Binomial Theorem

INTRODUCTION

Any algebraic expression consisting of only two different terms is known as binomial expression. The terms may consist of variables x, y etc. or constants or their mixed combinations.

For example: $2x + 3y$, $4xy + 5$ etc.

Terminology used in binomial theorem :

Factorial notation : $\underline{|n}$ or n! is pronounced as factorial n and is defined as

$$n! = \begin{cases} n(n-1)(n-2)........3.2.1 & ; \text{ if } n \in N \\ 1 & ; \text{ if } n = 0 \end{cases}$$

Note : $n! = n \,.\, (n-1)!$; $n \in N$

Mathematical meaning of ^nC_r : The term ^nC_r denotes number of combinations of r things choosen from n distinct things mathematically,

$$^nC_r = \frac{n!}{(n-r)!\,r!}, \; n \in N, r \in W, 0 \le r \le n$$

Note : Other symbols of of ^nC_r are $\binom{n}{r}$ and C(n, r).

Properties related to ^nC_r :

(i) $^nC_r = {^nC_{n-r}}$

Note : If $^nC_x = {^nC_y} \Rightarrow$ Either $x = y$ or $x + y = n$

(ii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

(iii) $\dfrac{^nC_r}{^nC_{r-1}} = \dfrac{n-r+1}{r}$

(iv) $^nC_r = \dfrac{n}{r}\,{^{n-1}C_{r-1}} = \dfrac{n(n-1)}{r(r-1)}\,{^{n-2}C_{r-2}} = ...$

$$= \frac{n(n-1)(n-2).........(n-(r-1))}{r\,(r-1)(r-2).......2\,.1}$$

(v) If n and r are relatively prime, then ^nC_r is divisible by n. But converse is not necessarily true.

BINOMIAL THEOREM FOR POSITIVE INDEX

Binomial theorem gives a formula for the expansion of a binomial expression raised to any positive integral power. In general for a positive integer n

$(x + y)^n = {^nC_0}x^n + {^nC_1}x^{n-1}y^1 + {^nC_2}x^{n-2}y^2 + ... + {^nC_n}x^0y^n$,

where $^nC_r = \dfrac{n!}{(n-r)!r!}$

for $r = 0, 1, 2, ..., n$ is called binomial coefficient.

Observations :

(a) The number of terms in the expansion is (n + 1) i.e. one more than the index.

(b) The sum of the indices of x & y in each term is n.

(c) The binomial coefficients of the terms ($^nC_0$, $^nC_1$.........) equidistant from the beginning and the end are equal.

Proof of binomial theorem

The Binomial theorem can be proved by mathematical induction

Let P(n) stands for the mathematical statement

$(x + a)^n = x^n + {^nC_1}\,x^{n-1}\,a + {^nC_2}\,x^{n-2}\,a^2 + ...+ {^nC_r}\,x^{n-r}\,a^r + ...+ a^n$ (i)

Note that there are (n + 1) terms in R.H.S. and all the terms are of the same degree in x and a together.

When n = 1, L.H.S. = x + a and R.H.S. = x + a (there are only 2 terms)

$\therefore$ P (1) is verified to be true

Assume P(m) to be true

i.e., $(x + a)^m = x^m + {^mC_1}\,x^{m-1}\,a + {^mC_2}\,x^{m-2}\,a_2 + ...+ {^mC_r}\,x^{m-r}\,a^r + ...+ a^m$ ...(ii)

Multiplying equation (ii) by (x + a), we have

$(x + a)^m\,(x + a) = (x + a)\,\{x^m + {^mC_1}\,x^{m-1}\,a + {^mC_2}\,x^{m-2}\,a^2 + ... + {^mC_r}\,x^{m-r}\,a^r + ...+ a^m\}$

i.e., $(x+a)^{m+1} = x^{m+1} + ({^mC_1} + 1)x^m a + ({^mC_2} + {^mC_1})x^{m-1}a^2 + ...$

$$...... + ({^mC_r} + {^mC_{r-1}})x^{m-r+1}a^r + ... + a^{m+1}$$

$= x^{m+1} + {^{(m+1)}C_1}\,x^m\, a + {^{(m+1)}C_2}\,x^{m-1}\,a^2 + ...$
$+ {^{m+1}C_r}\,x^{m+1-r}\,a^r + ...\, a^{m+1}$ (iii)

(using the formula $^nC_r + {^nC_{r-1}} = {^{(n+1)}C_r}$)

Equation (iii) implies that P (m + 1) is true and hence by induction P (n) is true.

Alternative method

By choosing x from all the brackets we get the term x^n. Choosing x from (n – 1) factors and 'a' from the remaining factor we get $x^{n-1}\,a$. But the number of ways of doing this is equal to the number of ways of choosing one factor from n factors (i.e.,) $^nC_1$. Choosing x from (n – 2) factor and a from the remaining two factors, we get $x^{n-2}\,a^2$. But the number of ways of doing this is equal to the number of ways of choosing two factors from n factors. i.e., $^nC_2$. Finally choosing 'a' from all the factors we get the term a^n.

$\therefore$ $(x + a)^n = x^n + {^nC_1}\,x^{n-1}\,a + {^nC_2}\,x^{n-2}\,a^2 + ... + {^nC_r}\,x^{n-r}\,a^r + ... + a^n$

SOLVED EXAMPLE

Example-1

The value of

$$\frac{(18^3+7^3+3.18.7.25)}{3^6+6.243.2+15.81.4+20.27.8+15.9.16+6.3.32+64}$$

is

Sol. The numerator is of the form

$a^3+b^3+3ab\,(a+b)=(a+b)^3$ where $a=18$ and $b=7$

$\therefore$ Nr $=(18+7)^3=(25)^3$.

Denominator can be written as

$$3^6+{}^6C_1.3^5.2^1+{}^6C_2.3^4.2^2+{}^6C_3.3^3.2^3+$$

$${}^6C_4 3^2.2^4+{}^6C_5 3.2^5+{}^6C_6 2^6=(3+2)^6=5^6=(25)^3$$

$$\therefore\ \frac{\text{Nr}}{\text{Dr}}=\frac{(25)^3}{(25)^3}=1$$

Example-2

Expand $\left(x-\frac{1}{x}\right)^6$

Sol. $$\left(x-\frac{1}{x}\right)^6={}^6C_0x^6+{}^6C_1x^5\left(\frac{-1}{x}\right)+{}^6C_2x^4\left(\frac{-1}{x}\right)^2$$

$$+{}^6C_3x^3\left(\frac{-1}{x}\right)^3+{}^6C_4x^2\left(\frac{-1}{x}\right)^4$$

$$+{}^6C_5x\left(\frac{-1}{x}\right)^5+{}^6C_6x^0\left(\frac{-1}{x}\right)^6$$

$$=x^6-6x^4+15x^2-20+\frac{15}{x^2}-\frac{6}{x^4}+\frac{1}{x^6}$$

GENERAL TERM IN THE BINOMIAL EXPANSION

The general term in the expansion of $(x+y)^n$ is $(r+1)^{th}$ term, given by $t_{r+1}={}^nC_r x^{n-r} y^r$ where $r=0, 1, 2 \ldots\ldots n$.

- Every term in the expansion is of n^{th} degree in variables x and y.
- The total number of terms in the expansion is $n+1$.
- Binomial expansion can also be expressed as $$(x+y)^n=\sum_{r=0}^{n}{}^nC_r x^{n-r}y^r.$$
- The binomial coefficients of the expansion equidistant from the beginning and the end are equal. In other words ${}^nC_r={}^nC_{n-r}$.
- **TERM INDEPENDENT OF x :** Term independent of x contains no x ; Hence find the value of r for which the exponent of x is zero.

SOLVED EXAMPLE

Example-3

Find the 11^{th} term in the expansion of $\left(3x-\frac{1}{x\sqrt{3}}\right)^{20}$.

Sol. The general term

$$=t_{r+1}=(-1)^r\,{}^{20}C_r\,(3x)^{20-r}\left(\frac{1}{x\sqrt{3}}\right)^r$$

For the 11th term, we must take $r=10$

$$\therefore\ t_{11}=t_{10+1}=(-1)^{10}\,{}^{20}C_{10}(3x)^{20-10}\left(\frac{1}{x\sqrt{3}}\right)^{10}$$

$$={}^{20}C_{10}3^{10}x^{10}\frac{1}{x^{10}\left(\sqrt{3}\right)^{10}}={}^{20}C_{10}\left(\sqrt{3}\right)^{10}={}^{20}C_{10}\,3^5$$

Example-4

Find the term independent of x in $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

Sol. The general term $={}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(\frac{-1}{3x}\right)^r$

$$=(-1)^r\ {}^9C_r\ \frac{3^{9-2r}}{2^{9-r}}.x^{18-3r}$$

The term independent of x, (or the constant term) corresponds to x^{18-3r} being

x^0 or $18-3r=0 \quad \Rightarrow r=6$

$\therefore$ the term independent of x is the 7^{th} term and its value is

$$(-1)^6\ {}^9C_6\frac{3^{9-12}}{2^{9-6}}={}^9C_3\frac{3^{-3}}{2^3}=\frac{9.8.7}{3.2.1}\cdot\frac{1}{(6)^3}=\frac{7}{18}$$

Example-5

Find the coefficient of x^7 in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$

Sol. In the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$, the general terms is

$$T_{r+1}={}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r={}^{11}C_r.\frac{a^{11-r}}{b^r}.x^{22-3r}$$

putting $22-3r=7\Rightarrow 3r=15\Rightarrow r=5$

$$\therefore T_6={}^{11}C_5\frac{a^6}{b^5}.x^7$$

Hence the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is ${}^{11}C_5\,a^6b^{-5}$.

Example-6

Find the number of rational terms in the expansion of $(9^{1/4}+8^{1/6})^{1000}$.

Sol. The general term in the expansion of $(9^{1/4}+8^{1/6})^{1000}$ is

$$T_{r+1} = {}^{1000}C_r\left(9^{\frac{1}{4}}\right)^{1000-r}\left(8^{\frac{1}{6}}\right)^{r} = {}^{1000}C_r 3^{\frac{1000-r}{2}} 2^{\frac{r}{2}}$$

The above term will be rational if exponent of 3 and 2 are integers. i.e. $\frac{1000-r}{2}$ and $\frac{r}{2}$ must be integers

The possible set of values of r is {0, 2, 4, ...1000}.
Hence, number of rational terms is 501

MIDDLE TERMS OF THE EXPANSION

In the binomial expansion of $(x+y)^n$

WHEN n IS ODD

There are $(n+1)$ i.e. even terms in the expansion and hence two middle terms are given by

$$t_{\frac{n+1}{2}} = {}^nC_{\frac{n-1}{2}}X^{\frac{n+1}{2}}y^{\frac{n-1}{2}} \text{ for } r=\frac{n-1}{2}$$

and $t_{\frac{n+3}{2}} = {}^nC_{\frac{n+1}{2}}X^{\frac{n-1}{2}}y^{\frac{n+1}{2}}$ for $r=\frac{n+1}{2}$

WHEN n IS EVEN

There are odd terms in the expansion and hence only one middle term is given by

$$t_{\frac{n}{2}+1} = {}^nC_{n/2}\,x^{n/2}y^{n/2} \text{ for } r=\frac{n}{2}$$

Note : The greatest binomial coefficient in the expansion is always the binomial coefficient of middle term/terms.

Example-7

Find the middle term in the expression of $(1-2x+x^2)^n$.

Sol. $(1-2x+x^2)^n=[(1-x)^2]^n=(1-x)^{2n}$

Here $2n$ is even integer, therefore, $\left(\frac{2n}{2}+1\right)^{\text{th}}$

i.e. $(n+1)^{\text{th}}$ term will be the middle term.

Now $(n+1)^{\text{th}}$ term in $(1-x)^{2n}={}^{2n}C_n(1)^{2n-n}(-x)^n$

$$= {}^{2n}C_n(-x)^n = \frac{(2n)!}{n!n!}(-x)^n$$

Example-8

Find the middle term in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$

Sol. The number of terms in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$ is 10 (even). So there are two middle terms.

i.e. $\left(\frac{9+1}{2}\right)$th and $\left(\frac{9+3}{2}\right)$th two middle terms. They are given by T_5 and T_6

$$\therefore\ T_5=T_{4+1}={}^9C_4(3x)^5\left(-\frac{x^3}{6}\right)^4$$

$${}^9C_4 3^5x^5.\frac{x^{12}}{6^4}=\frac{9\cdot8\cdot7\cdot6}{1\cdot2\cdot3\cdot4}\cdot\frac{3^5}{2^43^4}x^{17}=\frac{189}{8}x^{17}$$

and $T_6=T_{5+1}={}^9C_5(3x)^4\left(-\frac{x^3}{6}\right)^5=-{}^9C_4 3^4.x^4\frac{x^{15}}{6^5}$

$$=\frac{-9.8.7.6}{1.2.3.4}.\frac{3^4}{2^5.3^5}x^{19}=-\frac{21}{16}x^{19}$$

Example-9

The term independent of x in $\left[\sqrt{\left(\frac{3}{2x^2}\right)}+\sqrt{\frac{x}{3}}\right]^{10}$ is

Sol. General term in the expansion is

$${}^{10}C_r\left(\frac{x}{3}\right)^{\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{10-r}{2}} = {}^{10}C_r x^{\frac{3r}{2}-10}.\frac{3^{5-r}}{2^{\frac{10-r}{2}}}$$

For constant term, $\frac{3r}{2}=10$

$\Rightarrow r=\frac{20}{3}$ which is not an integer. Therefore, there will be no constant term.

