4 Work, Energy and Power

QUICK LOOK

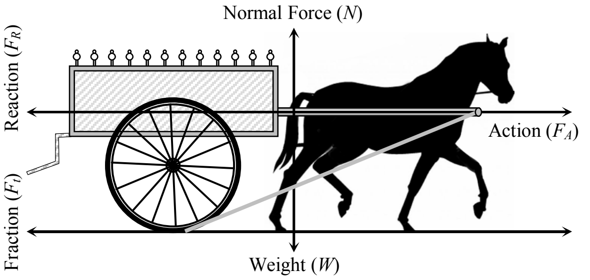


Figure: 4.1

- **Work** $(W) = \vec{F}\cdot\vec{s} = \vec{F}\cos\theta$

  Where θ = angle between force F and displacements s. Unit of work in M.K.S. or S.I. system is joule.
- Work done by a force is positive of $\theta < 90°$
- Work done by a force is zero if $\theta = 90°$
- Work done by a force is negative if $180° > \theta > 90°$
- **Power** $P = \frac{\text{Work}}{\text{Time}} = \frac{W}{t} = \vec{F}\cdot\vec{\upsilon} = F\upsilon\cos\theta$

  Unit of power in S.I. system is watt
- **Kinetic Energy** $(T) = \frac{1}{2}m\upsilon^2$ Kinetic energy is never negative.
- **Potential Energy** $U = -\int_{r_0}^{r}\vec{F}\cdot\vec{dr}$ Where r_0 is reference position for zero potential energy. Referred to zero potential energy at earth's surface $U = mgh$. Referred to zero potential energy at ∞, $U = \frac{-GM_e m}{r}$ where M_e = mass of earth and r= distance of body of mass m from earth's centre. Potential energy may be positive or negative.
- Elastic potential energy $U = \frac{1}{2}kx^2$
- Mechanical energy $E = T + U$
- Under conservation force $E = K + U$ = constant
- Under non-conservative forces, total energy of universe remains constant.

Work Energy Theorem

Work = gain in kinetic energy $= \frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2$

Stopping distance of a vehicle on a rough surface. $s = \frac{\upsilon^2}{2\mu g}$

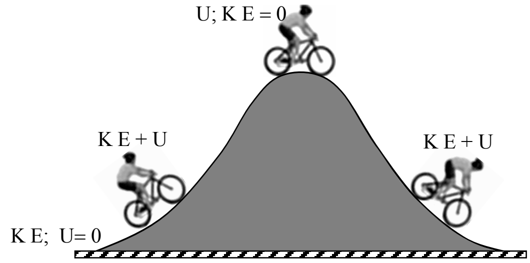


Figure: 4.2

Conservation of Momentum

- **Recoiling of a Gun**

  By the law of conservation of linear momentum:

  $m_G\vec{v}_G + m_B\vec{v}_B = 0$

  So, recoil velocity $\vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$

Figure: 4.3

- If 'n' bullets each of mass m are fired per unit time form a machine gun, then the force required to hold the gun $= v\left(\frac{dm}{dt}\right) = v(mn) = mnv$

Rocket Propulsion: When external force is zero net linear momentum of system is constant i.e., If $F_{ext} = 0,\ \vec{P} = 0.$ Rocket is based on the principle of conversation of linear momentum. Let

m_0 = initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

m_r = residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\frac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption = rate of ejection of the fuel.

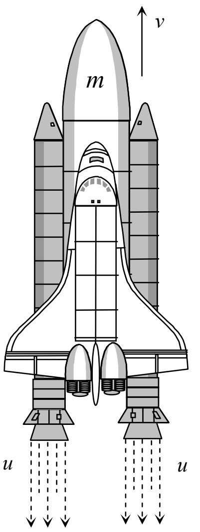


Figure: 4.4

- Thrust on the rocket: $F = -u\dfrac{dm}{dt} - mg$

  $F = -u\dfrac{dm}{dt}$ (if effect of gravity is neglected)

  Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

- Acceleration of the rocket: $a = \dfrac{u}{m}\dfrac{dm}{dt} - g$ and if effect of gravity is neglected $a = \dfrac{u}{m}\dfrac{dm}{dt}$

  Instantaneous velocity of the rocket $v = u\log_e\left(\dfrac{m_0}{m}\right) - gt$

  and $v = u\log_e\left(\dfrac{m_0}{m}\right) = 2.303\log_{10}\left(\dfrac{m_0}{m}\right)$ (neglecting gravity)

- Burnt out speed of the rocket: $v_b = v_{\max} = u\log_e\left(\dfrac{m_0}{m_r}\right)$

  The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

Collision: In elastic collision, kinetic energy is conserved while in an inelastic collision, kinetic energy may increase or decrease. In an elastic collision if $F_{ext} = 0$.

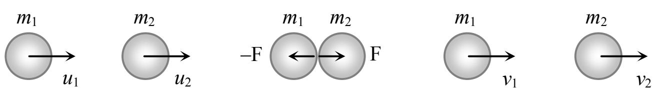


Figure: 4.5

$m_1u_1 + m_2u_1 = m_1v_1 + m_2v_2$ and $v_1 - v_2 = -(u_1 - u_2)$

or $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2$

After collision velocities are

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

- In an inelastic collision: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ and $v_1 - v_2 = -e(u_1 - u_2)$

  For a perfectly inelastic collision $e = 0, v_1 = v_2 = v$ say

$\therefore$ $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

- Coordinates of centre of mass are $x_{cm} = \dfrac{\Sigma mx}{\Sigma m}, y_{cm} = \dfrac{\Sigma my}{\Sigma m}$

  $z_{cm} = \dfrac{\Sigma mz}{\Sigma m}$

  The centre of mass may be outside or inside the material of body. If $F_{ext} = 0$, velocity of centre of mass remains constant.

- **Coefficient of restitution:** The coefficient of restitution in a collision of two bodies is defined as:

  $$e = \frac{\text{velocity of speparation}}{\text{velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2} = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

- For a perfectly elastic collision, $e = 1$
- For a perfectly inelastic collision, $e = 0$
- For an elastic collision: $0 < e < 1$

Bouncing of Ball: Inelastic collision of a ball with the earth: Let h_0 be the initial height of the ball w.r.t. earth. Since the earth is massive, the initial and final velocities of the earth can be assumed to be zero,

When the ball hits the ground first time, its velocity before collision is $u_1 = \sqrt{2gh_0}$.

After collision, $v_1 = eu_1$. The height h_1 attained after first impact should be $h_1 = \left(\dfrac{v_1^2}{2g}\right) = e^2h_0$ $\left(e = \sqrt{\dfrac{h_1}{h_0}}\right)$

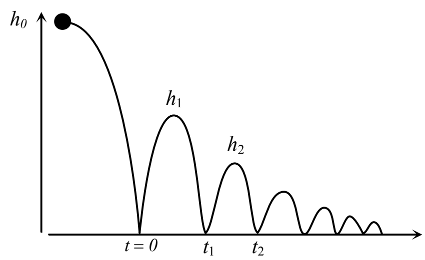


Figure: 4.6

- The velocity just after second impact, $v_2 = ev_1 = e^2 u_1$ and height attained after second impact, $h_2 = \left(\frac{v_2^2}{2g}\right) = e^2 h_0$.

  Hence the height attained after *n* impacts, $h_n = e^2 h_0$
- Total distance traveled by the ball before coming to rest is

  $s = h_0 + 2h_1 + 2h_2 = h_0 + 2e^2 h_0 + 2e^4 h_0 + \ldots$

  or $s = 2h_0\left(1 + e^2 + e^4 + \ldots\right) - h_0 = h_0\left[\frac{1+e^2}{1-e^2}\right]$
- If the ball is released at $t = 0$, at a height h_0, then the time t_1 after which first impact occurs on earth's surface is

  $t_1 = \sqrt{\frac{2h_0}{g}}$

  The second impact occurs after an additional time t_2.

  $t_2 = 2\sqrt{\frac{2h_1}{g}} = 2e\sqrt{\frac{2h_0}{g}} = 2et_1$

  The third impact occurs after a further additional time t_3.

  $t_3 = 2\sqrt{\frac{2h_2}{g}} = 2e^2\sqrt{\frac{2h_0}{g}} = 2e^2 t_1$

  Hence, the total time in which ball comes to rest is

  $T = t_1 + t_2 + t_3 + \ldots = t_1\left(1 + 2e + 2e^2 + \ldots\right)$

  $= t_1\left[1 + 2e\left(1 + e + e^2 + \ldots\right)\right] = t_1\left[1 + 2e\left(\frac{1}{1-e}\right)\right]$

  $T = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$
- Momentum transfer to floor: Let the momentum of the ball when it hits the floor first time is $p = mu_1$ (downwards). Then momentum of the rebounding ball is $p_1 = mv_{1=ep}$ (upwards). Thus momentum change (transfer) in first (one) collision is $\Delta p_1 = p - (-e) = p(1+e)$

  Similarly of the second impact, momentum transfer is:

  $\Delta p_2 = ep - \left(-e^2 p\right) = ep(1+e)$
- Therefore the total momentum transfer is

  $\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3 + \ldots$

  $= p(1+e) + ep(1+e) + e^2 p(1+e) + \ldots = p\left(\frac{1+e}{1-e}\right)$
- Average force exerted by the ball is

  $$= F_{av}\frac{\Delta p}{T} = \frac{p\frac{1+e}{1-e}}{\sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)} = \frac{mu_1}{\sqrt{\frac{2h_0}{g}}} = \frac{m\sqrt{2gh_0}}{\sqrt{\frac{2h_0}{g}}} = mg$$

Elastic Collisions: The total KE in an elastic collision is conserved. However, individual particles may gain or lose KE. Suppose a particle of mass m_1 moving with velocity u_1 collides with a particle of mass m_2 at rest.

The KE of particle 1 before collision is: $K_i = \frac{1}{2}m_1 u_1^2$

The KE of particle 1 after collision is: $K_f = \frac{1}{2}m_1 u_1^2$

But $v_1 = \left[\frac{(m_1 - m_2)}{(m_1 + m_2)}\right]u_1$

So $K_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 K_i$

$\therefore \frac{K_f}{K_i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

or $\frac{K_i - K_f}{K_i} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$

The loss is maximum (=100%) when $m_1 = m_2$

Note

- Above formula can be applied to perfectly inelastic collision with $e = 0$
- In general, if both mass m_1 and m_2 have initial velocities u_1 and u_2, then the energy loss in an inelastic collision is $\Delta K_{\text{lost}} \frac{1}{2} = \frac{m_1 m_2}{(m_1 + m_2)}(u_1 - u_2)^2(1 - e^2)$

Inelastic Collision: Let e = coefficient of restitution. Conservation of momentum and definition of e gives (with $u_2 = 0$)

$m_1 u_1 = m_1 v_1 + m_2 v_2$ and $eu_1 = v_2 = v_1$

Solving, we get $v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1$ and $v_2 = \frac{m_1(1-e)}{m_1 + m_2}u_1$

Initial *KE* of the system $K_i = \left(\frac{1}{2}\right)m_1 u_1^2$.

