DPP - Daily Practice Problems

Date : ______ Start Time : ______ End Time : ______

MATHEMATICS CM03

SYLLABUS : Trigonometric Functions

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. If $y = \cos^2 x + \sec^2 x$, then

(a) $y \le 2$ (b) $y \le 1$

(c) $y \ge 2$ (d) $1 < y < 2$

2. Period of $\dfrac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta}$ is

(a) 2π (b) π

(c) $\dfrac{2\pi}{3}$ (d) $\dfrac{\pi}{3}$

3. If an angle θ is divided into 2 parts A and B such that $A - B = k$ and $A + B = \theta$ and $\tan A : \tan B = k : 1$, then the value of $\sin k$ is :

(a) $\dfrac{k+1}{k-1}\sin\theta$ (b) $\dfrac{k}{k+1}\sin\theta$

(c) $\dfrac{k-1}{k+1}\sin\theta$ (d) None of these

4. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then $x^2 + 4y^2 =$

(a) 4 (b) -4

(c) ± 4 (d) None of these

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. The equation $\sin^4 x + \cos^4 x = a$ has a solution for

(a) all of values of a (b) $a = -1$

(c) $a = -\frac{1}{2}$ (d) $\frac{1}{2} \le a \le 1$

6. If for $n \in N$, $f_n(\theta) = \tan \theta/2\,(1 + \sec \theta)\,(1 + \sec 2\theta)\,(1 + \sec 4\theta)\ \ldots\ (1 + \sec 2^n\theta)$, then correct statement is

(a) $f_2(\pi/16) = 1$ (b) $f_3(\pi/32) = 1$

(c) $f_4(\pi/64) = 1$ (d) All of these

7. The expression $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(a) $\cos 2x$ (b) $2\cos x$

(c) $\cos^2 x$ (d) $1 + \cos x$.

8. If $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\sin(\alpha + \beta + \gamma)}{\sin\alpha + \sin\beta + \sin\gamma}$ is

(a) < 1 (b) > 1

(c) $= 1$ (d) None of these

9. The value of

$\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right)$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$

(c) $\frac{1}{32}$ (d) None of these

10. If $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$, then $\cos A + \sqrt{6}\sin A$ is equal to

(a) $\sqrt{6}\sin A$ (b) $\sqrt{7}\sin A$

(c) $\sqrt{6}\cos A$ (d) $\sqrt{7}\cos A$

11. General solution of the equation $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$ is

(a) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$ (b) $n\pi + (-1)^n \frac{\pi}{2}$

(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$ (d) None

12. The least positive non-integral solution of the equation $\sin \pi(x^2 + x) = \sin \pi x^2$ is

(a) rational

(b) irrational of the form $\sqrt{p}$

(c) irrational of the form $\frac{\sqrt{p} - 1}{4}$, where p is an odd integer

(d) irrational of the form $\frac{\sqrt{p} + 1}{4}$, where p is an even integer

13. If A and B are positive acute angles satisfying $3\cos^2 A + 2\cos^2 B = 4$ and $\frac{3\sin A}{\sin B} = \frac{2\cos B}{\cos A}$,

Then the value of $A + 2B$ is equal to :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

14. The greatest and least value of sin x cos x are

(a) $1, -1$ (b) $\frac{1}{2}, -\frac{1}{2}$

(c) $\frac{1}{4}, -\frac{1}{4}$ (d) $2, -2$

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. If tan (cot x) = cot (tan x), then

(a) $\sin 2x = \frac{2}{(2n+1)\pi}$ (b) $\sin x = \frac{4}{(2n+1)\pi}$

(c) $\sin 2x = \frac{4}{(2n+1)\pi}$ (d) None of these

16. $\sin\theta = \frac{1}{2}\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)$ necessarily implies :

