0$ are non real
(D) If $f'(10) = 20,$ then $f'(-2) = 20$

3. The value of x in $\left(0, \frac{\pi}{2}\right)$ satisfying equation $\frac{\sqrt{5}-1}{\sin x} + \frac{\sqrt{10+2\sqrt{5}}}{\cos x} = 8$ is
(A) $\frac{\pi}{10}$ (B) $\frac{3\pi}{10}$ (C) $\frac{9\pi}{10}$ (D) $\frac{7\pi}{10}$

4. Point M moved on the circle $(x-4)^2 + (y-8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle cut the x – axis at point (–2, 0). The coordinates of the point on the circle at which the moving point broke away is
(A) $\left(\frac{42}{5}, \frac{36}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{44}{5}\right)$ (C) (6, 4) (D) (2, 4)

Space For Rough Work

5. If f is an odd continuous function in [–1, 1] and differentiable in $(-1, 1)$ then which of the following statement(s) is (are) correct?

(A) $f'(a) = f(1)$ for some $a \in (-1, 0)$

(B) $f'(b) = f(1)$ for some $b \in (0, 1)$

(C) $n(f(\alpha))^{n-1} f'(\alpha) = (f(1))^n$ for some $\alpha \in (-1, 0)$ and $\forall n \in N$

(D) $n(f(\beta))^{n-1} f'(\beta) = (f(1))^n$ for some $\beta \in (0, 1)$ and $n \in N$

6. Two soldiers X and Y shoot each other. If one or both are hit then shooting is over. If both shoot miss then they repeat the process. Suppose the results of shots are independent and that each shot of X will hit Y with probability $\frac{2}{5}$ and each shot of Y will hit X with probability $\frac{1}{5}$.

(A) The probability that X is not hit is $\frac{8}{13}$

(B) The probability that X is not hit is $\frac{8}{25}$

(C) The probability that both are hit is $\frac{2}{25}$

(D) The probability that both are hit is $\frac{2}{13}$

7. Let P be the plane consisting of all points that are equidistant from A (–4, 2, 1) and B (2, –4, 3) and Q denotes the plane $x - y + Cz = 1$, where $C \in R$.

(A) If $C = \frac{1}{3}$ then plane P will be parallel to Q

(B) If C = –1 then plane P will be perpendicular to Q

(C) Volume of tetrahedron formed by plane P with coordinate planes will be $\frac{2}{81}$

(D) If line $L: \frac{x-1}{1} = \frac{y+2}{3} = \frac{z-7}{-1}$ intersect plane P at $R(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma = 12$

8. Let $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ where $x, y, z \in N$ if $|adj(adj(adj(adj\,A)))| = 4^8.5^{16}$ then the number of such matrix is p and value of $\int_{-\pi/2}^{\pi/2} 2^{\sin x}dx + \int_{5/2}^{4} \sin^{-1}(\log_2(x-2))dx = \frac{q\pi}{4}$, then $p + q$ is

(A) 40 (B) 41 (C) 42 (D) 44

Space For Rough Work

This section contains **2 List-Match Sets**, each List-Match set has **2 Multiple Choice Questions**. Each List-Match set has two lists: List-I and List-II. Four options are given in each Multiple Choice Question based On List-I and List-II and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

List Match Set (09-10)

Consider two circles C_1 of radius a and C_2 of radius b (b > a) both lying in the first quadrant and touching the coordinate axes. In each of conditions listed in list– I, the ratio of $\frac{b}{a}$ is given in list II

	List – I		**List – II**
(I)	C_1 and C_2 touch each other.	(P)	$2+\sqrt{2}$
(II)	C_1 and C_2 are orthogonal	(Q)	3
(III)	C_1 and C_2 intersect so that common chord is longest	(R)	$2+\sqrt{3}$
(IV)	C_1 passes through center of C_2	(S)	$3+2\sqrt{2}$
		(T)	$3-2\sqrt{2}$

9. Which is correct option?

(A) I $\rightarrow$ R
(B) II $\rightarrow$ T
(C) III $\rightarrow$ Q
(D) IV $\rightarrow$ S

Consider two circles C_1 of radius a and C_2 of radius b (b > a) both lying in the first quadrant and touching the coordinate axes. In each of conditions listed in list– I, the ratio of $\frac{b}{a}$ is given in list II

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(IV)	C_1 passes through center of C_2	(S)	$3+2\sqrt{2}$
		(T)	$3-2\sqrt{2}$

10. Which is correct option?

(A) I $\rightarrow$ Q
(B) II $\rightarrow$ S
(C) III $\rightarrow$ T
(D) IV $\rightarrow$ P

Space For Rough Work

List Match Set (11-12)

Consider square matrices of order 2 which has it is elements 0, 1, 2 and 4. N denotes the number of such matrices all elements of which are different

