Chapter 1. Rational Numbers

Question 1

Name the property under multiplication used in each of the following:

$(i)\frac{-8}{9}\times 1=1\times\frac{-8}{9}=\frac{-8}{9}$

$(ii)\frac{-21}{23}\times\frac{-3}{7}=\frac{-3}{7}\times\frac{-21}{23}$

$(iii)\frac{-17}{25}\times\frac{25}{-17}=1$

Solution:

$(i)\frac{-8}{9}\times 1=1\times\frac{-8}{9}=\frac{-8}{9}(\text{The role of 1})$

$(ii)\frac{-21}{23}\times\frac{-3}{7}=\frac{-3}{7}\times\frac{-21}{23}(\text{Using property of commutativity})$

$(iii)\frac{-17}{25}\times\frac{25}{-17}=1\left(\text{multiplicative inverse of }\frac{-17}{25}\text{ is }\frac{25}{-17}\right)$

Question 2

Multiply $\frac{4}{7}$ by the reciprocal of $\frac{1}{63}$.

Solution:

$\frac{4}{7}\times\frac{1}{\frac{1}{63}}$

$=\frac{4}{7}\times 63=36$

Question 3

Write five rational numbers greater than - 5 with common denominators.

Solution:

$-5=\frac{-5\times5}{1\times5}=\frac{-25}{5}$

- 5 is same as $\frac{-25}{5}$

Consider a rational number between $\frac{-25}{5}\&1$

Which is $\frac{\frac{-25}{5}+\frac{1}{1}}{2}=\frac{\frac{-25+5}{5}}{2}=\frac{\frac{-20}{5}}{2}=\frac{-20}{10}$

To make denominator as 5 divide both the numerator & denominator by 2.

$\therefore$we get $\frac{-10}{5}$ is a rational number between $\frac{-25}{5}\&1$

i.e., $\frac{-25}{5}<\frac{-10}{5}<1-----(1)$

Between $\frac{-25}{5}\&\frac{-10}{5}$

i.e., between - 5 & - 2 we have - 4

$\frac{-4}{1}=\frac{-4\times5}{1\times5}=\frac{-20}{5}----(2)$

From (1) & (2)

$\frac{-25}{5}<\frac{-20}{5}<\frac{-10}{5}<1$

$\therefore\frac{-25}{5}<\frac{-24}{5}<\frac{-23}{5}<\frac{-20}{5}<\frac{-10}{5}<1$

$\therefore$ Five rational numbers that are greater than - 5 are

$\frac{-24}{5},\frac{-23}{5},\frac{-20}{5},\frac{-10}{5}\&\frac{1}{5}$

Question 4

Write an equivalent rational number for each of the following rational numbers. Fill in the blanks shown below with the corresponding letter found below each answer. What you will find is the name of a famous author of children's books who was born almost a hundred years ago.

N	B	Y	L	E
$\frac{3}{8}=\frac{?}{16}$	$\frac{14}{20}=\frac{?}{10}$	$\frac{3}{6}=\frac{?}{12}$	$\frac{1}{3}=\frac{?}{6}$	$\frac{6}{8}=\frac{?}{24}$
I	T	D	O	N
$\frac{5}{10}=\frac{?}{20}$	$\frac{7}{8}=\frac{?}{16}$	$\frac{8}{10}=\frac{4}{?}$	$\frac{1}{4}=\frac{4}{?}$	$\frac{8}{36}=\frac{32}{?}$

___	___	___	___	___	___	___	___	___	___
$\frac{18}{24}$	$\frac{6}{16}$	$\frac{10}{20}$	$\frac{4}{5}$	$\frac{7}{10}$	$\frac{2}{6}$	$\frac{6}{12}$	$\frac{14}{16}$	$\frac{4}{16}$	$\frac{36}{162}$

Solution:

$N \rightarrow \frac{3}{8}\times\frac{2}{2}=\frac{6}{16}$

$B \rightarrow \frac{14}{20}\div\frac{2}{2}=\frac{7}{10}$

$Y \rightarrow \frac{3}{6}\times\frac{2}{2}=\frac{6}{12}$

$L \rightarrow \frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$

$E \rightarrow \frac{6}{8}\times\frac{3}{3}=\frac{18}{24}$

$I \rightarrow \frac{5}{10}\times\frac{2}{2}=\frac{10}{20}$

$T \rightarrow \frac{7}{8}\times\frac{2}{2}=\frac{14}{16}$

$D \rightarrow \frac{8}{10}\div\frac{2}{2}=\frac{4}{5}$

$O \rightarrow \frac{1}{4}\times\frac{4}{4}=\frac{4}{16}$

$N \rightarrow \frac{8}{36}\times\frac{4}{4}=\frac{32}{144}$

_E_	N	I	D	B	_L_	_Y_	T_	_O	_N
$\frac{18}{24}$	$\frac{6}{16}$	$\frac{10}{20}$	$\frac{4}{5}$	$\frac{7}{10}$	$\frac{2}{6}$	$\frac{6}{12}$	$\frac{14}{16}$	$\frac{4}{16}$	$\frac{36}{162}$

Question 5

Write four rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution:

The two numbers are $\frac{1}{3}$ and $\frac{1}{2}$.

Make the denominators same for the given rational number

$\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$ $\quad$ $\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$

The mean of $\frac{1}{3}$ and $\frac{1}{2}$ = $\frac{\frac{2}{6}+\frac{3}{6}}{2}=\frac{5}{12}$

$\therefore \frac{2}{6}<\frac{5}{12}<\frac{3}{6}$

The mean of $\frac{2}{6}$ and $\frac{5}{12}$ = $\frac{\frac{2}{6}+\frac{5}{12}}{2}=\frac{24+5}{24}=\frac{29}{24}$

The mean of $\frac{5}{12}$ and $\frac{3}{6}$ = $\frac{\frac{5}{12}+\frac{3}{6}}{2}=\frac{5+6}{24}=\frac{11}{24}$

Question 6

What must be added to $\frac{-3}{8}$ to get $\frac{5}{16}$?

Solution:

Let the number to be added = *y*.

Question 6

What must be added to $\frac{-3}{8}$ to get $\frac{5}{16}$?

Solution:

Let the number to be added = *y*.

$$\therefore -\frac{3}{8}+y=\frac{5}{16}$$

$$y=\frac{5}{16}+\frac{3}{8}$$

$$=\frac{5+6}{16}=\frac{11}{16}$$

The number to be added = $\frac{11}{16}$

Question 7

What must be subtracted from $\frac{-2}{3}$ to get $\frac{3}{5}$?

Solution:

Let the number to be subtracted = y.

$$-\frac{2}{3}-y=\frac{3}{5}$$

$$-\frac{2}{3}-\frac{3}{5}=y$$

$$\frac{-10-9}{15}=y$$

$$y=\frac{-19}{15}$$

The number to be subtracted is $\frac{-19}{15}$

Question 8

Write a rational number equivalent to $\frac{4}{7}$ with

(i) numerator 20

(ii) denominator 28

Solution:

$$(i)\frac{4}{7}\times\frac{5}{5}=\frac{20}{35}$$

$$(ii)\frac{4}{7}\times\frac{4}{4}=\frac{16}{28}$$

Question 9

Write the following numbers in descending order: $\frac{1}{3}, \frac{5}{6}, \frac{7}{9}, \frac{10}{27}, \frac{-5}{12}, \frac{-2}{3}$.

Solution:

Find the LCM of 3,6,9,27,12 and 3.