(a) $x > y$ (b) $x < y$

(c) $x = y$

(d) both x and y are purely imaginary

17. If $p_n = \cos^n\theta + \sin^n\theta$, then $p_n - p_{n-2} = kp_{n-4}$, where:

(a) $k = 1$ (b) $k = -\sin^2\theta\cos^2\theta$

(c) $k = \sin^2\theta$ (d) $k = \cos^2\theta$

18. If $f(x) = \cos(\log x)$ then

$f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\}$ is equal to :

(a) 0 (b) 1

(c) –1 (d) none of these

19. **Statement-1 :** The maximum and minimum values of the function

$f(x) = \frac{1}{6\sin x - 8\cos x + 5}$ does not exist

Statement-2 : The given function is an unbounded function.

(a) Statement - 1 is false, Statement-2 is true

(b) Statement - 1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1

(c) Statement - 1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement-1

(d) Statement - 1 is true, Statement-2 is false

20. If θ is an angle given by $\cos\theta = \frac{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}{\sin^2\alpha + \sin^2\beta + \sin^2\gamma}$

where α, β, γ are the equal angles made by a line with the positive directions of the axes, then the measure of θ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

21. sin 12° sin 24° sin 48° sin 84° =

(a) cos 20° cos 40° cos 60° cos 80°

(b) sin 20° sin 40° sin 60° sin 80°

(c) $\frac{3}{15}$

(d) None of these

22. If $S_n = \cos^n\theta + \sin^n\theta$ then the value of $3S_4 - 2S_6$ is given by

(a) 4 (b) 0

(c) 1 (d) 7

23. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by

(a) $\left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (b) $\left(-\frac{\pi}{10}, \pi\right)$

(c) $(-\pi, \pi)$ (d) $\left(-\pi, \frac{3\pi}{10}\right)$

24. Let $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} - \frac{\cos x}{\sqrt{1+\cot^2 x}}$ then range of $f(x)$ is

(a) [–1, 0] (b) [0, 1]

(c) [–1, 1] (d) none of these

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ is equal to

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$

(c) ab (d) None of these

26. **Statement-1:** If α and β are two distinct solutions of the equation $a\cos x+b\sin x=c$, then $\tan\left(\frac{\alpha+\beta}{2}\right)$ is independent of c.

Statement-2: Solution of $a\cos x+b\sin x=c$ is possible, if $-\sqrt{(a^2+b^2)}\le c\le\sqrt{(a^2+b^2)}$

(a) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(b) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(c) Statement -1 is false, Statement-2 is true

(d) Statement -1 is true, Statement-2 is false

27. The value of $\tan^2\theta\sec^2\theta(\cot^2\theta-\cos^2\theta)$ is

(a) 0 (b) 1

(c) –1 (d) $\frac{1}{2}$

28. If $\cos\theta+\cos 2\theta+\cos 3\theta=0$, then the general value of θ is :

(a) $\theta=2m\pi\pm 2\pi/3$ (b) $\theta=2m\pi\pm\pi/4$

(c) $\theta=m\pi+(-1)^n 2\pi/3$ (d) $\theta=m\pi+(-1)^n\pi/3$

29. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ is in the interval $\left(0,\frac{\pi}{2}\right)$ if the value of x is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

30. If α, β, γ, δ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive number x, then the value of

$4\sin\frac{\alpha}{2}+3\sin\frac{\beta}{2}+2\sin\frac{\gamma}{2}+\sin\frac{\delta}{2}$ is equal to

(a) $2\sqrt{1-x}$ (b) $2\sqrt{1+x}$

(c) $2\sqrt{x}$ (d) None of these

RESPONSE GRID	25.ⓐⓑⓒⓓ 26.ⓐⓑⓒⓓ 27.ⓐⓑⓒⓓ 28.ⓐⓑⓒⓓ 29.ⓐⓑⓒⓓ 30.ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 3 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	40	Qualifying Score	58
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. (c) Given $y = \cos^2 x + \sec^2 x$

