MOTION

PHYSICAL SCIENCE

This science deals with the properties and Behaviour of nonliving things.

(a) Physics (in Greek Nature):

It is the branch of science which deals with the study of the natural laws and their manifestation in the natural phenomenon.

Mechanics (oldest branch) :

If deals with the conditions of rest or motion of the material objects around us.

Statics :

It deals with the study of object at rest or in equilibrium, even when they are under the action of several forces (measurement of time is not essential).

Kinematics : If deals with the study of motion of objects without considering the cause of motion measurement of time is essential). $\left[\text{Kinematics}\frac{\text{Greek}}{\text{Word}}\text{Kinema}\rightarrow\text{motion}\right]$

Dynamics : It deal with the study of objects taking into consideration the cause of their motion.

$\left(\text{Dnamics}\frac{\text{Greek}}{\text{Work}}\text{Dynamis}\rightarrow\text{power}\right)$

Rest : An object is said to be at rest if it does not change its position w.r.t. its surroundings with the passage of time.

Motion :A body is said to be in motion if its position changes continuously w.r.t. the surroundings (or with respect to an observer) with the passage of time.

REST AND MOTION ARE RELATIVE TERMS

Eg. : 1 A, B and C are three persons. B and C are sitting in the car and A is standing outside it. When car starts to move, B and C are changing their position with respect to A so B and C are in motion with respect to A but B is not changing its position with time with respect to C, so B is at rest with respect to C (same for C). Therefore motion depends on the position of the observer , hence motion is relative.

Eg. :2 We know that the earth is rotating about its axis and revolving around the sun. The stationary objects like your classroom, a tree and the lamp posts etc., do not change their position with respect to each other i.e. they are at rest. Although earth is in motion. To an observer situated outside the earth, say in a space ship, our classroom, trees etc. would appear to be in motion. Therefore, all motions are relative. There is nothing like absolute motion.

(a) Concept of a Point Object :

In mechanics while studying the motion of an object, sometimes it dimension are of no importance and the object may be treated as point object without much error. When the size of the object is much less in comparison to the distance covered by the object then the object is considered as a point object.

Eg. : 1 If one travels by a car from one place to another far away place, then length of the car is ignored as compared to distance traveled.

Eg. : 2 Earth can be regarded as a point object for studying its motion around the sun.

(b) Frame of Reference :

To locate the position of object we need a frame of reference. A convenient way to set up a frame of reference is to choose three mutually perpendicular axis and name them x-y-z axis. The coordinates (x, y, z) of the particle then specify the position of object w.r.t. that frame. If any one o more coordinates change with time, then we say that the object is moving w.r.t. this frame.

MOTIONS IN ONE, TWO AND THREE DIMENSIONS (TYPE OF MOTION)

As position of the object may change with time due to change in one or two or all the three coordinates, so we have classified motion as follows :

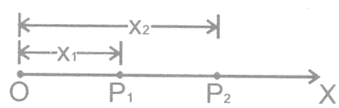
(a) Motion in 1-D:

If only one of the three co-ordinates specifying the position of object changes w.r.t. time. In such a case the object moves along a straight line and the motion therefore is also known as rectilinear or linear motion.

Eg. : (i) Motion of train along straight railway track.

(ii) An object falling freely under gravity.

(iii) When a particle moves from P_1 to P_2 along a straight line path only the x-co-ordinate changes.



(b) Motion in 2-D:

If two of the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called two dimensional. In such a motion the object moves in a plane.

Eg. : (i) Motion of queen on carom board.

(ii) An insect crawling on the floor of the room.

(iii) Motion of object in horizontal and vertical circles etc.

(iv) Motion of planets around the sun.

(v) A car moving along a zigzag path on a level road.

(c) Motion is 3-D:

If all the three co-ordinates specifying the position of object changes w.r.t. time, then the motion of object is called 3-D. In such a motion the object moves in a space.

Eg.: (i) A bird flying in the sky (also kite).

(ii) Random motion of gas molecules.

(iii) Motion of an aeroplane in space.

TYPES OF MOTION

(i) Linear motion (or translatory motion) : The motion of a moving car, a person running, a stone being dropped.

(ii) Rotational motion : The motion of an electric fan, motion of earth about its own axis.

(iii) Oscillatory motion : The motion of a simple pendulum, a body suspended from a spring (also called to and fro motion).

SCLALER AND VECTOR QUNTITY

Physically quantities (i.e. quantities of physics) can be divided into two types :

(i) Scalar quantity : Any physical quantity, which can be completely specified by its magnitude alone, is a scalar quantity or a scalar.

Eg.: Charge, distance, area, speed, time temperature, density, volume, work, power, energy, pressure, potential etc.

(ii) **Vector quantity :** Any physical quantity, which requires direction in addition to its magnitude is known as a vector.

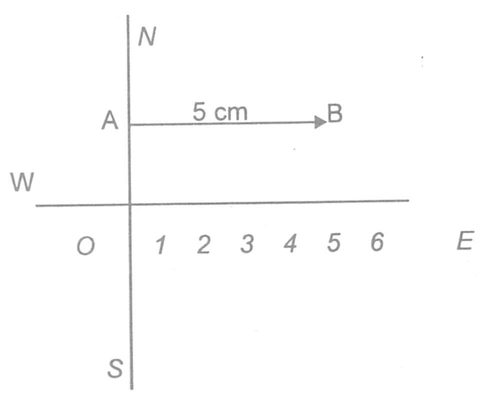
Eg. : Displacement, velocity, acceleration, force, momentum, weight and electric field etc.

(a) Representation of a vector :

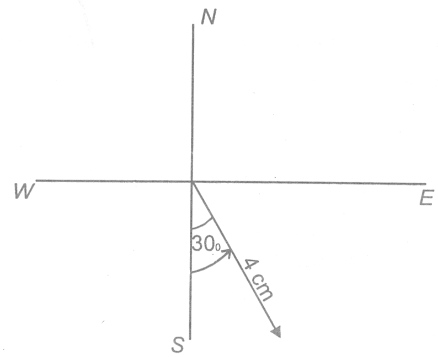
A vector is represented a directed line segment drawn in the given direction on a certain scale.

Tail ⟶ *head* (symbolic representation)

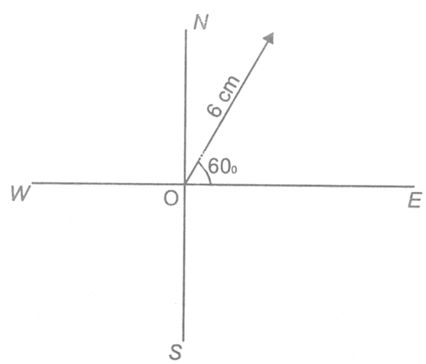
Eg.: To represent a displacement of 50 m towards east. Take 10 m = 1 cm (Scale)



Eg.: To represent a velocity of 20 k m/h towards 30^0 east of south. (Scale 5 km/h = 1 cm.)