Example-10

If the coefficient of $(2r+4)^{\text{th}}$ term and $(r-2)^{\text{th}}$ term in the expansion of $(1+x)^{18}$ are equal, find r.

Sol. Since coefficient of $(2r+4)^{\text{th}}$ term in $(1+x)^{18}={}^{18}C_{2r+3}$.

Coefficient of $(r-2)^{\text{th}}$ term $={}^{18}C_{r-3}$

$\Rightarrow\ {}^{18}C_{2r+3}={}^{18}C_{r-3}$

$\Rightarrow\ 2r+3+r-3=18$

$\Rightarrow\ 3r=18$

$\Rightarrow\ r=6.$

GREATEST TERM IN THE EXPANSION

The numerically greatest term (absolute value) in the expansion of $(1 + x)^n$ is determined by the following process:

Let t_{r+1} be the numerically greatest term, then $|t_r| \le |t_{r+1}| \ge |t_{r+2}|$

Considering $\left|\frac{t_{r+1}}{t_r}\right| \ge 1 \Rightarrow \left|\frac{n-r+1}{r}x\right| \ge 1$

$\Rightarrow \quad \frac{n-r+1}{r}|x| \ge 1 \Rightarrow (n-r+1)|x| \ge r$

$\Rightarrow r \le (n+1)\frac{|x|}{1+|x|} \qquad \ldots(i)$

Also $\left|\frac{t_{r+1}}{t_{r+2}}\right| \ge 1 \Rightarrow \left|\frac{r+1}{n-r}\frac{1}{x}\right| \ge 1 \Rightarrow r+1 \ge (n-r)|x|$

$\Rightarrow \quad r \ge \frac{n|x|-1}{1+|x|}$

$\Rightarrow \quad r \ge (n+1)\frac{|x|}{1+|x|} - 1 \qquad \ldots(ii)$

Thus from (i) and (ii) for t_{r+1} to be the greatest term,

$r \le m$ and $r \ge m-1$ where $m = (n+1)\frac{|X|}{1+|X|}$

For expansion of $(a+b)^n$

Case - I When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is an integer (say m), then

(i) $T_{r+1} > T_r$ when $r < m$ $(r = 1, 2, 3 \ldots, m-1)$
i.e. $T_2 > T_1, T_3 > T_2, \ldots, T_m > T_{m-1}$

(ii) $T_{r+1} = T_r$ when
$r = m$
i.e. $T_{m+1} = T_m$

(iii) $T_{r+1} < T_r$ when
$r > m$ $(r = m+1, m+2, \ldots n)$
i.e. $T_{m+2} < T_{m+1}$, $T_{m+3} < T_{m+2}$, $\ldots T_{n+1} < T_n$

Conclusion :

When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is an integer, say m, then T_m and T_{m+1} will be numerically greatest terms (both terms are equal in magnitude)

Case - II

When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is not an integer (Let its integral part be m), then

(i) $T_{r+1} > T_r$ when $r < \frac{n+1}{1+\left|\frac{a}{b}\right|}$ $(r = 1, 2, 3, \ldots, m-1, m)$

i.e. $T_2 > T_1$, $T_3 > T_2$, ..., $T_{m+1} > T_m$

(ii) $T_{r+1} < T_r$ when $r > \frac{n+1}{1+\left|\frac{a}{b}\right|}$ $(r = m+1, m+2, \ldots n)$

i.e. $T_{m+2} < T_{m+1}$, $T_{m+3} < T_{m+2}$, ..., $T_{n+1} < T_n$

Conclusion :

When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is not an integer and its integral part is m, then T_{m+1} will be the numerically greatest term.

Note : (i) In any binomial expansion, the middle term(s) has greatest binomial coefficient.

In the expansion of $(a+b)^n$

If n	No. of greatest binomial coeff.	Greatest binomial coefficient
Even	1	${}^nC_{n/2}$
Odd	2	${}^nC_{(n-1)/2}$ and ${}^nC_{(n+1)/2}$

(Values of both these coefficients are equal)

(ii) In order to obtain the term having numerically greatest coefficient, put a = b = 1, and proceed as discussed above.

In general, for the expansion $(a + x)^n$ or $a^n\left(1+\frac{x}{a}\right)^n$

we can consider $m = (n+1)\frac{|x/a|}{1+|x/a|}$.

SOLVED EXAMPLE

Example-11

Find the greatest term in the expansion of $(4 + 3x)^7$ when $x = \frac{2}{3}$.

Sol. Here the greatest term means the numerically greatest term.

$$\left|\frac{t_r+1}{t_r}\right| = \frac{{}^7C_r 4^{7-r}(3x)^r}{{}^7C_{r-1}4^{8-r}(3x)^{r-1}} = \frac{8-r}{r}\frac{3x}{4} = \frac{8-r}{2r} \text{ since } x = 2/3$$

Now $|t_{r+1}| \ge |t_r|$ if $8 - r \ge 2r$ or $\frac{8}{3} \ge r$

This inequality is valid only for $r = 1$ or 2

Thus for $r = 1, 2$; $|t_{r+1}| > |t_r|$ and

for $r = 3, 4$; $|t_{r+1}| < |t_r|$

$\therefore \quad |t_1| < |t_2| < |t_3| > |t_4| > |t_5| > \ldots\ldots$

greatest term $= |t_3| = {}^7C_2 4^5.(3x)^2$ where $x = \frac{2}{3}$

$= 21 \times 4^5 \times 2^2 = 86016$

SUMMATION OF SERIES INCLUDING BINOMIAL COEFFICIENTS

In the binomial expansion of $(1 + x)^n$, let us denote the coefficients

${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ……, ${}^nC_r$,….., ${}^nC_n$ by $C_0, C_1, C_2, \ldots\ldots, C_r, \ldots\ldots, C_n$ respectively.

- **The sum of the binomial coefficients in the expansion of $(1 + x)^n$ is 2^n :**

$\because$ $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$

Putting $x = 1$

$\therefore$ $2^n = C_0 + C_1 + C_2 + \ldots\ldots + C_n$ …(i)

or $\sum_{r=0}^{n} C_r = 2^n$.

- **The sum of the coefficients of the odd terms in the expansion of $(1 + x)^n$ is equal to the sum of the coefficients of the even terms and each is equal to 2^{n-1}**

Since $(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots + C_nx^n$

Putting $x = -1$,

$0 = C_0 - C_1 + C_2 - C_3 + \ldots\ldots + (-1)^n C_n$

and $2^n = C_0 + C_1 + C_2 + C_3 + \ldots\ldots + C_n$ {from (i)}

Adding and subtracting these two equations, we get

$2^n = 2\,(C_0 + C_2 + C_4 + \ldots\ldots)$

and $2^n = 2(C_1 + C_3 + C_5 + \ldots..)$

$\therefore$ $C_0 + C_2 + C_4 + \ldots\ldots = C_1 + C_3 + C_5 + \ldots. = 2^{n-1}$

sum of coefficients of odd terms = sum of coefficients of even terms = 2^{n-1}

Some Results on Binomial Coefficients

(a) $C_0 + C_1 + C_2 + \ldots\ldots\ldots + C_n = 2^n$

(b) $C_0 + C_2 + C_4 + \ldots\ldots\ldots = C_1 + C_3 + C_5 + \ldots\ldots = 2^{n-1}$

(c) $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots\ldots + C_n^2 = {}^{2n}C_n = \dfrac{(2n)!}{n!n!}$

(d) $C_0.C_r + C_1.C_{r+1} + C_2.C_{r+2} + \ldots + C_{n-r}C_n = \dfrac{(2n)!}{(n+r)!(n-r)!}$

Remember : $(2n)! = 2^n.\, n!\, [1.3.5\ldots\ldots(2n-1)]$

SOLVED EXAMPLE

Example-12

Prove that the sum of the coefficients in the expansion of $(1 + x - 3x^2)^{2163}$ is -1.

Sol. Putting $x = 1$ in $(1 + x - 3x^2)^{2163}$, the required sum of coefficients

$= (1 + 1 - 3)^{2163} = (-1)^{2163} = -1.$

Example-13

If the sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficients in the expansion of $(x - \alpha y)^{35}$, then find the value of α.

Sol. Sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$

= Sum of the coefficients in the expansion of $(x - \alpha y)^{35}$

Putting $x = y = 1$

$\therefore$ $(\alpha - 1)^{35} = (1 - \alpha)^{35}$

$\Rightarrow$ $(\alpha - 1)^{35} = -(\alpha - 1)^{35}$

$\Rightarrow$ $2(\alpha - 1)^{35} = 0$

$\therefore$ $\alpha - 1 = 0$ $\therefore \alpha = 1$

- **Differentiation can be used to solve series in which each term is a product of an integer and a binomial coefficient i.e. in the form $k\,.\,{}^nC_r$.**

Example-14

Show that $3\,.\,C_0 + 7\,.\,C_1 + 11\,.\,C_2 + \ldots.. + (4n + 3)\,C_n = (2n + 3)\,2^n$.

Sol. This problem can be done be differentiating the expansion of $x^3(1 + x^4)^n$ and putting $x = 1$.

$x^3\,(1 + x^4)^n = x^3\,(C_0 + C_1x^4 + C_2x^8 + \ldots.. + C_nx^{4n})$

$= C_0x^3 + C_1x^7 + C_2x^{11} + \ldots.. + C_nx^{4n+3}$

Differentiating we get,

$\Rightarrow$ $3x^2\,(1 + x^4)^n + x^3n\,(1 + x^4)^{n-1}\,4x^3$

$= 3x^2C_0 + 7x^6C_1 + 11x^{10}C_2 + \ldots. + (4n + 3)x^{4n+2}C_n$

Now substituting $x = 1$ in both sides.

$\Rightarrow$ $3C_0 + 7C_1 + 11C_2 + \ldots\ldots + (4n + 3)\,C_n$

$= 3(2^n) + 4n\,(2)^{n-1} = (3 + 2n)\,2^n$

Example-15

Show that

$C_1 - 2\,.\,C_2 + 3\,.\,C_3 - 4\,.\,C_4 + \ldots.. + (-1)^{n-1}\,n\,.\,C_n = 0.$

Sol. The problem can be done by differentiating the expansion of $(1 + x)^n$ and then putting $x = -1$.

2nd Method :

L.H.S. $= {}^nC_1 - 2.\,{}^nC_2 + 3.\,{}^nC_3 - \ldots\ldots + (-1)^{n-1}.\,n.\,{}^nC_n$

$= n\,\{1 - {}^{(n-1)}C_1 + {}^{(n-1)}C_2 - {}^{n-1}C_3 + \ldots. + (-1)^{n-1}\,{}^{(n-1)}C_{n-1}\}$

$= n\,(1 - 1)^{n-1} = n \times 0 = 0$

- **Integration can be used to solve series in which each term is a binomial coefficient divided by an integer i.e. in the form $\frac{^nC_r}{k}$.**

Example-16

Show that

$$2.C_0 + 2^2.\frac{C_1}{2} + 2^3.\frac{C_2}{3} + ... + 2^{n+1}\frac{C_n}{n+1} = \frac{3^{(n+1)}-1}{n+1}$$

Sol. Integrating the expansion of $(1+x)^n$ between the limits 0 to 2.

$$\int_0^2 (1+x)^n\,dx = \int_0^2 (C_0 + C_1x + ... + C_nx^n)\,dx$$

$$\Rightarrow \left.\frac{(1+x)^{n+1}}{n+1}\right|_0^2 = \left. C_0x + C_1\frac{x^2}{2} + ... + C_n\frac{x^{n+1}}{n+1}\right|_0^2$$

$$\Rightarrow 2.C_0 + 2^2.\frac{C_1}{2} + 2^3.\frac{C_2}{3} + ... + 2^{n+1}\frac{C_n}{n+1} = \frac{3^{(n+1)}-1}{n+1}$$

2nd Method : L.H.S.

$$= \frac{1}{n+1}\left\{^{(n+1)}C_1.2 + ^{(n+1)}C_2.2^2 + ^{(n+1)}C_3.2^3 + ... + ^{(n+1)}C_{n+1}.2^{(n+1)}\right\}$$

$$= \frac{1}{n+1}\left\{1 + ^{(n+1)}C_1.2 + ^{(n+1)}C_2.2^2 + ... + ^{(n+1)}C_{n+1}.2^{(n+1)} - 1\right\}$$

$$= \frac{1}{(n+1)}\left\{(1+2)^{n+1} - 1\right\} = \frac{3^{n+1}-1}{n+1}$$

- **Product of two expansions can be used to solve some problems related to series of binomial coefficients in which each term is a product of two binomial coefficients.**

Example-17

If $(1+x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$ then prove that $\sum\sum_{0\le i<j\le n} (C_i + C_j)^2 = (n-1)^{2n}C_n + 2^{2n}$

Sol. L.H.S $\sum\sum_{0\le i<j\le n} (C_i + C_j)^2 = (C_0 + C_1)^2 + (C_0 + C_2)^2 + ...$

$.....+ (C_0 + C_n)^2 + (C_1 + C_2)^2 + (C_1 + C_3)^2 + ...+ (C_1 + C_n)^2$
$+ (C_2 + C_3)^2 + (C_2 + C_4)^2 + ...+ (C_2 + C_n)^2 + ...+ (C_{n-1} + C_n)^2$

$$= n\,(C_0^2 + C_1^2 + C_2^2 + ...+ C_n^2) + 2\sum\sum_{0\le i<j\le n} C_iC_j$$

$$= n.^{2n}C_n + 2.\left\{2^{2n-1} - \frac{2n!}{2.n!n!}\right\}$$

$$= n.\,^{2n}C_n + 2^{2n} - {}^{2n}C_n = (n-1).\,^{2n}C_n + 2^{2n} = \text{R.H.S.}$$

IMPORTANT RESULTS

If $\left(\sqrt{P}+Q\right)^n = I + f$ where *I* and *n* are positive integers, *n* being odd, and $0 \le f < 1$, then show that $(I + f)\,f = k^n$ where $P - Q^2 = k > 0$ and $\sqrt{P} - Q < 1$.