$\Rightarrow \; y = \cos^2 x + \dfrac{1}{\cos^2 x} \; \left(\because \cos x = \dfrac{1}{\sec x}\right)$

$\Rightarrow \; y = \cos^2 x + \dfrac{1}{\cos^2 x} + 2 - 2$

$\Rightarrow \; y = \left(\cos x - \dfrac{1}{\cos x}\right)^2 + 2$

$\Rightarrow \; y = (\cos x - \sec x)^2 + 2$

As $(\cos x - \sec x)^2 = 0$ or positive

$\therefore \; y = 2$ or $y \geq 2$

2. (c) $\dfrac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta} = \dfrac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\left(\dfrac{3\theta}{2}\right)$

Hence period $= \dfrac{2\pi}{3}$

3. (c) Given an angle θ which is divided into two parts A and B such that $A - B = k$ and $A + B = \theta$,

and $\tan A : \tan B = k : 1$, i.e. $\dfrac{\tan A}{\tan B} = \dfrac{k}{1}$

$\Rightarrow \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{k+1}{k-1}$

(by componendo and dividendo)

$\Rightarrow \; \dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{k+1}{k-1} \qquad \Rightarrow \; \dfrac{\sin\theta}{\sin k} = \dfrac{k+1}{k-1}$

$\Rightarrow \; \sin k = \dfrac{k-1}{k+1}\sin\theta$

4. (a) Given that $2y\cos\theta = x\sin\theta$ ...(i)

and $2x\sec\theta - y\,\mathrm{cosec}\,\theta = 3$ ...(ii)

$\Rightarrow \; \dfrac{2x}{\cos\theta} - \dfrac{y}{\sin\theta} = 3$

$\Rightarrow \; 2x\sin\theta - y\cos\theta - 3\sin\theta\cos\theta = 0$ ...(iii)

Solving (i) and (iii), we get $y = \sin\theta$ and $x = 2\cos\theta$

Now, $x^2 + 4y^2 = 4\cos^2\theta + 4\sin^2\theta$

$= 4(\cos^2\theta + \sin^2\theta) = 4$

5. (d) The given equation can be written as

$1 - 2\sin^2 x\cos^2 x = a$

$\Rightarrow \sin^2 2x = 2(1-a) \Rightarrow 2(1-a) \leq 1$

and $2(1-a) \geq 0 \Rightarrow 1/2 \leq a \leq 1$

6. (d) Consider $\tan\dfrac{\theta}{2}(1 + \sec\theta) = \tan\dfrac{\theta}{2}\left(\dfrac{1+\cos\theta}{\cos\theta}\right)$

$= \dfrac{\sin\theta/2}{\cos\theta/2}.\dfrac{2\cos^2\theta/2}{\cos\theta} \ldots = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ ...(1)

$\therefore f_1(\theta) = \tan\theta/2\,(1+\sec\theta)(1+\sec 2\theta)$

$= [\tan\theta/2\,(1+\sec\theta)](1+\sec 2\theta)$

$= (\tan\theta)(1+\sec 2\theta)$ [from(1)]

$= \tan 2\theta$ [replacing θ by 2θ as above]

$\Rightarrow f_1(\theta) = \tan 2^1\theta$ ...(2)

Similarly, $f_2(\theta) = \tan 2^2\theta, f_3(\theta)$

$= \tan 2^3\theta, f_4(\theta) = \tan 2^4\theta$ etc.