Eg. : 6 m displacement, 60^0 north - east (north of east) (Scale 1 m = 1 cm)



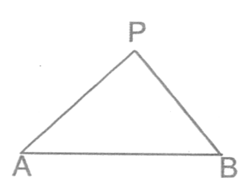
(b) Difference between Scalar and Vector :

Scalar	Vector
1. They have a magnitude only.	1. They have magnitude as well as direction.
2. They are added or subtracted arithmetically like 3 kg + 5 kg = 8 kg	2. They are added or subtracted by the process of vector addition.

DISTANCE AND DISPLACEMENT

(a) Distance :

Consider a body traveling from A to B along any path between A & B. The actual length of the path that a body travels between A and B is known as the distance. The distance traveled is different for different path between A and B. It is a scalar quantity. According to figure distance at path APB is AP + PB and at path AB is AB.



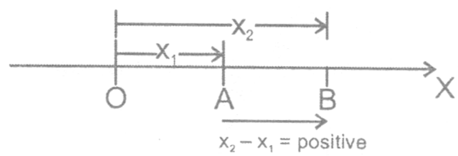
(b) Displacement :

The distance traveled in a given direction is the displacement. Thus displacement is the shortest distance between the given points. It is a vector quantity. S.I. unit of distance or displacement is metre.

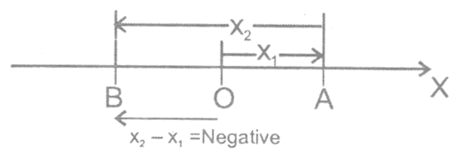
❖ **NOTE :** If a body travels in such a way that it comes back to its starting position, then the displacement is zero. However, distance traveled is never zero.

Eg. :

(i) When an object moves towards right from origin to in time t_1 to t_2, its displacement is positive.



(ii) When an object moves towards left in time t_1 to t_2, its displacement is negative.



(iii) When an object remains stationary or it moves first towards right and then an equal distance towards left, its displacement is zero.

(iv) Shifting origin causes no change in displacement.

(c) Difference between Distance and Displacement :

Distance	Displacement
1. Distance is the length of the path actually traveled by a body in any direction.	1. Displacement is the shortest distance between the initial and the final positions of a body in the direction of the point of the final position.
2. Distance between two given points depends upon the path chosen.	2. Displacement between two points is measured by the straight path between the points.
3. Distance is always positive.	3. Displacement may be positive as well as negative and even zero.
4. Distance is scalar quantity.	4. Displacement is a vector quantity
5. Distance will never decrease	5. Displacement may decrease.

EXERCISE

OBJECTIVE DPP - 1.1

1. A body whose position with respect to surrounding does not change, is said to be in a state of :

(A) Rest (B) Motion (C) Vibration (D) Oscillation

2. In case of a moving body :

(A) Displacement > Distance (B) Displacement < Distance

(C) Displacement $\geq$ Distance (D) Displacement $\leq$ Distance

3. Vector quantities are those which have :

(A) Only direction (B) Only Magnitude

(C) Magnitude and direction both (D) None of these

4. What is true about scalar quantities ?

(A) Scalars quantities have direction also. (B) Scalars can be added arithmetically.

(C) There are special law to add scalars. (D) Scalars have special method to represent.

5. A body is said to be in motion if :

(A) Its position with respect to surrounding objects remains same

(B) Its position with respect to surrounding objects keep on changing

(C) Both (A) and (B)

(D) Neither (A) nor (B)

6. A distance is always :

(A) shortest length between two points (B) path covered by an object between two points

(C) product of length and time (D) none of the above

7. A displacement :

(A) is always positive (B) is always negative

(C) may be positive as well as negative (D) is neither positive nor negative

8. Examples of vector quantities are :

(A) velocity, length and mass (B) speed, length and mass

(C) time, displacement and mass (D) velocity, displacement and force

9. Which of the following is not characteristic of displacement ?

(A) It is always positive.

(B) Is has both magnitude and direction.

(C) It can be zero.

(D) Its magnitude is less than or equal the actual path length of the object.

10. S.I. unit of displacement is :

(A) m (B) ms^{-1} (C) ms^{-2} (D) None of these

11. Which of the following is not a vector ?

(A) Speed (B) Velocity (C) Weight (D) Acceleration

12. Time is an example of :

(A) Scalar (B) Vector

(C) Scalar or vector (D) Neither scalar nor vector

13. In five minutes distance between a pole and a car changes progressively. What is true about the car ?

(A) Car is at rest (B) Car is in motion

(C) Nothing can be said with this information (D) None of the above

14. A distance :

(A) Is always positive (B) Is always negative

(C) May be positive as well as negative (D) Is neither positive nor negative

SUBJECTIVE DPP - 1.2

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1. Is absolute rest possible ?

2. Are distance and displacement equal in magnitude ?

3. Is distance a vector quantity ?

4. Define scalar quantity and give two examples.

5. Define rest and motion and give two examples of each.

6. A runner running along a circle, runs the circle completely. What is his displacement ? What distance has be run ?

7. Distinguish between rest and motion.

8. Write difference between distance and displacement.

9. Can a body be at rest and motion at the same time ? Explain.

10. When do we say that body is at rest and when do we say that it is moving ? Explain.

11. Give two examples to explain that motion is relative.

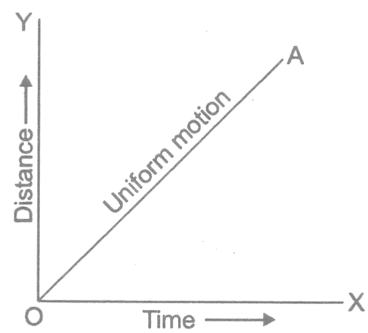
MOTION

PL - 2

UNIFORM AND NON UNOFORM MOTION

(a) Uniform Motion :

A body has a uniform motion if it travels equal distances in equal intervals of time, no matter how small these time intervals may be. For example, a car running at a constant speed of say, 10 meters per second, will cover equal distances of 10 metres every second, so its motion will be uniform. Please note that the distance-time graph for uniform motion is a straight line (as shown in the figure).



(b) Non-Uniform Motion :

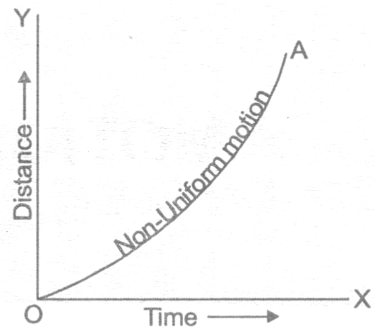
body has a non-uniform if it travels unequal distances in equal intervals of time. For example, if we drop a ball from the roof of a building, we will find that it covers unequal distances in equal intervals of time. It covers :

4.9 metres in the 1st second,

14.7 metres in the 2nd second,

24.5 metres in the 3rd second, and so on.