Final *KE* of the system, $K_f = \left(\frac{1}{2}\right)m_1 u_1^2 + \left(\frac{1}{2}\right)m_2 u_2^2$

Loss in *KE* in inelastic collision is

$K_i - K_f = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)u_1^2\left(1 - e^2\right)$

The fraction of *KE* lost is $\frac{\Delta K_{lost}}{K_i} = \frac{m_2\left(1 - e^2\right)}{(m_1 + m_2)}$

MULTIPLE CHOICE QUESTIONS

Work Done by a Constant Force

1. A force $F = (5\hat{i} + 3\hat{j})\,N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ metres. The work done on the particle is:

a. $-7\,J$ **b.** $+13\,J$

c. $+7\,J$ **d.** $+11\,J$

2. A box of mass 1 *kg* is pulled on a horizontal plane of length 1 *m* by a force of 8 *N* then it is raised vertically to a height of $2m$, the net work done is:

a. 28 *J*   **b.** 8 *J*

c. 18 *J*   **d.** None of above

3. A 10 *kg* satellite completes one revolution around the earth at a height of 100 *km* in 108 minutes. The work done by the gravitational force of earth will be:

a. $108 \times 100 \times 10\,J$ **b.** $\frac{108 \times 10}{100}\,J$

c. $\frac{100 \times 10}{108}\,J$ **d.** Zero

Work Done by a Variable Force

4. A position dependent force $\vec{F} = (7 - 2x + 3x^2)\,N$ acts on a small abject of mass 2 kg to displace it from $x = 0$ to $x = 5m$. The work done in joule is:

a. 70 *J*   **b.** 270 *J*

c. 35 *J*   **d.** 135 *J*

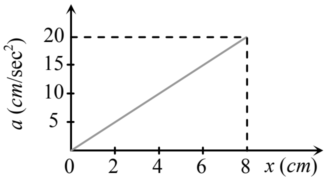
5. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is:

a. Cx_1^2 **b.** $\frac{1}{2}Cx_1^2$

c. Cx_1 **d.** Zero

Work Done Calculation by Force Displacement Graph

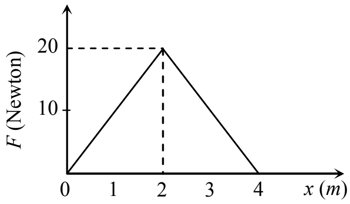
6. A 10 *kg* mass moves along *x*-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from $x = 0$ to $x = 8$ cm:



a. $8 \times 10^{-2}\,J$ **b.** $16 \times 10^{-2}\,J$

c. $4 \times 10^{-4}\,J$ **d.** $1.6 \times 10^{-3}\,J$

7. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 *kg* and initial velocity is 2 *m/s*. When the distance covered by the body is 5*m*, its kinetic energy would be:

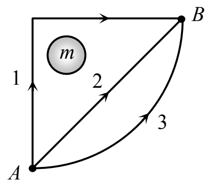


a. 50 *J*   **b.** 40 *J*

c. 20 *J*   **d.** 10 *J*

Work Done in Conservative and Non-Conservative Field

8. If W_1, W_2 and W_3 represent the work done in moving a particle from *A* to *B* along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass *m*, find the correct relation:



a. $W_1 > W_2 > W_3$ **b.** $W_1 = W_2 = W_3$

c. $W_1 < W_2 < W_3$ **d.** $W_2 > W_1 > W_3$

9. A particle of mass 0.01 *kg* travels along a curve with velocity given by $4\hat{i} + 16\hat{k}\,ms^{-1}$. After some time, its velocity becomes $8\hat{i} + 20\hat{j}\,ms^{-1}$ due to the action of a conservative force. The work done on particle during this interval of time is:

a. 0.32 *J*   **b.** 6.9 *J*

c. 9.6 *J*   **d.** 0.96 *J*

Energy

10. An ice cream has a marked value of 700 kcal. How many kilowatt hour of energy will it deliver to the body as it is digested?

a. 0.81 *kWh*   **b.** 0.90 *kWh*

c. 1.11 *kWh*   **d.** 0.71 *kWh*

11. A metallic wire of length *L* metres extends by l metres when stretched by suspending a weight *Mg* to it. The mechanical energy stored in the wire is:

a. $2Mgl$ **b.** Mgl

c. $\frac{Mgl}{2}$ **d.** $\frac{Mgl}{4}$

Kinetic Energy

12. A body of mass 10 *kg* at rest is acted upon simultaneously by two forces 4N and 3N at right angles to each other. The kinetic energy of the body at the end of 10 sec is:

a. 100 *J* **b.** 300 *J*
c. 50 *J* **d.** 125 *J*

13. If the momentum of a body is increased by 100 %, then the percentage increase in the kinetic energy is:

a. 150 % **b.** 200 %
c. 225 % **d.** 300 %

14. A body of mass 2 kg is thrown upward with an energy 490 J. The height at which its kinetic energy would become half of its initial kinetic energy will be: [$g = 9.8\,m/s^2$]

a. 35 *m* **b.** 25 *m*
c. 12.5 *m* **d.** 10 *m*

15. A body of mass 5 *kg* is moving with a momentum of 10 *kg-m/s*. A force of 0.2 *N* acts on it in the direction of motion of the body for 10 seconds. The increase in its kinetic energy is:

a. 2.8 J **b.** 3.2 J
c. 3.8 J **d.** 4.4 J

Stopping of Vehicle by Retarding Force

16. An unloaded bus and a loaded bus are both moving with the same kinetic energy. The mass of the latter is twice that of the former. Brakes are applied to both, so as to exert equal retarding force. If x_1 and x_2 be the distance covered by the two buses respectively before coming to a stop, then:

a. $x_1 = x_2$ **b.** $2x_1 = x_2$
c. $4x_1 = x_2$ **d.** $8x_1 = x_2$

17. A bus can be stopped by applying a retarding force F when it is moving with a speed *v* on a level road. The distance covered by it before coming to rest is *s*. If the load of the bus increases by 50% because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be:

a. 1.5 *s* **b.** 2 *s*
c. 1 *s* **d.** 2.5 *s*

18. A vehicle is moving on a rough horizontal road with velocity *v*. The stopping distance will be directly proportional to:

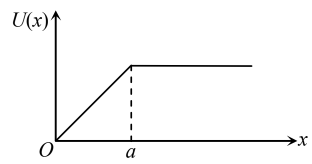
a. $\sqrt{v}$ **b.** v
c. v^2 **d.** v^3

Potential Energy

19. The potential energy of a body is given by $A - Bx^2$ (where x is the displacement). The magnitude of force acting on the particle is:

a. Constant
b. Proportional to x
c. Proportional to x^2
d. Inversely proportional to x

20. The potential energy of a system is represented in the first figure. The force acting on the system will be represented by:



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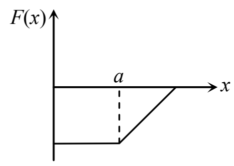

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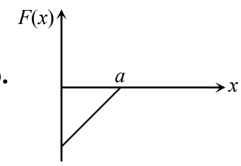
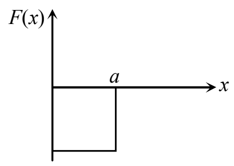
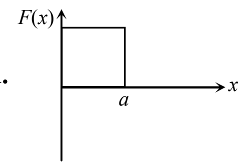


c.


d.


21. A particle moves in a potential region given by $U = 8x^2 - 4x + 400\,J$. Its state of equilibrium will be:

a. $x = 25\,m$ **b.** $x = 0.25\,m$
c. $x = 0.025\,m$ **d.** $x = 2.5\,m$

Elastic Potential Energy

22. Two equal masses are attached to the two ends of a spring of spring constant *k*. The masses are pulled out symmetrically to stretch the spring by a length *x* over its natural length. The work done by the spring on each mass is:

a. $\frac{1}{2}kx^2$ **b.** $-\frac{1}{2}kx^2$
c. $\frac{1}{4}kx^2$ **d.** $-\frac{1}{4}kx^2$

23. A long spring is stretched by 2 cm, its potential energy is U. If the spring is stretched by 10 cm, the potential energy stored in it will be:

a. $\frac{U}{25}$ **b.** $\frac{U}{5}$
c. 5 *U* **d.** 25 *U*

24. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length x. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case?

a. x **b.** $2x$ **c.** $3x$ **d.** $x/2$

Electrical Potential Energy

25. $_{80}Hg^{208}$ nucleus is bombarded by α-particles with velocity 10^7 m/s. If the α-particle is approaching the Hg nucleus head-on then the distance of closest approach will be:

a. $1.115\times10^{-13}\,m$ **b.** $11.15\times10^{-13}\,m$

c. $111.5\times10^{-13}\,m$ **d.** Zero

26. A charged particle *A* moves directly towards another charged particle *B*. For the (*A*+*B*) system, the total momentum is *P* and the total energy is *E*:

a. *P* and *E* are conserved if both *A* and *B* are free to move

b. *A* is true only if *A* and *B* have similar charges

c. If *B* is fixed, *E* is conserved but not *P*

d. If *B* is fixed, neither *E* nor *P* is conserved

Gravitational Potential Energy

27. A rod of mass m and length l is lying on a horizontal table. The work done in making it stand on one end will be:

a. mgl **b.** $\frac{mgl}{2}$

c. $\frac{mgl}{4}$ **d.** 2mgl

28. The mass of a bucket containing water is 10 kg. What is the work done in pulling up the bucket from a well of depth 10 m if water is pouring out at a uniform rate from a hole in it and there is loss of 2 kg of water from it while it reaches the top? $(g=10\,m/\sec^2)$

a. 1000 J **b.** 800 J

c. 900 J **d.** 500 J

29. A uniform chain of length *L* and mass *M* is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is:

a. MgL **b.** $\frac{MgL}{3}$

c. $\frac{MgL}{9}$ **d.** $\frac{MgL}{18}$

Law of Conservation of Energy

30. Two stones each of mass $5kg$ fall on a wheel from a height of $10m$. The wheel stirs $2kg$ water. The rise in temperature of water would be:

a. $2.6^o\,C$ **b.** $1.2^o\,C$

c. $0.32^o\,C$ **d.** $0.12^o\,C$

31. A $2kg$ block is dropped from a height of 0.4 m on a spring of force constant $K=1960\,Nm^{-1}$. The maximum compression of the spring is:

a. 0.1 m **b.** 0.2 m

c. 0.3 m **d.** 0.4 m

32. A stone projected vertically upwards from the ground reaches a maximum height *h*. When it is at a height $\frac{3h}{4}$, the ratio of its kinetic and potential energies is:

a. 3 : 4 **b.** 1 : 3

c. 4 : 3 **d.** 3 : 1

Position and Velocity of an Automobile w.r.t Time

33. A car of mass 'm' is driven with acceleration '*a*' along a straight level road against a constant external resistive force '*R*'. When the velocity of the car is '*v*', the rate at which the engine of the car is doing work will be:

a. Rv **b.** mav

c. $(R+ma)v$ **d.** $(ma-R)v$

34. A bus weighing 100 quintals moves on a rough road with a constant speed of $72km/h$. The friction of the road is 9% of its weight and that of air is 1% of its weight. What is the power of the engine? Take $g = 10m/s^2$

a. 50 kW **b.** 100 kW

c. 150 kW **d.** 200 kW

35. A constant force F is applied on a body. The power (P) generated is related to the time elapsed (t) as:

a. $P\propto t^2$ **b.** $P\propto t$

c. $P\propto\sqrt{t}$ **d.** $P\propto t^{3/2}$

36. A particle moves with a velocity $\vec{v}=5\hat{i}-3\hat{j}+6\hat{k}\,ms^{-1}$ under the influence of a constant force $\vec{F}=10\hat{i}+10\hat{j}+20\hat{k}\,N$. The instantaneous power applied to the particle is:

a. 200 $J\text{-}s^{-1}$ **b.** 40 $J\text{-}s^{-1}$

c. 140 $J\text{-}s^{-1}$ **d.** 170 $J\text{-}s^{-1}$

37. Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is:

a. $\frac{15}{11}$ b. $\frac{11}{15}$

c. $\frac{11}{9}$ d. $\frac{9}{11}$

Perfectly Elastic Head on Collision

38. n small balls each of mass m impinge elastically each second on a surface with velocity u. The force experienced by the surface will be:

a. mnu **b.** $2\ mnu$

c. $4\ mnu$ **d.** $\frac{1}{2}mnu$

39. A ball of mass m moving with velocity V, makes a head on elastic collision with a ball of the same mass moving with velocity $2V$ towards it. Taking direction of V as positive velocities of the two balls after collision are:

a. $-V$ and $2V$ **b.** $2V$ and $-V$

c. V and $-2V$ **d.** $-2V$ and V

40. A smooth sphere of mass M moving with velocity u directly collides elastically with another sphere of mass m at rest. After collision their final velocities are V and v respectively. The value of v is:

a. $\frac{2uM}{m}$ **b.** $\frac{2um}{M}$

c. $\frac{2u}{1+(m/M)}$ **d.** $\frac{2u}{1+(M/m)}$

Head on Inelastic Collision

41. A body of mass 40 kg having velocity 4 m/s collides with another body of mass $60kg$ having velocity 2 m/s. If the collision is inelastic, then loss in kinetic energy will be:

a. 440 J **b.** 392 J

c. 48 J **d.** 144 J

42. One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is $\frac{1}{2}$, the ratio of their speeds after collision shall be:

a. 1 : 2 **b.** 2 : 1

c. 1 : 3 **d.** 3 : 1

Rebounding of Ball after Collision With Ground

43. The change of momentum in each ball of mass $60gm$, moving in opposite directions with speeds $4m/s$ collide and rebound with the same speed, is:

a. $0.98\,kg\text{-}m/s$ **b.** $0.73\,kg\text{-}m/s$

c. $0.48\,kg\text{-}m/s$ **d.** $0.22\,kg\text{-}m/s$

44. A body falling from a height of $20m$ rebounds from hard floor. If it loses 20% energy in the impact, then coefficient of restitution is:

a. 0.89 **b.** 0.56

c. 0.23 **d.** 0.18

45. A rubber ball is dropped from a height of 5 m on a planet where the acceleration due to gravity is not known. On bouncing, it rises to $1.8m$. The ball loses its velocity on bouncing by a factor of:

a. 16/25 **b.** 2/5

c. 3/5 **d.** 9/25

Perfectly Inelastic Collision

46. A neutron having mass of $1.67\times10^{-27}kg$ and moving at $10^8 m/s$ collides with a deutron at rest and sticks to it. If the mass of the deutron is $3.34\times10^{-27}kg$; the speed of the combination is:

a. $2.56\times10^3 m/s$ **b.** $2.98\times10^5 m/s$

c. $3.33\times10^7 m/s$ **d.** $5.01\times10^9 m/s$

47. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northward with the same speed v. The two particles coalesce on collision. The new particle of mass $2m$ will move in the north-easterly direction with a velocity:

a. $v/2$ **b.** $2v$

c. $v/\sqrt{2}$ **d.** v

48. A particle of mass 'm' moving with velocity 'v' collides inelastically with a stationary particle of mass '$2m$'. The speed of the system after collision will be:

a. $\frac{v}{2}$ **b.** $2v$

c. $\frac{v}{3}$ **d.** $3v$

Collision Between Bullet and Vertically Suspended Block

49. A bullet of mass m moving with velocity v strikes a block of mass M at rest and gets embeded into it. The kinetic energy of the composite block will be:

a. $\frac{1}{2}mv^2\times\frac{m}{(m+M)}$ **b.** $\frac{1}{2}mv^2\times\frac{M}{(m+M)}$

c. $\frac{1}{2}mv^2\times\frac{(M+m)}{M}$ **d.** $\frac{1}{2}Mv^2\times\frac{m}{(m+M)}$

50. A wooden block of mass M is suspended by a cord and is at rest. A bullet of mass m, moving with a velocity v pierces through the block and comes out with a velocity $v/2$ in the same direction. If there is no loss in kinetic energy, then upto what height the block will rise:

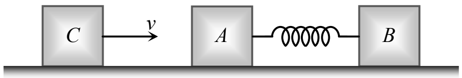
a. $m^2v^2/2M^2g$ **b.** $m^2v^2/8M^2g$

c. $m^2v^2/4Mg$ **d.** $m^2v^2/2Mg$

NCERT EXEMPLAR PROBLEMS

More than One Answer

51. Two blocks A and B each of mass m, are connected by a masseless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in figure.



A third identical block C, also of mass m moves on the floor with a speed υ along the line joining A and B and collides with A; then:

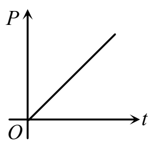
a. the KE of the AB system at maximum compression of the spring is zero

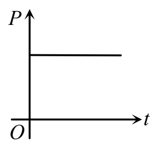
b. the KE of the AB system is at maximum compression is $1/4m\upsilon^2$

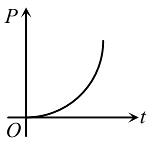
c. the maximum compression of the spring is $\upsilon\sqrt{\frac{m}{k}}$

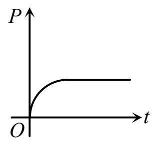
d. the maximum compression of the spring is $\upsilon\sqrt{\frac{m}{2k}}$

52. A vehicle is drive along a straight horizontal track by a motor which exerts a constant driving force. The vehicle starts form rest at $t=0$ and effects of friction and air resistance are negligible. If kinetic energy of vehicle at time t is E and power developed by the motor is P, which of the following graphs are correct?

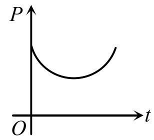
a.


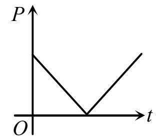
b.


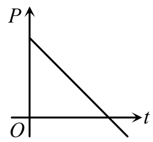
c.


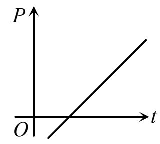
d.


53. A stone is projected at time $t=0$ with a speed V_0 and at an angle θ with the horizontal in a uniform gravitational field. The rate of work done (P) by the gravitational force plotted against time (t) will not be represented by which of the following graphs?

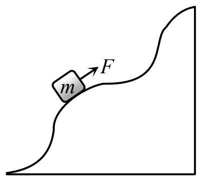
a.


b.


c.


d.


54. A body of mass M was slowly hauled up the rough hill by a force F which at each point was directed along a tangent to the hill. Work done by the force:



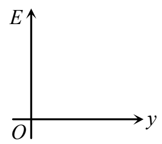
a. is independent of shape of trajectory

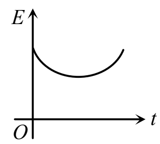
b. depends upon vertical component of displacement but in dependent of horizontal component

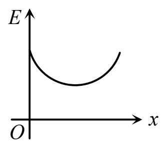
c. depends upon both the components

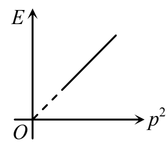
d. does not depend upon coefficient of friction

55. A particle is projected form a point at an angle with the horizontal at $t=0$. At an instant 't', if p is linear momentum, x horizontal displacement, y is vertical displacement and E is kinetic energy of the particle, then which of the following graphs are correct?

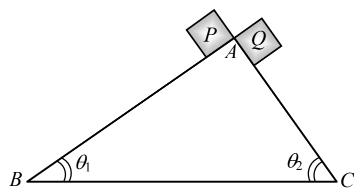
a.


b.


c.


d.


56. Two inclined frictionless tracks of different inclinations meet at A from where two blocks P and Q of different masses are allowed to slide down form rest at the same time, one on each track, as shown in figure. Then:



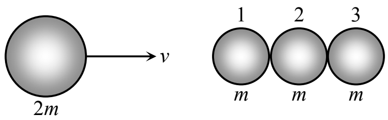
a. both blocks will reach the bottom at the same time

b. block Q will reach the bottom earlier than block P

c. both block reach the bottom with the same speed

d. block Q will reach the bottom with a higher speed than block P

57. A steel ball of mass 2 m surface one-dimensional elastic collision with a row of three steel balls, each of mass m. If mass 2 m has collided with velocity υ and the three balls numbered 1, 2, 3 were in initially at rest, then after the collision:



a. balls 1, 2 and 3 would start moving to the right, each with velocity $\frac{\upsilon}{3}$

b. balls 2 and 3 would start moving to the right, each with a velocity $\frac{\upsilon}{2}$

c. balls 2 and 3 would start moving to the right, each with a velocity υ

d. ball 1 and ball of mass 2 m would remain at rest

58. A ball A of mass m with velocity υ collides head-on with a stationary ball B of mass m. If e be the coefficient of restitution, then which of the following are correct?

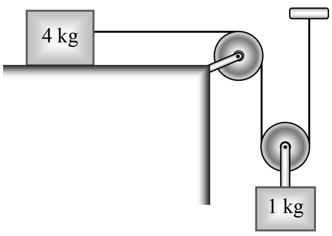
a. The ratio of velocities of balls A and B after the collision is $(1+e)/(1-e)$

b. The ratio of the final and initial velocities of ball A is $(1-e)/2$

c. The ratio of velocities of balls A and B after the collision is $(1-e)/(1+e)$

d. The ratio of the final and initial velocities of ball B is $(1+e)/2$

59. Consider the situation shown in figure.



The block of mass 1.0 kg is released form rest and it is found to have a speed of 0.3 m s^{-1} after it has descended through a distance of 1 m. Which of the following are correct? (Take g = 10 m s^{-2})

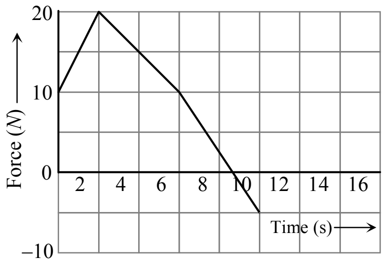
a. Loss in gravitational potential energy is 10 J

b. Kinetic energy of 1 kg block is 0.045 J

c. 4 kg block travels a distance of 2 m to acquire a velocity of 0.6 m s^{-1}

d. Work done against friction in 80μ joule where μ is coefficient of kinetic friction.