	List – I		List – II
(I)	Possible non negative values of det (A) is	(P)	2
(II)	Sum of values of determinants corresponding to N matrices is	(Q)	4
(III)	If absolute value of (det A) is least then possible value of $\left\|\left(\text{adj}\left(\text{adj}\left(\text{adj}\right)\right)\right)\right\|$ is	(R)	–2
(IV)	If $(\det A)$ is algebraically least, then possible values of $\det\left(4A^{-1}\right)$ is	(S)	0
		(T)	8

11. Which is correct option?

(A) I $\rightarrow$ Q, R
(B) II $\rightarrow$ S
(C) III $\rightarrow$ Q, P
(D) IV $\rightarrow$ T

Consider square matrices of order 2 which has it is elements 0, 1, 2 and 4. N denotes the number of such matrices all elements of which are different

	List – I		List – II
(I)	Possible non negative values of det(A) is	(P)	2
(II)	Sum of values of determinants corresponding to N matrices is	(Q)	4
(III)	If absolute value of (det A) is least then possible value of $\left\|\left(\text{adj}\left(\text{adj}\left(\text{adj}\right)\right)\right)\right\|$ is	(R)	–2
(IV)	If $(\det A)$ is algebraically least, then possible values of $\det\left(4A^{-1}\right)$ is	(S)	0
		(T)	8

12. Which is correct option?

(A) I $\rightarrow$ P, Q, T
(B) II $\rightarrow$ R
(C) III $\rightarrow$ P, S
(D) IV $\rightarrow$ Q

Space For Rough Work

(PART – B)

(Numerical Type)

Part-B (01-06) contains six (06) Numerical based questions, the answer of which maybe positive or negative numbers or decimals to **Two decimals Places** (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30)**.**

1. A 3×3 determinant has entries either 1 or –1. Let S be the set of all determinants such that the product of elements of any row or column is –1. For example $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix}$ is an element of S and number of elements in S is m. Let $P=\begin{bmatrix} 3 & -2 & 3 \\ 2 & -2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$ and trace of the matrix $\text{adj}(\text{adj}P)$ is n then find the value of $\frac{m}{n}$.

2. Let z_1 and z_2 be two complex numbers such that $|z_2|=1$ and $\frac{z_1-1}{z_1+1}=\left(\frac{z_2-1}{z_2+1}\right)^2$, then find the value of $|z_1-1|+|z_1+1|$.

3. Let $<a_n>$ is an infinite geometric sequence with first term $2\cot x$ and common ratio $\sin^2 x$ and $<b_n>$ is an infinite geometric sequence with first term $\sin 2x$ and common ratio $\sin^2 x$. If $x\in\left(0,\frac{\pi}{4}\right]$, then the minimum value of $\sum_{i=1}^{\infty} a_i-\sum_{j=1}^{\infty} b_j$ is

4. Let $\vec{a}=(2+\sin\theta)\hat{i}+\cos(\theta)\hat{j}+\sin(2\theta)\hat{k}$

$$\vec{b}=\sin\left(\theta+\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta+\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta+\frac{4\pi}{3}\right)\hat{k}$$

$$\vec{c}=\sin\left(\theta-\frac{2\pi}{3}\right)\hat{i}+\cos\left(\theta-\frac{2\pi}{3}\right)\hat{j}+\sin\left(2\theta-\frac{4\pi}{3}\right)\hat{k}$$

be three vectors where $\theta\in\left(0,\frac{\pi}{2}\right)$. Let V is the maximum volume of tetrahedron whose coterminous edges are represented by the vector $2\vec{b}\times\vec{c}, 3\vec{c}\times\vec{a}$ and $\vec{a}\times 4\vec{b}$ then $\frac{V}{6}=$

Space For Rough Work

5. Considering the curve $xy = 15!$. Let n be the number of points (α, β) lying on it, where $\alpha, \beta \in I^{+}$ and $\text{HCF}(\alpha, \beta) = 1$ then $\left[\frac{n}{13}\right]$ equals (where $[.]$ represents G.I.F.)

6. Let $g : R - \left\{\frac{3}{2}\right\} \to R - \left\{\frac{3}{2}\right\}$ be defined as $g(x) = \frac{3x+5}{2x-3}$. If $g_{2014}(x) = \frac{px-q}{-rx+s}$, where $g_{2014}(x) = \underbrace{g(g(g(...)))}_{2014 \text{ times}}(x)$, then find the value of $(p+q+r+s)$.