Thus, a freely falling ball covers smaller distance in the initial '1 second' interval and larger distance in the later '1 second' interval. From this discussion we conclude that the motion of a freely falling body is an example of non-uniform motion. The motions of a train starting from the railway station is also an example of non-uniform motion. This is because when the train starts from a s station, if moves a very small distance in the 'first' second. The train moves a little more distance in the '2nd' second and so on. And when the train approaches the next station, the distance traveled by it per second decreases.



Please note that the distance-time graph for a body having non-uniform motion is curved line (as shown in the figure). Thus, in order to find out whether a body has uniform motion or non-uniform motion, we should draw the distance-time graph for it. If the distance time graph is straight line, the motion will be uniform and if the distance -time graph is a curved line, the motion will be non-uniform. It should be noted that non-uniform motion is also called accelerated motion.

SPEED

The distance traveled by a body in unit time is called its peed. Therefore,

speed = $\frac{\text{Distance}}{\text{Time}}$ or s = $\frac{d}{t}$. S.I. unit of speed or average speed is m/sec. It is a scalar quantity,

(a) Average Speed :

For an object moving with variable speed, it is the total distance traveled by the object divided by the total time taken to cover that distance.

Average speed = $\frac{\text{total distance travelled}}{\text{total time taken}}$

(b) Uniform Speed (or Constant Speed):

When an object covers equal distance in equal intervals of time, it is said to move with uniform speed.

Eg. A car moves 10 m is every one second so it motion is uniform.

(c) Variable Speed (Non-Uniform Speed) :

If a body covers unequal distance in equal intervals of time, its motion is said to be non-uniform.

Eg. Falling of a apple from a tree, a cyclist moving on a rough road, an athlete running a race, vehicle starting from rest, the motion of freely falling body etc.

(d) Instantaneous Speed :

The speed of an object at any particular instant of time or at particular point of its path is called the instantaneous speed of the object. it is measure red by speedometer in an automobile.

VELOCITY

It is the rate of change of displacement.

Therefore, velocity = $\frac{\text{displacement}}{\text{time}}$ or it is the distance traveled in unit time in a given direction.

$$\text{velocity} = \frac{\text{distance travelled in a given direction}}{\text{time taken}}$$

S.I. unit of velocity is m/s. If is a vector quantity.

(Magnitude of the velocity is known as speed) 1 km/h = 5/18 m/s.

Speed	Velocity
1. It is a scalar quantity. 2. Speed = $\frac{\text{dis tan ce travelled}}{\text{time}}$ 3. It is rate of change of position of an object.	1. It is a vector quanity. 2. Velocity = $\frac{\text{displacement}}{\text{time}}$ 3. It is rate of change of position of an object in specific direction.

(a) Uniform Velocity (Constant Velocity) :

If a body covers equal distance in equal intervals of time in a given direction then it is said to be moving with constant velocity.

(b) Non-Uniform Velocity :

When a body does not cover not cover equal distances in equal intervals of time, in a given direction (in this case speed is not constant), then it is known as non uniform velocity. If speed is constant then also body can have a non-uniform velocity.

Eg : A car moving on a circular road with constant speed.

(c) Average Velocity :

If initial velocity of body is u and final velocity is v then the arithmetic means of velocity is called average velocity and is given as $v_{2v} = \frac{u+v}{2}$. Where, u = initial velocity and v = final velocity. Also for an object moving with variable velocity it is defined as the ratio of its total displacement to the total time interval in which the displacement occurs. Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$. If x_1 & x_2 are the positions of an object at times t_1 & t_2 then, $\vec{v}_{av} = \frac{\vec{x}_2 - \vec{x}_1}{\Delta t} = \frac{\vec{\Delta x}}{\Delta t}$ $\Delta t = t_2 - t_1$

(d) Instantaneous Velocity :

The velocity of an object at any given instant of time at particular point of its path is called its instantaneous velocity.

$$\vec{V} = _{\lim \Delta t \to 0} \frac{\Delta \vec{x}}{Xt} = \frac{d\vec{x}}{dt}$$

Ex. When is the average speed of an object equal to the magnitude of its average velocity ? Give reason also.

Sol. As average speed = $\frac{\text{total pathlength}}{\text{time int erval}}$ also, average velocity = $\frac{\text{Displacement}}{\text{time int erval}}$. When an object moves along a straight line and in the same direction its total path length is equal to the magnitude of its displacement. Hence average speed is equal to the magnitude of its average velocity.

FEATURE OF UNIFORM MOTION

(i) The velocity in uniform motion does not depend on the choice of origin.

(ii) The velocity in uniform motion does not depend on the choice of the time interval ($t_2 - t_1$).

(iii) For uniform motion along a straight line in the same direction, the magnitude of the displacement is equal to the actual distance covered by the object.

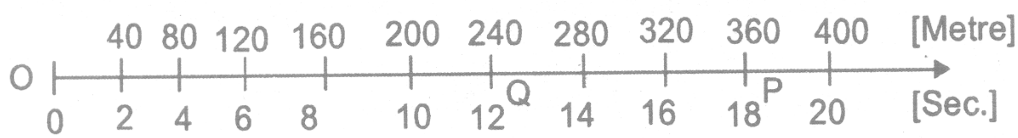
(iv) The velocity is positive if the object is moving towards the right of the origin and negative if the object is moving towards the left of the origin.

(v) For an object is uniform motion no force is required to maintain its motion.

(vi) In uniform motion, the instantaneous velocity is equal to the average velocity at all time because velocity remains constant at each instant, at each point of the path.

ILLUSTRATIONS

1. A car is moving along x-axis. As shown in figure it moves from O to P in 18 s and returns from P to Q in 6 second. What is the average velocity and average speed of the car in going from (i) O to P and (ii) from O to P and back to Q.



Sol. (i) Average velocity = $\dfrac{\text{path lenght}}{\text{time interval}} = \dfrac{360\text{m}}{18} = 20\,\text{ms}^{-1}$

Average speed = $\dfrac{\text{path length}}{\text{time interval}} = \dfrac{360\text{m}}{18} = 20\,\text{ms}^{-1}$

(ii) From O to P and back to Q

Average velocity = $\dfrac{OQ}{18+6} = \dfrac{240\text{m}}{24} = 10\,\text{ms}^{-1}$

Average speed = $\dfrac{\text{path length}}{\text{time interval}} = \dfrac{OP+PQ}{18+6} = \dfrac{360+120}{24} = 20\,\text{ms}^{-1}$

2. A car covers the 1st half of the distance between two places at a speed of 40 km h^{-1} and the 2nd half at 60 km h^{-1}. What is the average speed of the car ?