60. Following figure gives force *versus* time graph. The force is acting on a particle of mass 2.0 kg at rest at t = 0 and particle is moving in one-dimension. Which of the following are correct?



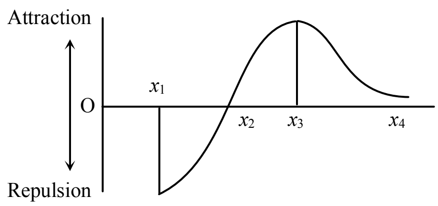
a. The impulse of the force in the time interval $t = 8$ s to $t = 12$ s is –10 N-s

b. The velocity change in the interval $t = 8$ s to $t = 10$ s is –15 m s^{-1}

c. The kinetic energy of the particle at t = 6 s is 2500 J

d. The kinetic energy of the particle at t = 6 s is 500 J

61. The figure given below shows how the net interaction force between two particles A and B is related to the distance between them. When the distance between them varies from x_1 to x_4, then:



a. Potential energy of the system increases from x_1 to x_2

b. Potential energy of the system increases from x_2 to x_3

c. Potential energy of the system increases from x_3 to x_4

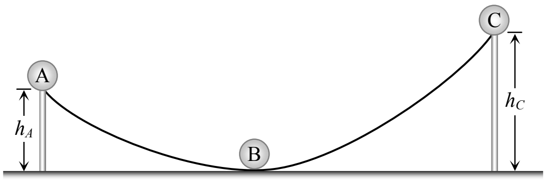
d. Kinetic energy increases from x_1 to x_2 and decreases from x_2 to x_3

62. If m, p, and t denote respectively the mass, linear momentum and angular momentum of a particle moving on a circle of radius r, then the kinetic energy of the particle can be expressed as:

a. $\frac{p^2}{2m}$ **b.** $\frac{l^2}{2m}$

c. $\frac{p^2}{2mr^2}$ **d.** $\frac{l^2}{2mr^2}$

63. A ball moves over a fixed track as shown in figure. From *A* to *B*, the ball rolls without slipping. Surface BC is frictionless K_A, K_B and K_C are kinetic energies of the ball at *A, B* and *C* respectively. Then:



a. $h_A > h_c; K_B > K_C$ **b.** $h_A > h_C; K_C > K_A$

c. $h_A = h_C; K_B = K_C$ **d.** $h_A < h_C; K_B > K_C$

64. A rope ladder with a length *l* carrying a man with a mass m at its end is attached to the basket of balloon with a mass *M*. The entire system is in equilibrium in the air. As the man climbs up the ladder into the balloon, the balloon descends by a height *h*. Then the potential energy of the man:

a. increases by $mg(l-h)$ **b.** increases by $mg\ l$

c. increases by $mg\ h$ **d.** increases by $mg\ (2l-h)$

65. A stone tied to a string of length *L* is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a spend *u*. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is:

a. $\sqrt{u^2-2gL}$ **b.** $\sqrt{2gL}$

c. $\sqrt{u^2-gL}$ **d.** $\sqrt{2(u^2-gL)}$

66. A skier starts at the top of a snowball with negligible speed and skies straight down the side. At what point does he lose contact with the snow-ball?

a. $\theta = \sin^{-1}\left(\frac{2}{3}\right)$ **b.** $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

c. $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ **d.** none of these

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

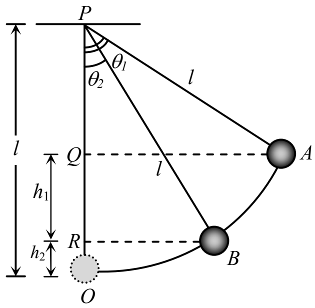
b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

66. **Assertion:** A simple pendulum of length *l* is displaced from its mean position *O* to position *A* so that the string makes an angle θ_1 with the vertical and then released. If air resistances is neglected, the speed of the bob when the string makes an angle θ_2 with the vertical is $v = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$.

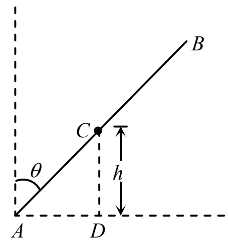


Reason: The total momentum of a system is conserved if no external force acts on it.

68. **Assertion:** A uniform rod of mass *m* and length *l* is held at an angle θ with the vertical. The potential energy of the rod in this position is $\frac{1}{2}mgl\cos\theta$.

Reason: The entire mass of the rod can be assumed to be concentrated at its centre of mass.

69. **Assertion:** A block of mass *m* starts moving on a rough horizontal surface with a velocity *v*. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity *v*. The decrease in the mechanical energy in the second situation is smaller than in the first situation.



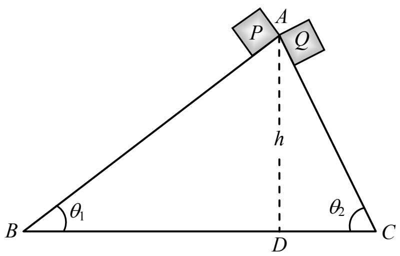
Reason: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

70. **Assertion:** A man carrying a bucket of water and walking on a rough level road with a uniform velocity does no work while carrying the bucket. 1

Reason: The work done on a body by a force F in giving it a displacement S is defined as $W = F \cdot S = FS\cos\theta$. Where θ is the angle between vectors F and S.

71. **Assertion:** A crane P lifts a car up to a certain height in 1 min. Another crane Q lifts the same car up to the same height in 2 min. Then crane P consumes two times more fuel than crane Q.
Reason: Crane P supplies two times more power than crane Q.

72. **Assertion:** Two inclined frictionless tracks of different inclinations θ_1 and θ_2 meet at A from where two blocks P and Q of different masses m_1 and m_2 are allowed to slide down from rest, one on each track as shown in Fig.



Then blocks P and Q will reach the bottom with the same speed.
Reason: Blocks P and Q have equal accelerations down their respective tracks.

73. **Assertion:** In above question, block P will take a longer time to reach the bottom than block Q.
Reason: Block Q has a greater acceleration down the track than block P.

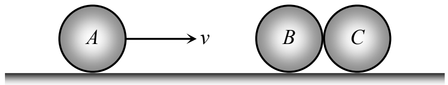
74. **Assertion:** Comets move around the sun in highly elliptical orbits. The work done by the gravitational force of the sun on a comet over a complete orbit is zero.
Reason: The gravitational force is conservative.

75. **Assertion:** The total energy of a system is always conserved irrespective of whether external forces act on the system.
Reason: If external forces act on a system, the total momentum and energy will increase.

76. **Assertion:** The rate of change of the total linear momentum of a system consisting of many particles is proportional to the vector sum of all the internal forces due to inter-particle interactions.
Reason: The internal forces can change the kinetic energy of the system of particles but not the linear momentum of the system.

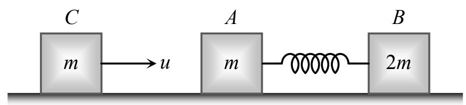
77. **Assertion:** An elastic spring of force constant k is stretched by a small length x. The work done in extending the spring by a further length x is $2kx^2$.
Reason: The work done in extending an elastic spring by a length x is proportional to x^2.

78. **Assertion:** Two identical balls B and C lie on a horizontal smooth straight groove so that they are touching. A third identical ball A moves at a speed υ along the groove and collides with B (see Fig).



If the collisions are perfectly elastic, then after the collision, balls A and B will come to rest and ball C moves with velocity υ to the right.
Reason: In an elastic collision, linear momentum and kinetic energy are both conserved.

79. **Assertion:** Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with a velocity u and collides elastically with A as shown in Fig.



At a certain instant t_0 after the collision, it is found that the velocities of A and B are the same $= \frac{u}{3}$.
Reason: In an elastic collision, the kinetic energy of the system is conserved.

80. **Assertion:** In an inelastic collision between two bodies, the total energy does not change after the collision but the kinetic energy of the system decreases.
Reason: The loss of kinetic energy appears as heat in the system.

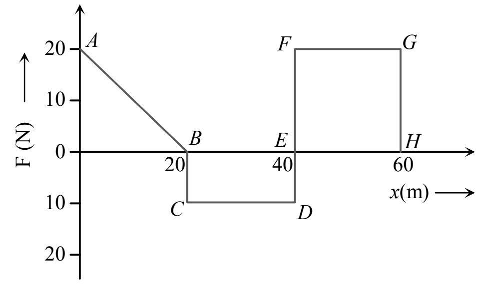
81. **Assertion:** In a collision between two bodies, each body exerts and equal and opposite force on the other at each instant of time during the collision.
Reason: The total energy of the system is conserved.

82. **Assertion:** The term 'collision' between two bodies does not necessarily mean that the two bodies actually strike against each other.
Reason: In physics, a collision is said to take place if the one body influences the motion of the other.

Comprehension Based

Paragraph –I

The work done is defined as $W = \int \vec{F} d\vec{x}$. It can be measured as area between force displacement graph.



It is a scalar quantity. The graph below shows the variation of force and displacement. The word done in different regions is:

83. Total work done is:

a. 800 J **b.** 200 J **c.** 600 J **d.** 400 J

84. Work done is zero upto displacement:

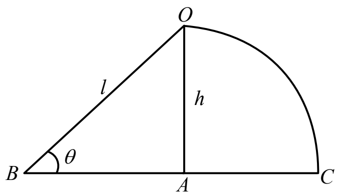
a. 20 m **b.** 40 m
c. 60 m **d.** cannot be known

85. Positive work done is during displacement:

a. 0 to 80 m **b.** 0 to 80 m and 40 to 60 m
c. 0 to 40 m **d.** 20 to 40 m

Paragraph –II

Bodies are falling under gravity along different paths. The paths may be smooth or rough. Bodies cover different distances.



86. The work done by force of gravity along the paths *OA*, *OB*, *OC* if smooth, are in the order:

a. $W_{OA} < W_{OC} < WO_{OB}$ **b.** $W_{OA} < W_{OB} < WO_{OC}$
c. $W_{OB} < W_{OC} < WO_{OA}$ **d.** $W_{OA} = W_{OB} = WO_{OC}$

87. If surface is smooth the work done under gravity in moving a body along the path *OCAO* is W_1 and along the path *OA, BO* is W_2, then:

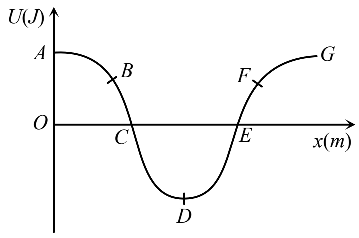
a. $W_1 = W_2$ **b.** $W_1 > W_2$
c. $W_1 < W_2$ **d.** zero in both cases

88. If surface *OB* is rough and μ is coefficient of friction θ is angle of repose, the work done in taking a body form *OB* and back *BO* is:

a. zero **b.** $2mgh$ **c.** $2mgl$ **d.** $2\mu mgh$

Paragraph –III

The potential energy of a particle varies with displacement *x* as shown in figure when the particle is acted upon by a conservative force.