Space For Rough Work

Answers & Solutions

SECTION – I : PHYSICS

(PART – A)

1.	AC	2.	B	3.	A	4.	AC
5.	B	6.	AD	7.	C	8.	B
9.	C	10.	A	11.	B	12.	C

(PART – B)

1.	0.67	2.	7	3.	3	4.	0.50
5.	1	6.	5				

SECTION – II : CHEMISTRY

(PART – A)

1.	ABC	2.	BC	3.	ABCD	4.	CD
5.	AD	6.	BD	7.	ABD	8.	ABD
9.	C	10.	A	11.	A	12.	D

(PART – B)

1.	416	2.	Zero	3.	12	4.	3
5.	50	6.	2.5				

SECTION – III : MATHEMATICS

(PART – A)

1.	D	2.	ABD	3.	AB	4.	BC
5.	ABD	6.	AD	7.	AD	8.	B
9.	C	10.	D	11.	B	12.	A

(PART – B)

1.	8	2.	2	3.	2	4.	2
5.	4	6.	2				

Answers & Solutions

SECTION – I : PHYSICS

(PART – A)

1. **AC**

Sol. $\frac{GMm}{r^2} = \frac{mv^2}{r}$

If G keeps decreasing, earth will attract satellite with smaller force, hence leading to spiral outward path.

2. **B**

Sol. R to be maximum.

$Y = \frac{3H}{2} = \frac{\text{Total height}}{2}$.

3. **A**

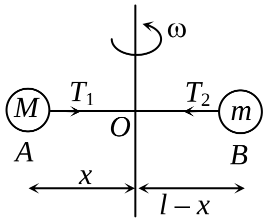
Sol. $T_1 = T_2$

$\Rightarrow \quad M\omega^2 x = m\omega^2(l - x)$

$x = \frac{ml}{M+m}$

$l - x = \frac{Ml}{M+m}$

$\frac{AO}{OB} = \frac{x}{l-x} = \frac{m}{M} = \frac{3}{9} = \frac{1}{3}$



4. **AC**

Sol. $\frac{dx}{dt} = t^2$ …(i)

$y = \frac{1}{2}\frac{t^3}{3}$

$\frac{dy}{dt} = \frac{t^2}{2}$ …(ii)

$t = 1$, $v_x = 1$, $v_y = \frac{1}{2}$; $\vec{v} = \hat{i} + \frac{1}{2}\hat{j}$

$\frac{d^2x}{dt^2} = 2t$ …(iii)

$\frac{d^2y}{dt^2} = t$ …(iv)

at t = 1 s $a_x = 2$ and $a_y = 1$

$\therefore \quad \vec{a} = 2\hat{i} + \hat{j}$.

5. **B**

Sol. After brakes applied,

Common velocity of car and boat = $\left(\frac{mv_o}{m+M}\right)$

Time taken to hit wall = $\frac{L(M+m)}{mv_o}$

6. **AD**

Sol. Draw F.B.D. and apply Newton's law of motion.

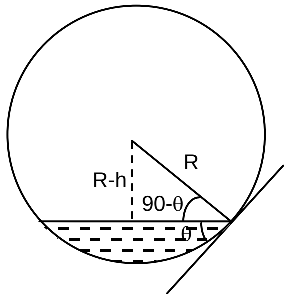
7. **C**

Sol. Work done by gas = $\int P dv$ = Area under P–V graph.

$\therefore\ W_1 > W_2 > W_3$

8. **B**

Sol. $\sin(90-\theta) = \dfrac{R-h}{R}$



9. **C**

Sol. Just after cutting force of spring will be zero whereas the force of other spring will be unchanged.

10. **A**

Sol. Just after cutting force of spring will be zero whereas the force of other spring will be unchanged.

11. **B**

Sol. Apply principle of superposition of electric fields.

12. **C**

Sol. Apply principle of superposition of electric fields.

(PART – B)

1. **0.67**

Sol. Loudness (dB) = $10 \log_{10}\left(\dfrac{I}{I_o}\right)$ ($I_o \to$ Minimum audible intensity)

$30 = 10 \log_{10}\left(\dfrac{I}{I_o}\right)$

$\dfrac{I}{I_o} = 1000$

$\Rightarrow\ I = (1000)I_o$

For point source, intensity $\propto \dfrac{1}{(\text{Distance})^2}$

$(1000)I_o \propto \dfrac{1}{(20)^2}$

$\Rightarrow\ I_o \propto \dfrac{1}{(20)^2\,(1000)}$

$\Rightarrow$ Required distance = $20\sqrt{1000} = 200\sqrt{10}$

2. **7**

Sol. $\dfrac{P^2}{2m} = KE = \dfrac{9\times10^9 \times 120e^2}{10\times10^{-15}}$

$\lambda = \dfrac{h}{P} = \dfrac{h}{e\sqrt{360\times10^{-3}}} = \dfrac{4.2\times10^{-15}}{0.6} = 7.$

3. **3**

Sol. Let T be the tension in the ideal string and 'a' be the acceleration of the blocks at the instant of release. For the block on the left, the upward acceleration may be found from

$T + k_1x - mg = ma$

For the block on the right, the downward acceleration may be found from

$k_2x + mg - T = ma$

Adding the equations gives the acceleration of the blocks as

$a = (k_1 + k_2)x/(2m)$

However, subtracting the equations gives

$T = mg - (k_1 - k_2)x/2$

for maximum value of k_1T will be zero.