Proof. Given $\sqrt{P} - Q < 1$

$\therefore\quad 0 < (\sqrt{P} - Q)^n < 1$

Now let $\left(\sqrt{P} - Q\right)^n = f'$ where $0 < f' < 1$

$\therefore\quad I + f - f' = (\sqrt{P}+Q)^n - \left(\sqrt{P}-Q\right)^n$

$\because\quad$ RHS contains even powers of $\sqrt{P}$ ($\because$ *n* is odd)

$\Rightarrow\quad$ RHS is an integer

Since RHS and I are integers,

$\therefore\quad f - f'$ is also integer.

$\therefore\quad \Rightarrow f - f' = 0$

$\because -1 < f - f' < 1$

or $\quad f = f'$

$$\therefore\ (I+f)f = (I+f)f' = (\sqrt{P}+Q)^n(\sqrt{P}-Q)^n = (P-Q^2)^n = k^n$$

If $\left(\sqrt{P}+Q\right)^n = I + f$ where *I* and *n* are positive integer, *n* being even, and $0 \le f < 1$, then show that $(I + f)(1 - f) = k^n$ where $P - Q^2 = k > 0$ and $\sqrt{P} - Q < 1$.

Proof : If *n* is an even integer then

$$\left(\sqrt{P}+Q\right)^n + (\sqrt{P}-Q)^n = I + f + f'$$

Hence LHS and I are integer.

$\therefore\quad f + f'$ is also integer.

$\Rightarrow\quad f + f' = 1 \quad \because 0 < f + f' < 2$

$\therefore\quad f' = (1 - f)$

Hence

$$(I+f)(1-f) = (I+f)f' = (\sqrt{P}+Q)^n\left(\sqrt{P}-Q\right)^n$$

$= (P - Q^2)^n = k^n$.

SOLVED EXAMPLE

Example-18

If $\left(2+\sqrt{3}\right)^n = I + f$ where I and n are positive integers and $0 < f < 1$, show that

(i) I is an odd integer and

(ii) $(I + f)(1 - f) = 1$.

Sol. (i) Now $0 < 2 - \sqrt{3} < 1$, since $2 - \sqrt{3} = 0.268$ (approx.)

$\therefore \quad 0 < (2-\sqrt{3})^n < 1;$

we can take $\left(2-\sqrt{3}\right)^n$ as f '.

Now $(2+\sqrt{3})^n + (2-\sqrt{3})^n = I + f + f'$

But L.*H*.S.

$= 2\left\{2^n + {}^nC_2 2^{n-2}(\sqrt{3})^2 + {}^nC_4 2^{n-4}(\sqrt{3})^4 + ...\right\}$ = an integer

(in fact an even integer)

$\therefore$ RHS. = I + f + f' = an even integer

Also f + f' = 1, since f and f ' are both positive proper fractions.

$\therefore$ I = an even integer – 1 = an odd integer.

(ii) $(1+f)(1-f) = (1+f)(f') = (2+\sqrt{3})^n.(2-\sqrt{3})^n$

$= (4-3)^n = 1^n = 1$

MULTINOMIAL EXPANSION

If $n \in N$, then the general term of the multinomial expansion $(x_1 + x_2 + x_3 + ... + x_k)^n$ is

$$\frac{n!}{a_1!a_2!a_3!.....ak!} x_1^{a_1}.x_2^{a_2}.x_3^{a_3}.....x_k^{a_k}$$

where $a_1 + a_2 + a_3 + ... + a_k = n$

and $0 \le a_i \le n, i = 1, 2, 3, ... k.$

and the number of terms in the expansion are ${}^{n+k-1}C_{k-1}$.

The greatest coefficient in the expansion of

$\left(a_1 + a_2 + a_3 + ... + a_m\right)^n$ is $\dfrac{n!}{(q!)^{m-r}\left((q+1)!\right)^r}$

where q is the quotient and r is the remainder when n is divided by m.

SOLVED EXAMPLE

Example-19

Find the co-efficient of $a^2b^3c^5$ in the expansion of $\left(a+b+c\right)^{10}$.

Sol. General term is $\dfrac{10!}{a_1!a_2!a_3!}(a)^{a_1}(b)^{a_2}(c)^{a_3}$

$\therefore \quad a_1 = 2, a_2 = 3, a_3 = 5$

$\therefore$ Co-efficient of at $a^2b^3c^5 = \dfrac{10!}{2!3!5!}$.

BINOMIAL THEOREM FOR ANY INDEX

If $n \in R, -1 < x < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ... + \frac{n(n-1)(n-2)....(n-r+1)}{r!}x^r + ...\infty$$

Note:

(i) In the expansion if $n \in \boldsymbol{R}$ and $n > 0$ then $-1 < x < 1$.

(ii) ${}^nC_r$ can not be used because it is defined only for natural number.

(iii) If x be so small then its square and higher powers may be neglected, then approximate value of $(1+x)^n = 1 + nx$.

Important results:

- $(1+x)^{-1} = 1 - x + x^2 - x^3 + ... + (-1)^r x^r + ...$
- $(1-x)^{-1} = 1 + x + x^2 + x^3 + ... + x^r + ...$
- $(1+x)^{-2} = 1 - {}^2C_1x + {}^3C_2X^2 - {}^4C_3x^3 + ... + (-1)^r {}^{r+1}C_rx^r + ...$
- $(1-x)^{-2} = 1 + {}^2C_1x + {}^3C_2x^2 + {}^4C_3x^3 + ... + {}^{r+1}C_rx^r + ...$
- $(1+x)^{-3} = 1 - {}^3C_1x + {}^4C_2x^2 - {}^5C_3x^3 + ... + (-1)^r {}^{r+2}C_rx^r + ...$
- $(1-x)^{-3} = 1 + {}^3C_1x + {}^4C_2x^2 + {}^5C_3x^3 + ... + {}^{r+2}C_rx^r + ...$

In general, the coefficient of x^n in $(1-x)^{-k}$ is ${}^{n+k-1}C_{k-1}$

APPROXIMATIONS

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 + ...$$

If x < 1, the terms of the above expansion go on decreasing and if x be very small, a stage may be reached when we may neglect the terms containing higher powers of x in the expansion. Thus, if x be so small that its squares and higher powers may be neglected then $(1+x)^n = 1 + nx$, approximately,

This is an approximate value of $(1+x)^n$

SOLVED EXAMPLE

Example-20

If x is so small such that its square and higher powers may be neglected then find the approximate value of

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{(4+x)^{1/2}}$$

Sol. $\dfrac{(1-3x)^{1/2} + (1-x)^{5/3}}{(4+x)^{1/2}} = \dfrac{1 - \frac{3}{2}x + 1 - \frac{5x}{3}}{2\left(1+\frac{x}{4}\right)^{1/2}}$

$$= \frac{1}{2}\left(2 - \frac{19}{6}x\right)\left(1+\frac{x}{4}\right)^{-1/2} = \frac{1}{2}\left(2-\frac{19}{6}x\right)\left(1-\frac{x}{8}\right)$$

$$= \frac{1}{2}\left(2 - \frac{x}{4} - \frac{19}{6}x\right) = 1 - \frac{x}{8} - \frac{19}{12}x = 1 - \frac{41}{24}x$$

Example-21

The value of cube root of 1001 upto five decimal places is

Sol. $(1001)^{1/3}=(1000+1)^{1/3}=10\left(1+\frac{1}{1000}\right)^{1/3}$

$$=10\left\{1+\frac{1}{3}.\frac{1}{1000}+\frac{1/3(1/3-1)}{2!}\frac{1}{1000^2}+......\right\}$$

$= 10\ \{1+0.0003333-0.00000011+......\}=10.00333$

EXPONENTIAL SERIES

(a) e is an irrational number lying between 2.7 & 2.8. Its value correct upto 10 places of decimal is 2.7182818284.

(b) Logarithms to the base 'e' are known as the Napierian system, so named after Napier, their inventor. They are also called **Natural Logarithm.**

(c) $e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+......\infty$; where x may be any real or complex number & $e=\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

(d) $a^x=1+\frac{x}{1!}\ell na+\frac{x^2}{2!}\ell n^2a+\frac{x^3}{3!}\ell n^3a+.........\infty$ where $a>0$

(e) $e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+......\infty$

LOGARITHMIC SERIES

(a) $\ell n\ (1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+........\infty$ where $-1<x\le 1$

(b) $\ell n(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+........\infty$ where $-1\le x<1$

Remember : (i) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+........\infty=\ell n\,2$

(ii) $e^{\ell n x}=x$ **(iii)** $\ell n2=0.693$ **(iv)** $\ell n\,10=2.303$

EXERCISE-I

General Term

Q.1 6^{th} term in expansion of $\left(2x^2-\frac{1}{3x^2}\right)^{10}$ is

(1) $\frac{4580}{17}$ (2) $-\frac{896}{27}$

(3) $\frac{5580}{17}$ (4) None of these

Q.2 If the coefficients of r^{th} term and $(r+4)^{th}$ term are equal in the expansion of $(1+x)^{20}$, then the value of r will be

(1) 7 (2) 8 (3) 9 (4) 10

Q.3 16^{th} term in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is

(1) $136xy^7$ (2) $136xy$

(3) $-136xy^{15/2}$ (4) $-136xy^2$

Q.4 In $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$ if the ratio of 7^{th} term from the beginning to the 7th term from the end is 1/6 , then n is equal to

(1) 7 (2) 8 (3) 9 (4) 5

Q.5 If x^4 occurs in the r^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then r =

(1) 7 (2) 8 (3) 9 (4) 10

Q.6 If the third term in the binomial expansion of $(1+x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m is

(1) 2 (2) 1/2 (3) 3 (4) 4

Q.7 Coefficient of x in the expansion of $\left(x^2+\frac{a}{x}\right)^5$ is

(1) $9a^2$ (2) $10a^3$ (3) $10a^2$ (4) $10a$

Q.8 In the expansion of $\left(x-\frac{1}{x}\right)^6$, the constant term is

(1) –20 (2) 20 (3) 30 (4) –30

Q.9 In the expansion of $\left(x^2-2x\right)^{10}$, the coefficient of x^{16} is

(1) –1680 (2) 1680

(3) 3360 (4) 6720

Q.10 The coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is

(1) ${}^{15}C_5$ (2) ${}^{15}C_6$ (3) ${}^{15}C_4$ (4) ${}^{15}C_7$

Q.11 If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is

(1) 56 (2) 55 (3) 45 (4) 15

Q.12 In the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$, the term independent of x is

(1) ${}^9C_3\cdot\frac{1}{6^3}$ (2) ${}^9C_3\left(\frac{3}{2}\right)^3$

(3) ${}^9C_3$ (4) ${}^9P_2\left(\frac{3}{2}\right)^3$

Q.13 The term independent of x in the expansion of $\left(x^2-\frac{3\sqrt{3}}{x^3}\right)^{10}$ is

(1) 153090 (2) 150000 (3) 150090 (4) 153180

Q.14 The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

(1) $C_0^2+2C_1^2+....+(n+1)C_n^2$

(2) $(C_0+C_1+....+C_n)^2$

(3) $C_0^2+C_1^2+.....+C_n^2$

(4) $C_0^2+2C_1^2+....+nC_n^2$

Middle Term / Numerically greatest term and Algebrically greatest & least term

Q.15 If the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then $n=$

(1) 10 (2) 12 (3) 14 (4) 16

Q.16 The middle term in the expansion of $(1+x)^{2n}$ is

(1) $\frac{(2n)!}{n!}x^2$ (2) $\frac{(2n)!}{n!(n-1)!}x^{n+1}$

(3) $\frac{(2n)!}{(n!)^2}x^n$ (4) $\frac{(2n)!}{(n+1)!(n-1)!}x^n$

Q.18 The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$, is

(1) $\frac{1.3.5....(2n-3)}{n!}$ (2) $\frac{1.3.5....(2n-1)}{n!}$

(3) $\frac{1.3.5....(2n+1)}{n!}$ (4) $\frac{1.3.5....(2n+3)}{n!}$

Q.19 In the expansion of $(1+3x+2x^2)^6$ the coefficient of x^{11} is

(1) 144 (2) 288

(3) 216 (4) 576

Properties of Binomial Coefficient

Q.20 $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+....+C_{n-r}C_n=$

(1) $\frac{(2n)!}{(n-r)!(n+r)!}$ (2) $\frac{n!}{(-r)!(n+r)!}$

(3) $\frac{n!}{(n-r)!}$ (4) $\frac{2n!}{(n-r)!}$

Q.21 $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}+....+\frac{C_n}{n+1}=$

(1) $\frac{2^n}{n+1}$ (2) $\frac{2^n-1}{n+1}$

(3) $\frac{2^{n+1}-1}{n+1}$ (4) $\frac{2^{n+1}+1}{n+1}$

Q.22 $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+....=$

(1) $\frac{2^n}{n!}$; for all even values of n

(2) $\frac{2^{n-1}}{n!}$; for all values of n *i.e.*, all even odd values

(3) 0

(4) $\frac{2n}{n!}$; for all odd value of n

Q.23 The sum of all the coefficients in the binomial expansion of $(x^2+x-3)^{319}$ is

(1) 1 (2) 2 (3) – 1 (4) 0

Q.24 The sum of the coefficients of even power of x in the expansion of $(1+x+x^2+x^3)^5$ is