$\Rightarrow \; f_2\left(\dfrac{\pi}{16}\right) = \tan\left(2^2\dfrac{\pi}{16}\right) = \tan\dfrac{\pi}{4} = 1$

$f_3\left(\dfrac{\pi}{32}\right) = \tan\left(2^3\dfrac{\pi}{32}\right) = \tan\dfrac{\pi}{4} = 1$

$f_4\left(\dfrac{\pi}{64}\right) = \tan\left(2^4\dfrac{\pi}{64}\right) = \tan\dfrac{\pi}{4} = 1$

7. (b) The given expression can be written as

$\dfrac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$

$= \dfrac{2\cos 5x\cos x + 5.2\cos 3x\cos x + 10.2\cos^2 x}{\cos 5x + 5\cos 3x + 10\cos x}$

$= \dfrac{2\cos x(\cos 5x + 5\cos 3x + 10\cos x)}{\cos 5x + 5\cos 3x + 10\cos x} = 2\cos x$

8. **(a)** We have $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$
$= \sin\alpha + \sin\beta + \sin\gamma - \sin\alpha\cos\beta\cos\gamma$
$- \cos\alpha\sin\beta\cos\gamma - \cos\alpha\cos\beta\sin\gamma$
$+ \sin\alpha\sin\beta\sin\gamma$
$= \sin\alpha(1-\cos\beta\cos\gamma) + \sin\beta(1-\cos\alpha\cos\gamma)$
$+ \sin\gamma(1-\cos\alpha\cos\beta) + \sin\alpha\sin\beta\sin\gamma > 0$
$\therefore\ \sin\alpha + \sin\beta + \sin\gamma > \sin(\alpha+\beta+\gamma)$
Trick :Put $\alpha = 30, \beta = 30, \gamma = 60$ and check...
$\Rightarrow\ \dfrac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma} < 1$

9. **(b)** The expression

$$= \left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)$$

$$\left[\because \cos\frac{7\pi}{10} = \cos\left(\pi-\frac{3\pi}{10}\right) = -\cos\frac{3\pi}{10}\right.$$

$$\left.\text{and } \cos\frac{9\pi}{10} = \cos\left(\pi-\frac{\pi}{10}\right) = -\cos\frac{\pi}{10}\right]$$

$$= \left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right) = \sin^2\frac{\pi}{10}.\sin^2\frac{3\pi}{10}$$

$$= \sin^2 18°.\sin^2 54° = \left(\frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4}\right)^2 = \frac{1}{16}$$

10. **(b)** Consider $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$

$\Rightarrow \sin A = (\sqrt{7}+\sqrt{6})\cos A$

$\Rightarrow \sin A = \dfrac{(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})}{(\sqrt{7}-\sqrt{6})}\cos A$

$\Rightarrow \sqrt{7}\sin A = \cos A + \sqrt{6}\sin A$

11. **(a)** Let $\sqrt{3}+1 = r\cos\alpha$, and $\sqrt{3}-1 = r\sin\alpha$
$\therefore\ r^2 = \left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2 = 8$ i.e. $\alpha = \pi/12$
From the equation, $r\cos(\theta-\alpha) = 2$
$\Rightarrow\ \cos(\theta-\pi/12) = 1/\sqrt{2} = \cos(\pi/4)$
$\therefore\ \theta = 2n\pi \pm \pi/4 + \pi/12$

12. **(a)** We have, $\sin\pi(x^2+x) = \sin\pi x^2$

$\Rightarrow\ \pi(x^2+x) = n\pi + (-1)^n\pi x^2$

$\therefore$ Either $x^2+x = 2m + x^2 \Rightarrow x = 2m \in I$

or $x^2+x = k - x^2$, where k is an odd integer

$\Rightarrow\ 2x^2+x-k = 0 \Rightarrow x = \dfrac{-1\pm\sqrt{1+8k}}{4}$

For least positive non-integral solution

is $x = \dfrac{1}{2}$, when $k = 1$

13. **(b)** Given, $3\cos^2 A + 2\cos^2 B = 4$

$\Rightarrow 2\cos^2 B - 1 = 4 - 3\cos^2 A - 1$

$\Rightarrow \cos 2B = 3(1-\cos^2 A) = 3\sin^2 A$ ...(1)

and $2\cos B\sin B = 3\sin A\cos A$

$\sin 2B = 3\sin A\cos A$ ...(2)