$mg = \left(\frac{k_1 - k_2}{2}\right)x$; $k_1 = 300$.

4. **0.50**

Sol. Let the length of each side of the cube is x, $x^2 \times 2 \times 1 \times g = 200 \times g$

$\Rightarrow$ x = 10 cm. $\therefore$ $k = \frac{1}{2}$.

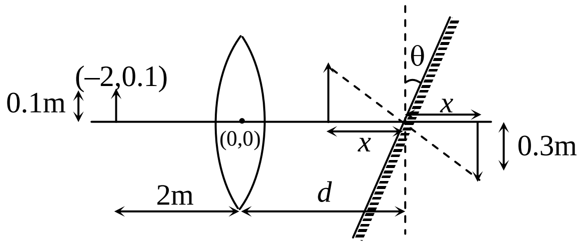
5. **1**

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi\rho r^2}}$

$\therefore \quad \frac{v_1}{v_2} = \frac{1}{2}$; $\frac{f_1}{f_2} = \frac{v_1/2L_1}{V_2/2L_2} = \frac{v_1}{v_2} \times \frac{L_2}{L_1} = \frac{1}{2} \times 2 = 1$.

6. **5**

Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\frac{1}{v} - \frac{1}{-2} = \frac{1}{1.5}$; v = 6m

$m = \frac{v}{u} = -3$

$x = 6 - d$; $\tan\theta = \frac{0.3}{x}$

$\Rightarrow$ d = 5m.

SECTION – II : CHEMISTRY

(PART – A)

1. ABC

Sol. Bridge bonding

$Al_2Cl_6 \Rightarrow$ 3c – 4e$^-$ Bond

2. BC

Sol. $|\psi|^2 dv$

3. ABCD

Sol. uf $\rightarrow$ f

$\Delta G = \Delta H - T\Delta S$

For spontaneous ΔG = -ve

4. CD

Sol. 'VBT' with SFL pairing occur.

5. AD

Sol. Ogranometallic $\Rightarrow$Carbon – Metal bond.

6. BD

Sol. Theoretical & Conceptual

7. ABD

Sol. Non-mirror images, stereoisomer.

8. ABD

Sol. "Counter ion effect + solution effect" to opposite charges electrodes.

9. C

10. A

Sol.9 & 10: $W_{Expansion} < 0$ $\Delta U = \Delta H = 0$(for isothermal process)

$W_{Compression} > 0$

11. A

12. D

Sol.12 & 13:

$$CO_3^{2-} + H^+ \longrightarrow HCO_3^-$$

$$HCO_3^- + H^+ \longrightarrow H_2CO_3$$

Before first equivalent point $\Rightarrow$ CO_3^{2-} + HCO_3^-

At first equivalent point $\Rightarrow$ HCO_3^-

After first equivalent point $\Rightarrow HCO_3^-$ + H_2CO_3

(PART – B)

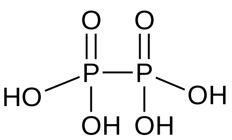
1. 416

Sol. $CH_4(g) \longrightarrow C(g) + 4H(g)$

$$\Delta H_{Rxn} = 1664,\ \ E_{C-H} = \frac{1664}{4} = 416$$

2. Zero

Sol.



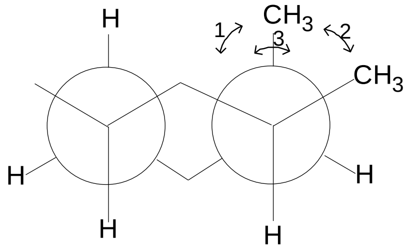
$H_4P_2O_6 \Rightarrow n_f = 4$

3. 12

Sol. $B_{12}H_{12}^{2-}$

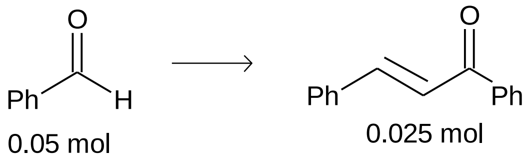
4. 3

Sol.



5. 50

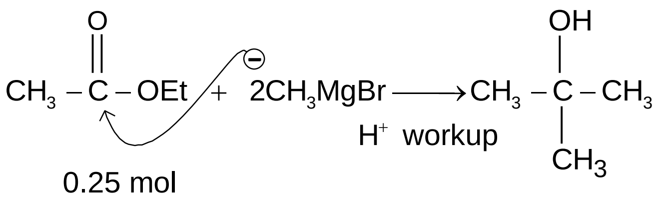
Sol.



$$\frac{0.025}{0.05} \times 100 = 50\%$$

6. 2.5

Sol.