Sol. Suppose the total distance covered is 2S.

Then time taken to cover first distance with speed 40 km/h,

$t_1 = \dfrac{S}{40}\,h$

Time taken to cover second S distance with speed 60 km/h,

$$t_2 = \frac{S}{60} h$$

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{S+S}{\left(\frac{S}{40}+\frac{S}{60}\right)}$$

$$V_{av} = \frac{2S}{\left(\frac{3S+2S}{120}\right)} = \frac{2S}{5S}\times 120$$

$$\Rightarrow \quad V_{av} = 48 km/h$$

3. A non-stop bus goes from one station to another station with a speed of 54 km/h, the same bus returns from the second station to the first station with a speed of 36 km/h. Find the average speed of the bus for the entire journey.

Sol. Suppose the distance between the stations is S. Time taken in reaching from one station to another station.

$$t_1 = \frac{S}{54} h$$

Time taken in returning back,

$$t_2 = \frac{S}{36} h$$

Total $t = t_1 + t_2$

$$t = \frac{S}{54}+\frac{S}{36} = \frac{2S+3S}{108} = \frac{5S}{108} h$$

Average speed $V_{av} = \frac{\text{Total distance}}{\text{Total time}}$

$$V_{av} = \frac{2S}{5S}\times 108$$

$$V_{av} = \frac{216}{5} = 43.2 \, km/h$$

EXERCISE

OBJECTIVE DPP - 2.1

1. When a body covers equal distance in equal intervals of time, its motion is said to be :

(A) Non-uniform (B) Uniform (C) Accelerated (D) Back and forth

2. The motion along a straight line is called :

(A) Vibratory (B) Stationary (C) Circular (D) Linear

3. A particle is traveling with a constant speed. This means :

(A) Its position remains constant as time passes.

(B) It covers equal distance in equal interval of time

(C) Its acceleration is zero

(D) It does not change its direction of motion

4. The rate of change of displacement is :

(A) Speed (B) Velocity (C) Acceleration (D) Retardation

5. Speed is never :

(A) zero (B) Fraction (C) Negative (D) Positive

6. The motion of a body covering different distances in same intervals of time is said to be :

(A) Zig - Zag (B) Fast (C) Slow (D) Variable

7. Unit of velocity is :

(A) ms (B) ms^{-1} (C) ms^2 (D) none of these

8. A speed :

(A) is always positive (B) is always negative

(C) may be positive as well as negative (D) is neither zero nor negative

9. A particle moves with a uniform velocity :

(A) The particle must be at rest (B) The particle moves along a curved path

(C) The particle moves along a circle (D) The particle moves along a straight line

10. A quantity has value of $-6.0\ ms^{-1}$. It may be the :

(A) Speed of a particle (B) Velocity of a particle

(C) Position of a particle (D) Displacement of a particle

11. In 10 minutes, a car with speed of $60\ kmh^{-1}$ travels a distance of :

(A) 6 km (B) 600 km (C) 10 km (D) 7 km

12. A particle covers equal distances in equal intervals of times, it is said to be moving with uniform :

(A) Speed (B) Velocity (C) Acceleration (D) Retardation

13. The SI unit of the average velocity is :

(A) m/s (B) km/s (C) cm/s (D) mm/s

14. Mere per second is not the unit of :

(A) Speed (B) Velocity (C) Displacement (D) None of them

SUBJECTIVE DPP - 2.2

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1. What is the S.I. unit of velocity ?

2. Which is vector, speed or velocity ?

3. Give the name of the physical quantity that corresponds to the rate of change of displacement ?

4. Apart from velocity name two other quantities which are vector ?

5. When is a body said to have uniform velocity ?

6. A particle is moving with uniform velocity. it is necessary moving with uniform speed ? Is it necessary that it is moving along a straight line ?

7. Write difference between sped and velocity.

8. A train covers 80 km in 2 hours. Find its average speed in kmh^{-1}, $m\ min^{-1}$ and ms^{-1}.

9. Which one of the following have maximum and the least average speed ?

(i) Sanjeev moving with 12 kmh^{-1}

(ii) Rajeev running with 5 ms^{-1}

(iii) Kabir moving with 150 $m\ min^{-1}$.

10. (a) Uniform motion (b) Non uniform motion

11. (a) Average speed (b) Velocity

MOTION

PL - 3

ACCELERATION

Mostly the velocity of a moving object changes either in magnitude or in direction or in both when the object moves. The body is then said to have acceleration. So it is the rate of change of velocity i.e. change in velocity in unit time to the acceleration (it is a vector quantity). Its S.I. unit is m/sec^2 and c.g.s unit is c $m/sec.^2$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}$$

(a) Uniform Acceleration (Uniformly Accelerated Motion):

If a body travels in a straight line and its velocity increases in equal amounts in equal intervals of time. Its motion is known as uniformly accelerated motion.

Eg.1 Motion of a freely falling body is an example of uniformly accelerated motion (or motion of a body under the gravitational pull of the earth).

Eg.2 Motion of a bicycle going down the slope of a road when the rider is not pedaling and wind resistance is negligible.

(b) None-Uniform Acceleration :

If during motion of a body its velocity increases by unequal amounts in equal intervals of time, then its motion is known as non uniform accelerated motion.

Eg.1 Car moving in a crowded street.

Eg.2 Motion of a train leaving or entering the platform.

TYPES OF ACCELERATIO

(i) Positive acceleration : If the velocity of an object increases in the same direction, the object has a positive acceleration.

(ii) Negative acceleration (retardation): If the velocity of a body decreases in the same direction, the body has negative acceleration or it is said to be retarding.

Eg. A train slows down.

EQUATIONS OF UNIFORMLY ACCELERATION MOTION

(a) 1st Equation of Motion :

Consider a body having initial velocity 'u'. Suppose it is subjected to a uniform acceleration 'a' so that after time 't' its final velocity becomes 'v'. Now we now,

$$\text{Acceleration} = \frac{\text{change in velcity}}{\text{time}}$$

$$a = \frac{v-u}{t}$$

or $v = u + at$ or $v = at + u$ .....(i)

(b) 2nd Equation of Motion :

Suppose a body has an initial velocity 'u' and uniform acceleration 'a' for time 't' so that its final velocity becomes 'v'. The distance traveled by moving body in time 't' is 's' then the average velocity = (v + u)/2. Distance traveled = Average velocity × time

$$s = \left(\frac{u+v}{2}\right)t \quad \Rightarrow \quad s = \left(\frac{u+u+at}{2}\right)t \qquad (\text{as } u = v + at)$$

$$s = \left(\frac{2u+at}{2}\right)t \quad \Rightarrow \quad s = \frac{2ut+at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$

(c) 3rd Equation of Motion

Distance traveled = Average velocity × time

$$s = \left(\frac{u+v}{2}\right)t \qquad ......(ii)$$

from equation (i) $t = \frac{v-u}{a}$

Substituting the value of t in equation (iii), we get $s = \left(\frac{v-u}{a}\right)\left(\frac{v+u}{2}\right)$

$$s = \left(\frac{v^2-u^2}{2a}\right) \quad \Rightarrow \quad 2as = v^2 - u^2 \text{ or } v^2 = u^2 + 2as \qquad ....(iv)$$

(d) Distance covered in nth second :

$S = ut + \frac{1}{2}at^2$ is the distance covered by a body in t s.