Now, $\cos(A+2B) = \cos A\cos 2B - \sin A\sin 2B$
$= \cos A(3\sin^2 A) - \sin A(3\sin A\cos A) = 0$
[using eqs. (1) and (2)]

$\Rightarrow\ A + 2B = \dfrac{\pi}{2}$

14. **(b)** Let $f(x) = \sin x\cos x = \dfrac{1}{2}\sin 2x$

We know $-1 \le \sin 2x \le 1 \Rightarrow -\dfrac{1}{2} \le \dfrac{1}{2}\sin 2x \le \dfrac{1}{2}$

Thus the greatest and least value of of f(x) are

$\dfrac{1}{2}$ and $-\dfrac{1}{2}$ respectively

15. **(c)** $\tan(\cot x) = \cot(\tan x) = \tan\left(\dfrac{\pi}{2} - \tan x\right)$

$\Rightarrow\ \cot x = n\pi + \dfrac{\pi}{2} - \tan x$

$[\because\ \tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha]$

$\Rightarrow\ \cot x + \tan x = n\pi + \dfrac{\pi}{2}$

$\Rightarrow\ \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow\ \dfrac{1}{\sin x\cos x} = (2n+1)\dfrac{\pi}{2}$

$\Rightarrow\ \dfrac{1}{\sin 2x} = \dfrac{(2n+1)\pi}{4}$

$\Rightarrow\ \sin 2x = \dfrac{4}{(2n+1)\pi}$

16. **(c)** Put $t = \sqrt{\dfrac{x}{y}} > 0$,

Consider $t + \dfrac{1}{t} = \left(\sqrt{t} - \dfrac{1}{\sqrt{t}}\right)^2 + 2 \ge 2$ equality holding

iff $t = 1$

Also, $t + \dfrac{1}{t} = 2\sin\theta \le 2$, so that t should necessarily

be 1, i.e., $x = y$.

17. **(b)** $p_n - p_{n-2} = (\cos^n\theta + \sin^n\theta) - (\cos^{n-2}\theta + \sin^{n-2}\theta)$

$= \cos^{n-2}\theta(\cos^2\theta - 1) + \sin^{n-2}\theta(\sin^2\theta - 1)$

$= -\sin^2\theta\cos^{n-2}\theta - \cos^2\theta\sin^{n-2}\theta$

$= -\sin^2\theta\cos^2\theta(\cos^{n-4}\theta + \sin^{n-4}\theta)$

$= -\sin^2\theta\cos^2\theta p_{n-4} = kp_{n-4}$

$\Rightarrow k = -\sin^2\theta\cos^2\theta$

18. **(a)** Given $f(x) = \cos(\log x)$
$\therefore\ f(xy) = \cos(\log xy)$
$f(xy) = \cos[\log x + \log y]$ ....(i)

And $f\left(\dfrac{x}{y}\right) = \cos\left(\log\dfrac{x}{y}\right)$

$$f\left(\frac{x}{y}\right) = \cos(\log x - \log y) \quad \text{....(ii)}$$

Adding (i) and (ii), we get

$$f(xy) + f\left(\frac{x}{y}\right) = \cos(\log x + \log y) + \cos(\log x - \log y)$$

$$= 2\cos(\log x).\cos(\log y)$$

$$\Rightarrow f(xy) + f\left(\frac{x}{y}\right) = 2f(x).f(y)$$

Then the value of $f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\}$

$$= f(x)\,f(y) - \frac{1}{2}.2\{f(x).f(y)\} = 0$$

19. (b) Let $g(x) = 6\sin x - 8\cos x + 5$

Max. value of $g(x) = \sqrt{6^2 + 8^2} + 5 = 5 + 10 = 15$

Min. value of $g(x) = -\sqrt{6^2 + 8^2} + 5 = 5 - 10 = -5$

$\therefore$ The range of $f(x) = \frac{1}{g(x)}$ is $R - \left(-\frac{1}{5}, \frac{1}{15}\right)$

$\Rightarrow$ it is an unbounded function.