89. The force acting on particle is maximum in magnitude at:

a. *B, C* **b.** *A, G*
c. *B, F* **d.** *D*

90. Particle is in equilibrium at points:

a. *A, D, G* **b.** *C, E*
c. *B, F* **d.** *A, G*

91. Body is in stable equilibrium at:

a. *A* and *G* **b.** *D*
c. *A, D, G* **d.** *C* and *E*

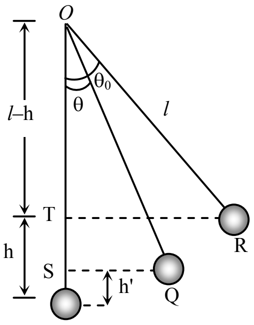
Match the Column

92. Work is defined as dot product of force and displacement $W = \int dW = \int \vec{F} \cdot \vec{dS}$. It is a scalar quantity. The total work done will depend on the displacement and the force, which may be constant or variable. Thus in different situations of variable force applied in column I, the final expression for work done can be expressed as in column II.

Column I	Column II
(A) Force constant in magnitude acts at constant angle θ with direction of motion	**1.** $\frac{1}{2}kx^2 \cos\theta$
(B) Force constant in magnitude acts at angle θ which varies as $\theta = kx$	**2.** $\frac{F}{K}\sin\theta$
(C) Force varies with distance *x* as $F = kx$ but angle θ is constant	**3.** $Fx\sin\theta$
(D) Force is constant in magnitude but changes in direction with changing angle and always acts along radius of circular path	**4.** zero

a. A → 1, B → 2, C → 3, D → 4
b. A → 3, B→2, C → 1, D → 4
c. A → 4, B → 3, C → 2, D → 1
d. A → 2, B → 1, C → 4, D → 3

93. A simple pendulum is vibrating with maximum angle of reflection θ_0. If l is length of pendulum and m is mass of bob, then match the statements in column I with corresponding expressions in column II for any deflection $\theta < \theta_0$.



Column I	Column II
(A) Potential energy	**1.** $mg(\cos\theta - \cos g\theta_0)$
(B) Kinetic energy	**2.** $mgl(1-\cos\theta)$
(C) Momentum	**3.** $m\sqrt{2gl(\cos\theta - \cos\theta_0)}$
(D) Momentum at mid point	**4.** $m\sqrt{2gh}$

a. A → 2, B → 1, C → 3, D → 4
b. A → 4, B → 3, C → 2, D → 1
c. A → 1, B → 2, C → 3, D → 4
d. A → 3, B → 4, C → 2, D → 1

94. There is a definite relation between velocity, mass and acceleration of body to know about the work done by the forces applied on body. Column I have some statements showing some relations which are related with some statements in column II. Match the correct options.

Column I	Column II
(A) If kinetic energy is K the momentum P is 0	**1.** zero
(B) If momentum p is zero the kinetic energy is 0	**2.** $\sqrt{2mk}$
(C) If different masses have same kinetic energy the ratio of their momenta is 0	**3.** $\frac{m_2}{m_1}$
(D) If different masses have same momentum, the ratio of their kinetic energy is	**4.** $\sqrt{\frac{m_1}{m_2}}$

a. A → 1, B → 2, C → 3, D → 4
b. A → 2, B → 1, C → 4, D → 3
c. A → 4, B → 3, C → 2, D → 1
d. A → 3, B → 2, C → 1, D → 4

95. Compare the units of work in column I with their value in terms of S.I. units and branch of physics in which they are used in column II.

Column I	Column II
(A) kWh	**1.** $3.6\times10^6 J$ (B.O.T. for electricity)
(B) Calorie	**2.** 4.2 J unit of heat
(C) Erg	**3.** 10^{-7}, C.G.S, unit of mechanical work
(D) Electron volt	**4.** $1.6\times10^{-19} J$. atomic physics

a. A → 4, B → 3, C → 2, D → 1
b. A → 1, B → 2, C → 3, D → 4
c. A → 4, B → 2, C → 3, D → 1
d. A → 3, B → 4, C → 1, D → 2

Integer

96. A bullet fired into a fixed target loses half of its velocity after penetrating through 3cm. How much further will it penetrate before coming to rest, assuming that it faces constant resistance to motion?

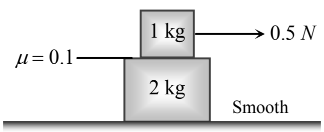
97. A particle moves with a velocity of $(2\hat{i} - 3\hat{j} + \hat{k})$ under the influence of a constant force $\vec{F}(5\hat{i} + 2\hat{j} - 2\hat{k})N$. The instantaneous power applied to the particle is:

98. The potential energy of a 2 kg particle free to move along the x-axis is given by

$$V(x) = \left[\frac{x^4}{4} - \frac{x^2}{2}\right] \text{Joule}\cdot$$

The total mechanical energy of the particle is 0.75 J. The maximum speed of the particle is:

99. A force of 0.5 N is applied on upper block as shown in Fig. The work done by upper block on lower block for a displacement of 3 m is:



100. A long spring, when stretched by x cm has a potential energy u. On increasing the length of spring by stretching it to 3 x cm, the potential energy stored will be n times, where n is:

ANSWER

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	a	d	d	b	a	d	b	d	a
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	d	d	c	d	a	a	c	d	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
b	d	d	b	a	a,c	b	c	d	d
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
a	b	c	d	b	c	a	b	d	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
c	c	c	a	c	c	c	c	a	b
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
b,d	a,c	a,b,c	a,c	all	b,c	c,d	b,c,d	all	a,b,c
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
b,c,d	a,b	a,b	a	d	b	b	a	c	a
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
d	c	a	a	d	d	a	a	b	a
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
b	a	d	b	b	d	d	b	a	a
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
b	b	a	b	b	1	2	1	1	9

SOLUTION

Multiple Choice Questions

1. **(c)** Work done $= \vec{F}.\vec{r} = (5i + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7J$

2. **(a)** Work done to displace it horizontally
$= F \times s = 8 \times 1 = 8J$

Work done to raise it vertically
$F \times s = mgh = 1 \times 10 \times 2 = 20J$

$\therefore$ Net work done = 8 + 20 = 28 J

3. **(d)** Work done by centripetal force in circular motion is always equal to zero.

4. **(d)** Work done
$$= \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)dx$$
$$= [7x - x^2 + x^3]_0^5$$
$$= 35 - 25 + 125 = 135J$$

5. **(b)** Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}C x_1^2$

6. **(a)** Work done on the mass = mass × covered area between the graph and displacement axis on a-t graph.
$$= 10 \times \frac{1}{2}(8 \times 10^{-2}) \times 20 \times 10^{-2}$$
$$= 8 \times 10^{-2} J.$$

7. **(d)** Initial kinetic energy of the body
$$= \frac{1}{2}mu^2 = \frac{1}{2} \times 25 \times (2)^2 = 50J$$
Final kinetic energy = Initial energy – work done against resistive force (Area between graph and displacement axis)
$$= 50 - \frac{1}{2} \times 4 \times 20 = 50 - 40 = 10J.$$

8. **(b)** As gravitational field is conservative in nature. So work done in moving a particle from A to B does not depends upon the path followed by the body. It always remains same.

9. **(d)** $v_1 = \sqrt{4^2 + 16^2} = \sqrt{272}$ and $v_2 = \sqrt{8^2 + 20^2} = \sqrt{464}$

Work done = Increase in kinetic energy
$$= \frac{1}{2}m[v_2^2 - v_1^2] = \frac{1}{2} \times 0.01[464 - 272] = 0.96J.$$

10. **(a)** 700 k cal $= 700 \times 10^3 \times 4.2\,J$
$$= \frac{700 \times 10^3 \times 4.2}{3.6 \times 10^6} = 0.81kWh \text{ [As } 3.6 \times 10^6 J = 1kWh\text{]}$$

11. **(c)** Elastic potential energy stored in wire
$$U = \frac{1}{2}Fx = \frac{Mgl}{2}.$$

12. **(d)** As the forces are working at right angle to each other therefore net force on the body $F = \sqrt{4^2 + 3^2} = 5N$

Kinetic energy of the body = work done = F × s
$$= F \times \frac{1}{2}at^2 = F \times \frac{1}{2}\left(\frac{F}{m}\right)t^2$$
$$= 5 \times \frac{1}{2}\left(\frac{5}{10}\right)(10)^2 = 125J.$$

13. **(d)** $E = \frac{P^2}{2m}$

$\Rightarrow$ $\frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{2P}{P}\right)^2 = 4$

$E_2 = 4E_1 = E_1 + 3E_1 = E_1 + 300\%$ of E_1.

14. **(c)** If the kinetic energy would become half, then Potential energy $= \frac{1}{2}$ (Initial kinetic energy)

$\Rightarrow$ $mgh = \frac{1}{2}[490]$

$\Rightarrow$ $2 \times 9.8 \times h = \frac{1}{2}[490]$

$\Rightarrow$ $h = 12.5\,m$

15. **(d)** Change in momentum $= P_2 - P_1 = F \times t$

$\Rightarrow$ $P_2 = P_1 + F \times t = 10 + 0.2 \times 10 = 12 kg\text{-}m/s$

Increase in kinetic energy $E = \frac{1}{2m}[P_2^2 - P_1^2]$

$= \frac{1}{2m}[(12)^2 - (10)^2] = \frac{1}{2 \times 5}[144 - 100] = \frac{44}{10} = 4.4 J.$

16. **(a)** If the vehicle stops by retarding force then the ratio of stopping distance $\frac{x_1}{x_2} = \frac{E_1}{E_2}$.

But in the given Illustration kinetic energy of bus and car are given same i.e. $E_1 = E_2$.

$\therefore$ $x_1 = x_2$.

17. **(a)** Retarding force (F) × distance covered (x)

= Kinetic energy $\left(\frac{1}{2}mv^2\right)$

If v and F are constants then $x \propto m$

$\therefore$ $\frac{x_2}{x_1} = \frac{m_2}{m_1} = \frac{1.5m}{m} = 1.5$

$\Rightarrow$ $x_2 = 1.5 s.$

18. **(c)** As $s = \frac{v^2}{2a}$

$\therefore$ $s \propto v^2$.

19. **(b)** $F = \frac{-dU}{dx} = -\frac{d}{dx}(A - Bx^2) = 2Bx$

$\therefore$ $F \propto x$.

20. **(c)** As slope of graph is positive and constant upto distance a then it becomes zero. Therefore from $F = -\frac{dU}{dx}$ we can say that upto distance a force will be constant (negative) and suddenly it becomes zero.