(1) 256 (2) 128

(3) 512 (4) 64

Q.25 The value of ${}^{15}C_0^2-{}^{15}C_1^2+{}^{15}C_2^2-....-{}^{15}C_{15}^2$ is

(1) 15 (2) – 15 (3) 0 (4) 51

Application of Binomial theorem

Q.26 The greatest integer which divides the number $101^{100}-1$, is

(1) 100 (2) 1000 (3) 10000 (4) 100000

Q.27 The last digit in 7^{300} is

(1) 7 (2) 9 (3) 1 (4) 3

Multinomial theorem and other index

Q.28 The number 111......1 (91 times) is

(1) Not a prime (2) An even number

(3) Not an odd number (4) Prime Number

Q.29 In the expansion of $\left(\frac{1+x}{1-x}\right)^2$, the coefficient of x^n will be [where $|x|<1$]

(1) $4n$ (2) $4n-3$

(3) $4n+1$ (4) $4n-1$

Q.30 The coefficient of two consecutive terms in the expansion of $(1+x)^n$ will be equal, if

(1) n is any integer (2) n is an odd integer

(3) n is an even integer (4) n is prime number

EXERCISE-II

Q.1 The $(m+1)^{th}$ term of $\left(\frac{x}{y}+\frac{y}{x}\right)^{2m+1}$ is:

(1) independent of x
(2) a constant
(3) depends on the ratio x/y and m
(4) none of these

Q.2 The total number of terms in the expansion of, $(x+a)^{100}+(x-a)^{100}$ after simplification is :

(1) 50 (2) 202
(3) 51 (4) none of these

Q.3 If the 6^{th} term in the expansion of the binomial $\left[\frac{1}{x^{8/3}}+x^2\log_{10}x\right]^8$ is 5600, then x =

(1) 10 (2) 8 (3) 11 (4) 9

Q.4 If the second term of the expansion $\left[a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is:

(1) 4 (2) 3 (3) 12 (4) 6

Q.5 The co-efficient of x in the expansion of $(1-2x^3+3x^5)\left(1+\frac{1}{x}\right)^8$ is :

(1) 56 (2) 65 (3) 154 (4) 62

Q.6 In the expansion of $(7^{1/3}+11^{1/9})^{6561}$, the number of terms free from radicals is:

(1) 730 (2) 729 (3) 725 (4) 750

Q.7 The term independent of x in the expansion of $\left(x-\frac{1}{x}\right)^4\left(x+\frac{1}{x}\right)^3$ is:

(1) – 3 (2) 0 (3) 1 (4) 3

Q.8 The value of m, for which the coefficients of the $(2m+1)^{th}$ and $(4m+5)^{th}$ terms in the expansion of $(1+x)^{10}$ are equal, is

(1) 3 (2) 1 (3) 5 (4) 8

Q.9 If the coefficients of x^7 & x^8 in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is

(1) 15 (2) 45 (3) 55 (4) 56

Q.10 Number of rational terms in the expansion of $\left(\sqrt{2}+\sqrt[4]{3}\right)^{100}$ is

(1) 25 (2) 26 (3) 27 (4) 28

Q.11 If $n \in N$ & n is even, then

$$\frac{1}{1.(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\ldots\ldots+\frac{1}{(n-1)!1!}$$

equals

(1) 2^n (2) $\frac{2^{n-1}}{n!}$
(3) $2^n n!$ (4) none of these

Q.12 If $k \in R$ and the middle term of $\left(\frac{k}{2}+2\right)^8$ is 1120, then value of k is:

(1) 3 (2) 2 (3) – 3 (4) – 4

Q.13 The sum of the binomial coefficients of $\left[2x+\frac{1}{x}\right]^n$ is equal to 256. The constant term in the expansion is

(1) 1120 (2) 2110 (3) 1210 (4) none

Q.14 The sum of the co-efficients in the expansion of $(1-2x+5x^2)^n$ is 'a' and the sum of the co-efficients in the expansion of $(1+x)^{2n}$ is b. Then

(1) a = b (2) $a = b^2$ (3) $a^2 = b$ (4) ab = 1

Q.15 The binomial expansion of $\left(x^k+\frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ contains a term independent of x

(1) only if k is an integer
(2) only if k is a natural number
(3) only if k is rational
(4) for any real k

Q.16 If the expansion of $(3x + 2)^{-1/2}$ is valid in ascending powers of x, then x lies in the interval.
(1) $(-2/3, 2/3)$ (2) $(-3/2, 3/2)$
(3) $(-1/3, 2/3)$ (4) $(-\infty, -3/2) \cup (3/2, \infty)$

Q.17 Coefficient of α^t in the expansion of $(\alpha+p)^{m-1} + (\alpha+p)^{m-2}(\alpha+q) + (\alpha+p)^{m-3}(\alpha+q)^2 + ...(\alpha+q)^{m-1}$ where $\alpha \neq -q$ and $p \neq q$ is
(1) $\frac{{}^mC_t(p^t - q^t)}{p-q}$ (2) $\frac{{}^mC_t(p^{m-t} - q^{m-t})}{p-q}$
(3) $\frac{{}^mC_t(p^t + q^t)}{p-q}$ (4) $\frac{{}^mC_t(p^{m-t} + q^{m-t})}{p-q}$

Q.18 The greatest terms of the expansion $(2x + 5y)^{13}$ when x = 10, y = 2 is
(1) ${}^{13}C_5 . 20^8 . 10^5$ (2) ${}^{13}C_6 . 20^7 . 10^4$
(3) ${}^{13}C_4 . 20^9 . 10^4$ (4) none of these

Q.19 Find numerically greatest term in the expansion of $(2 + 3x)^9$, when $x = 3/2$.
(1) ${}^9C_6 . 2^9 . (3/2)^{12}$ (2) ${}^9C_3 . 2^9 . (3/2)^6$
(3) ${}^9C_5 . 2^9 . (3/2)^{10}$ (4) ${}^9C_4 . 2^9 . (3/2)^8$

Q.20 The numerically greatest term in the expansion of $(2x + 5y)^{34}$, when x = 3 & y = 2 is :
(1) T_{21} (2) T_{22} (3) T_{23} (4) T_{24}

Q.21 $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} =$
(1) $\frac{n}{2}$ (2) $\frac{n+1}{2}$
(3) $(n+1)\frac{n}{2}$ (4) $\frac{n(n-1)}{2(n+1)}$

Q.22 Set of values of r for which, ${}^{18}C_{r-2} + 2.{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ contains
(1) 4 elements (2) 5 elements
(3) 7 elements (4) 10 elements

Q.23 $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_{10}}{11} =$
(1) $\frac{2^{11}}{11}$ (2) $\frac{2^{11}-1}{11}$ (3) $\frac{3^{11}}{11}$ (4) $\frac{3^{11}-1}{11}$

Q.24 The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to:
(1) ${}^{47}C_5$ (2) ${}^{52}C_5$
(3) ${}^{52}C_4$ (4) ${}^{49}C_4$

Q.25 The value of $\binom{50}{0}\binom{50}{1} + \binom{50}{1}\binom{50}{2} + ... + \binom{50}{49}\binom{50}{50}$ is, where ${}^nC_r = \binom{n}{r}$
(1) $\binom{100}{50}$ (2) $\binom{100}{51}$ (3) $\binom{50}{25}$ (4) $\binom{50}{25}^2$

Q.26 The remainder when 2^{2003} is divided by 17 is :
(1) 1 (2) 2
(3) 8 (4) none of these

Q.27 The last two digits of the number 3^{400} are:
(1) 81 (2) 43 (3) 29 (4) 01

Q.28 The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is
(1) 196 (2) 197 (3) 198 (4) 199

Q.29 Let $\left(5 + 2\sqrt{6}\right)^n = p + f$, where $n \in N$ and $p \in N$ and $0 < f < 1$, then the value of $f^2 - f + pf - p$ is :
(1) a natural number (2) a negative integer
(3) a prime number (4) an irrational number

Q.30 The co-efficient of x^4 in the expansion of $(1 - x + 2x^2)^{12}$ is
(1) ${}^{12}C_3$ (2) ${}^{13}C_3$
(3) ${}^{14}C_4$ (4) ${}^{12}C_3 + 3\,{}^{13}C_3 + {}^{14}C_4$

EXERCISE-III

MCQ/COMPREHENSION/MATCHING/NUMERICAL

Q.1 In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$
(A) the number of irrational terms is 19
(B) middle term is irrational
(C) the number of rational terms is 2
(D) 9th term is rational

Q.2 In the expansion of $\left(x^{2/3} - \frac{1}{\sqrt{x}}\right)^{30}$, a term containing the power x^{13}
(A) does not exist
(B) exists & the co-efficient is divisible by 29
(C) exists & the co-efficient is divisible by 63
(D) exists & the co-efficient is divisible by 65

Q.3 In the expansion of $\left(x^3 + 3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11}$

(A) there appears a term with the power x^2
(B) there does not appear a term with the power x^2
(C) there appears a term with the power x^{-3}
(D) the ratio of the co-efficient of x^3 to that of x^{-3} is $\frac{1}{3}$

Q.4 If the 6^{th} term in the expansion of $\left(\frac{3}{2}+\frac{x}{3}\right)^n$ when x = 3 is numerically greatest then the possible integral value(s) of n can be
(A) 11 (B) 12 (C) 13 (D) 14

Q.5 Let $(1 + x^2)^2 (1 + x)^n = A_0 + A_1x + A_2x^2 + \ldots\ldots\ldots$ If A_0, A_1, A_2 are in A.P. then the value of n is
(A) 2 (B) 3 (C) 5 (D) 7

Q.6 If $\left(9+\sqrt{80}\right)^n$ = I + f, where I, n are integers and 0 < f < 1, then :
(A) I is an odd integer (B) I is an even integer
(C) (I + f) (1 – f) = 1 (D) $1 - f = \left(9-\sqrt{80}\right)^n$

Q.7 The number $101^{100} - 1$ is divisible by
(A) 100 (B) 1000 (C) 10000 (D) 100000

Q.8 $7^9 + 9^7$ is divisible by :
(A) 16 (B) 24 (C) 64 (D) 72

Q.9 If recursion polynomials $P_k(x)$ are defined as $P_1(x) = (x-2)^2$, $P_2(x) = ((x-2)^2 - 2)^2$
$P_3(x) = ((x-2)^2 - 2)^2 - 2)^2$ .... (In general $P_k(x) = (P_{k-1}(x) - 2)^2$, then the constant term in $P_k(x)$ is
(A) 4 (B) 2
(C) 16 (D) a perfect square

Q.10 If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$, then :
(A) $a_1 = 20$ (B) $a_2 = 210$
(C) $a_4 = 8085$ (D) $a_{20} = 2^2 . 3^7 . 7$

Comprehension # 1 (Q. No. 11 to 13)

If m, n, r are positive integers and if r < m, r < n, then
${}^mC_r + {}^mC_{r-1} . {}^nC_1 + {}^mC_{r-2} . {}^nC_2 + \ldots + {}^nC_r$
= Coefficient of x^r in $(1 + x)^m (1 + x)^n$
= Coefficient of x^r in $(1 + x)^{m+n}$
$= {}^{m+n}C_r$

On the basis of the above information, answer the following questions.

Q.11 The value of ${}^nC_0 . {}^nC_n + {}^nC_1 . {}^nC_{n-1} + \ldots + {}^nC_n . {}^nC_0$ is
(A) ${}^{2n}C_{n-1}$ (B) ${}^{2n}C_n$ (C) ${}^{2n}C_{n+1}$ (D) ${}^{2n}C_2$

Q.12 The value of r for which ${}^{30}C_r . {}^{20}C_0 + {}^{30}C_{r-1} . {}^{20}C_1 + \ldots + {}^{30}C_0 . {}^{20}C_r$ is maximum, is
(A) 10 (B) 15 (C) 20 (D) 25

Q.13 The value of r$(0 \le r \le 30)$ for which ${}^{20}C_r . {}^{10}C_0 + {}^{20}C_{r-1} . {}^{10}C_1 + \ldots + {}^{20}C_0 . {}^{10}C_r$ is minimum, is
(A) 0 (B) 1 (C) 5 (D) 15

Comprehension # 2 (Q. No. 14 to 16)

We know that if ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots\ldots, {}^nC_n$ be binomial coefficients, then $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3x^3 + \ldots\ldots + C_n x^n$. Various relations among binomial coefficients can be derived by putting x = 1, – 1, i, $\omega\left(\text{where } i=\sqrt{-1}, \omega = -\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$.