$0.5 = 0.2 \times V$

$V = 2.5$ mL

SECTION – III : MATHEMATICS

(PART – A)

1. D

Sol. $f(x) = xe^x + (xe^x)^{-1}$

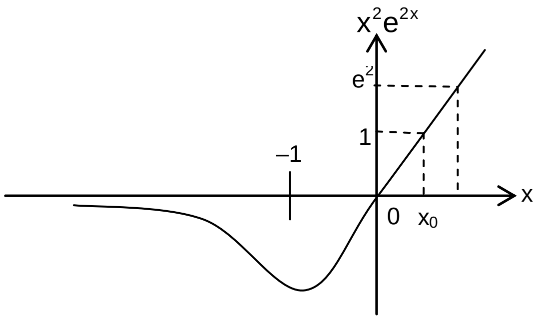
$= x e^x + \frac{1}{x} e^{-x}$

$f'(x) = (x+1)e^x - \frac{1}{x^2} e^{-x} - \frac{1}{x} e^{-x}$

$= (x+1)e^x - \frac{e^{-x}}{x^2}(1+x)$

$= \frac{(1+x)}{x^2} e^{-x}\left[x^2 e^{2x} - 1\right]$

$f'(x) = 0 \Rightarrow 1 + x = 0$ or $x^2e^{2x} - 1 = 0$



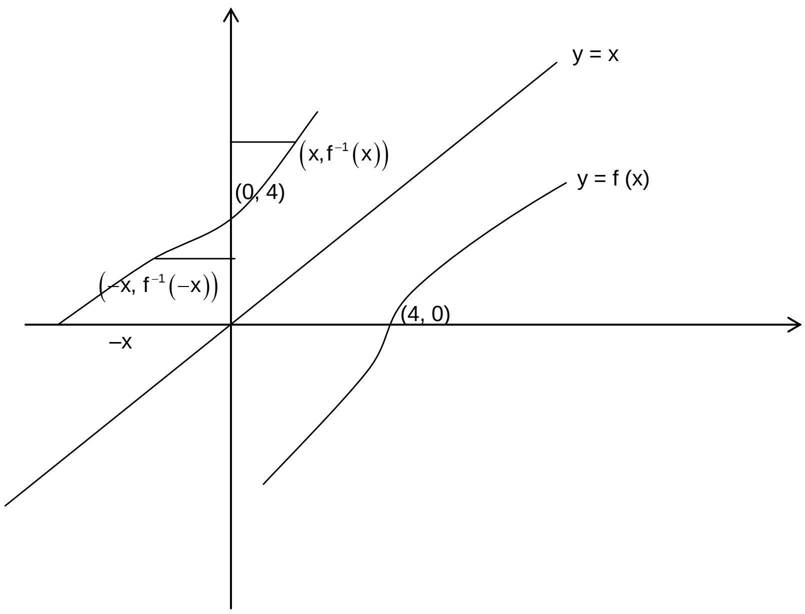
From figure $x^2e^{2x} - 1 = 0$

at $x = x_0 \in (0, 1)$

So $x = -1$ and x_0 are two points of extrema. At $x = x_0$ $x^2e^{2x} - 1$ changes sign from –ive to +ive $\Rightarrow x_0$ is point of minima. At $x = -1$ sign of $f(x)$ changes from time to + ive to –ive $\Rightarrow x = -1$ is point of maxima.

2. ABD

Sol. $f(x)$ is symmetric about (4, 0) $\Rightarrow f^{-1}(x)$ is symmetric about (0, 4)



(a) Mid point of $\left(x, f^{-1}(x)\right)$ and $\left(-x, f^{-1}(-x)\right)$ is (0, 4)

$$\Rightarrow \left(\frac{x-x}{2}, \frac{f^{-1}(x)+f^{-1}(x)}{2}\right)=(0,4)$$

$$\Rightarrow \frac{f^{-1}(x)+f^{-1}(-x)}{2}=4 \Rightarrow f^{-1}(x)+f^{-1}(-x)=8$$

(b) $f(x)$ is symmetric about (4, 0)

$\Rightarrow f(4+x)=-f(4-x)$

$x \to 4-x$

$\Rightarrow f(8-x)=-f(x)$

$\Rightarrow f(x)+f(8-x)=0$ ………(1)

$I=\int_{-2010}^{2018} f(x)\,dx$ ………(2)

$iI=\int_{-2010}^{2018} f(2018-2010-x)\,dx=\int_{-2010}^{2018} f(8-x)\,dx$ ………(3)

(2) + (3)

$2I=\int_{-2010}^{2018}\left[f(x)+f(8-x)\right]dx=0$

$\Rightarrow I=0$

(c) Since $f(x)$ is one to one and $f'(-100)>0 \Rightarrow f(x)$ is increasing function

So $f'(10)>0$

$D=\left(f'(10)^2-4x_1\left[-f'(10)\right]\right)=\left[f'(10)\right]^2+4f'(10)>0$

$\Rightarrow$ roots are always real.