$\Rightarrow$ f(x) has no maximum and no minimum values.

20. (a) Since $\alpha = \beta = \gamma \Rightarrow \cos^2\alpha = \cos^2\beta = \cos^2\gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 3\cos^2\alpha = 3\cos^2\beta = 3\cos^2\gamma = 1$

$\Rightarrow \cos^2\alpha = \cos^2\beta = \cos^2\gamma = \frac{1}{3}$

$\therefore \sin^2\alpha = \sin^2\beta = \sin^2\gamma = \frac{2}{3}$

$\therefore \cos\theta = \frac{3(1/3)}{3(2/3)} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

21. (a) $\sin 12° \sin 24° \sin 48° \sin 84°$

$= \frac{1}{4}(2\sin 12° \sin 48°)(2\sin 24° \sin 84°)$

$= \frac{1}{2}(\cos 36° - \cos 60°)(\cos 60° - \cos 108°)$

$= \frac{1}{4}\left(\cos 36° - \frac{1}{2}\right)\left(\frac{1}{2} + \sin 18°\right)$

$= \frac{1}{4}\left\{\frac{1}{4}\left(\sqrt{5}+1\right) - \frac{1}{2}\right\}\left\{\frac{1}{2} + \frac{1}{4}\left(\sqrt{5}-1\right)\right\} = \frac{1}{16}$

and $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}[\cos(60° - 20°)\cos 20° \cos(60° + 20°)]$

$= \frac{1}{2}\left[\frac{1}{4}\cos 3(20°)\right] = \frac{1}{8}\cos 60° = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}.$

22. (c) Let $S_n = \cos^n\theta + \sin^n\theta$; $S_4 = \cos^4\theta + \sin^4\theta$

$S_6 = \cos^6\theta + \sin^6\theta = (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta + \sin^2\theta)(\cos^4\theta + \sin^4\theta - \cos^2\theta \times \sin^2\theta)$

$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$\therefore \quad 3S_4 - 2S_6$

$= 3(\cos^4\theta + \sin^4\theta) - 2(\cos^4\theta + \sin^4\theta - \cos^2\theta\sin^2\theta)$

$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta$

$= (\cos^2\theta + \sin^2\theta)^2 = (1)^2 = 1$

23. (a) We have $|4\sin x - 1| < \sqrt{5}$

$\Rightarrow -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$

$\Rightarrow -\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \frac{\sqrt{5}+1}{4}$

$\Rightarrow \sin\left(\frac{-\pi}{10}\right) < \sin x < \sin\left(\frac{3\pi}{10}\right)$

$\Rightarrow x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ $\quad [\because x \in (-\pi, \pi)]$

24. (c) $f(x) = \frac{\sin x}{|\sec x|} - \frac{\cos x}{|\operatorname{cosec} x|}$

Range $f(x) = \sin x\,.\,|\cos x| - \cos x\,|\sin x| = ?$

$$f(x) = \begin{cases} 0 & x \in \left[0, \frac{\pi}{2}\right] \\ -\sin 2x & x \in \left(\frac{\pi}{2}, \pi\right) \\ 0 & x \in \left(\pi, \frac{3\pi}{2}\right) \\ \sin 2x & x \in \left(\frac{3\pi}{2}, 2\pi\right) \end{cases}$$

– sin 2x	0
0	sin 2x

So range is [–1, 1]

25. (b) Let $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

If $\frac{a}{b} = \frac{c}{d}$, then by componendo and dividendo, we have $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

Applying componendo and dividendo, we get

$$\frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{2a}{2b}$$

[using sin (A + B) and sin (A – B)]