Q.14 The value of ${}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \ldots\ldots$ must be
(A) $2^{n/2} \cos\frac{n\pi}{2}$ (B) $2^{n/2} \sin\frac{n\pi}{2}$
(C) $2^{n/2} \cos\frac{n\pi}{4}$ (D) $2^{n/2} \sin\frac{n\pi}{4}$

Q.15 The value of expression $({}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \ldots\ldots)^2 + ({}^nC_1 - {}^nC_3 + {}^nC_5 \ldots\ldots\ldots)^2$ must be
(A) 2^{2n} (B) 2^n
(C) 2^{n^2} (D) None of these

Q.16 The value of ${}^nC_0 + {}^nC_3 + {}^nC_6 + \ldots\ldots$ must be
(A) $\frac{2^n}{3}$ (B) $\frac{1}{3}\left(2^n + \cos\frac{n\pi}{3}\right)$
(C) $\frac{1}{3}\left(2^n + 2\cos\frac{n\pi}{3}\right)$ (D) $\frac{1}{3}\left(2^n + 2\sin\frac{n\pi}{3}\right)$

Q.17 MATCH THE COLUMN:

Column – I	Column – II
(A) If $(r + 1)^{th}$ term is the first negative term in the expansion of $(1 + x)^{7/2}$, then the value of r (where $\|x\| < 1$) is	(p) divisible by 2
(B) The coefficient of y in the expansion of $(y^2 + 1/y)^5$ is	(q) divisible by 5
(C) ${}^nC_r$ is divisible by n, $(1 < r < n)$ if n is	(r) divisible by 10
(D) The coefficient of x^4 in the expression $(1 + 2x + 3x^2 + 4x^3 + ...\text{up to } \infty)^{1/2}$ is c, $(c \in N)$, then $c + 1$ (where $\|x\| < 1$) is	(s) a prime number

Q.18 MATCH THE COLUMN:

Column - I	Column - II
(A) If $x = \left(7 + 4\sqrt{3}\right)^{2n} = [x] + f$, then $x(1 - f) =$	(p) 6
(B) If second, third and fourth terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively, then n is equal to	(q) 1
(C) value of ${}^4C_0\,{}^4C_4 - {}^4C_1\,{}^4C_3 + {}^4C_2\,{}^4C_2 - {}^4C_3\,{}^4C_1 + {}^4C_4\,{}^4C_0$ is	(r) 2
(D) If x is very large as compare to y, then the value of k in $\sqrt{\dfrac{x}{x+y}}\,\sqrt{\dfrac{x}{x-y}} = 1 + \dfrac{y^2}{kx^2}$	(s) 5

INTEGER TYPE

Q.19 If the 6^{th} term in the expansion of $\left[\dfrac{1}{x^{8/3}} + x^2\log_{10} x\right]^8$ is 5600, then x =

Q.20 The value of p, for which coefficient of x^{50} in the expression $(1 + x)^{1000} + 2x\,(1 + x)^{999} + 3x^2\,(1 + x)^{998} + ... + 1001\,x^{1000}$ is equal to ${}^{1002}C_p$, is :

Q.21 The index 'n' of the binomial $\left(\dfrac{x}{5} + \dfrac{2}{5}\right)^n$ if the 9^{th} term of the expansion has numerically the greatest coefficient $(n \in N)$, is :

Q.22 The number of values of 'r' satisfying the equation, ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is :

Q.23 2^{60} when divided by 7 leaves the remainder

Q.24 The last digit of the number 3^{400} is equal to ...

Q.25 If n is even then the middle term in the expansion of $\left(x^2 + \dfrac{1}{x}\right)^n$ is $924x^6$, then n is equal to

Q.26 The term independent of x in $\left(\dfrac{3}{2}x^2 - \dfrac{1}{3x}\right)^9$ is

Q.27 The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q} = 0$ if $P < q$) is maximum when m is

Q.28 How many term's in the expansion of $\left(y^{\frac{1}{3}} + x^{\frac{1}{10}}\right)^{55}$ are free from radical sign.

EXERCISE-IV

JEE-MAIN
PREVIOUS YEAR'S

Q.1 If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n, x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is: **[JEE Main-2016]**
(1) 64 (2) 2187 (3) 243 (4) 729

Q.2 The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is - **[JEE Main-2017]**
(1) $2^{20} - 2^{10}$ (2) $2^{21} - 2^{11}$ (3) $2^{21} - 2^{10}$ (4) $2^{20} - 2^{9}$

Q.3 The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x>1)$ is - **[JEE Main-2018]**
(1) 0 (2) 1 (3) 2 (4) –1

Q.4 If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to : **[JEE Main - 2019 (January)]**
(1) 6 (2) 8 (3) 4 (4) 14

Q.5 The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is **[JEE Main - 2019 (January)]**
(1) 12 (2) 15 (3) 10 (4) 14

Q.6 If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$, then k equals: **[JEE Main - 2019 (January)]**
(1) 400 (2) 50 (3) 200 (4) 100

Q.7 If the third term in the binomial expansions of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is: **[JEE Main - 2019 (January)]**
(1) $\frac{1}{4}$ (2) $4\sqrt{2}$ (3) $\frac{1}{8}$ (4) $2\sqrt{2}$

Q.8 The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is : **[JEE Main - 2019 (January)]**
(1) 4 (2) $2\sqrt{2}$ (3) $\sqrt{5}$ (4) 3

Q.9 If $\sum_{r=0}^{25}\left\{{}^{50}C_r.{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to **[JEE Main - 2019 (January)]**
(1) $(25)^2$ (2) $2^{25} - 1$ (3) 2^{24} (4) 2^{25}

Q.10 Let $S_n = 1 + q + q^2 + ...+q^n$ and $T_n = 1+\left(\frac{q+1}{2}\right)+\left(\frac{q+1}{2}\right)^2+...+\left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 . S_1 + ...+{}^{101}C_{101} . S_{100} = \alpha T_{100}$ then α is equal to : **[JEE Main - 2019 (January)]**
(1) 2^{99} (2) 202 (3) 200 (4) 2^{100}

Q.11 Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1 x + a_2x^2 + ... + a_{50} x^{50}$, for $x \in R$: then $\frac{a_2}{a_0}$ is equal to: **[JEE Main - 2019 (January)]**
(1) 12.50 (2) 12.00 (3) 12.25 (4) 12.75

Q.12 The value of r for which ${}^{20}C_r{}^{20}C_0 + {}^{20}C_{r-1}{}^{20}C_1 + {}^{20}C_{r-2}{}^{20}C_2 +.....{}^{20}C_0{}^{20}C_r$ is maximum is : **[JEE Main - 2019 (January)]**
(1) 15 (2) 20 (3) 11 (4) 10

Q.13 The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :- **[JEE Main - 2019 (January)]**
(1) 0 (2) 6 (3) 4 (4) 8

Q.14 A ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is : **[JEE Main - 2019 (January)]**
(1) $1:2(6)^{\frac{1}{3}}$ (2) $1:4(16)^{\frac{1}{3}}$
(3) $4(36)^{\frac{1}{3}}:1$ (4) $2(36)^{\frac{1}{3}}:1$

Q.15 The total number of irrational terms in the binomial expansion of $\left(7^{1/5} - 3^{1/10}\right)^{60}$ is :

[JEE Main - 2019 (January)]

(1) 55 (2) 49 (3) 48 (4) 54

Q.16 If ${}^nC_4, {}^nC_5$, and ${}^nC_6$ are in A.P., then n can be :

[JEE Main - 2019 (January)]

(1) 9 (2) 14 (3) 11 (4) 12

Q.17 The sum of the series $2\,.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots\ldots + 62\,.{}^{20}C_{20}$ is equal to : **[JEE Main-2019(April)]**

(1) 2^{24} (2) 2^{25} (3) 2^{26} (4) 2^{23}

Q.18 The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$, $(x > 1)$ is equal to :

[JEE Main-2019(April)]

(1) 32 (2) 26 (3) 29 (4) 24

Q.19 If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is : **[JEE Main-2019(April)]**

(1) 10^3 (2) 100 (3) 10^4 (4) 10

Q.20 If the fourth term in the binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6 (x > 0)$ is 20×8^7, then a value of x

[JEE Main-2019(April)]

(1) 8 (2) 8^2 (3) 8^{-2} (4) 8^3

Q.21 If some three consecutive terms in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :- **[JEE Main-2019(April)]**

(1) 964 (2) 625 (3) 227 (4) 232

Q.22 The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2 + \frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

[JEE Main-2019(April)]

(1) 35 (2) 38 (3) 23 (4) 58

Q.23 If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

[JEE Main-2019(April)]

(1) (28, 315) (2) (–54, 315)
(3) (–21, 714) (4) (24, 861)

Q.24 The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is :

[JEE Main-2019(April)]

(1) –84 (2) 84 (3) 126 (4) –126

Q.25 The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right)\cdot\left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to:

[JEE Main-2019(April)]

(1) 36 (2) –108 (3) –72 (4) –36

Q.26 If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$ is 61, then n is equal to

[JEE Main-2020 (January)]

Q.27 The number of ordered pairs (r, k) for which $6.\,{}^{35}C_r = (k^2 - 3).\,{}^{36}C_{r+1}$, where k is an integer, is:

[JEE Main-2020 (January)]

(1) 3 (2) 6 (3) 4 (4) 2

Q.28 The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots\ldots + x^{10}$ is:

(1) 210 (2) 330 (3) 420 (4) 120

Q.29 If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then

[JEE Main-2020 (January)]

(1) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$ (2) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$

(3) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (4) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$

Q.30 The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is….. **[JEE Main-2020 (January)]**

Q.31 In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if I_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and I_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $I_2 : I_1$ is equal to :

[JEE Main-2020 (January)]

(1) 1 : 8 (2) 1 : 16 (3) 8 : 1 (4) 16 : 1

Q.32 If $C_r = {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to .... **[JEE Main-2020 (January)]**

Q.33 For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to ____. **[JEE Main-2020 (September)]**

Q.34 Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{1/9} + \beta x^{-1/6}\right)^{10}$ is 10 K, then K is equal to : **[JEE Main-2020 (September)]**

(1) 84 (2) 176 (3) 336 (4) 352

Q.35 If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to :

(1) 9 (2) 11 (3) 5 (4) 7

Q.36 If the number of integral terms in the expansion of $\left(3^{1/2} + 5^{1/8}\right)^n$ is exactly 33, then the least value of n is: **[JEE Main-2020 (September)]**

(1) 264 (2) 128 (3) 256 (4) 248

Q.37 The value of $(2.{}^1P_0 - 3.{}^2P_1 + 4.{}^3P_2 - ...$ up to 51^{th} term) + $(1! - 2! + 3! - ...$ up to 51^{th} term) is equal to : **[JEE Main-2020 (September)]**

(1) 1 (2) 1 + (52)! (3) 1 – 51(51)! (4) 1 + (51)!

Q.38 If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is : **[JEE Main-2020 (September)]**

(1) 252 (2) 462 (3) 792 (4) 330

Q.39 The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to : **[JEE Main-2020 (September)]**

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{50}C_7 - {}^{30}C_7$

(3) ${}^{51}C_7 + {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Q.40 The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______. **[JEE Main-2020 (September)]**

Q.41 The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ________. **[JEE Main-2020 (September)]**

Q.42 If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals : **[JEE Main-2020 (September)]**

(1) 2 (2) 3 (3) 9 (4) 1

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to **[IIT JEE - 2010]**

(A) $B_{10} - C_{10}$ (B) $A_{10}(B^2_{10} - C_{10}A_{10})$

(C) 0 (D) $C_{10} - B_{10}$

Q.2 The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = **[JEE Advanced-2013]**

Q.3 Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is **[JEE Advanced-2014]**

(A) 1051 (B) 1106 (C) 1113 (D) 1120

Q.4 The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) ......(1 + x^{100})$ is **[JEE Advanced-2015]**

Q.5 Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + ... + (1+x)^{49} + (1+mx)^{50}$ is $(3n + 1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is **[JEE Advanced-2016]**

Q.6 Let $X = \left({}^{10}C_1\right)^2 + 2\left({}^{10}C_2\right)^2 + 3\left({}^{10}C_3\right)^2 + ... + 10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r, r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is .... **[JEE Advanced-2018]**

Q.7 Suppose

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k K & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$$

, holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals **[JEE Advanced -2019]**

Q.8 For non-negative integers s and r, let

$$\binom{s}{r} = \begin{cases} \dfrac{s!}{r!(s-r)!} & \text{if } r \le s, \\ 0 & \text{if } r > s. \end{cases}$$

For positive integers m and n, let

$$(m,n)\sum_{p=0}^{m+n} \frac{f(m,n,p)}{\binom{n+p}{p}}$$

where for any nonnegative integer p,

$$f(m,n,p) = \sum_{i=0}^{p} \binom{m}{i}\binom{n+i}{p}\binom{p+n}{p-i}$$

[JEE(Advanced) - 2020]

Then which of the following statements is/are TRUE?

