Session 2

Ordered Pair, Definition of Relation, Ordered Relation, Composition of Two Relations

Ordered Pair

If A be a set and $a, b \in A$, then the ordered pair of elements a and b in A denoted by (a, b), where a is called the first coordinate and b is called the second coordinate.

Remark

1. Ordered pairs (a, b) and (b, a) are different.
2. Ordered pairs (a, b) and (c, d) are equal iff $a = c$ and $b = d$.

Cartesian Product of Two Sets

The cartesian product to two sets A and B is the set of all those ordered pairs whose first coordinate belongs to A and second coordinate belongs to B. This set is denoted by $A \times B$ (read as 'A cross B' or 'product set of A and B').

Symbolically, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

or $\quad A \times B = \{(a, b) : a \in A \wedge b \in B\}$

Thus, $\quad (a, b) \in A \times B \Leftrightarrow a \in A \wedge b \in B$

Similarly, $\quad B \times A = \{(b, a) : b \in B \wedge a \in A\}$

Remark

1. $A \times B \neq B \times A$
2. If A has p elements and B has q elements, then $A \times B$ has pq elements.
3. If $A = \phi$ and $B = \phi$, then $A \times B = \phi$.
4. Cartesian product of n sets $A_1, A_2, A_3, \ldots, A_n$ is the set of all ordered n-tuples $(a_1, a_2, \ldots, a_n)$ $a_i \in A_i$, $i = 1, 2, 3, \ldots, n$ and is denoted by $A_1 \times A_2 \times \ldots \times A_n$ or $\prod_{i=1}^{n} A_i$.

Example 11. ***If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, find $A \times B$, $B \times A$ and show that $A \times B \neq B \times A$.***

Sol. $A \times B = \{1, 2, 3\} \times \{4, 5\} = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

and $B \times A = \{4, 5\} \times \{1, 2, 3\} = \{(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$

It is clear that $A \times B \neq B \times A$.

Example 12. If A and B be two sets and $A \times B = \{(3, 3), (3, 4), (5, 2), (5, 4)\}$, find A and B.

Sol. A = First coordinates of all ordered pairs = $\{3, 5\}$

and B = Second coordinates of all ordered pairs = $\{2, 3, 4\}$

Hence, $A = \{3, 5\}$ and $B = \{2, 3, 4\}$

Important Theorems on Cartesian Product

If A, B and C are three sets, then

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

(iv) $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$, where S and T are two sets.

(v) If $A \subseteq B$, then $(A \times C) \subseteq (B \times C)$

(vi) If $A \subseteq B$, then $(A \times B) \cap (B \times A) = A^2$

(vii) If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

Example 13. ***If A and B are two sets given in such a way that $A \times B$ consists of 6 elements and if three elements of $A \times B$ are (1, 5), (2, 3) and (3, 5), what are the remaining elements?***

Sol. Since, $(1, 5), (2, 3), (3, 5) \in A \times B$, then clearly $1, 2, 3 \in A$ and $3, 5 \in B$.

$A \times B = \{1, 2, 3\} \times (3, 5)$

$= (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)$

Hence, the remaining elements are (1, 3), (2, 5), (3, 3).

Relations

Introduction of Relation

We use sentences depending upon the relationship of an object to the other object in our daily life such as

(i) 'Ram, Laxman, Bharat, Shatrughan' were the sons of Dashrath.
(ii) 'Sita' was the wife of Ram.
(iii) 'Laxman' was the brother of Ram.
(iv) 'Dashrath' was the father of Ram.
(v) 'Kaushaliya' was the mother of Ram.

If Ram, Laxman, Bharat, Shatrughan, Sita, Kaushaliya and Dashrath are represented by a, b, c, d, e, f and y respectively and A represents the set, then

$$A = \{a, b, c, d, e, f, y\}$$

Here, we see that any two elements of set A are related many ways, i.e. a, b, c, d are sons of y. 'a' is the son of y is represented by aRy. Similarly, b is the son of y, c is the son of y and d is also son of y are represented as $b\,R\,y$, $c\,R\,y$ and $d\,R\,y$, respectively.