3	3, 6, 9, 27, 12, 3
2	1, 2, 3, 9, 4, 1
2	1, 2, 3, 9, 2, 1
3	1, 1, 3, 9, 1, 1
	1, 1, 1, 3, 1, 1

$$\begin{aligned} \text{L.C.M.} &= 3 \times 3 \times 2 \times 2 \times 3 \\ &= 9 \times 4 \times 3 \\ &= 36 \times 3 \\ &= 108 \end{aligned}$$

Since LCM is 108, equivalent fraction with denominators 108 is found out for all the above fractions.

$$\frac{1}{3} \times \frac{36}{36} = \frac{36}{108}$$

$$\frac{5}{6} \times \frac{18}{18} = \frac{90}{108}$$

$$\frac{7}{9} \times \frac{12}{12} = \frac{84}{108}$$

$$\frac{10}{27} \times \frac{4}{4} = \frac{40}{108}$$

$$\frac{-5}{12} \times \frac{9}{9} = \frac{-45}{108}$$

$$\frac{-2}{3} \times \frac{36}{36} = \frac{-72}{108}$$

$$\frac{-72}{108} < \frac{-45}{108} < \frac{36}{308} < \frac{40}{108} < \frac{84}{108} < \frac{90}{108}$$

$\therefore$Descending order is

$$\frac{-2}{3} < \frac{-5}{12} < \frac{1}{3} < \frac{10}{27} < \frac{7}{9} < \frac{5}{6}$$

Question 10

Compare the following pairs of rational numbers. $(i)\frac{2}{5}$ and $\frac{3}{4}$

Solution:

$(i)\frac{2}{5}$ *and* $\frac{3}{4}$

L.C.M. of the denominator is 20

$\frac{2}{5}=\frac{2}{5}\times\frac{4}{4}=\frac{8}{20}$

$\frac{3}{4}=\frac{3}{4}\times\frac{5}{5}=\frac{15}{20}$

$\therefore\frac{2}{5}<\frac{3}{4}$

Question 11

Find the missing entries in the following table:

Fraction in Standard form	Numerator	Denominator	Sign of the rational number
1. $\frac{-3\times3}{8\times3}$	_____	24	_____
2. $\frac{6}{7}$	84	____	Positive
3. ____	- 5	- 8	_____
4. $\frac{-21\times-2}{71\times-2}$	_____	- 142	Negative

Solution:

Fraction in Standard form	Numerator	Denominator	Sign of the rational number
1. $\frac{-3\times3}{8\times3}$	**- 9**	24	**negative**
2. $\frac{6}{7}$	84	**98**	Positive
3. $\frac{-5}{-8}$	- 5	- 8	**Positive**
4. $\frac{-21\times-2}{71\times-2}$	**42**	- 142	Negative

Question 12

State whether each of the following is true or false. Correct the statements which are false

1. $\frac{-3}{8} \geq 0.$

2. If $\frac{1}{2} > \frac{1}{3}$, **then** $\frac{1}{2} - \frac{1}{3}$ **is positive**

Solution:

1. $\frac{-3}{8} \geq 0.$ False

Negative numbers are always lesser than 0.

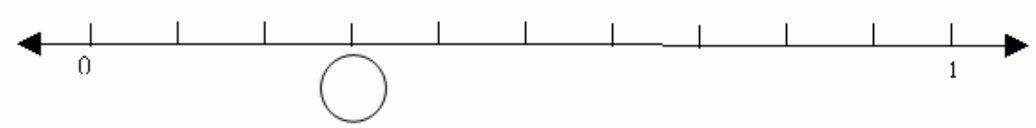
2. If $\frac{1}{2} > \frac{1}{3}$, then $\frac{1}{2} - \frac{1}{3}$ is positive - True

Question 13

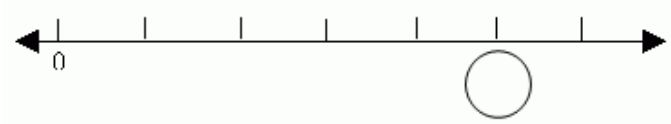
Represent the following numbers on the number line $(i)\frac{3}{10}\ (ii)\frac{5}{6}$

Solution:

$(i)\frac{3}{10}$



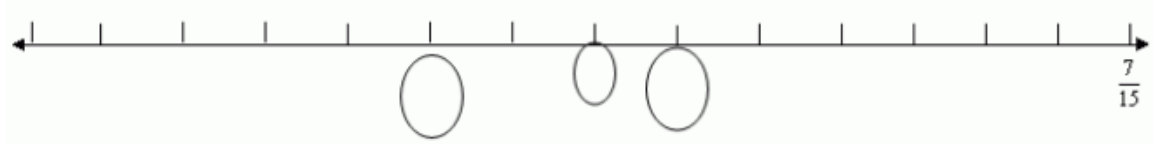
$(ii)\frac{5}{6}$



Question 14

Represent $\frac{-2}{15}, \frac{1}{15}, 0$ **on the number line**

Solution:



Question 15

Represent 4 rational numbers between $\frac{3}{4}$ & 1.

Solution:

We find the mean of the given rational number.

Mean is $\frac{1+\frac{3}{4}}{2}=\frac{4+3}{4}\times\frac{1}{2}=\frac{7}{4}\times\frac{1}{2}=\frac{7}{8}$

$\therefore \frac{3}{4}<\frac{7}{8}<1$

Now find another rational number between $\frac{3}{4}$ & $\frac{7}{8}$

For this again we find the mean of $\frac{3}{4}$ & $\frac{7}{8}$

Mean is $\frac{\frac{3}{4}+\frac{7}{8}}{2}=\frac{\frac{6+7}{8}}{\frac{2}{1}}=\frac{13}{8}\times\frac{1}{2}=\frac{13}{16}$

$\therefore \frac{3}{4}<\frac{13}{16}<\frac{7}{8}<1$

Now find the Mean of $\frac{7}{8}$ & 1

We have $\left(\frac{7}{8}+1\right)\div 2=\frac{7+8}{8}=\frac{15}{8\times 2}=\frac{15}{16}$

Thus we get

$\frac{3}{4}<\frac{13}{16}<\frac{7}{8}<\frac{15}{16}<1$

Now find the mean of $\frac{3}{4}$ & $\frac{13}{16}$

We have, $\left(\frac{3}{4}+\frac{13}{16}\right)\div 2=\frac{\frac{12+13}{16}}{\frac{2}{1}}=\frac{25}{16}\times\frac{1}{2}=\frac{25}{32}$

Thus we get $\frac{3}{4}<\frac{25}{32}<\frac{13}{16}<\frac{7}{8}<\frac{15}{16}<1$

Question 16

A Dutch mathematician and engineer first wrote fractional numbers in the form we now use. To find his name, match each rational number or mixed number with its equivalent in the simplest form. Then write the corresponding letter in the space provided.

$(i)\frac{6}{100}$	$(ii)\frac{14}{56}$	$(iii)\frac{10}{16}$	$(iv)\frac{34}{12}$	$(v)\frac{48}{60}$	$(vi)\frac{32}{20}$	$(vii)\frac{16}{24}$	$(viii)3\frac{145}{435}$	$(ix)\frac{44}{12}$	$(x)\frac{78}{10}$	$(xi)\frac{84}{100}$
T	S	I	O	S	M	V	N	N	I	E
$\frac{2}{3}$	$1\frac{3}{5}$	$7\frac{4}{5}$	$2\frac{5}{6}$	$\frac{3}{50}$	$\frac{5}{8}$	$3\frac{2}{3}$	$\frac{4}{5}$	$\frac{21}{25}$	$\frac{1}{4}$	$3\frac{1}{3}$
$\overline{(i)}$	$\overline{(ii)}$	$\overline{(iii)}$	$\overline{(iv)}$	$\overline{(v)}$	$\overline{(vi)}$	$\overline{(vii)}$	$\overline{(viii)}$	$\overline{(ix)}$	$\overline{(x)}$	$\overline{(xi)}$

Solution:

$(i)\frac{6}{100}\div\frac{2}{2}=\frac{3}{50}$

$(ii)\frac{14}{56}\div\frac{2}{2}=\frac{7}{28}\div\frac{7}{7}=\frac{1}{4}$

$(iii)\frac{10}{16}\div\frac{2}{2}=\frac{5}{8}$

$(iv)\frac{34}{12}\div\frac{2}{2}=\frac{17}{6}=2\frac{5}{6}$

$(v)\frac{48}{60}\div\frac{4}{4}=\frac{12}{15}\div\frac{3}{3}=\frac{4}{5}$

$(vi)\frac{32}{20}\div\frac{2}{2}=\frac{16}{10}\div\frac{2}{2}=\frac{8}{5}=1\frac{3}{5}$

$(vii)\frac{16}{24}\div\frac{4}{4}=\frac{4}{6}\div\frac{2}{2}=\frac{2}{3}$

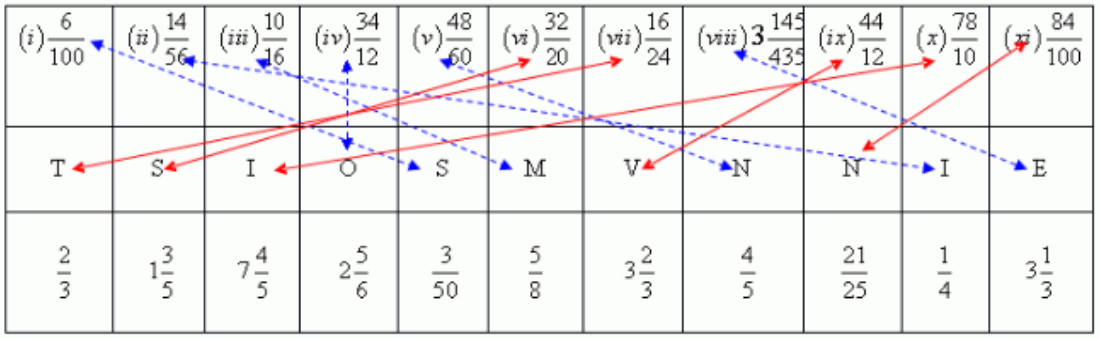
$(viii)3\frac{145}{435}=3\frac{145}{435}\div\frac{145}{145}=3\frac{1}{3}$

$(ix)\frac{44}{12}\div\frac{4}{4}=\frac{11}{3}=3\frac{2}{3}$

$(x)\frac{78}{10}\div\frac{2}{2}=\frac{39}{5}=7\frac{4}{5}$

$(xi)\frac{84}{100}\div\frac{4}{4}=\frac{21}{25}$

$(i)\frac{6}{100}$	$(ii)\frac{14}{56}$	$(iii)\frac{10}{16}$	$(iv)\frac{34}{12}$	$(v)\frac{48}{60}$	$(vi)\frac{32}{20}$	$(vii)\frac{16}{24}$	$(viii)3\frac{145}{435}$	$(ix)\frac{44}{12}$	$(x)\frac{78}{10}$	$(xi)\frac{84}{100}$
T	S	I	O	S	M	V	N	N	I	E
$\frac{2}{3}$	$1\frac{3}{5}$	$7\frac{4}{5}$	$2\frac{5}{6}$	$\frac{3}{50}$	$\frac{5}{8}$	$3\frac{2}{3}$	$\frac{4}{5}$	$\frac{21}{25}$	$\frac{1}{4}$	$3\frac{1}{3}$



$\underset{(i)}{S}\ \underset{(ii)}{I}\ \underset{(iii)}{M}\ \underset{(iv)}{O}\ \underset{(v)}{N}\quad \underset{(vi)}{S}\ \underset{(vii)}{T}\ \underset{(viii)}{E}\ \underset{(ix)}{V}\ \underset{(x)}{I}\ \underset{(xi)}{N}$

The name of the Dutch Mathematician is **SIMON STEVIN**