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}.$$

26. **(b)** $\because \quad a\cos x + b\sin x = c$

$$\Rightarrow \quad a\left(\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right)+b\left(\frac{2\tan(x/2)}{1+\tan^2(x/2)}\right)=c$$

$$\Rightarrow \quad (a+c)\tan^2\left(\frac{x}{2}\right)-2b\tan\left(\frac{x}{2}\right)+(c-a)=0$$

$$\therefore \quad \tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=\frac{2b}{(a+c)}$$

and $\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)=\frac{c-a}{a+c}$

Now, $\tan\left(\frac{\alpha+\beta}{2}\right)=\dfrac{\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)}{1-\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\beta}{2}\right)}$

$$=\frac{\frac{2b}{a+c}}{1-\left(\frac{c-a}{a+c}\right)}=\frac{b}{a}=\text{Independent of } c$$

Also,

$$-\sqrt{(a^2-b^2)}\le a\cos x+b\sin x\le\sqrt{(a^2+b^2)}$$

$$\therefore \quad -\sqrt{(a^2+b^2)}\le c\le\sqrt{(a^2+b^2)}$$

27. **(b)** $\tan^2\theta\sec^2\theta\,(\cot^2\theta-\cos^2\theta)$

$=\sec^2\theta\,(\tan^2\theta\cot^2\theta-\tan^2\theta\cos^2\theta)$

$$=\sec^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right)=\sec^2\theta(1-\sin^2\theta)$$

$=\sec^2\theta.\cos^2\theta=1$

28. **(a)** Given $\cos\theta+\cos2\theta+\cos3\theta=0$

$\Rightarrow \quad (\cos3\theta+\cos\theta)+\cos2\theta=0$

$\Rightarrow \quad 2\cos2\theta.\cos\theta+\cos2\theta=0$

$\Rightarrow \quad \cos2\theta.(2\cos\theta+1)=0$

we have, $\cos\theta=\cos\alpha\Rightarrow\theta=2n\pi\pm\alpha$

$\therefore$ For general value of θ, $\cos2\theta=0$

$\Rightarrow \cos2\theta=\cos\frac{\pi}{2} \Rightarrow 2\theta=2m\pi\pm\frac{\pi}{2}$

$\Rightarrow \theta=m\pi\pm\frac{\pi}{4}$ or $2\cos\theta+1=0$;

$\Rightarrow \cos\theta=\frac{-1}{2} \Rightarrow \cos\theta=\cos\frac{2\pi}{3}$

So, $\theta=2m\pi\pm\frac{2\pi}{3}$

29. **(b)** Let, $y=\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$

$$=\sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\left(x+\frac{\pi}{6}\right)+\frac{1}{\sqrt{2}}\cos\left(x+\frac{\pi}{6}\right)\right]$$

$$=\sqrt{2}\left[\sin\frac{\pi}{4}\sin\left(x+\frac{\pi}{6}\right)+\cos\frac{\pi}{4}\cos\left(x+\frac{\pi}{6}\right)\right]$$

$$=\sqrt{2}\left[\cos\left(x+\frac{\pi}{6}-\frac{\pi}{4}\right)\right]=\sqrt{2}\left[\cos\left(x-\frac{\pi}{12}\right)\right]$$

$\Rightarrow \quad x-\frac{\pi}{12}=0 \qquad [\because y \text{ to be max.}]$

$\Rightarrow \quad x=\frac{\pi}{12}$

30. **(b)** If α is the smallest positive angle for which $\sin\alpha=x$, then $\beta=\pi-\alpha$, $\gamma=2\pi+\alpha$ and $\delta=3\pi-\alpha$

So, $4\sin\frac{\alpha}{2}+3\sin\frac{\beta}{2}+2\sin\frac{\gamma}{2}+\sin\frac{\delta}{2}$

$$=4\sin\frac{\alpha}{2}+3\cos\frac{\alpha}{2}-2\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}$$

$$=2\sin\frac{\alpha}{2}+2\cos\frac{\alpha}{2}=2\sqrt{1+\sin\alpha}=2\sqrt{1+x}$$