If we write here $y\,R\,a$ it means that y is the son of a which is impossible, since a is the son of y. Hence, y and a cannot be related like this. Its generally represented as $y \not R a$. Hence, we can say that a and y are in definite order. a comes before R and y after R. Therefore, aRy may be represented as a order pair (a, y). Similarly, bRy, cRy and dRy are represented by $(b, y), (c, y)$ and (d, y), respectively. If all pairs will represented by a set, then we see that first element of each pair is the son of second element. Hence, the set of these pairs may be represented by set R, then

$$R = \{(a, y), (b, y), (c, y), (d, y)\}$$

Symbolically, $R = \{(x, y) : x, y \in A$, where x is son of $y\}$

It is clear that R is subset of $A \times A$

i.e., $R \subseteq A \times A$

Corollary In above example, if

$A = \{a, b, c, d\}$ and $B = \{e, f, y\}$, then

$R = \{(x, z) : x \in A, z \in B$, where x is son of $z\}$

It is clear that $R \subseteq A \times B$.

Definition of Relation

A relation (or binary relation) R, from a non-empty set A to another non-empty set B, is a subset of $A \times B$.

i.e., $R \subseteq A \times B$ or $R \subseteq \{(a, b) : a \in A, b \in B\}$

Now, if (a, b) be an element of the relation R, then we write aRb (read as 'a is related to b')

i.e., $(a, b) \in R \Leftrightarrow aRb$

and if (a, b) is not an element of the relation R, then we write $a \not R b$ (read as 'a is not related to b'),

i.e. $(a, b) \notin R \Leftrightarrow a \not R b$.

Remark

1. Any subset of $A \times A$ is said to be a relation on A.
2. If A has m elements and B has n elements, then $A \times B$ has $m \times n$ elements and total number of different relations from A to B is 2^{mn}.
3. If $R = A \times B$, then Domain $R = A$ and Range $R = B$.
4. The domain as well as range of the empty set ϕ is ϕ.
5. If A = Dom R and B = Ran R, then we write $B = R\,[A]$.

For example,

Let $A = \{1, 2, 3\}$ and $B = \{3, 5, 7\}$, then
$A \times B = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7)\}$.
But $R \subseteq A \times B$
i.e., every subset of $A \times B$ is a relation from A to B. If we consider the relation, $R = \{(1, 5), (1, 7), (3, 5), (3, 7)\}$
Then, $1\,R\,5$; $1\,R\,7$; $3\,R\,5$; $3\,R\,7$
Also, $1 \not R 3$; $2 \not R 3$; $2 \not R 5$; $2 \not R 7$; $3 \not R 3$;
Clearly, Domain $R = \{1, 3\}$ and Range $R = \{5, 7\}$

For example,

Let $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, then number of different relations from A to B is $2^{3\times2} = 2^6 = 64$ because A has 3 elements and B has 2 elements.

Types of Relations from One Set to Another Set

1. Empty Relation

A relation R from A to B is called an empty relation or a void relation from A to B if $R = \phi$.

For example,

Let $A = \{2, 4, 6\}$ and $B = \{7, 11\}$
Let $R = \{(a, b) : a \in A, b \in B$ and $a - b$ is even$\}$
As, none of the numbers $2 - 7, 2 - 11, 4 - 7, 4 - 11, 6 - 7, 6 - 11$ is an even number, $R = \phi$.
Hence, R is an empty relation.

2. Universal Relation

A relation R from A to B is said to be the universal relation, if $R = A \times B$.

For example, Let $A = \{1, 2\}$, $B = \{1, 3\}$ and

$$R = \{(1, 1), (1, 3), (2, 1), (2, 3)\}$$

Here, $R = A \times B$
Hence, R is the universal relation from A to B.

Types of Relations on a Set

1. Empty Relation

A relation R on a set A is said to be an empty relation or a void relation, if $R = \phi$.