(d) $f(x)+f(8-x)=0$

Diff. w.r.t. $\Rightarrow f'(x)-f'(8-x)=0$ ……….(4)

Put $x=10$

3. AB

Sol. $$\frac{\left(\sqrt{5}-1\right)\cos x}{\sqrt{\left(\sqrt{5}-1\right)^2+10+2\sqrt{5}}}+\frac{\sqrt{10+2\sqrt{5}}\sin x}{\sqrt{\left(\sqrt{5}-1\right)^2+10+2\sqrt{5}}}=\frac{8\sin x\cos x}{\sqrt{(5-1)^2+10+2\sqrt{5}}}$$

$$\left(\frac{\sqrt{5}-1}{4}\right)\cos x+\frac{\sqrt{10+2\sqrt{5}}}{4}\sin x=\frac{8}{4}\sin x\cos x$$

$$\sin\frac{\pi}{10}\cos x+\cos\frac{\pi}{10}\sin x=\sin 2x$$

$$\sin\left(x+\frac{\pi}{10}\right)=\sin 2x$$

$$\sin 2x=\sin\left(x+\frac{\pi}{10}\right)$$

$$\Rightarrow 2x=x+\frac{\pi}{10} \text{ or } \pi-\left(x+\frac{\pi}{10}\right)$$

$$\Rightarrow x=\frac{\pi}{10} \text{ or } \frac{3\pi}{10}$$

4. BC

Sol. A and B are intersection of chord of contact with circle.

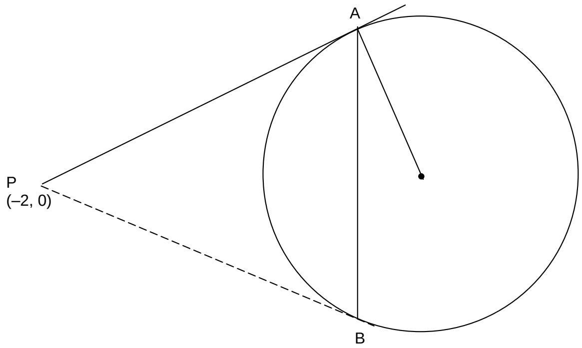
$x^2+y^2-8x-16y+60=0$ ......(1)

Equation of AB.

$-2x+0y+4(x-2)-8(y+0)+60'=0$

$-6x-8y+68=0 \Rightarrow 3x+4y-34=0$ ...(2)

Now solve (1) and (2)



5. ABD

Sol. Clearly, $f(-1)=-f(1),\ f(0)=0$

For option (A) and (B), apply LMVT for the function $y=f(x)$ in $[-1,0]$ and [0, 1] respectively

For option (D), apply LMVT to the function $y=(f(x))^n$ in $[0,1]$

6. AD

Sol. Probability that X is not hit is $=\frac{2}{5}.\frac{4}{5}+\frac{3}{5}.\frac{4}{5}.\frac{2}{5}.\frac{4}{5}+......=\frac{\frac{2}{5}.\frac{4}{5}}{1-\frac{3}{5}.\frac{4}{5}}=\frac{8}{13}$

Probability that both are hit is $=\frac{2}{5}.\frac{1}{5}+\frac{3}{5}.\frac{4}{5}.\frac{2}{5}.\frac{1}{5}+....=\frac{\frac{2}{5}.\frac{1}{5}}{1-\frac{3}{5}.\frac{4}{5}}=\frac{2}{13}$

7. AD

Sol. Clearly plane P will be perpendicular bisector of line joining A and B.

$\therefore$ It passes through (–1, –1, 2) and drs of its normal are (3, –3, 1)

$\therefore$ Equation of plane P is

$3(x+1)-3(y+1)+1(z-2)=0$

$\Rightarrow 3x-3y+z=2$ . ...(1)

(a) $\because$ P is parallel to Q

$\therefore\ \frac{1}{3}=\frac{1}{3}=\frac{C}{1} \qquad \Rightarrow \qquad C=\frac{1}{3}$

(b) If P is perpendicular to Q then $1.3-1.(-3)+1.C=0 \Rightarrow C=-6$

(c) $\because$ Plane P intersects coordinate axes at points with position vectors $\frac{2}{3}\hat{i},-\frac{2}{3}\hat{j}$ and $2\hat{k}$

$\therefore$ volume of tetrahedron $=\frac{1}{6}\left|\left[\frac{2}{3}\hat{i}\ -\frac{2}{3}\hat{j}\ \ 2\hat{k}\right]\right|=\frac{4}{27}$

(d) Point on line L can be taken as $(\lambda+1,\ 3\lambda-2,\ -\lambda+7)$

$\therefore$ It is on plane P.