21. **(b)** $F = -\frac{dU}{dx} = -\frac{d}{dx}(8x^2 - 4x + 400)$

For the equilibrium condition $F = -\frac{dU}{dx} = 0$

$\Rightarrow$ $16x - 4 = 0$

$\Rightarrow$ $x = 4/16$

$\therefore$ $x = 0.25 m.$

22. **(d)** If the spring is stretched by length x, then work done by two equal masses $= \frac{1}{2}kx^2$

So, Work done by each mass on the spring $= \frac{1}{4}kx^2$

$\therefore$ Work done by spring on each mass $= -\frac{1}{4}kx^2$.

23. **(d)** Elastic potential energy of a spring $U = \frac{1}{2}kx^2$

$\therefore$ $U \propto x^2$

So $\frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2$

$\Rightarrow$ $\frac{U_2}{U} = \left(\frac{10 cm}{2 cm}\right)^2 \Rightarrow U_2 = 25U$

24. **(b)** When spring is gradually lowered to it's equilibrium position $kx = mg$

$\therefore$ $x = \frac{mg}{k}$

When spring is allowed to fall suddenly it oscillates about it's mean position Let y is the amplitude of vibration then at lower extreme, by the conservation of energy

$\Rightarrow$ $\frac{1}{2}ky^2 = mgy$

$\Rightarrow$ $y = \frac{2mg}{k} = 2x$

25. **(a)** When α particle moves towards the mercury nucleus its kinetic energy gets converted in potential energy of the system. At the distance of closest approach

$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\Rightarrow$ $\frac{1}{2} \times (1.6 \times 10^{-27})(10^7)^2$

$= 9 \times 10^9 \frac{(2.e)(80e)}{r}$

$\Rightarrow$ $r = 1.115 \times 10^{-13} m.$

26. **(a, c)** If A and B are free to move, no external forces are acting and hence P and E both are conserved but when B is fixed (with the help of an external force) then E is conserved but P is not conserved.

27. **(b)** When the rod is lying on a horizontal table, its potential energy = 0

But when we make its stand vertical its centre of mass rises upto high $\frac{l}{2}$. So it's potential energy $= \frac{mgl}{2}$

$\therefore$ Work done = charge in potential energy

$= mg\frac{l}{2} - 0 = \frac{mgl}{2}$.

28. **(c)** Gravitational force on bucket at starting position

$= mg = 10 \times 10 = 100$ N

Gravitational force on bucket at final position = 8 × 10 = 80 N

So, the average force throughout the vertical motion

$= \frac{100+80}{2} = 90\,N$

∴ Work done = Force × displacement = 90 × 10 = 900 J.

29. **(d)** As 1/3 part of the chain is hanging from the edge of the table.

So, by substituting n = 3 in standard expression

$W = \frac{MgL}{2n^2} = \frac{MgL}{2(3)^2} = \frac{MgL}{18}$

30. **(d)** For the given condition potential energy of the two masses will convert into heat and temperature of water will increase $W = JQ$

⇒ $2m \times g \times h = J(m_w S\,\Delta t)$

⇒ $2 \times 5 \times 10 \times 10 = 4.2(2 \times 10^3 \times \Delta t)$

∴ $\Delta t = \frac{1000}{8.4 \times 10^3} = 0.119^o C = 0.12^o C$

31. **(a)** When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring get compressed due to this energy.

∴ Gravitational potential energy of block = Elastic potential energy of spring

⇒ $mgh = \frac{1}{2}Kx^2$

⇒ $x = \sqrt{\frac{2mgh}{K}} = \sqrt{\frac{2 \times 2 \times 10 \times 0.4}{1960}} = 0.09\,m \simeq 0.1\,m.$

32. **(b)** At the maximum height, Total energy = Potential energy = mgh

At the height $\frac{3h}{4}h$, Potential energy $= mg\frac{3h}{4} = \frac{3}{4}mgh$

and Kinetic energy = Total energy – Potential energy

$= mgh - 3\frac{mgh}{4} = \frac{1}{4}mgh$

∴ $\frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{3}.$

33. **(c)** The engine has to do work against resistive force R as well as car is moving with acceleration a. Power = Force × velocity = $(R + ma)v$.

34. **(d)** Weight of a bus = mass × g

$= 100 \times 100\,kg \times 10\,m/s^2 = 10^5\,N$

Total friction force = 10% of weight = $10^4\,N$

∴ Power = Force × velocity

$= 10^4\,N \times 72\,km/h$

$= 10^4 \times 20\,watt$

$= 2 \times 10^5\,watt = 200\,kW$

35. **(b)** $F = \frac{mdv}{dt}$

∴ $F\,dt = mdv \Rightarrow v = \frac{F}{m}t$

Now $P = F \times v = F \times \frac{F}{m}t = \frac{F^2 t}{m}$

If force and mass are constants then $P \propto t$.

36. **(c)** $P = \vec{F}.\vec{v}$

$= (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k})$

$= 50 - 30 + 120 = 140\,J\text{-}s^{-1}$

37. **(a)** Power (P) = $\frac{mgh}{t}$ or $P \propto \frac{m}{t}$

⇒ $\frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{t_2}{t_1} = \left(\frac{5}{3}\right)\left(\frac{9}{11}\right) = \frac{45}{33} = \frac{15}{11}$ (g and h are constants)

38. **(b)** As the ball rebounds with same velocity therefore change in velocity = $2u$ and the mass colliding with the surface per second = nm

Force experienced by the surface $F = m\frac{dv}{dt}$

∴ $F = 2\,mnu.$

39. **(d)** Initial velocities of balls are $+V$ and $-2V$ respectively and we know that for given condition velocities get interchanged after collision. So the velocities of two balls after collision are $-2V$ and V respectively.

40. **(c)** Final velocity of the target

$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

As initially target is at rest so by substituting $u_2 = 0$

we get $v_2 = \frac{2Mu}{M+m} = \frac{2u}{1 + \frac{m}{M}}$.

41. **(c)** Loss of K.E. in inelastic collision

$\Delta K = \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}(u_1 - u_2)^2$

$= \frac{1}{2}\frac{40 \times 60}{(40+60)}(4-2)^2 = \frac{1}{2}\frac{2400}{100}4 = 48\,J.$

42. **(c)** $\frac{v_1}{v_2} = \frac{1-e}{1+e} = \frac{1-1/2}{1+1/2} = \frac{1/2}{3/2} = \frac{1}{3}$.

43. **(c)** Momentum before collision = mv,

Momentum after collision = $-mv$

∴ Change in momentum

$= 2mv = 2 \times 60 \times 10^{-3} \times 4 = 480 \times 10^{-3}\, kg\text{-}m/s = 0.48\, kg\text{-}m/s$

44. **(a)** It loses 20% energy in impact and only 80% energy remains with the ball

So ball will rise upto height $h_2 = 80\%$ of $h_1 = \frac{80}{100} \times 20 = 16\,m$

Now, coefficient of restitution $e = \sqrt{\frac{h_2}{h_1}}$

$= \sqrt{\frac{16}{20}} = \sqrt{0.8} = 0.89$.

45. **(c)** If ball falls from height h_1, then it collides with ground with speed

$v_1 = \sqrt{2gh_1}$ . . . (*i*)

and if it rebound with velocity v_2, then it goes upto height h_2 from ground,

$v_2 = \sqrt{2gh_2}$ . . . (*ii*)

From (*i*) and (*ii*) $\frac{v_2}{v_1} = \sqrt{\frac{2gh_2}{2gh_1}} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

46. **(c)** $m_1 = 1.67 \times 10^{-27}\, kg$,

$u_1 = 10^8\, m/s$,

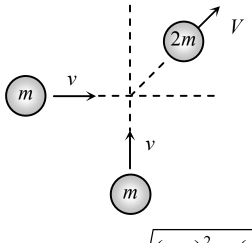
$m_2 = 3.34 \times 10^{-27}\, kg$ and $u_2 = 0$

Speed of the combination $V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

$= \frac{1.67 \times 10^{-27} \times 10^8 + 0}{1.67 \times 10^{-27} + 3.34 \times 10^{-27}}$

$= 3.33 \times 10^7\, m/s$.

47. **(c)** Initially both the particles are moving perpendicular to each other with momentum mv.



So, the net initial momentum $= \sqrt{(mv)^2 + (mv)^2} = \sqrt{2}\, mv$.

After the inelastic collision both the particles (system) moves with velocity V,

So, linear momentum = $2mV$

By the law of conservation of momentum $\sqrt{2}\, mv = 2mV$

∴ $V = v/\sqrt{2}$.

48. **(c)** By the conservation of momentum

$mv + 2m \times 0 = 3mV$

∴ $V = \frac{v}{3}$.

49. **(a)** By conservation of momentum,

Momentum of the bullet (mv) = momentum of the composite block $(m + M)V$

⇒ Velocity of composite block $V = \frac{mv}{m+M}$

∴ Kinetic energy $= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$

$= \frac{1}{2}\frac{m^2v^2}{m+M} = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$.

50. **(b)** By the conservation of momentum Initial momentum = Final momentum $mv + M \times 0 = m\frac{v}{2} + M \times V$

⇒ $V = \frac{m}{2M}v$

If block rises upto height h then

$h = \frac{V^2}{2g} = \frac{(mv/2M)^2}{2g} = \frac{m^2v^2}{8M^2g}$

NCERT Exemplar Problems

More than One Answer

51. **(b, d)** Initially, there will be collision between C and A which is elastic; so by conservation of momentum, we have

$m\upsilon_0 = m\upsilon_A + m\upsilon_C$ or $\upsilon_0 = \upsilon_A + \upsilon_C$ . . .(*i*)

and as in elastic collision KE after collision is same as before collision; hence

$\frac{1}{2}m\upsilon_0^2 = \frac{1}{2}m\upsilon_A^2 + \frac{1}{2}m\upsilon_C^2$ *i.e.,* $\upsilon_0^2 = \upsilon_A^2 + \upsilon_C^2$ . . .(*ii*)

Subtracting eq. (*ii*) from the square of eq. (*i*), we have $2\upsilon_A\upsilon_C = 0$ So, either $\upsilon_A = 0$ or $\upsilon_C = 0$

$\upsilon_A = 0$ corresponds to interaction between A and B, so the only physically possible solution is $\upsilon_C = 0$, which in the light of eq. (*i*) gives $\upsilon_A = \upsilon_0$, *i.e.,* after collision C stops and A starts moving with velocity υ_0. Now A will move and compress the spring which in true accelerations B and retards A and finally both A and B will move with same velocity (say V). In this situation, compression of the spring will be maximum.