For example,

Let $A = \{1, 3\}$ and $R = \{(a, b) : a, b \in A$ and $a + b$ is odd$\}$
Hence, R contains no element, therefore R is an empty relation on A.

2. Universal Relation

A relation R on a set A is said to be universal relation on A, if $R = A \times A$.

For example,

Let $A = \{1, 2\}$
and $R = [(1, 1), (1, 2), (2, 1), (2, 2)]$
Here, $R = A \times A$
Hence, R is the universal relation on A.

3. Identity Relation

A relation R on a set A is said to be the identity relation on A, if

$$R = [(a, b) : a \in A, b \in A \text{ and } a = b]$$

Thus, identity relation, $R = [(a, a) : \forall a \in A]$
Identity relation on set A is also denoted by I_A.
Symbolically, $I_A = [(a, a) : a \in A]$

For example,

Let $A = \{1, 2, 3\}$
Then, $I_A = \{(1, 1), (2, 2), (3, 3)\}$

Remark

In an identity relation on A every element of A should be related to itself only.

4. Inverse Relation

If R is a relation from a set A to a set B, then inverse relation of R to be denoted by R^{-1}, is a relation from B to A.

Symbolically, $R^{-1} = \{(b, a) : (a, b) \in R\}$

Thus, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}, \forall a \in A, b \in B.$

Remark

1. Dom (R^{-1}) = Range (R) and Range (R^{-1}) = Dom (R)
2. $(R^{-1})^{-1} = R$

For example,

If $R = \{(1, 2), (3, 4), (5, 6)\}$, then
$R^{-1} = \{(2, 1), (4, 3), (6, 5)\}$
$\therefore (R^{-1})^{-1} = \{(1, 2), (3, 4), (5, 6)\} = R$

Here, dom $(R) = \{1, 3, 5\}$, range $(R) = \{2, 4, 6\}$
and dom $(R^{-1}) = \{2, 4, 6\}$, range $(R^{-1}) = \{1, 3, 5\}$
Clearly, dom (R^{-1}) = range (R)
and range (R^{-1}) = dom (R).

Various Types of Relations

1. Reflexive Relation

A relation R on a set A is said to be reflexive, if $a R a, \forall a \in A$
i.e., if $(a, a) \in R, \forall a \in A$

For example,

Let $A = \{1, 2, 3\}$
$R_1 = \{(1, 1), (2, 2), (3, 3)\}$
$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\}$
and $R_3 = \{(1, 1), (2, 2), (2, 3), (3, 2)\}$

Here, R_1 and R_2 are reflexive relations on A, R_3 is not a reflexive relation on A as $(3, 3) \notin R_3$, i.e. $3 \not R_3\, 3$.

Remark

The identity relation is always a reflexive relation but a reflexive relation may or may not be the identity relation. It is clear in the above example given, R_1 is both reflexive and identity relation on A but R_2 is a reflexive relation on A but not an identity relation on A.

2. Symmetric Relation

A relation R on a set A is said to be symmetric relation, if

$$a R b \Rightarrow b R a, \forall a, b \in A$$

i.e., if $(a, b) \in R \Rightarrow (b, a) \in R, \forall a, b \in A$

For example,

Let $A = \{1, 2, 3\}$
$R_1 = \{(1, 2), (2, 1)\}$
$R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$
and $R_3 = \{(2, 3), (1, 3), (3, 1)\}$

Here, R_1 and R_2 are symmetric relations on A but R_3 is not a symmetric relation on A because $(2, 3) \in R_3$ and $(3, 2) \notin R_3$.

