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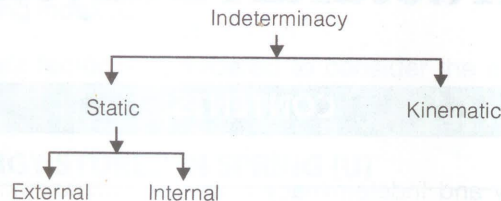
DETERMINACY AND INDETERMINACY

STATICALLY DETERMINATE STRUCTURES

Conditions of equilibrium are sufficient to analyse the structure. Bending moment and shear force is independent of the cross-sectional areas of the components and flexural rigidity of the material. No stresses are caused due to temperature changes. No stresses are caused due to lack of fit.

STATICALLY INDETERMINATE STRUCTURES

Additional compatibility conditions are required. Bending moment and shear force depends upon the cross-sectional area and EI of the material. Stresses are caused due to temperature variation. Stresses are caused due to lack of fit.



STATIC INDETERMINACY

If a structure can not be analyzed for external and internal reactions using equilibrium conditions alone then such a structure is called indeterminate structure.

$$(i) \quad D_s = D_{se} + D_{si}$$

where, D_s = Degree of static indeterminacy

D_{se} = External static indeterminacy

D_{si} = Internal static indeterminacy

- **External static indeterminacy:**

It is related with the support system of the structure and it is equal to number of external reaction components in addition to number of equilibrium conditions.

$$(ii) \quad D_{se} = r_e - 3 \quad \rightarrow \text{For 2D}$$

$$D_{se} = r_e - 6 \quad \rightarrow \text{For 3D}$$

where, r_e = total external reactions

- **Internal static indeterminacy:**

It refers to the geometric stability of the structure. If after knowing the external reactions it is not possible to determine all internal forces/internal reactions using equilibrium conditions alone then the structure is said to be internally indeterminate.

For geometric stability sufficient number of members are required to preserve the shape of rigid body without excessive deformation.

$$(iii) \quad D_{si} = 3C - r_r \quad \dots \text{For 2D}$$

$$D_{si} = 6C - r_r \quad \dots \text{For 3D}$$

where, C = number of closed loops.

and r_r = released reaction.

$$(iv) \quad r_r = \Sigma(m_{j'} - 1) \quad \dots \text{For 2D}$$

$$r_r = 3\Sigma(m_{j'} - 1) \quad \dots \text{For 3D}$$

where $m_{j'}$ = number of member connecting with J' number of joints.

and J' = number of hybrid joint.

$$(v) \quad D_s = m + r_e - 2j \quad \dots \text{For 2D truss}$$

$$D_{se} = r_e - 3 \quad \& \quad D_{si} = m - (2j - 3)$$

$$(vi) \quad D_s = m + r_e - 3j \quad \dots \text{For 3D truss}$$

$$D_{se} = r_e - 6 \quad \& \quad D_{si} = m - (3j - 6)$$

$$(vii) \quad D_s = 3m + r_e - 3j - r_r \quad \dots \text{2D Rigid frame.}$$

$$(viii) \quad D_s = 6m + r_e - 6j - r_r \quad \dots \text{3D rigid frame.}$$

$$(ix) \quad D_s = (r_e - 6) + (6C - r_r) \quad \dots \text{3D rigid frame}$$

KINEMATIC INDETERMINACY

If the number of unknown displacement components are greater than the number of compatibility equations, for these structures additional equations based on equilibrium must be written in order to obtain sufficient number of equations for the determination of all the unknown displacement components. The number of these additional equations necessary is known as degree of kinematic indeterminacy or degree of freedom of the structure.

A fixed beam is kinematically determinate and a simply supported beam is kinematically indeterminate.

- (i) Each joint of plane pin jointed frame has 2 degree of freedom.
- (ii) Each joint of space pin jointed frame has 3 degree of freedom.
- (iii) Each joint of plane rigid jointed frame has 3 degree of freedom
- (iv) Each joint of space rigid jointed frame has 6 degree of freedom.

Degree of kinematic indeterminacy is given by:

- (i) $D_k = 3j - r_e$...For 2D Rigid frame when all members are axially extensible.
- (ii) $D_k = 3j - r_e - m$...For 2D Rigid frame if 'm' members are axially rigid/inextensible.
- (iii) $D_k = 3(j + j') - r_e - m + r_r$... For 2D Rigid frame when $J' =$ Number of Hybrid joints is available.
- (iv) $D_k = 6(j + j') - r_e - m + r_r$...For 3D Rigid frame.
- (v) $D_k = 2(j + j') - r_e - m + r_r$... For 2D Pin jointed truss.
- (vi) $D_k = 3(j + j') - r_e - m + r_r$... For 3D Pin jointed truss.