$S_n - un + \frac{1}{2}an^2$ ....(v) [distance covered by a body along a straight line in n second.

$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$ ....(vi) [distance covered by a body along a straight line in (n-1) sec.]

$\therefore$ The distance covered by the body in n^{th} second will be -

$S_{nth} = S_n - S_{n-1}$

$\therefore \quad S_{nth} = un + \frac{1}{2}an^2 - \{u(n-1) + \frac{1}{2}a(n-1)^2\}$

$S_{nth} = un + \frac{1}{2}an^2 - \{nu - u + \frac{1}{2}a(n^2 + 1 - 2n)\}$

$S_{nth} = un + \frac{1}{2}an^2 - \{un - u + \frac{an^2}{2} + \frac{a}{2} - an\}$

$S_{nth} = un + \frac{1}{2}an^2 - un + u - \frac{an^2}{2} - \frac{a}{2} + an$

$S_{nth} = u + a\left(n - \frac{1}{2}\right)$

$S_{nth} = u + a\left(\frac{2n-1}{2}\right)$

$S_{nth} = u + \frac{a}{2}(2n-1)$ ....(vii)

TO SOLVE NUMERIAL PROBLEMS

(i) If a body is dropped from a height then its initial velocity u = 0 but has acceleration (acting). If a body starts from rest its initial velocity u = 0.

(ii) If a body comes to rest, its final velocity v = 0 or, if a body reached the highest point after being thrown upwards its final velocity v = 0 but has acceleration (acting).

(iii) if a body moves with uniform velocity, its acceleration is zero i.e. a = 0.

(iv) Motion of body is called free fall if only force acting on it is gravity (i.e. earth's attraction).

MOTION UNDER GRAVITY (UNIFORM ACCELERATED MOTION)

The acceleration with which a body travels under gravity is called acceleration due to gravity 'g'. Its value is 9.8 m/s^2 (or $\approx$ 10 m/s^2). If you have to take g = 10 m/s^2 then it must be mentioned in the question otherwise take g = 9.8 m/s^2.

(i) If a body moves upwards (or thrown up) g is taken negative (i.e. motion is against gravitation of earth). So we can form the equation of motion like.

$v = u - gt$, $s = ut - \frac{1}{2}gt^2$, $v^2 - u^2 = -2gh$.

(ii) If a body travels downwards (towards earth) then g is taken + ve. So equations of motion becomes $v = u + gt$, $s = ut + \frac{1}{2}gt^2$, $v^2 - u^2 = 2gh$.

(iii) if a body is projected vertically upwards with certain velocity then it returns to the same point of projection with the same velocity in the opposite direction.

(iv) The time for upward motion is the same as for the downward motion.

ILLUSTRATION

1. A car is moving at a speed of 50 km/h. Two seconds there after it is moving at 60 km/h. Calculate the acceleration of the car.

Sol. Here u = 50 km/h = $50 \times \frac{5}{18}$ m/s = $\frac{250}{18}$ m/s

and v = 60 km/h = $60 \times \frac{5}{18} = \frac{300}{18}$ m/s

Since a $= \frac{v-u}{t} = \frac{\frac{300}{18} - \frac{250}{18}}{2} = \frac{\frac{50}{18}}{2} = \frac{50}{36} = 1.39\,m/s^2$

2. A car attains 54 km/h in 20 s after it starts. Find the acceleration of the car.

Sol. u = 0 (as car starts from rest)

v = 54 km/h = $54 \times \frac{5}{18}$ = 15 m/s

As, $a = \frac{v-u}{t} \therefore a = \frac{15-0}{20} = 0.75\,m/s^2$

3. A ball is thrown vertically upwards with a velocity of 20 m/s. How high did the ball go ? (take g = 9.8 m/s^2).

Sol. u = 20 m/s, a = - g = - 9.8 m/s^2 (moving against gravity)

s = ? v = 0 (at highest point)

$v^2 - u^2 = 2as$

$(o)^2 - (20)^2 = 2(-g)\ s$

- 400 = 2 (-9.8) s

- 400 = - 19.6 s

$\frac{400}{19.6} = s \Rightarrow s = 20.4$ m.

EXERCISE

OBJECTIVE DPP 3.1

1. A car accelerated uniformly from 18 km/h to 36 km/h in 5 s. The accelerating is ms^{-2} is :

(A) 1 (B) 2 (C) 3 (D) 4

2. Out of energy and acceleration which is vector ?

(A) Acceleration (B) Energy (C) Both (D) None of these

3. C.G.S. unit of acceleration is :

(A) ms^{-2} (B) $cm\ s^{-2}$ (C) ms^2 (D) $cm\ s^2$

4. A train starting from a railway station and moving with inform acceleration, attains a speed of 40 kmh^{-1} in 10 minutes, Is acceleration is :

(A) 18.5 ms^{-2} (B) 1.85 $cm\ s^{-2}$ (C) 18.5 cms^{-2} (D) 1.85 $m\ s^{-2}$

5. The brakes applied to a cap produce a negative acceleration of 6ms^{-2}. If the car stops after 2 seconds, the initial velocity of the car is :

(A) 6 ms^{-1} (B) 12 ms^{-1} (C) 24 ms^{-1} (D) zero

6. A body is moving with uniform velocity of 10 ms^{-1}. The velocity of the body after 10 s is :

(A) 100 ms^{-1} (B) 50 ms^{-1} (C) 10 ms^{-1} (D) 5 ms^{-1}

7. In 12 minutes a car whose speed is 35 kmh^{-1} travels of distance of :

(A) 7 km (B) 3.5 km (C) 14 km (D) 28 km

8. A body is moving along a straight line at 20 ms^{-1} undergoes an acceleration of 4 ms^{-2}. After 2 s, its speed will be:

(A) 8 ms^{-2} (B) 12 ms^{-1} (C) 16 ms^{-2} (D) 28 ms^{-2}

9. A car increase its speed from 20 kmh^{-1} to 50 kmh^{-1} is 10 sec., its acceleration is :

(A) 30 ms^{-1} (B) 3 ms^{-1} (C) 18 ms^{-1} (D) 0.83 ms^{-1}

10. When the distance travelled by an object is directly proportional to the time, it is said to travel with :

(A) zero velocity (B) constant speed (C) constant acceleration (D) uniform velocity

11. A body freely failing from rest has a velocity V after it falls through a height h. The distance it has to fall further for its velocity to be come double is :

(A) 3 h (B) 6 h (C) 8 h (D) 10 h

12. The velocity of bullet is reduced from 200m/s to 100 m/s while traveling through a wooden block of thickness 10 cm. The retardation, assuming it to be uniform will be :

(A) 10×10^4 m/s^2 (B) 1.2×10^4 m/s^2 (C) 13.5×10^4 m/s^2 (D) 15×10^4 m/s^2

13. A body starts falling from height 'h' and travels distance h/2 during the last second of motion. The find of travel (in sec.) is :

(A) $\sqrt{2}-1$ (B) $2+\sqrt{2}$ (C) $\sqrt{2}+\sqrt{3}$ (D) $\sqrt{3}+2$

SUBJECTIVE DPP - 3.2

1. Find the formula for the distance covered by a body in n^{th} s.
2. How is the position of a moving particle along a straight line described by a number ? How is the direction of motion specified by the number describing position ?
3. A ball is thrown vertically upward from the ground with a velocity 39.2 ms^{-1}. Calculate :

   (i) the maximum height to which the ball rises and

   (ii) the time taken by the ball to reach the highest point.
4. A body standing near the edge of a cliff 125 m above a river throws a stone downward with a speed of 10 ms^{-1} Find :

   (i) with what speed will the stone hit water and

   (ii) how long will it take to descend ?
5. A stone is dropped from the top of a building 200 m high and at the same time another stone is projected vertically upward from the ground with a velocity of 50 ms^{-1}. Find where and when the two stone will meet.
6. A ball thrown vertically upward reached a height of 80 m. Calculate :

   (i) the time to reach the highest point

   (ii) the spend of the ball upon arrival on the ground.

MOTION

PL - 4

DISTANCE (DISPLACEMENT) FROM SPEED (VELOCITY) TIME GRAPH

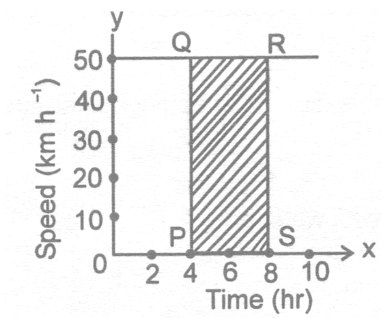
A distance (displacement = speed (velocity) x time, so the distance (displacement) can be calculated (computed) with speed (velocity) - time graph.

Case (i) : When speed (velocity) is uniform (constant):

Figure shows the speed - time graph of a car (taxi) moving with a uniform speed of 50 km h^{-1}. It is a straight line parallel to X - axis (time axis). Distance covered by this taxi from time t_1 = 4h at P to time t_2 = 8 h at S, is given by distance = 50 × (t_2 - t_1)

= 50 (8 - 4)

= 50 × 4 = 200 km

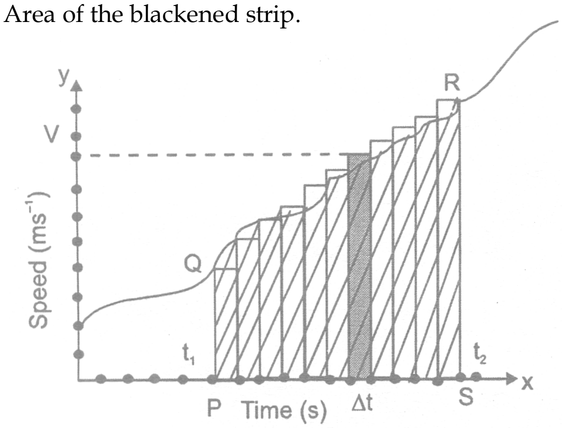


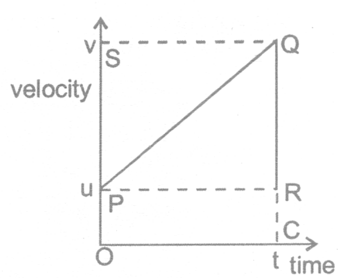
In figure, PQ = 50, SP = (t_1 - t_1)

Hence distance = PQ × SP = Area of rectangle PSRQ

Case (ii) : When speed (velocity) as well as acceleration is non-uniform (variable)

Figure shows the speed- time graph of a body moving with variable speed and acceleration. Over a small interval of time Δt , the speed can be taken as constant. For this small time interval, distance $\Delta S = v\Delta t$ = Area of the blackened strip.





For whole time-interval between t_1 and t_2

distance = sum of area of all the strips between t_1 and t_2 = Area of shaded figure PQRS.

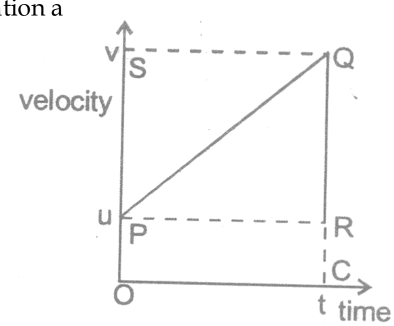
GRAPHICAL DERIVATION OF EQUATIONS OF MOTION

(a) First Equations :

v = u + at

It can be derived from v - t graph, as shown is figure

From line PQ, the slope of the line = acceleration a



$$a = \frac{QR}{RP} = \frac{SP}{RP}$$

$\therefore$ SP = v - u

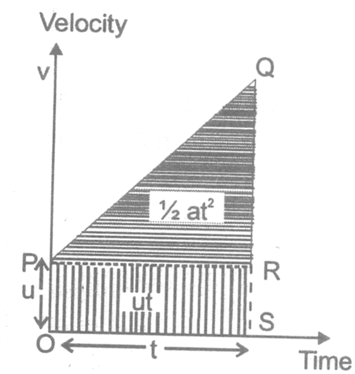
So $a = \frac{v - u}{t}$

or $v \ u + at$

(b) Second Equation :

$$s = ut + \frac{1}{2}at^2$$

It can also be derived from v - t graph as shown in figure.