As external force is zero, momentum of the system ($A + B$ + spring) is conserved, *i.e.,* $m\upsilon = (m+m)V$

i.e., $V = \frac{\upsilon}{2}$

So, KE of the AB system at maximum compression

$= \frac{1}{2} \times 2m \times \left(\frac{\upsilon}{2}\right)^2 = \frac{1}{4} m\upsilon^2$

By conservation of mechanical energy

$\frac{1}{2} m\upsilon^2 = \frac{1}{2}(m+m)V^2 + \frac{1}{2} kx_0^2$

or $m\upsilon^2 - 2m(\upsilon/2)^2 = kx_0^2$

i.e., $k = \frac{m\upsilon^2}{2x_0^2}$, or $x_0 = \upsilon\sqrt{m/2k}$

52. **(a, c)** Since, force on the vehicle is constant, therefore, it will move with a constant acceleration, Let this acceleration be a, Then at time t, its velocity will be equal to $\upsilon = at$. Hence, at time t, the kinetic energy,

$\text{KE} = \frac{1}{2} m\upsilon^2 = \frac{1}{2} ma^2t^2$.

It means, graph between E and t will be a parabola of increasing slope. Hence option (c) is correct.

The power associated with the force is equal to $P = Fu = Fat$. Hence, the graph between power and time will be a straight line passing through the origin. Hence option (a) is also correct.

53. **(a, b, c)** Rate of work done is the power associated with the force. It means rate of work done by the gravitational force is the power associated with the gravitational force. Gravitational force acting on the block is equal to its weight mg which acts vertically downwards. Velocity of the particle, at time t, has two components

(a) horizontal component $\upsilon = \cos\theta$ and (b) a vertically upward component $(\upsilon_0 \sin\theta - gt)$

Hence, the power associated with the weight mg will be equal to $P = m\vec{g} \cdot \vec{\upsilon} = -mg(\upsilon_0 \sin\theta - gt)$

This shows that the curve between power and time will be a straight line having positive slope but negative intercept on y-axis. Hence, only option (d) is correct.

54. **(a, c)** If the body slips over a rough surface such that normal reaction of the surface has to balance only the normal component of weight of the body, then energy lost against friction depends only upon the horizontal component of displacement and is equal to μmgx. It does not depend upon shape of the surface.

If vertical component of displacement of the body is equal to y, increase in its gravitational potential energy will be equal to mgy. Hence total work done by the force will be equal to $(\mu mgx + mgy)$.

55. **(a, b, c, d)** When the particle rise, its potential energy increase. At height y from the ground, potential energy of the particle is equal to mgy. But total energy of the particle will remain conserved. Hence, at height y, kinetic energy of the particle will be equal to: $E = (initial\ KE - mgy)$

It means graph between E and y will be a straight line having a negative slope and having an intercept on y-axis equal to initial KE. Therefore option (a) is correct.

At time t, height of the particle from the ground will be equal to: $y = [(u\sin\theta)t - 1/2gt^2]$

Hence, at time t, potential energy of the particle will be equal to $mg\left[(u\sin\theta)t - 1/2\, gt^2\right]$.

Therefore, at time t, KE of the particle will be equal to

$E = [Initial\ KE - mg(u\sin 0.t - 1/2gt^2)]$

This shows the graph between t and E will be a parabola. Hence, option (b) is correct. Since, horizontal displacement of the particle $x = (u\cos\theta)t$

therefore, $x \propto t$

Hence, graph between E and x will have the same shape as graph between E and t has, therefore, option (c) is correct.

KE of the particle is equal to $\frac{1}{2} m\upsilon^2$ while momentum p is equal to $m\upsilon$.

From this equation $E = \frac{p^2}{2m}$. It means, $E \propto p^2$. Hence, the graph between E and p^2 will be a straight line which passes through the origin. In fact, the minimum magnitude of momentum of the particle is equal to $mu\cos\theta$. Therefore, the graph between E and p^2 should start form minimum possible value of p^2. Dotted portion of straight line curve (d) shows that this portion does not exist. Hence, option (d) is correct.

56. **(b, c)** The acceleration of block P and Q are:

$a_1 = \frac{m_1 g\sin\theta_1}{m_1} = g\sin\theta_1$ and $a_2 = \frac{m_2 g\sin\theta_2}{m_2} = g\sin\theta_2$

Since, $\theta_2 > \theta_1; a_2 > a_1$.

Now PE of block P at $A = m_1 gh$.

Its KE on reaching the bottom $=\frac{1}{2}m_1\upsilon_1^2$

Equation the both, we get; $\frac{1}{2}m_1\upsilon_1^2=m_1gh$ or $\upsilon_1=\sqrt{2gh}$

Similarly, for block Q, $\upsilon_2=\sqrt{2gh}$

Since, $\upsilon_1=\upsilon_2$, both blocks will reach the bottom with the same speed, Now, $\upsilon=a_1t_1$. ($\because u=0$) and $\upsilon_2=a_2t_2$.

But $\upsilon_1=\upsilon_2$

Therefore $a_1t_1=a_2t_2$ or, $(t_1/t_2)=(a_2/a_1)$

Since, $a_2>a_1; t_1>t_2$; *i.e.,* block P takes a longer time to reach the bottom. Hence, the correct choices are (b) and (c).

57. **(c, d)** (a) Neither momentum nor kinetic energy are conserved so collision is not elastic. Hence, option (a) is not correct. (b) Same difficulty lies with option (b).

58. **(b, c, d)** Using definition of coefficient of restitution,

$e=\frac{\upsilon_2-\upsilon_1}{\upsilon}$ or $\upsilon_2-\upsilon_1=e\upsilon$ . . .(*i*)

Applying law of conservation of momentum,

$m\upsilon=m\upsilon_1+m\upsilon_2$ or $\upsilon_2+\upsilon_1=\upsilon$ . . .(*ii*)

Adding eq. (*i*) and (*ii*)

$2\upsilon_2=(1-e)\upsilon$ or $\upsilon_2=\frac{1+e}{2}\upsilon$ . . .(*iii*)

Subtracting eq. (*i*) from eq. (*ii*), we get

$2\upsilon_1=(1-e)\upsilon$ or $\upsilon_1=\frac{1-e}{2}\upsilon$ . . .(iv)

Dividing eq. (*iv*) by (*iii*), give $\frac{\upsilon_1}{\upsilon_2}=\frac{1-e}{1+e}$

So, option (c) is correct,

From equation (*iv*), $\frac{\upsilon_1}{\upsilon_2}=\frac{1-e}{2}$

So, option (b) is correct

From equation (*iii*), $\frac{\upsilon_2}{\upsilon}=\frac{1+e}{2}$

So, option (c) is correct.

59. **(a, b, c, d)** (a) Loss of gravitational PE = mgh

$=1.0\times10\times1=10$ J

(b) KE of 1 kg block $=(1/2)m\upsilon^2$

$=(1/2)\times1\times(0.3)^2=0.045$ J

(b) work done against friction $=\mu mgs$

$=\mu\times4.0\times10\times2=80\,\mu$ joule

(c) Loss gravitation PE – Work done against friction = KE of two blocks $10-80\,\mu=0.045+\frac{1}{2}\times4\times(0.6)^2$

$10-0.045-80\mu=(1/2)\times4\times0.6\times0.6$

Solving we get $\mu=0.12$

60. **(a, b, c)** Each square has a value $2s\times5N=10N_{-s}$

(a) Area under the curve between 8 s and 12 s = – 1 square = – $10N_{-s}$

(b) Area for $t=8$ s to $t=16s$ = – 3 square $=-30\,N_{-s}$

As mass = 2 kg, hence velocity change

$=\frac{-30\,N_{-s}}{2\,kg}=-15$ m s^{-1}

(c) Area for $r=0$ to $t=6s$ = 10 square $=100\,N_{-s}$

Kinetic energy $=\frac{p^2}{2m}=\frac{100\times100}{2\times2}=2500\,j$

63. **(a, b)** At point A, potential energy of the ball $=mgh_A$

At point B, potential energy of the ball = 0

At point C, potential energy of the ball $=mgh_C$

Total energy at point A, $E_A=K_A+mgh_A$

Total energy at point B, $E_B=K_B$

Total energy at point C, $E_C=K_C+mgh_C$

According to the law of conservation of energy

$E_A=E_B=E_C$

$E_A=E_B\Rightarrow K_B>K_A$ . . .(*i*)

$E_B=E_C\Rightarrow K_B>K_C$ . . .(*ii*)

$E_A=E_C\Rightarrow K_A+mgh_A=K_C+mgh_C$

or $h_A-h_C=\frac{K_C-K_A}{mg}$ . . .(*iii*)

If $h_A>h_C$; then $K_C>K_A$ . . .(*iv*)

Hence option (b) is correct.

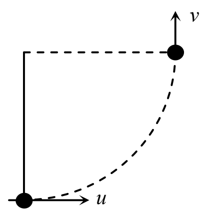
From eqs. (*i*), (*ii*) and (*iv*), we get

If $h_A>h_C$; then $K_B>K_C$ So, option (a) is also correct.

64. **(a)** The increase in height of man relative to earth = $(l-h)$

$\therefore$ gain in P.E. of man $=mg(l-h)$

65. **(d)** From energy conservation $v^2=u^2-2gL$ . . . (*i*)



Now, since the two velocity vectors shown in figure are mutually perpendicular, hence the magnitude of change of velocity will be given by $|\Delta\vec{v}| = \sqrt{u^2 + v^2}$

Substituting value of v^2 from Eq. (*i*)

$|\Delta\vec{v}| = \sqrt{u^2 + u^2 - 2gL} = \sqrt{2(u^2 - gL)}$

66. **(b)** Gain in KE = Loss in GPE

$\frac{1}{2}m\upsilon^2 = mgh = mgR(1 - \cos\theta)$

or, $\frac{m\upsilon^2}{R} = 2mg(-\cos\theta)$

Condition of losing contact $\frac{m\upsilon^2}{R} = mg\cos\theta$

Hence, $mg\cos\theta = 2mg(1 - \cos\theta)$

$\therefore \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$

Assertion and Reason

67. **(b)** It is clear from Fig. that $PQ = l\cos\theta_1$ and $PR = l\cos\theta_2$. Therefore, $h_1 = l - l\cos\theta_1 = l(1 - \cos\theta_1)$ and $h_2 = l(1 - \cos\theta_2)$. Let m be the mass of the bob and υ be its speed when it reaches position B. Then, from the principle of conservation of energy, K.E. at B = loss of P.E. as the bob moves from A to B.

Hence $\frac{1}{2}m\upsilon^2 = mgh_1 - mgh_2$

$= mg[l(1 - \cos\theta_1) - l(1 - \cos\theta_2)]$

$= mgl(\cos\theta_2 - \cos\theta_1)$

$\Rightarrow \quad \upsilon = \sqrt{2gl(\cos\theta_2 - \cos\theta_1)}$

68. **(a)** Let C be the centre of mass of the rod AB so that $AC = l/2$. Let h be the height of C above the ground. In triangle ACD, we have $CD = AC\sin(90^\circ - \theta)$.

Or $h = \frac{1}{2}\cos\theta$. Since the entire mass of the rod can be assumed to be concentrated at the centre of mass, therefore, potential energy = work done to raise the rod from horizontal position on the ground to the position shown in the figure $= mgh = \frac{1}{2}mgl\cos\theta$.