3. Anti-symmetric Relation

A relation R on a set A is said to be anti-symmetric,
if $a R b, b R a \Rightarrow a = b, \forall a, b \in A$
i.e., $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b, \forall a, b \in A$

For example,

Let R be the relation in N (natural number) defined by, "x is divisor of y", then R is anti-symmetric because x divides y and y divides $x \Rightarrow x = y$

4. Transitive Relation

A relation R on a set A is said to be a transitive relation, if $a\,R\,b$ and $b\,R\,c \Rightarrow aRc, \forall\ a, b, c \in A$

i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R, \forall\ a, b, c \in A$

For example,

Let $A = \{1, 2, 3\}$

$$R_1 = \{(1,2), (2,3), (1,3), (3,2)\}$$
$$R_2 = \{(2,3), (3,1)\}$$
$$R_3 = \{(1,3), (3,2), (1,2)\}$$

Then, R_1 is not transitive relation on A because $(2,3) \in R_1$ and $(3,2) \in R_1$ but $(2,2) \notin R_1$. Again, R_2 is not transitive relation on A because $(2,3) \in R_2$ and $(3,1) \in R_2$ but $(2,1) \notin R_2$. Finally R_3 is a transitive relation.

Example 14. ***Let*** $A = \{1, 2, 3\}$ ***and*** $R = \{(a, b) : a, b \in A$***,*** a ***divides*** b ***and*** b ***divides*** $a\}$***. Show that*** R ***is an identity relation on*** A***.***

Sol. Given, $A = \{1, 2, 3\}$

$a \in A$, $b \in B$, a divides b and b divides a.

$\Rightarrow \quad a = b$

$\therefore \quad R = \{(a, a), a \in A\} = \{(1, 1), (2, 2), (3, 3)\}$

Hence, R is the identity relation on A.

Example 15. Let $A = \{3, 5\}$, $B = \{7, 11\}$.
Let $R = \{(a, b) : a \in A, b \in B, a - b$ is even$\}$.
Show that R is an universal relation from A to B.

Sol. Given, $A = \{3, 5\}$, $B = \{7, 11\}$.

Now, $R = \{(a, b) : a \in A, b \in B$ and $a - b$ is even$\}$
$= \{(3, 7), (3, 11), (5, 7), (5, 11)\}$

Also, $A \times B = \{(3, 7), (3, 11), (5, 7), (5, 11)\}$

Clearly, $R = A \times B$

Hence, R is an universal relation from A to B.

Example 16. Prove that the relation R defined on the set N of natural numbers by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0$ is not symmetric but it is reflexive.

Sol. (i) $2x^2 - 3x \cdot x + x^2 = 0, \forall\ x \in N$.

$\therefore\ x\ R\ x, \forall\ x \in N$, i.e. R is reflexive.

(ii) For $x = 1, y = 2, 2x^2 - 3xy + y^2 = 0$

$\therefore\ 1R2$

But $2 \cdot 2^2 - 3 \cdot 2 \cdot 1 + 1^2 = 3 \neq 0$

So, 2 is not related to 1 i.e., $2 \not R 1$

$\therefore\ R$ is not symmetric.

Example 17. Let N be the set of natural numbers and relation R on N be defined by $x R y \Leftrightarrow x$ divides y, $\forall\ x, y \in N$.
Examine whether R is reflexive, symmetric, anti-symmetric or transitive.

Sol. (i) x divides x i.e., $x R x, \forall\ x \in N$

$\therefore\ R$ is reflexive.

(ii) 1 divides 2 i.e., $1\ R\ 2$ but $2 \not R 1$ as 2 does not divide 1.

(iii) x divides y and y divides $x \Rightarrow x = y$

i.e., $x R y$ and $y R x \Rightarrow x = y$

$\therefore\ R$ is anti-symmetric relation.

(iv) $x R y$ and $y R z \Rightarrow x$ divides y and y divides z.

$\Rightarrow kx = y$ and $k' y = 2$, where k, k' are positive integers.

$\Rightarrow\ kk' x = z\ \Rightarrow\ x$ divides $z\ \Rightarrow\ x Rz$

$\therefore\ R$ is transitive.

Equivalence Relation

A relation R on a set A is said to be an equivalence relation on A, when R is (i) reflexive (ii) symmetric and (iii) transitive. The equivalence relation denoted by ~.