$\therefore\ 3(\lambda+1)-3(3\lambda-2)+(-\lambda+7)=2$

$\Rightarrow\ 7\lambda=14 \Rightarrow \lambda=2$

$\therefore$ point of int. = (3, 4, 5)

$\therefore\ \alpha+\beta+\gamma=12$

8. B

Sol. $|A| = x + y + z$

$\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}\right)\right)\right)\right| = |A|^{(n-1)^4} = |A|^{16}$

$|A|^{16} = 4^8 . 5^{16} = 2^{16} . 5^{16}$

$|A| = 2 \times 5 = 10 = x + y + z$

$x + y + z = 10$

Number of natural numbers solutions $= {}^{10-1}C_{3-1} = {}^9C_2 = 36$

$\Rightarrow p = 36$

$\int_{-\pi/2}^{\pi/2} 2^{\sin x}\, dx + \int_{5/2}^{4} \sin^{-1}\left(\log_2(x-2)\right) dx$

sub $x - 2 = t$

$+\int_{1/2}^{2} \sin^{-1}(\log_2 t)\, dt$

$\int_{-\pi/2}^{\pi/2} 2^{\sin x}\, dx + \int_{1/2}^{2} \sin^{-1}(\log_2 x)\, dx = \frac{\pi}{2} 2^{\sin \pi/2} - \left(\frac{-\pi}{2}\right) 2^{\sin(-\pi/2)}$

$= \frac{\pi}{2}.2 + \frac{\pi}{2}.\frac{1}{2}$

$= \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

So, $p = 36,\ q = 5$

9. C

10. D

(Sol. 9 & 10)

For C_1: Center (a, a), radius = a

C_2: Center (b, b), radius = b

(i) $C_1C_2 = r_1r_2 \Rightarrow \sqrt{(a-b)^2 + (a-b)^2} = (a+b)$

$\Rightarrow \sqrt{2}(b-a) = a + b$

$\Rightarrow \frac{b}{a} = 3 + 2\sqrt{2}$

(ii) $C_1C_2^2 = r_1^2 + r_2^2$

$(a-b)^2 + (a-b)^2 = a^2 + b^2 \Rightarrow a^2 - 4ab + b^2 = 0$

$\Rightarrow \frac{b^2}{a^2} - \frac{4b}{a} + 1 = 0$

$\frac{b}{a} = 2 \pm \sqrt{3}$ but $\frac{b}{a} \geq 1$

$\Rightarrow \frac{b}{a} = 2 + \sqrt{3}$

(iii) Common chord is diameter of smaller circle

$x^2 + y^2 - 2bx - 2by + b^2 = 0$ ……….(i)

$x^2 + y^2 - 2ax - 2ay + a^2 = 0$ ……….(ii)

(2) – (1)

$2x + 2y = a + b$

passes through (a, a)

$2a + 2a = a + b$

$\Rightarrow \frac{b}{a} = 3$

(iv) $(x-b)^2 + (y-b)^2 = b^2$ passes through $(a_1 a)$

11. B

12. A

(Sol. 11 & 12)

Total matrices $= 4! = 24$

For non negative $\det(A)$ possibilities

$\begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix} = 2, \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = 4, \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} = 8$

So, (i) P, Q, T

(iii) least absolute value of $\det A = 2$

$|\text{adj adj adj}| = |A|^{(n-1)^3} = |A|^{(2-1)^3} = |A| = 2 \Rightarrow (P)$

(ii) Possible value of $\det A$ are $\pm 2, \pm 4, \pm 8$

So sum = 0 $\Rightarrow (S)$

(iv) $(\det A)\text{least} = -8$ so $\det(4A^{-1}) = 4^2 \frac{1}{(\det A)}$

$= \frac{16}{-8} = -2 \Rightarrow (R)$

(PART – B)

1. 8

Sol. $\because$ Elements of determinant can be 1 or –1

$\therefore$ First two places of first row can be filled in $2 \times 2 = 4$ ways and last place can be filled in only one way to product be 1.

$\therefore$ Second row can be made in 4 ways and third row can be made in 1 way only.