69. **(c)** Statement-1 is true. The decrease in mechanical energy is smaller when the block is made to go up on the inclined surface because some part of the kinetic energy is converted into gravitational potential energy. Statement-2 is false. The coefficient of friction does not depend on the angle of inclination of the plane.

70. **(a)** Since the velocity is uniform, the man exerts no net force on the bucket in the direction of motion. The only force he exerts on the bucket is against gravity (to overcome) the weight *mg* of the bucket) and this force is perpendicular to the displacement (*i.e.* θ = 90°). Hence $W = FS\cos 90° = 0$.

71. **(d)** The two cranes do the same amount of work = *mgh*. Hence they consume the same amount of fuel. Crane *P* does the same amount of work in half the time. Hence crane *P* supplies two times more power than crane *Q*.

72. **(c)** The acceleration of blocks *P* and *Q* respectively are

$a_1 = \frac{m_1 g\sin\theta_1}{m_1} = g\sin\theta_1$

$a_2 = \frac{m_2 g\sin\theta_2}{m_2} = g\sin\theta_2$

Since $\theta_2 > \theta_1$; $a_2 > a_1$. The potential energy of block *P* at $A = m_1gh$. When it reaches the bottom *B*, its kinetic energy is $\frac{1}{2}m_1\upsilon_1^2$ where υ_1 is its speed when it reaches B. Now P.E at A = K.E. at B. Hence

$m_1gh = \frac{1}{2}m_1\upsilon_1^2 \Rightarrow \upsilon_1 = \sqrt{2gh}$.

Similarly $m_2gh = \frac{1}{2}m_2\upsilon_2^2 \Rightarrow \upsilon_2 = \sqrt{2gh} = \upsilon_1$.

73. **(a)** If t_1 and t_2 are the times taken by *P* and *Q* to reach the bottom, then

$\upsilon_1 = u_1 + a_1t_1 = a_1t_1 \qquad (\because u_1 = 0)$

and $\upsilon_2 = u_2 + a_2t_2 = a_2t_2 \qquad (\because u_2 = 0)$

Now, $\upsilon_1 = \upsilon_2$. Hence $a_1t_1 = a_2t_2$.

Thus $\frac{t_1}{t_2} = \frac{a_2}{a_1}$ Since $a_2 > a_1; t_1 > t_2$.

74. **(a)** For a conservation force, the work done in moving a body from one point to another does not depend on the nature of the path and the work done over a closed path is zero, irrespective or the nature of the path.

75. **(d)** Statement-1 is false; the total energy of an isolated system is conserved. Statement-2 is true.

76. **(d)** Statement-1 is false and Statement-2 is true. The rate of change of momentum is proportional to the net external force acting on the system.

77. **(a)** Potential energy stored in the spring when it is extended by x is $U_1 = \frac{1}{2}kx^2$

Potential energy stored in the spring when it is further extended by x is

$U_2 = \frac{1}{2}k(x+x)^2 = 2kx^2$

$\therefore$ Work done = gain in potential energy = $U_2 - U_1$

$= 2kx^2 - \frac{1}{2}kx^2 = \frac{1}{2}kx^2$

78. **(a)** Linear momentum will be conserved if A comes to rest and B and C move to the right with a velocity $\upsilon/2$ each or A, B and C all move to the right with velocity $\upsilon/3$ each. It is easy to see that in these two cases, the kinetic energy is not conserved. Hence the only result of the collision is the one given in Assertion.

89. **(b)** Let C collide with A at t = 0. Since the collision in elastic and A and C have equal masses, C will come to rest and A will move to the right with velocity u and at this instant the spring is uncompressed and B is at rest. Hence at t = 0, the momentum of the system = mu. When A moves to the right, it compresses the spring and as a result B beings to move to the right. Let υ be the common velocity of A and B at time t_0. From the principle of conservation of linear momentum, we have Momentum of C before collision = momentum of A after collision + momentum of B after collision or

$mu = m\upsilon + (2m)\upsilon \Rightarrow \upsilon = \frac{u}{3}$.

80. **(a)** The total energy (which includes all forms of energy) is conserved in any process.

81. **(b)** Assertopm follows from Newton's third law of motion.

Comprehension Based

83. **(d)** Net area of graph = Area AOB – $Area\ BCDE$ + Area $EFGH$ $= \frac{1}{2} \times 20 \times 20 - 10 \times 20 + 20 \times 20 = 400\,\text{J}$

84. **(b)** From 0 to 20 m, W_1 = Area AOB = + ve = + 200 J

From 20 to 40 m W_2 = Area AOB = – ve = – 200 J

Total work done upto 40 m displacement $W_1 + W_2 = 0$

85. **(b)** During displacement 0 to 20 m and 40 to 60 m. The force acting is accelerating force hence work done is positive. During 20 m to 40 m the force acting is retarding force.

86. **(d)** Work done under conservative force is independent of path followed and gravitational force is conservative force.

87. **(d)** Work done is zero in both cases because work done is zero over a closed path under conservative forces.

88. **(b)** Work is done against friction. If μ is coefficient of friction then work done is

$W = 2F_\mu \times l = 2mg\sin\theta \times l = 2mgh.$

$(\therefore F_\mu = \mu R = \mu mg\cos\theta = mg\sin\theta \text{ and } \sin\theta = \frac{h}{l})$

89. **(a)** For conservation force $F = -\frac{dU}{dx}$

Thus where slope is $\frac{dU}{dx} = -\upsilon e$,

$F = +\upsilon e$, *i.e.,* at B and C

90. **(a)** For particle to be in equilibrium sum of all forces is zero which is possible if $F = \left|\frac{dU}{dx}\right| = 0$ or U is maximum or minimum which is corresponding to point A, D, G.

91. **(b)** Anybody is in stable equilibrium if it is in state of minimum potential energy.

Match the Column

92. **(b)** $W = \vec{F}\cdot\vec{S} = Fx\cos\theta$

$\therefore$ Displacements $\vec{S} = \vec{x}$ while angle θ between $\vec{F}$ and $\vec{x}$ is constant $\theta = kx$, hence $\frac{d\theta}{dx} = K$ or $dx = \frac{d\theta}{k}$

$\therefore$ $\int dW = \int \vec{F}\cdot\overrightarrow{dS}$

$= \int \vec{F}\cdot\overrightarrow{dx} = \int F\cdot\cos\theta\cdot\frac{d\theta}{K} = \frac{F}{K}\sin\theta$

$W = \int dW = \int \vec{F}\cdot\overrightarrow{dx} = \int \overrightarrow{Kx}\cdot\overrightarrow{dx}$

$= K\int x\,dx\cos\theta = \frac{1}{2}Kx^2\cos\theta$

$\vec{F} \perp \vec{x}$ at every point

$\therefore$ $dW = \vec{F}\cdot\overrightarrow{dx} = 0$

$\therefore$ $\theta = 90° = Fdx\cos 90° = 0$

93. **(a)** Potential energy at Q relative to Q is mgh'

Here $PS = h'$

$\therefore$ $OS = l - h'OQ\cos\theta = l\cos\theta$

or, $h' = l - l\cos\theta - l(1-\cos\theta)$

$\therefore$ $mgh' = mgl(1-\cos\theta)$

Total energy of bob at extreme position

$R = mgl(1-\cos\theta_0)$

= Maximum potential energy = Kinetic energy at mean position

= Kinetic energy at any point Q + Potential energy at Q

$\therefore$ K.E. at $Q = T.E. - P.E.$

$= mgl(1-\cos\theta_0) - mgl(1-\cos\theta)$

$= mgl(1-\cos\theta-\cos\theta_0) = \frac{1}{2}m\upsilon^2$

$p = \sqrt{2m \times \text{K.E.}} = \sqrt{2m \times mgl(\cos\theta - \cos\theta_0)}$

$p = \sqrt{2m \times K.E.} = \sqrt{2 \times m \times mgh} = m\sqrt{2gh}$

94. **(b)** Kinetic energy $K = \frac{1}{2}m\upsilon^2$

or, $K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$

If $K = 0$, $p = 0$ as above

For different mass m_1 and m_2 if $E_1 = E_2$

$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2}$ or $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

If $p_1 = p_2$ for different masses.

$\frac{E_1}{E_2} = \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2} = \frac{m_2}{m_1}$

95. **(b)** This is a knowledge based question. Units are defined as practical units with their suitability of use in different branches of physics.

Integer

96. **(1 cm)** Let m be the mass of bullet and υ be its initial velocity. If F is constant resistance to motion, then

(i) Work done = loss in KE = force × distance

$\frac{1}{2}m\left(\upsilon^2 - \frac{\upsilon^2}{4}\right) = F \times 3$

or $\frac{1}{2}m\left(\frac{3\upsilon^2}{4}\right) = F \times 3$ . . . (*i*)

(ii) Work done = loss in KE = force × distance

$= \frac{1}{2}m\left(\frac{\upsilon^2}{4} - 0\right) = F \times s$ . . . (*ii*)

Dividing (*ii*) by (*i*), we get $\frac{1}{3} = \frac{s}{3}$

$\Rightarrow$ $s = 1$ cm.

97. **(2 watt)** Here, $\upsilon = (2\hat{i} - 3\hat{j} + \hat{k})$ ms^{-1}

$\vec{F} = (5\hat{i} + 2\hat{j} - 2\hat{k})\ N$; $P = ?$

$P = \vec{F} \cdot \vec{\upsilon} = (5\hat{i} + 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= 10 - 6 - 2 = 2$ watt.

98. **(1 m/s)** When speed is maximum, KE is maximum; PE is minimum.

As $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J = minimum

$\therefore$ $\frac{dV(x)}{dx} = 0$

$4\frac{x^3}{4} - \frac{2x}{2} = 0$

$\therefore$ $x = 0$ or $x = \pm 1$

$V_{min}(\text{at } x = \pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} = -0 \cdot 25$ J

As $K_{max} + V_{min}$ = Mechanical energy

$\therefore$ $K_{max} = ME - V_{min}$

$\frac{1}{2}m\upsilon_{max}^2 = 0.75 - (-0.25) = 1J$

$\upsilon_{max} = \sqrt{\frac{2 \times 1}{m}} = \sqrt{\frac{2}{2}} = 1$ m/s

99. **(1 J)** Force of friction between the two blocks, Fig.

$f = \mu R = \mu mg = 0 \cdot 1 \times 1 \times 10 = 1N$

As applied force $F = 0 \cdot 5N > f \cdot$

$\therefore$ The two blocks move together.

Common acceleration, $a = \frac{F}{1+2} = \frac{0 \cdot 5}{3} = \frac{1}{6}$ m/s$^2 \cdot$

$\therefore$ Force applied by upper block on lower block

$F' = ma = 2a = 2 \times \frac{1}{6} = \frac{1}{3}N$

Work done by upper block on lower block for displacement of $3m = F3' \times s$

$= \frac{1}{3} \times 3 = 1J.$

100. **(9)** As P.E. $\propto$ (stretch)^2

$\therefore$ P.E. becomes 3^2 times *i.e.*, 9 times.

* * *