Example 18. N ***is the set of natural numbers. The relation*** R ***is defined on*** $N \times N$ ***as follows***

$$(a, b) R\ (c, d) \Leftrightarrow a + d = b + c$$

Prove that R ***is an equivalence relation.***

Sol. (i) $(a, b)\ R\,(a, b)\ \Rightarrow a + b = b + a$

$\therefore\ R$ is reflexive.

(ii) $(a, b)\ R\,(c, d)\ \Rightarrow a + d = b + c$

$\Rightarrow \quad c + b = d + a\ \Rightarrow\ (c, d)\ R\,(a, b)$

$\therefore\ R$ is symmetric.

(iii) $(a, b)\ R\,(c, d)$ and $(c, d)\ R\,(e, f) \Rightarrow a + d = b + c$
and $\quad c + f = d + e$

$\Rightarrow\ a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e\ \Rightarrow (a, b)\ R\,(e, f)$

$\therefore\ R$ is transitive.

Thus, R is an equivalence relation on $N \times N$.

Example 19. A relation R on the set of complex numbers is defined by $z_1 R z_2 \Leftrightarrow \dfrac{z_1 - z_2}{z_1 + z_2}$ is real, show that R is an equivalence relation.

Sol. (i) $z_1 R z_1 \Rightarrow \dfrac{z_1 - z_1}{z_1 + z_1}, \forall z_1 \in C\ \Rightarrow\ 0$ is real

$\therefore\ R$ is reflexive.

(ii) $z_1 R z_2 \Rightarrow \dfrac{z_1 - z_2}{z_1 + z_2}$ is real

$\Rightarrow \; -\left(\dfrac{z_2 - z_1}{z_1 + z_2}\right)$ is real $\Rightarrow \left(\dfrac{z_2 - z_1}{z_1 + z_2}\right)$ is real

$\Rightarrow \; z_2 R z_1, \forall\, z_1, z_2 \in C$

$\therefore$ R is symmetric.

(iii) $\because z_1 R z_2 \Rightarrow \dfrac{z_1 - z_2}{z_1 + z_2}$ is real

$$\Rightarrow \quad \overline{\left(\frac{z_1 - z_2}{z_1 + z_2}\right)} = -\left(\frac{z_1 - z_2}{z_1 + z_2}\right)$$

$$\Rightarrow \quad \left(\frac{\bar{z}_1 - \bar{z}_2}{\bar{z}_1 + \bar{z}_2}\right) + \left(\frac{z_1 - z_2}{z_1 + z_2}\right) = 0$$

$$\Rightarrow \quad 2(z_1\bar{z}_1 - z_2\bar{z}_2) = 0 \Rightarrow |z_1|^2 = |z_2|^2 \qquad \text{...(i)}$$

Similarly, $z_2 R z_2 \Rightarrow |z_2|^2 = |z_3|^2$ ...(ii)

From Eqs. (i) and (ii), we get

$$z_1 R z_2, z_2 R z_3$$

$$\Rightarrow \quad |z_1|^2 = |z_3|^2$$

$$\Rightarrow \quad z_1 R z_3$$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

Ordered Relation

A relation R is called ordered, if R is transitive but not an equivalence relation.

Symbolically, $\quad a R b, b R c \Rightarrow a R c, \forall\, a, b, c \in A$

For example,

Let $R = \{(1, 2), (2, 1), (2, 3), (3, 2), (1, 3)\}$.

Here, R is symmetric.

Since, $(1, 2) \in R \Rightarrow (2, 1) \in R, (2, 3) \in R \Rightarrow (3, 2) \in R$

and R is transitive.

Since, $(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$

but R is not reflexive.

Since, $(1, 1) \notin R, (2, 2) \notin R, (3, 3) \notin R$

Hence, R is not an equivalence relation.

$\therefore$ R is an ordered relation.

Partial Order Relation

A relation R is called partial order relation, if R is reflexive, transitive and anti-symmetric at the same time.