$\therefore$ Number of elements in S = 16 = m

$\because |P| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & -2 & 3 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -2 & 3 \\ 0 & -1 & 1 \end{vmatrix} = 1(-2+3) = 1$

$\because \text{adj}(\text{adj}P) = |P|^{n-2}.P = P$

$\Rightarrow \text{tr.}(\text{adj}(\text{adj}P)) = \text{tr}(P) = 2 = n$

$\therefore \frac{m}{n} = 8$

2. 2

Sol. $\because |z_2| = 1 \Rightarrow z_2 = e^{i\theta} = \cos\theta + i\sin\theta$

$\therefore \frac{z_2 - 1}{z_2 + 1} = \frac{\cos\theta - 1 + i\sin\theta}{\cos\theta + 1 + i\sin\theta}$

$= \frac{-2\sin^2\frac{\theta}{2} + i.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + i.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$$=\frac{i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}=i\tan\frac{\theta}{2}$$

$$\therefore \frac{z_1-1}{z_1+1}=-\tan^2\frac{\theta}{2}\Rightarrow\left(\frac{z_1-1}{z_1+1}\right)=-\tan^2\frac{\theta}{2}$$

$$\Rightarrow z_1=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\cos\theta \text{ which is purely real and lies in } [-1,1]$$

$$\therefore |z_1-1|+|z_1+1|=|\cos\theta-1|+|\cos\theta+1|=2$$

3. 2

Sol. $a_n=\frac{2\cot x}{1-\sin^2 x}=\frac{2\cot x}{\cos^2 x}=\frac{2}{\sin x\cos x}$

$$b_n=\frac{\sin 2x}{1-\sin^2 x}=\frac{2\sin x\cos x}{\cos^2 x}=2\tan x$$

$$a_n-b_n=\frac{2}{\sin x\cos x}-2\tan x$$

$$=\frac{2\left(1-\sin^2 x\right)}{\sin x\cos x}=\frac{2\cos^2 x}{\sin x\cos x}$$

$$=2\cot x$$

Since $\cot x\downarrow \text{in}\left(0,\frac{\pi}{4}\right]$

So, minimum $=2\cot\frac{\pi}{4}=2$

4. 2

Sol. $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=\begin{vmatrix}2+\sin\theta & \cos\theta & \sin 2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$

$R_1\to R_1+R_2$

$$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=\begin{vmatrix}2+\sin\theta & \cos\theta & \sin 2\theta\\ 2\sin\theta\cos\frac{2\pi}{3} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3}\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

$$=\begin{vmatrix}27\sin\theta & \cos\theta & \sin 2\theta\\ -\sin\theta & -\cos\theta & -\sin 2\theta\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

$R_1\to R_1+R_2$

$$\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]=\begin{vmatrix} 2 & 0 & 0 \\ -\sin\theta & -\cos\theta & -\sin 2\theta \\ \sin\left(\theta.\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right) \end{vmatrix}$$

$$=2\left(\sin 2\theta\cos\left(\theta-\frac{2\pi}{3}\right)-\cos\theta\sin\left(2\theta-\frac{4\pi}{3}\right)\right)$$

$$=2\left(\sin 2\theta\cos\left(\theta-\frac{2\pi}{3}\right)-\cos\theta\sin\left(2\theta-\frac{4\pi}{3}\right)\right)$$

$$=2\left\{\sin 2\theta\left[\cos\theta\left(\frac{1}{-2}\right)+(\sin\theta)\frac{\sqrt{3}}{2}\right]-\cos\theta\left[\sin 2\theta\left(-\frac{1}{2}\right)-(\cos 2\theta)\left(-\frac{\sqrt{3}}{2}\right)\right]\right\}$$

$$=\sin 2\theta\left[-\cos\theta+\sqrt{3}\sin\theta\right]-\cos\theta\left[\sqrt{3}\cos^2\theta-\sin^2\theta\right]$$

$$=\sqrt{3}\left(\sin 2\theta\sin\theta-\cos\theta\cos 2\theta\right)=-\sqrt{3}\cos 3\theta$$

$$v=\frac{1}{6}\left[2\vec{b}\times\vec{c}\quad 3\vec{c}\times\vec{a}\quad \vec{a}\times 4\vec{b}\right]=\frac{24}{6}[a,b,c]^2$$

$$=\frac{24\times 3}{6}\cos^2 3\theta$$

So, $V=12\cos^2 3\theta$

So $V\le 12$

Hence $V_{max}=12$

5. 4

Sol. $15!=2^{11}.3^6.7^3.11.13$

All the powers of single prime number has two possibilities either is α or β so total pair of $(\alpha,\beta)=2^5=32$

6. 2

Sol. $$gog=\frac{3\left(\frac{3x+5}{2x-3}\right)+5}{2\left(\frac{3x+5}{2x-3}\right)-3}=\frac{19x}{19}=x$$

$gogog=g(x)$, $gogogog=x$

$$\Rightarrow gogogog\_\_\_\_\text{up to n times}=\begin{cases} x, & \text{if n is even} \\ \frac{3x+5}{2x-3} & \text{if n is odd} \end{cases}$$

So, $g_{2014}(x)=x$ In comparing $p=1, q=0, r=0, s=1$