From relation,

Distance covered = Area under v - graph

s = Area of trapezium OPQS

= Area of rectangle OPRS + Area of triangle PQR

$= OP \times PR + \frac{RQ \times PR}{2}$

Putting values,

$$S = u \times t + \frac{1}{2}(v - u) \times t$$

$(\therefore RQ = v - u \,\&\, PR = OS = t)$

$$= u \times t + \frac{1}{2}at \times t$$

$(\therefore v - u = at)$

Pr $s = ut + \frac{1}{2}at^2$

(c) Third Equation :

$v^2 = u^2 + 2as$

From above graph OP = um SQ = v, OP + SQ = u + v

$a = \frac{QR}{PR}$ or $PR = \frac{QR}{a} = \frac{v - u}{a}$

S = Area of trapezium OPQS $= \frac{OP + SQ}{2} \times PR$

On putting the values,

$S = \frac{u + v}{2} \times \frac{v - u}{a} = \frac{v^2 - u^2}{2a}$ or $v^2 = u^2 + 2as$

EXERCISE

OBJECTIVE DPP 4.1

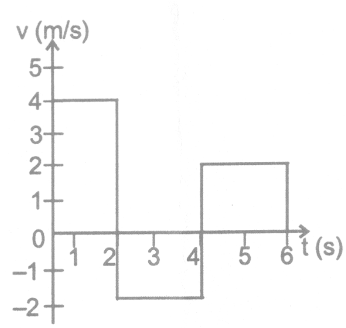
1. Area between speed - time graph and time axis gives :

(A) Distance (B) Velocity (C) Speed (D) None of these

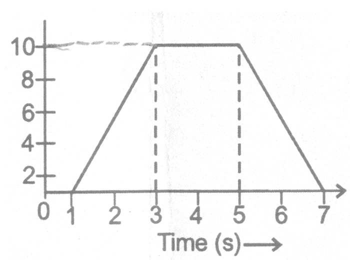
2. An object undergoes an acceleration of 8 ms^{-2} starting from rest. Distance traveled is 1 s is :

(A) 2 m (B) 4m (C) 6m (D) 8 m

3. The velocity-time graph of a body moving in a straight line is shown in figure. The displacement and distance travelled by the body is 6 seconds are respectively.

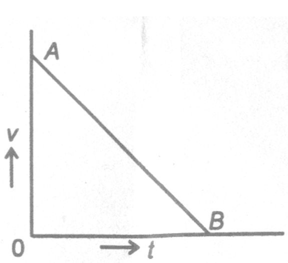


4. For the velocity time graph shown in figure, the distance covered by the body in the last two seconds of its motion is what fraction is of the total distance covered in all the seven seconds ?



(A) 1/2 (B) 1/4 (C) 1/3 (D) 2/3

5. Velocity-time graph AB (Figure) shows that the body has :



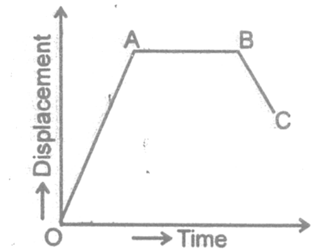
(A) A uniform acceleration

(B) A non-uniform retardation

(C) Uniform speed

(D) Initial velocity O*A* and is moving with uniform retardation

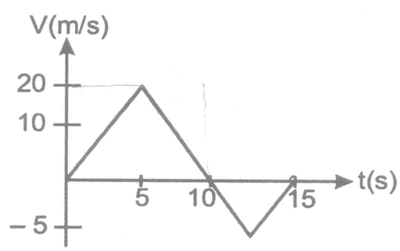
6. In figure BC represents a body moving :
(A) Backward with uniform velocity
(B) Forward with uniform velocity
(C) Backward with non-uniform velocity
(D) Forward with non-uniform velocity



7. Speedometer measures .... speeds.

SUBJECTIVE DPP - 4.2

1. A stone is thrown vertically upward which takes time 't' to reach to maximum height 'h'. After next 't' seconds it reached the ground from the maximum height. Draw (i) distance-time graph and (ii) displacement time graph for the motion of the stone.
2. Draw V-t graphs in the following cases : (i) uniform retardation (ii) non uniform acceleration
3. From the following (V-t) graph find :



(i) Distance and displacement in 10 second.
(ii) Distance and displacement in 15 second.

MOTION

PL - 5

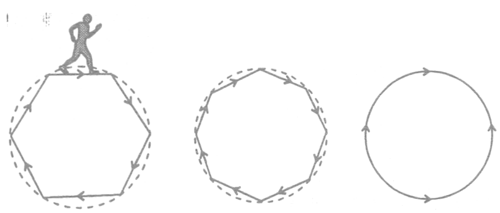
CIRCULAR MOTION

(a) Definition :

Motion of a particle (small body) along a circle (circular path), is called a circular motion. If the body covers equal distances along the circumference of the circle in equal intervals of time, the motion is said to be a uniform circular motion. A uniform circular motion is a motion in which speed remains constant but direction of velocity changes.

(b) Explanation :

Consider a boy running along a regular hexagonal track (path) as shown in figure. As the boy runs along the side of the hexagon at a uniform speed, he has to take turn at each corner changing direction but keeping the sped same. In one round he has to take six turns at regular intervals. If the same boy runs along the side of a regular octagonal track with same uniform speed, he will have to take eight turns in one round at regular intervals but the interval will become smaller.



By increasing the number of sides of the regular polygon, we find the number of turns per round becomes more and the interval between two turns become still shorter. A circle is a limiting case of polygon with an infinite number of sides. On the circular track, the turning becomes a continuous process without any gap in between. The boy running along the sides of such a track will be performing a circular motion. Hence, circular motion is the motion of a body along the sides of polygon of infinite number of sides with uniform speed, the direction changing continuously.

Eg. Example of uniform circular motion are :

(i) Motion of moon around the earth.

(ii) Motion of satellite around its planet.

(c) Nature of Circular of Motion :

Circular motion is an acceleration motion. Since, in a circular motion, velocity changes though in direction only, the motion is said to be accelerated.

DIFFERENCE BETWEEN UNIFORM LINEAR MOTION AND A UNIFOR CIRCULAR MOTION

Uniform linear motion	Uniform circular motion
1. The direction of motion does not changes	1. The direction of motion changes continuously.
2. The motion is non-accelerated.	2. The motion is accelerated.

RADIAN - (A UNIT FOR PLANE ANGLE)

It is a convenient unit for measuring angle in physics.

(a) Definition :

One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to its radius.

Eg. In figure, the arc AB of the circle has length ℓ and subtends an angle θ at the centre C.

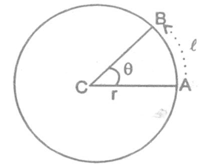
If $\angle ACB = \theta$ radians.

Then, $\theta = \frac{\ell}{r}$ radians.

[For $\ell = 1$, $\theta = 1$ radian]

Angle subtended by the circumference at the centre,

$\theta = \frac{2\pi r}{r} = 2\pi$ radians $\{$or $2\pi^c\}$



[^{c}] is symbol for radian, just as (^{0}) is symbol for degree.