(A) $(m,n)=g(n,m)$ for all positive integers m,n

(B) $(m,n+1)=g(m+1,n)$ for all positive integers m,n

(C) $(2m,2n)=2g(m,n)$ for all positive integers m,n

(D) $(2m,2n)=(g(m,n))^2$ for all positive integers m,n

Answer Key

EXERCISE-I

Q.1 (2)	**Q.2 (3)**	**Q.3 (3)**	**Q.4 (3)**	**Q.5 (3)**	**Q.6 (2)**	**Q.7 (2)**	**Q.8 (1)**	**Q.9 (3)**	**Q.10 (3)**
Q.11 (2)	**Q.12 (1)**	**Q.13 (1)**	**Q.14 (3)**	**Q.15 (2)**	**Q.16 (3)**	**Q.17 (2)**	**Q.18 (2)**	**Q.19 (4)**	**Q.20 (1)**
Q.21 (3)	**Q.22 (2)**	**Q.23 (3)**	**Q.24 (3)**	**Q.25 (3)**	**Q.26 (3)**	**Q.27 (3)**	**Q.28 (1)**	**Q.29 (1)**	**Q.30 (2)**

EXERCISE-II

Q.1 (3)	**Q.2 (3)**	**Q.3 (1)**	**Q.4 (1)**	**Q.5 (3)**	**Q.6 (1)**	**Q.7 (2)**	**Q.8 (2)**	**Q.9 (3)**	**Q.10 (2)**
Q.11 (2)	**Q.12 (2)**	**Q.13 (1)**	**Q.14 (1)**	**Q.15 (4)**	**Q.16 (1)**	**Q.17 (2)**	**Q.18 (3)**	**Q.19 (1)**	**Q.20 (2)**
Q.21 (1)	**Q.22 (3)**	**Q.23 (2)**	**Q.24 (3)**	**Q.25 (2)**	**Q.26 (3)**	**Q.27 (4)**	**Q.28 (2)**	**Q.29 (2)**	**Q.30 (4)**

EXERCISE-III

MCQ/COMPREHENSION/MATCHING/NUMERICAL

Q.1 (**A, B, C, D**) **Q.2 (B,C,D)** **Q.3 (B,C,D)** **Q.4 (B, C, D)** **Q.5 (A, B)** **Q.6** (**A, C, D**) **Q.7 (A,B,C)**

Q.8 (**A, C**) **Q.9 (A, D)** **Q.10 (A, B, C)** **Q.11** (B) **Q.12** (D) **Q.13** (A) **Q.14** (C) **Q.15** (B) **Q.16** (C)

Q.17 (A) $\to$ (q, s), (B) $\to$ (p, q, r), (C) $\to$ (s), (D) $\to$ (p, s) **Q.18** (A) $\to$ (q), (B) $\to$ (s), (C) $\to$ (p), (D) $\to$ (r)

Q.19 [10] **Q.20** [50] **Q.21** [n = 12] **Q.22** [2] **Q.23** [0001] **Q.24** [0001] **Q.25** [0012] **Q.26** [0007] **Q.27** [0015]

Q.28 [0006]

EXERCISE-IV

JEE-MAIN

PREVIOUS YEAR'S

Q.1 (Bonus)	**Q.2 (1)**	**Q.3** (3)	**Q.4 (2)**	**Q.5** (2)	**Q.6** (4)	**Q.7** (1)	**Q.8** (1)	**Q.9** (4)	**Q.10** (4)
Q.11 (3)	**Q.12** (2)	**Q.13** (1)	**Q.14** (3)	**Q.15** (4)	**Q.16** (2)	**Q.17 (2)**	**Q.18 (4)**	**Q.19** (Bonus)	**Q.20** (2)
Q.21 (4)	**Q.22** (2)	**Q.23** (1)	**Q.24 (2)**	**Q.25 (4)**	**Q.26 [30]**	**Q.27** (3)	**Q.28** (2)	**Q.29 (3)**	**Q.30 [615]**
Q.31 (4)	**Q.32** [51]	**Q.33** [118]	**Q.34** (3)	**Q.35** (4)	**Q.36** (3)	**Q.37** (2)	**Q.38** (2)	**Q.39** (1)	
Q.40 [120]	**Q.41** [13]	**Q.42** (2)							

JEE-ADVANCED

PREVIOUS YEAR'S

Q.1 (D)	**Q.2 [6]**	**Q.3 (C)**	**Q.4 [8]**	**Q.5 [5]**	**Q.6 [646]**	**Q.7** [6.20]	**Q.8** (A,B,D)

EXERCISE (Solution)

EXERCISE-I

Q.1 (2)

Applying $T_{r+1} = {}^nC_r x^{n-r} a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5 (2x^2)^5 \left(-\frac{1}{3x^2}\right)^5$

$$= -\frac{10!}{5!5!} 32 \times \frac{1}{243} = -\frac{896}{27}$$

Q.2 (3)

${}^{20}C_{r-1} = {}^{20}C_{r+3} \Rightarrow 20 - r + 1 = r + 3 \Rightarrow r = 9$.

Q.3 (3)

$T_{16} = {}^{17}C_{15} (\sqrt{x})^2 (-\sqrt{y})^{15}$

$= -\frac{17 \times 16}{2 \times 1} \times xy^{15/2} = -136xy^{15/2}$

Q.4 (3)

$\frac{1}{6} = \frac{{}^nC_6 (2^{1/3})^{n-6} (3^{-1/3})^6}{{}^nC_{n-6} (2^{1/3})^6 (3^{-1/3})^{n-6}}$ or $6^{-1} = 6^{-4}.6^{n/3} = 6^{n/3-4}$

$\therefore \frac{n}{3} - 4 = -1 \Rightarrow n = 9.$

Q.5 (3)

$T_r = {}^{15}C_{r-1} (x^4)^{16-r} \left(\frac{1}{x^3}\right)^{r-1} = {}^{15}C_{r-1} x^{67-7r}$

$\Rightarrow 67 - 7r = 4 \Rightarrow r = 9$.

Q.6 (2)

We have $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + ...$

By hypothesis, $\frac{m(m-1)}{2} x^2 = -\frac{1}{8} x^2$

$\Rightarrow 4m^2 - 4m = -1 \Rightarrow (2m-1)^2 = 0 \Rightarrow m = \frac{1}{2}$.

Q.7 (2)

In the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ the general term is

$T_{r+1} = {}^5C_r (x^2)^{5-r} \left(\frac{a}{x}\right)^r = {}^5C_r a^r x^{10-3r}$

Here, exponent of x is $10 - 3r = 1 \Rightarrow r = 3$

$\therefore T_{2+1} = {}^5C_3 a^3 x = 10a^3.x$

Hence coefficient of x is $10a^3$.

Q.8 (1)

In the expansion of $\left(x - \frac{1}{x}\right)^6$, the general term is

${}^6C_r x^{6-r} \left(-\frac{1}{x}\right)^r = {}^6C_r (-1)^r x^{6-2r}$

For term independent of x, $6 - 2r = 0 \Rightarrow r = 3$

Thus the required coefficient $= (-1)^3.{}^6C_3 = -20$.

Q.9 (3)

The coefficient of x^{16} in the expansion of $(x^2 - 2x)^{10}$

= The coefficient of x^{16} in $x^{10}(x-2)^{10}$

= The coefficient of x^6 in $(x-2)^{10}$ f

$= {}^{10}C_4.2^4$, $\left(\because T_{r+1} = {}^nC_r x^{n-r} a^r\right)$

$= 210 \times 16 = 3360$.

Q.10 (3)

Let T_{r+1} term containing x^{32}.

Therefore ${}^{15}C_r x^{4r} \left(\frac{-1}{x^3}\right)^{15-r}$

$\Rightarrow x^{4r} x^{-45+3r} = x^{32} \Rightarrow 7r = 77 \Rightarrow r = 11$.

Hence coefficient of x^{32} is ${}^{15}C_{11}$ or ${}^{15}C_4$

Q.11 (2)

x^7, x^8 will occur in T_8 and T_9.

Coefficients of T_8 and T_9 are equal.

$\therefore {}^nC_7 2^{n-7} \left(\frac{1}{3}\right)^7 = {}^nC_8 2^{n-8} \left(\frac{1}{3}\right)^8 \Rightarrow n = 55$.

Q.12 (1)

In the expansion of $\left(\frac{3x^2}{2} + \frac{1}{3x}\right)^9$, the general term is

$T_{r+1} = {}^9C_r . \left(\frac{3x^2}{2}\right)^{9-r} \left(-\frac{1}{3x}\right)^r = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For the term independent of x, $18-3r=0 \quad r=6$
This gives the independent term

$$T_{6+1} = {}^9C_6\left(\frac{3}{2}\right)^{9-6}\left(-\frac{1}{3}\right)^6 = {}^9C_3.\frac{1}{6^3}$$

Q.13 (1)

$$T_{r+1} = {}^{10}C_r(x^2)^{10-r}\left(\frac{-3\sqrt{3}}{x^3}\right)^r$$

For term independent of x, $20-2r-3r=0 \Rightarrow r=4$

$\therefore\ T_{4+1} = {}^{10}C_4(-3)^4(\sqrt{3})^4 = 153090.$

Q.14 (3)

As in Previous question, obviously the term independent of x will be

$${}^nC_0.{}^nC_0 + {}^nC_1.{}^nC_1 + ...{}^nC_n.{}^nC_n = C_0^2 + C_1^2 + ... + C_n^2.$$

Q.15 (2)

Since n is even therefore $\left(\frac{n}{2}+1\right)^{th}$ term is middle term,

hence ${}^nC_{n/2}(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2} = 924x^6$

$\Rightarrow\ x^{n/2} = x^6 \Rightarrow n = 12$ ·

Q.16 (3)

Middle term $= T_{\frac{2n+2}{2}} = T_{n+1} = {}^{2n}C_n x^n = \frac{2n!}{(n!)^2}.x^n$.

Q.17 (2)

Greatest coefficient of $(1+x)^{2n+2}$ is

$$= {}^{(2n+2)}C_{n+1} = \frac{(2n+2)!}{\{(n+1)!\}^2}$$

Q.18 **(2)**

Obviously the middle term

$$= {}^{2n}C_n(x)^n.\left(\frac{1}{2x}\right)^n = \frac{2n!}{n!.n!.2^n} = \frac{1.3.5....(2n-1)}{n!}.$$

Q.19 **(4)**

$$(1+3x+2x^2)^6 = [1+x(3+2x)]^6$$
$$= 1 + {}^6C_1x(3+2x) + {}^6C_2x^2(3+2x)^2$$
$$+ {}^6C_3x^3(3+2x)^3 + {}^6C_4x^4(3+2x)^4$$
$$+ {}^6C_5x^5(3+2x)^5 + {}^6C_6x^6(3+2x)^6$$

Only x^{11} gets from ${}^6C_6x^6(3+2x)^6$

$\because\ {}^6C_6x^6(3+2x)^6 = x^6(3+2x)^6$

$\therefore$ Coefficient of $x^{11} = {}^6C_5\,3^1 2^5 = 576.$

Q.20 **(1)**

We know that :

$(1+x)^n = {}^nC_0 + {}^nC_1x + ...{}^nC_nx^n$

Also, $\left(1+\frac{1}{x}\right)^n = {}^nC_0 + {}^nC_1\left(\frac{1}{x}\right) + ...{}^nC_n\left(\frac{1}{x}\right)^n$

The summation aksed above

$$= \text{coefficient of } x^r \text{ in } (1+x)^n\left(1+\frac{1}{x}\right)^n$$

$$= \text{coefficient of } x^r \text{ in } (1+x)^n\left(\frac{x+1}{x}\right)^n$$

$$= \text{coefficient of } x^r \text{ in } \frac{(1+x)^{2n}}{x^n}$$

$= \text{coefficient of } x^{r+1} \text{in } (1+x)^{2n}$

$= {}^{2n}_{n+r}C$

$$= \frac{2n!}{(n+r)!(2n-n-r)!}$$

$$= \frac{2n!}{(n+r)!(n-r)!}$$

Q.21 **(3)**

Proceeding as above and putting $n+1=N$.
So given term can be written as

$$\frac{1}{N}\left\{{}^NC_1 + {}^NC_2 + {}^NC_3 + ...\right\}$$

$$= \frac{1}{N}\left\{2^N - 1\right\} = \frac{1}{n+1}(2^{n+1}-1) \qquad (\because N = n+1)$$

Q.22 **(2)**

Multiplying each term by $n!$ the question reduces to

$$\frac{n!}{1!(n-1)!} + \frac{1}{3!}.\frac{n!}{(n-3)!} + \frac{1}{5!}.\frac{n!}{(n-5)!} + ...$$

$$= {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}.$$

Thus $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + ... = \frac{1}{n!}2^{n-1}$.

Q.23 **(3)**

Putting $x = 1$ in $(x^2 + x - 3)^{319}$

We get the sum of coefficient = $(1+1-3)^{319} = -1$

Q.24 **(3)**

$(1+x+x^2+x^3)^5 = (1+x)^5(1+x^2)^5$

$= (1+5x+10x^2+10x^3+5x^4+x^5)$

$\times(1+5x^2+10x^4+10x^6+5x^8+x^{10})$

Therefore the required sum of coefficients

$= (1+10+5).2^5 = 16\times 32 = 512$

Note : $2^n = 2^5$ = Sum of all the binomial coefficients in the 2nd bracket in which all the powers of x are even.

Q.25 **(3)**

As we know that

${}^nC_0^2 - {}^nC_1^2 + {}^nC_2^2 - {}^nC_3^2 + \dots + (-1)^n.{}^nC_n^2 = 0$

(if n is odd) and in the question n =15 (odd).

Q.26 **(3)**

$101^{100} - 1 = (1+100)^{100} - 1$

$= {}^{100}C_0 + {}^{100}C_1(100)^1 + {}^{100}C_2\,(100)^2 + \dots + {}^{100}C_{100}\,(100)^{100} - 1$

$= 100\times 100 + {}^{100}C_2\,(100)^2 + \dots + {}^{100}C_{100}\,100^{100}$

$= 10000\,[1 + {}^{100}C_2 + {}^{100}C_3(100) + \dots + {}^{100}C_{100}\,100^{98}]$

= 10000 [Integer]

Q.27 **(3)**

We have $7^2 = 49 = 50 - 1$

Now, $7^{300} = (7^2)^{150} = (50-1)^{150}$

$= {}^{150}C_0(50)^{150}(-1)^0 + {}^{150}C_1(50)^{149}(-1)^1 + \dots$

$+ {}^{150}C_{150}(50)^0(-1)^{150}$

Thus the last digits of 7^{300} are ${}^{150}C_{150}.1.1$ *i.e.,* 1.

Q.28 **(1)**

111…..1 (91 times)

$= 1+10+10^2+\dots+10^{90}$

$= \dfrac{10^{91}-1}{10-1} = \dfrac{(10^7)^{13}-1}{10-1} = \dfrac{t^{13}-1}{9}$, where $t = 10^7$

$= \left(\dfrac{t-1}{9}\right)(t^{12}+t^{11}+\dots+t+1)$

$= \left(\dfrac{10^7-1}{10-1}\right)(1+t+t^2+\dots+t^{12})$

$= (1+10+10^2+\dots+10^6)(1+t+t^2+\dots+t^{12})$

$\therefore$ 111.....1(91 times) is a composite number.

Q.29 **(1)**

Given term can be written as $(1+x)^2(1-x)^{-2}$

$= (1+2x+x^2)[1+2x+3x^2+\dots+(n-1)x^{n-2}$

$+nx^{n-1}+(n+1)x^n+\dots]$

$= x^n(n+1+2n+n-1)+\dots$

Therefore coefficient of x^n is $4n$.