For example,

Let $\quad R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$

$\therefore \quad R^{-1} = \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (3, 1)\}$

$R \cap R^{-1} = \{(1, 1), (2, 2), (3, 3)\} = \text{Identity}$

$\therefore$ R is anti-symmetric.

It is clear that R is reflexive.

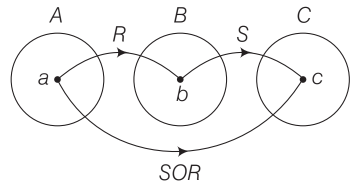
Since, $(1, 1) \in R, (2, 2) \in R, (3, 3) \in R$ and R is transitive.

Since, $(1, 2) \in R$ and $(2, 3) \in R \Rightarrow (1, 3) \in R$

Hence, R is partial order relation.

Composition of Two Relations

If A, B and C are three sets such that $R \subseteq A \times B$ and $S \subseteq B \times C$, then $(SOR)^{-1} = R^{-1} O S^{-1}$. It is clear that aRb, bSc $\Rightarrow aSORc$.



More generally,

$$(R_1 O R_2 O R_3 O \ldots O R_n)^{-1} = R_n^{-1} O \ldots O R_3^{-1} O R_2^{-1} O R_1^{-1}$$

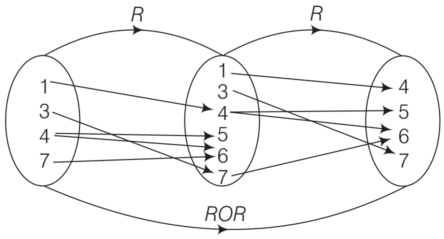
Example 20. ***Let R be a relation such that $R = \{(1, 4), (3, 7), (4, 5), (4, 6), (7, 6)\}$, find***

(i) $R^{-1} O R^{-1}$ and (ii) $(R^{-1} O R)^{-1}$.

Sol. (i) We know that, $(R O R)^{-1} = R^{-1} O R^{-1}$

Dom $(R) = \{1, 3, 4, 7\}$

Range $(R) = \{4, 5, 6, 7\}$



We see that,

$$1 \longrightarrow 4 \longrightarrow 5 \Rightarrow (1, 5) \in ROR$$
$$1 \longrightarrow 4 \longrightarrow 6 \Rightarrow (1, 6) \in ROR$$
$$3 \longrightarrow 7 \longrightarrow 6 \Rightarrow (3, 6) \in ROR$$