Relation

For complete circle at centre

$2\pi^c = 360^0$

Or $1^c = \left|\frac{360}{2\pi}\right| = 57.3^0$

ANGULAR DISPLACEMENT AND ANGULAR VELOCITY

(a) Definitions :

(i) Angular displacement : In a circular motion, the angular displacement of a body is the angle subtended by the body at the centre in a given interval of time. It is represented by the symbol θ (theta).

(ii) Angular velocity : The angular displacement per unit time is called the angular velocity. it is represented by the symbol ω (omega).

Eg. Let a body move along a circle of radius r and perform a uniform circular motion. Let the body be at point P to start with and reach point Q after time t. Then, angular displacement = $\angle PCQ = \theta$ and angular velocity

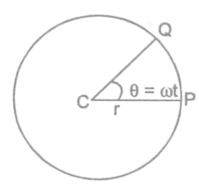
$= \omega = \frac{\theta}{t}$ (i.e. $\theta = \omega t$)

If the time period of the body is T (time taken in one complete round), the angular displacement = $2\pi^c$

Hence $\omega = \frac{2\pi}{T}$

But $\frac{1}{T}$ = N (frequency)

There $\omega = 2\pi N$



(b) Units for θ and ω :

The unit for angular displacement is radian (a supplementary quantity). The radian is defined at the angle subtended at the centre of a circle by an arc equal in length to its radius. The unit from angular velocity radian per second (rad/s).

(c) Relation between Linear and Angular Quantities :

For an arc of length ℓ

Linear displacement = ℓ

Angular displacement , $\theta = \frac{\ell}{r}$

Hence,

For a time interval t,

Linear velocity, $v = \frac{\ell}{t}$

Angular velocity $\omega = \frac{\theta}{t} = \frac{\ell}{rt} = \frac{v}{r}$

Hence $v = r\omega$

EXERCISE

OBJECTIVE DPP 5.1

1. 1^c is equal to :

(A) 57.3^0 (B) 573^0 (C) 180^0 (D) 360^0

2. An athlete complete one round of a circular track of diameter 200 m in 40 s. What will be the displacement at the end of 2 minutes 40 s.?

(A) 2200 m (B) 220 m (C) 22 m (D) Zero

3. What will be the distance in the above equation ?

(A) 2512 m (B) 2500 m (C) 2200 m (D) Zero

4. The distance traveled by a body is directly proportional to the time, then the body is said to have :

(A) Zero speed (B) Zero velocity (C) Constant speed (D) None of these

5. An athlete runs along a circular track of diameter 28m. The displacement of the athlete after he completes one circle is :

(A) 28 m (B) 88 m (C) 44 m (D) Zero

6. A boy is running along a circular track of radius 7 m. He completes one circle in 10 second. The average velocity of the boy is :

(A) 4.4 m^{-1} (B) 0.7 ms^{-1} (C) Zero (D) 70 ms^{-1}

7. A body is moving with a uniform speed of 5 ms^{-1} in a circular path of radius 5 m. The acceleration of the body is :

(A) 25 ms^{-2} (B) 15 ms^{-2} (C) 5 ms^{-2} (D) 1 ms^{-2}

8. Unit of angular velocity is :

(A) red (B) m/s (C) rad/s^2 (D) rad/s

9. The bodies in circular paths of radii 1 : 2 take same time to compete their circles. The ratio of their linear speeds is :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1

10. In a circular path of radius 1m, a mass of 2kg moves with a constant speed 10 ms^{-1}. The angular speed in radian/sec. is :

(A) 5 (B) 10 (C) 15 (D) 20

11. The relation among v, ω and r is :

(A) $\omega = \frac{v}{r}$ (B) $v = \frac{\omega}{r}$ (C) $\omega = \frac{r}{v}$ (D) None of these

12. Uniform circular motion is an example of :

(A) Variable acceleration (B) Constant acceleration

(C) A and B both (D) None of these

13. Rate of change of angular velocity refer to :

(A) angular speed (B) angular displacement

(C) angular acceleration (D) None of these

14. A car travels $\left(\frac{1}{4}\right)^{th}$ of a circle with radius r. The ratio of the distance to its displacement is :

(A) $1; \frac{\pi}{2\sqrt{2}}$ (B) $\frac{\pi}{2\sqrt{2}} : 1$ (C) $2\sqrt{2} : \pi$ (D) $\pi 2\sqrt{2} : 1$

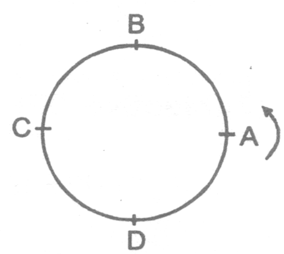
SUBJECTIVE DPP 5.2

1. The wheel of a cycle of radius 50 cm is moving with a speed 14 ms^{-1}. Calculate the angular velocity of the wheel.

2. An air craft completes a horizontal loop of radius 1 km with a uniform speed of 900 kmh^{-1}. Find the angular velocity of the air craft.

3. A artificial satellite takes 90 minutes to complete its revolution around the earth. Calculate the angular velocity of the satellite.

4. A particle moves along a circle of radius R as shown in figure. It starts from A and moves in anticlock-wise direction.



Calculate the distance traveled and displacement :

(i) From A to B (ii) From A to C (iii) From A to D

5. Name a physical quantity that (i) varies (ii) remains same in a circular motion.

6. Define angular speed write its S.I. unit.

7. Define the time period and find the relation between v and ω.

ANSWER KEY

(Objective DPP # 1.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	D	C	B	B	B	C	D	A	A	A	A	B	A

(Objective DPP # 2.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	B	D	B	B	C	D	B	A	D	B	C	A	A	C

(Subjective DPP # 2.2)

8. 40 kmh^{-1}, 666.7 m min^{-1}, 11.1 ms^{-1}

(Objective DPP # 3.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13
Ans.	A	A	B	B	B	C	A	D	D	B	A	D	B

(Subjective DPP # 3.2)

3. (i) 78.4 m (ii) 4 s **4.** (i) 5.5 ms^{-1} (ii) 4.13 s

5. After 4 second, it will be at a height of 121.6 m from the ground.

6. (i) 4.04s (ii) 39.59 ms^{-1}

(Objective DPP # 4.1)

Qus.	1	2	3	4	5	6
Ans.	A	B	A	B	D	A

7. **Instantaneous speed**

(Subjective DPP # 4.2)

3. (i) 100 m, 100 m (ii) 112.5 m, 87.5 m

(Objective DPP # 5.1)

Qus.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	A	D	A	C	D	C	C	D	A	B	A	B	C	B

(Subjective Dpp # 5.2)

1. 28 rad/s **2.** 0.25 rad/s **3.** $\frac{\pi}{2700}$ rad/d