Q.30 **(2)**

Let consecutive terms are ${}^nC_r$ and ${}^nC_{r+1}$

$\Rightarrow \dfrac{n!}{(n-r)!r!} = \dfrac{n!}{(n-r-1)!(r+1)!}$

$\Rightarrow \dfrac{1}{(n-r)(n-r-1)!r!} = \dfrac{1}{(n-r-1)!(r+1)r!}$

$\Rightarrow r+1 = n-r \Rightarrow n = 2r+1$. Hence n is odd.

EXERCISE-II

Q.1 **(3)**

$${}^{2m+1}C_m\left(\frac{x}{y}\right)^{m+1}\left(\frac{y}{x}\right)^m = {}^{2m+1}C_m\left(\frac{x}{y}\right)$$

Dependent upon the ratio $\dfrac{x}{y}$ and m.

Q.2 **(3)**

$(x+a)^{100} + (x-a)^{100}$

$= 2\left({}^{100}C_0\,x^{100} + {}^{100}C_2\,x^{98}a^2 + \dots + {}^{100}C_{100}\,a^{100}\right)$

Number of terms = 51 terms

Q.3 **(1)**

$$T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2\log_{10}x)^5 = 5600$$

$\Rightarrow \dfrac{1}{x^8}\,x^{10}(\log_{10}x)^5 = 100$

$\Rightarrow x = 10$

Q.4 **(1)**

$T_2 = {}^nC_1\,(a^{1/13})^{n-1}(a^{3/2}) = 14a^{5/2}$

$\Rightarrow n = 14$

$\therefore \dfrac{{}^nC_3}{{}^nC_2} = 4$

Q.5 **(3)**

$(1-2x^3+3x^5)\left(1+\dfrac{1}{x}\right)^8$

Co-efficient of $x = -2.{}^8C_2 + 3.{}^8C_4 = 154$

Q.6 **(1)**

$T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/9})^r$

Here r should be multiple of 9

$r = 0, 9, 18..............6561$

Number of terms = 730

Q.7 **(2)**

$\left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right)^3 = \left(x-\frac{1}{x}\right)({}^3C_0x^6 - {}^3C_1x^2 + {}^3C_2x^{-2} - {}^3C_3x^{-6})$

There is no term independent of x

Q.8 **(2)**

$\left.\begin{matrix} T_{2m+1} \Rightarrow {}^{10}C_{2m} \\ T_{4m+5} \Rightarrow {}^{10}C_{4m+4} \end{matrix}\right]$ equal

$2m + 4m + 4 = 10; 6m + 4 = 10$

$m = 1$

Q.9 **(3)**

$T_{r+1} = {}^nC_r (2)^{n-r}\left(\frac{x}{3}\right)^r = {}^nC_r \frac{2^{n-r}}{3^r} x^r$

Coeff $T_8 = T_9 \Rightarrow {}^nC_7 \frac{2^{n-7}}{3^7} = {}^nC_8 \frac{2^{n-8}}{3^8}$

$\Rightarrow \frac{{}^nC_8}{{}^nC_7} = \frac{3.2}{1} \Rightarrow \frac{n-8+1}{8} = 6$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

Q.10 **(2)**

G.T. is $T_{r+1} = {}^{100}C_r (2)^{\frac{100-r}{2}} (3)^{\frac{r}{4}}$

The above term will be rational if exponent of 2 & 3 are integers.

i.e. $\frac{100-r}{2}$ and $\frac{r}{4}$ must be integers

the possible set of r is = {0, 4, 8, 16,……, 100}

no. of rational terms is 26

Q.11 **(2)**

If $n \in N$ & n is even then

$$\frac{1}{1\cdot(n-1)!} + \frac{1}{3!(n-3)} + \frac{1}{5!(n-5)} + \dots + \frac{1}{(n-1)!\ 1!}$$

$$= \frac{1}{n!}\left[{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots\dots + {}^nC_{n-1}\right]$$

n is even $\Rightarrow$ n – 1 is odd

${}^nC_{n-1}$ second Binomail coeff. from the end

$$= \frac{1}{n!}[C_1 + C_3 + C_5 + \dots\dots + C_{n-1}]$$

$$= \frac{1}{n!}\cdot 2^{n-1} = \frac{2^{n-1}}{n!}$$

Q.12 **(2)**

middle term = T_5

$T_5 = T_{4+1} = {}^8C_4 . k^4 = 1120$

$\Rightarrow k = 2$

Q.13 **(1)**

$C_0 + C_1 + C_2 + \dots\dots + C_n = 2^n = 256$

$\Rightarrow 2^n = 2^8 \Rightarrow n = 8$

$T_{r+1} = {}^8C_r (2x)^{8-r}\left(\frac{1}{x}\right)^r = {}^8C_r 2^{8-r} x^{8-2r}$

For Constant term $\Rightarrow 8 - 2r = 0 \Rightarrow r = 4$

$$= {}^8C_4 2^4 = \frac{8\times7\times6\times5}{4\times3\times2}2^4 = 70\times16 = 1120$$

Q.14 **(1)**

sum of coeff of $(1 - 2x + 5x^2)^n = a$

sum of coeff of $(1 + x)^{2n} = b$

put x = y = 1

$a = (1 - 2 + 5)^n = 4^n$ & $b = (1 + 1)^{2n} = 2^{2n} = 4^n$

a = b

Q.15 **(4)**

$\left(x^k + \frac{1}{x^{2k}}\right)^{3n}$, $n \in N$ Independent of x

$T_{r+1} = {}^{3n}C_r \left(x^k\right)^{3n-r}\left(\frac{1}{x^{2k}}\right)^r$

$= {}^{3n}C_r x^{3nk - rk - 2kr} = {}^{3n}C_r x^{3k(n-r)}$

For Constant term $\Rightarrow 3k(n - r) = 0 \Rightarrow n = r$

$\therefore T_{r+1} = {}^{3n}C_n$ true for any real k or $K \in R$

Q.16 **(1)**

$(3x + 2)^{-1/2}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

Q.17 **(2)**

Coeff of α^t in

$(\alpha+p)^{m-1}+(\alpha+p)^{m-2}(\alpha+q)+(\alpha+p)^{m-3}(\alpha+q)^2 \ldots\ldots + (\alpha+q)^{m-1}$

$\because \ a \neq -q, p \neq q$

Let $\alpha + P = x$ & $\alpha + q = y$

$= x^{m-1} + x^{m-2}y + x^{m-3}y^2 + \ldots\ldots y^{m-1}$

$$= x^{m-1}\left[1-\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2+\ldots\ldots+\left(\frac{y}{x}\right)^{m-1}\right]$$

$$= x^{m-1}\frac{\left[1-\left(\frac{y}{x}\right)^m\right]}{\left(1-\frac{y}{x}\right)}$$

$$= \frac{x^{m-1}}{x^m}\frac{x^m-y^m}{x-y}\cdot x = \frac{(\alpha+p)^m-(\alpha+q)^m}{\alpha+p-\alpha-q}$$

$$= \frac{1}{(p-q)}\left[(\alpha+p)^m-(\alpha+q)^m\right]$$

$$= \text{coeff of } \alpha^t = \left(\frac{{}^mC_t\, p^{m-t} - {}^mC_t\, q^{m-t}}{p-q}\right)$$

Q.18 (3)

$(2x+5y)^{13}$ greatest form for $x = 10, y = 2$

$$\frac{n+1}{\left|\frac{x}{y}\right|+1}-1 \le r \le \frac{n+1}{\left|\frac{x}{y}\right|+1}$$

$$\Rightarrow \frac{14}{\left|\frac{2x}{5y}\right|+1}-1 \le r \le \frac{14}{\left|\frac{2x}{5y}\right|+1}$$

$$\Rightarrow \frac{14}{3}-1 \le r \le \frac{14}{3} \Rightarrow \frac{11}{3} \le r \le \frac{14}{3}$$

$\Rightarrow 3.66 \ldots\ldots\ r \le 4.666 \Rightarrow r = 4$

$\Rightarrow T_5 = {}^{13}C_4 (20)^9 (10)^4$

Q.19 **(1)**

$$\text{For numerically greatest term } r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right]$$

$$= \left[\frac{9+1}{1+\left|\frac{4}{9}\right|}\right] \qquad \Rightarrow r = 6$$

$$\text{Numerically greatest term } T_{r+1} = {}^9C_6 (2)^3 \left(\frac{9}{2}\right)^6$$

Q.20 **(2)**

$$\text{For numerically greatest term } r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] = \left[\frac{34+1}{1+\left|\frac{6}{10}\right|}\right]$$

$\Rightarrow r = 21.$

Q.21 **(1)**

$$\sum_{r=0}^{n-1}\frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} = \sum_{r=0}^{n-1}\frac{r+1}{n+1}$$

$$= \frac{1}{n+1}[1+2+\ldots\ldots\ldots+n] = \frac{1}{n+1}\times\frac{n(n+1)}{2} = \frac{n}{2}$$

Q.22 **(3)**

${}^{18}C_{r-2} + 2\cdot{}^{18}C_{r-1} + {}^{18}C_r \ge {}^{20}C_{13}$

$\Rightarrow {}^{18}C_{r-2} + {}^{18}C_{r-1} + {}^{18}C_{r-1} + {}^{18}C_r \ge {}^{20}C_{13}$

$\Rightarrow {}^{19}C_{r-1} + {}^{19}C_r \ge {}^{20}C_{13} \Rightarrow {}^{20}C_r \ge {}^{20}C_{13}$

$\therefore {}^{20}C_{10} > {}^{20}C_{11} > {}^{20}C_{12} > {}^{20}C_{13}$

& ${}^{20}C_{10} > {}^{20}C_9 > {}^{20}C_8 > {}^{20}C_7$

$r = 7, 8, 9, 10, 11, 12, 13 \Rightarrow$ Total 7 elements

Q.23 **(2)**

$$\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}+\ldots\ldots+\frac{C_{10}}{11}$$

$$\sum_{r=0}^{10}T_{r+1} = \sum_{r=0}^{10}\frac{{}^{10}C_r}{r+1}$$

$$= \sum_{r=0}^{10}\frac{1}{(10+1)}\cdot\frac{10+1}{r+1}\cdot{}^{10}C_r = \frac{1}{11}\sum_{r=0}^{10}{}^{11}C_{r+1}$$

$$= \frac{1}{11}\left[{}^{11}C_1 + {}^{11}C_2 + \ldots\ldots + {}^{11}C_{11}\right]$$

$$= \frac{1}{11}\left[{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \ldots + {}^{11}C_{11} - {}^{11}C_0\right]$$

$$= \frac{1}{11}\left[2^{11}-1\right] = \frac{2^{11}-1}{11}$$

Q.24 **(3)**

${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{52}C_4$

Q.25 **(2)**

${}^{50}C_0 \times {}^{50}C_1 + {}^{50}C_1 \times {}^{50}C_2 + \ldots\ldots\ldots\ldots + {}^{50}C_{49} \times {}^{50}C_{50}$

$= {}^{50}C_0 \times {}^{50}C_{49} + {}^{50}C_1 \times {}^{50}C_{48} + \ldots\ldots\ldots\ldots + {}^{50}C_{49} \times {}^{50}C_0$

= co-eff. of x^{49} in $(1 + x)^{100} = {}^{100}C_{49}$

Q.26 **(3)**

$2^{2003} = 8.(16)^{500}$

$= 8\,(17-1)^{500}$

$\therefore$ Remainder = 8

Q.27 **(4)**

$3^{400} = (10-1)^{200}$

${}^{200}C_0(10)^{200} + \ldots\ldots + {}^{200}C_{199}(10)(-1) + {}^{200}C_{200}$

Last two digits = 01

Q.28 **(2)**

T_{22} is the numerically greatest term.