$\therefore \quad ROR = \{(1, 5), (1, 6), (3, 6)\}$

Then, $R^{-1} O R^{-1} = (R O R)^{-1}$

$= \{(5, 1), (6, 1), (6, 3)\}$

(ii) We know that, $(R^{-1} O R)^{-1} = R^{-1} O (R^{-1})^{-1} = R^{-1} O R$

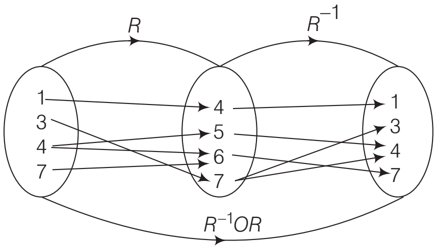
Since,

$R = \{(1, 4), (3, 7), (4, 5), (4, 6), (7, 6)\}$

$\therefore \quad R^{-1} = \{(4, 1), (7, 3), (5, 4), (6, 4), (6, 7)\}$

$\therefore$ Dom $(R) = \{1, 3, 4, 7\}$, Range $(R) = \{4, 5, 6, 7\}$

Dom $(R^{-1}) = \{4, 5, 6, 7\}$, Range $(R^{-1}) = \{1, 3, 4, 7\}$



We see that,

$$1 \xrightarrow{R} 4 \xrightarrow{R^{-1}} 1 \Rightarrow (1, 1) \in R^{-1} O R$$

$$3 \xrightarrow{R} 7 \xrightarrow{R^{-1}} 3 \Rightarrow (3, 3) \in R^{-1} O R$$

$$4 \xrightarrow{R} 5 \xrightarrow{R^{-1}} 4 \Rightarrow (4, 4) \in R^{-1} O R$$

$$4 \xrightarrow{R} 6 \xrightarrow{R^{-1}} 4 \Rightarrow (4, 4) \in R^{-1} O R$$

$$4 \xrightarrow{R} 6 \xrightarrow{R^{-1}} 7 \Rightarrow (4, 7) \in R^{-1} O R$$

$$7 \xrightarrow{R} 6 \xrightarrow{R^{-1}} 4 \Rightarrow (7, 4) \in R^{-1} O R$$

$$7 \xrightarrow{R} 6 \xrightarrow{R^{-1}} 7 \Rightarrow (7, 7) \in R^{-1} O R$$

$$\therefore \ R^{-1} O R = \{(1, 1), (3, 3), (4, 4), (7, 7), (4, 7), (7, 4)\}$$

Hence, $(R^{-1} O R)^{-1} = R^{-1} O R = \{(1, 1), (3, 3)$
$(4, 4), (7, 7), (4, 7), (7, 4)\}$

Theorems on Binary Relations

If R is a relation on a set A, then

(i) R is reflexive $\Rightarrow R^{-1}$ is reflexive.

(ii) R is symmetric $\Rightarrow R^{-1}$ is symmetric.

(iii) R is transitive $\Rightarrow R^{-1}$ is transitive.

Exercise for Session 2

1. If $A = \{2, 3, 5\}$, $B = \{2, 5, 6\}$, then $(A - B) \times (A \cap B)$ is

(a) $\{(3, 2), (3, 3), (3, 5)\}$ (b) $\{(3, 2), (3, 5), (3, 6)\}$
(c) $\{(3, 2), (3, 5)\}$ (d) None of these

2. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C)$ equals

(a) 1 (b) 2 (c) 17 (d) 288

3. The relation R defined on the set of natural numbers as $\{(a, b) : a$ differs from b by 3$\}$ is given by

(a) $\{(1, 4), (2, 5), (3, 6), \ldots\}$ (b) $\{(4, 1), (5, 2), (6, 3), \ldots\}$ (c) $\{(1, 3), (2, 6), (3, 9), \ldots\}$ (d) None of these

4. Let A be the non-void set of the children in a family. The relation 'x is a brother of y' on A, is

(a) reflexive (b) anti-symmetric (c) transitive (d) equivalence

5. Let $n(A) = n$, then the number of all relations on A, is

(a) 2^n (b) $2^{n!}$ (c) 2^{n^2} (d) None of these

6. If $S = \{1, 2, 3, \ldots, 20\}$, $K = \{a, b, c, d\}$, $G = \{b, d, e, f\}$. The number of elements of $(S \times K) \cup (S \times G)$ is

(a) 40 (b) 100 (c) 120 (d) 140

7. The relation R is defined on the set of natural numbers as $\{(a, b) : a = 2b\}$, then R^{-1} is given by

(a) $\{(2, 1) (4, 2) (6, 3), \ldots\}$ (b) $\{(1, 2) (2, 4) (3, 6), \ldots\}$ (c) R^{-1} is not defined (d) None of these

8. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

(a) reflexive but not symmetric (b) reflexive but not transitive
(c) symmetric and transitive (d) Neither symmetric nor transitive

9. The number of equivalence relations defined in the set $S = \{a, b, c\}$ is

(a) 5 (b) 3! (c) 2^3 (d) 3^3

10. If R be a relation $<$ from $A = \{1, 2, 3, 4\}$ to $B = \{1, 3, 5\}$, i.e. $(a, b) \in R \Leftrightarrow a < b$, then ROR^{-1}, is

(a) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$ (b) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$
(c) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$ (d) $\{(3, 3), (3, 4), (4, 5)\}$

Answers

Exercise for Session 2

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (c)
7. (b)	**8.** (a)	**9.** (a)	**10.** (c)		