$(\sqrt{2}+1)^6 = I + f$

$(\sqrt{2}-1)^6 = f'$

$2[{}^6C_0 + {}^6C_2 \cdot 2 + {}^6C_4\ (2)^2 + \ldots\ldots] = I + f + f'$

$f + f' = 1$ or $f' = 1 - f$

$I = 2\,[{}^6C_0 + {}^6C_2 . 2 + {}^6C_4 . 4 + {}^6C_6 . 8] - 1$

$I = 2\,[1 + 30 + 60 + 8] - 1 = 197$

Q.29 **(2)**

$\left(5+2\sqrt{6}\right)^n = p + f$

$\left(5-2\sqrt{6}\right)^n = f' \Rightarrow 0 < f + f' < 2$

$p + f + f' = 2$ [integer]

so $f + f' =$ integer $= 1$

$\because n \in N$

$(f-1)(p+f) = -f'(p+f) = -(+1)^n = -1$

Q.30 **(4)**

coef of x^4 in $(1 - x + 2x^2)^{12}$

$= {}^{12}C_0(1-x)^{12}(2x^2)^0 + {}^{12}C_1(1-x)^{11}(2x^2) + {}^{12}C_2(1-x)^{10}$

$(2x^2)^2$ + above x^4 powers terms of x^4

$= {}^{12}C_0 \cdot {}^{12}C_4(-x)^4 + {}^{12}C_1\ {}^{11}C_2(-x)^2\, 2x^2 + {}^{12}C_2\ {}^{10}C_0\, 4x^4$

$= {}^{12}C_4 + 12 \cdot {}^{11}C_2 \cdot 2 + {}^{12}C_2 \cdot 4$

$= {}^{12}C_4 + 2.3 \cdot \dfrac{12}{3}\ {}^{11}C_2 + {}^{12}C_2 \cdot 4$

$= {}^{12}C_3 + {}^{12}C_2 + 3\left({}^{12}C_2 + {}^{12}C_3\right) + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$

$= {}^{12}C_3 + 3({}^{12}C_2 + {}^{12}C_3) + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$

$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{13}C_3 + {}^{13}C_4$

$= {}^{12}C_3 + 3\,{}^{13}C_3 + {}^{14}C_4$

EXERCISE-III

Q.1 **A, B, C, D**

$\left(4^{1/3} + \dfrac{1}{6^{1/4}}\right)^{20}$

$T_{r+1} = {}^{20}C_r(4^{1/3})^{20-r}(6^{-1/4})^r$

For rational terms

$20 - r = 3k$ & $r = 4p$, where $k, p \in I$

$\Rightarrow r = 20$ & $r = 8$

$\therefore$ no. of rational terms = 2

$\therefore$ no. of irrational terms = 19

Q.2 **B,C,D**

$\left(x^{2/3} - \dfrac{1}{\sqrt{x}}\right)^{30}$ term x^{13}

$T_{r+1} = {}^{30}C_r x^{\frac{2}{3}(30-r)}(+x)^{-\frac{1}{2}r}(-1)^r = {}^{30}C_r(-1)^r\, x^{\frac{120-7r}{6}}$

$\dfrac{120-7r}{6} = 13 \Rightarrow \dfrac{120-78}{7} = r \Rightarrow r = 6$

$= {}^{30}C_6 = \dfrac{30\cdot29\cdot28\cdot27\cdot26\cdot25}{6\cdot5\cdot4\cdot3\cdot2}$

which is divisibe by 29, 63, 65

Q.3 **B,C,D**

$\left(x^3 + 3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11} = \left(x^3 + \dfrac{3}{x^3}\right)^{11}$

$T_{r+1} = {}^{11}C_r(x^3)^{11-r}\ 3^r\left(x^3\right)^{-r} = {}^{11}C_r\ 3^r(x^3)^{11-2r}$

$= {}^{11}C_r\, 3^r(x^3)^{11-2r} = {}^{11}C_r\, 3^r x^{33-6r}$

(A) $33 - 6r = 2 \Rightarrow \dfrac{31}{6} = r \Rightarrow$ Not possible

(B) x^2 doesn't appear

(C) $33 - 6r = -3 \Rightarrow 36 = 6r \Rightarrow r = 6$

(x^{-3}) term exist/appear in exp.

(D) for x^3, $r = 5$, & x^{-3}, $r = 6$

$\dfrac{T_6}{T_7} = \dfrac{{}^{11}C_5\ 3^5}{{}^{11}C_6\ 3^6} = \dfrac{1}{3}$

Q.4 **B, C, D**

6^{th} term in the Expension of $\left[\dfrac{3}{2} + \dfrac{x}{3}\right]^n$ for $x = 3$

is numerically greatest $\dfrac{n+1}{\left|\dfrac{x}{y}\right|+1} - 1 \leqslant r \leqslant \dfrac{n+1}{\left|\dfrac{x}{y}\right|+1}$

$\Rightarrow \dfrac{n+1}{\dfrac{3}{2}+1} - 1 \leqslant 5 \leqslant \dfrac{n+1}{\dfrac{3}{2}+1}$

$\Rightarrow \dfrac{2(n+1)}{5} - 1 \le 5 \le \dfrac{2(n+1)}{5}$

$\Rightarrow \dfrac{2(n+1)}{5} \leqslant 6$ and $\dfrac{2(n+1)}{5} \geqslant 5$

$\Rightarrow n + 1 \leqslant 15$ and $n + 1 \geqslant \dfrac{25}{2}$

$\Rightarrow n \leqslant 14$ and $n + 1 \geqslant \dfrac{23}{2}$

$\Rightarrow\ 11.4 \leqslant n \leqslant 14$

$n \in N \Rightarrow n = 12, 13, 14$

for these value of n 6^{th} term is greatest term

Q.5 **A, B**

$(1+x^2)^2(1+x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$\Rightarrow\ (1+x^4+2x^2)(1+x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$$\Rightarrow \left(1+x^4+2x^2\right)\left[1+nx+\frac{n(n-1)x^2}{2!}+\frac{n(n-1)(n-2)}{3!}x^3+\ldots\ldots\right]$$

$= A_0 + A_1x + A_2x^2 + \ldots.$

by comparism

$A_0 = 1$

$$A_1 = n \ \&\ A_2 = \frac{n(n-1)}{2!} + 2 = \frac{n^2-n+4}{2}$$

$2A_1 = A_0 + A_2$

$$\Rightarrow\ 2n = 1 + \frac{n^2-n+4}{2}$$

$\Rightarrow\ 4n = 2 + n^2 - n + 4$

$\Rightarrow\ n^2 - 5n + 6 = 0$

$\Rightarrow\ (n-2)(n-3) = 0$

$\Rightarrow\ n = 2$ or $n = 3$

Q.6 **A, C, D**

$\left(9+\sqrt{80}\right)^n = I + f$

$\left(9-\sqrt{80}\right)^n = f'$

$2[{}^nC_0(9)^n + {}^nC_2(9)^{n-2}(\sqrt{80})^2 + \ldots.] = I + f + f'$

$\therefore\ I = 2(\text{integer}) - 1 \qquad (\because f + f' = 1)$

$\therefore\ (I+f)(1-f) = 1$

Q.7 **A,B,C**

$101^{100} - 1 = (1+100)^{100} - 1$

$= {}^{100}C_0 + {}^{100}C_1(100)^1 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}(100)^{100} - 1$

$= 100 \times 100 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}\,100^{100}$

$= 10000\,[1 + {}^{100}C_2 + {}^{100}C_3(100) + \ldots + {}^{100}C_{100}\,100^{98}]$

$= 10000$ [Integer]

which is divisible by 100, 1000, 10000

Q.8 **A, C**

$7^9 + 9^7 = (8-1)^9 + (8+1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7$

$\ldots.. + {}^9C_8(8) - {}^9C_9 + {}^7C_0(8)^7 + \ldots\ldots + {}^7C_6(8) + {}^7C_7$

This is divisible by 64 & 16

Q.9 **A, D**

Constant term in $P_1(x)$ is 4

If the constant term in $P_k(x)$ is also 4, then

$P_k(x) = 4 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots$

and $P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + \ldots.+ 2)^2$

Q.10 **A, B, C**

$$\text{General term} = \frac{10!}{r_1!\,r_2!\,r_3!}(1)^{r_1}(2x)^{r_2}(3x^2)^{r_3}$$

$a_1 =$ Coeff. of x

$r_2 + 2r_3 = 1 \Rightarrow r_2 = 1, r_1 = 9, r_3 = 0$

$$\therefore\ a_1 = \frac{10!}{1!\,9!}(2)^1 = 20$$

$a_2 =$ Coeff. of x^2

$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2, r_1 = 8, r_3 = 0$

$r_2 = 0, r_1 = 9, r_3 = 1$

$$a_2 = \frac{10!}{2!\,8!}(2)^2 + \frac{10!}{9!\,1!}(3) = 210$$

$a_4 =$ coeff. of x^4

$r_2 + 2r_3 = 4 \Rightarrow r_2 = 4, r_1 = 6, r_3 = 0$

$r_2 = 2, r_1 = 7, r_3 = 1$

$r_2 = 0, r_1 = 8, r_3 = 2$

$$a_4 = \frac{10!}{4!\,6!}(2)^4 + \frac{10!}{2!\,7!\,1!}(2)^2(3) + \frac{10!}{8!\,2!}(3)^2$$

$= 8085$

$a_{20} = 3^{10}$

Q.11 **B**

Required sum = coefficient of x^n in

$(1+x)^n(1+x)^n$

= coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n$

Q.12 **D**

$\because\ {}^{30}C_r.\,{}^{20}X_0 + {}^{30}C_{r-1}.\,{}^{20}C_1. + \ldots.. + {}^{30}C_0.{}^{20}C_r$

= coefficient of x^r $(1+x)^{30}(1+x)^{20}$

= coefficient of x^r in $(1+x)^{50} = {}^{50}C_r$

$\because\ {}^{50}C_r$ is maximum

$$\therefore\ r = \frac{50}{2} = 25$$

Q.13 **A**

$\because\ {}^{20}C_r.\,{}^{10}C_0 + {}^{20}C_{r-1}.\,{}^{10}C_1 + \ldots + {}^{20}C_0.\,{}^{10}C_r$

= coefficient of x^r in$(1+x)^{20}(1+x)^{10}$

= coefficient of x^r $(1+x)^{30} = {}^{30}C_r$

$\because\ {}^{30}C_r$ is minimum

$\therefore\ r = 0$

Q.14 **C**

$(1+i)^n = C_0 + C_1 i + C_2 i^2 + C_3 i^3 + ...$

$\Rightarrow (1+i)^n = (C_0 - C_2 + C_4 - C_6 + ...)$
$+ i(C_1 - C_3 + C_5 - ....)$ ....(i)

$\Rightarrow C_0 - C_2 + C_4 - C_6 + ... =$ Real part of $(1+i)^n$

$=$ Real part of $\left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^n$

$=$ Real part of $2^{n/2}\left(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4}\right) = 2^{n/2}\cos\frac{n\pi}{4}$.

Q.15 **(B)**

From the previous question on taking modulus of equation .....(i)

$(C_0 - C_2 + C_4 - ......)^2 + (C_1 - C_3 + C_5 - ......)^2 = ((\sqrt{2})^n)^2$

$= 2^n$

Q.16 **(C)**

Add the expansion of $(1+1)^n$, $(1+\omega)^n$ and $(1+\omega^2)^n$

$(1+\omega)^n = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^n = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^n$

$(1+\omega^2)^n = \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^n = \left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^n$

Q.17 (A) $\to$ (q, s), (B) $\to$ (p, q, r), (C) $\to$ (s), (D) $\to$ (p, s)

(A) We have, T_{r+1}

$= \dfrac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right).....\left(\frac{7}{2}-r+1\right)x^r}{r!}$

This will be the first negative term when $\frac{7}{2} - r + 1$

< 0 i.e. $r > \frac{9}{2}$

Hence $r = 5$.

(B) We have, $T_{r+1} = {}^5C_r\,(y^2)^{5-r}\left(\frac{1}{y}\right)^r = {}^5C_r\,y^{10-3r}$

Now, $10 - 3r = 1 \Rightarrow r = 3$

So, coefficient of $y = {}^5C_3 = 10$

(C) Obviously a prime number.

(D) We have : $(1 + 2x + 3x^2 + 4x^3 + ...)^{1/2}$

$= [(1-x)^{-2}]^{1/2} = (1-x)^{-1} = 1 + x + x^2 + ... + x^n + ...$

Hence, coefficient of $x^4 = 1$ c = 1, hence c + 1 = 2

Q.18 (A) $\to$ (q), (B) $\to$ (s), (C) $\to$ (p), (D) $\to$ (r)

(A) $I + f = (7 + 4\sqrt{3})^{2n}$

Here $(7 - 4\sqrt{3})^{2n} = f' = 1 - f$

$(I + f)(1 - f) = 1$

(B) $T_2 = {}^nC_1\,(x)^{n-1}\,.\,a = 240$ ....(i)

$T_3 = {}^nC_2\,(x)^{n-2}\,a^2 = 720$ ....(ii)

$T_4 = {}^nC_3\,(x)^{n-3}\,a^3 = 1080$ ....(iii)

From (i) and (ii)

Here $\dfrac{{}^nC_1(x)^{n-1}a}{{}^nC_2 x^{n-2}a^2} = \dfrac{2x}{(n-1)a} = \dfrac{240}{720} = \dfrac{1}{3}$

$\Rightarrow 6x = (n-1)a$

From (ii) and (iii)

$9x = 2(n-2)\,a$

On dividing $\dfrac{3}{2} = \dfrac{2(n-2)}{(n-1)} \Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$

(C) $C_0C_4 - C_1C_3 + C_2C_2 - C_3C_1 + C_4C_0 = 2 - 2.4.4 + 6.6 = 6$

(D) $\sqrt{\dfrac{x}{x+y}}\ \sqrt{\dfrac{x}{x-y}} = \left(\dfrac{1}{1+\frac{y}{x}}\right)^{1/2}\left(\dfrac{1}{1-\frac{y}{x}}\right)^{1/2}$

$= \left(1 - \dfrac{y^2}{x^2}\right)^{-1/2} = 1 + \dfrac{1}{2}\,.\,\dfrac{y^2}{x^2}$ $\Rightarrow k = 2$

NUMERICAL VALUE BASED

Q.19 [10]

$T_6 = {}^8C_5\left(\dfrac{1}{x^{8/3}}\right)^3 (x^2\log_{10}x)^5 = 5600 \Rightarrow \dfrac{1}{x^8}\ x^{10}(\log_{10} x)^5 = 100$ $\Rightarrow x = 10$

Q.20 [50]

Co-efficient of x^{50}

$S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001x^{1000}$ ....(i)

$\dfrac{xS}{1+x} = x(1+x)^{999} + 2x^2(1+x)^{998}.... + 1000x^{1000} + \dfrac{1001x^{1001}}{(1+x)}$ ....(ii)

$\dfrac{S}{1+x} = (1+x)^{1000} + x(1+x)^{999} + ... + x^{1000} - \dfrac{1001\,x^{1001}}{1+x}$

$$\Rightarrow \frac{S}{1+x} = (1+x)^{1000}\left[\frac{1-\left(\frac{x}{1+x}\right)^{1001}}{1-\frac{x}{1+x}}\right] - \frac{1001\,x^{1001}}{(1+x)}$$

$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001\,x^{1001}$

Co-efficient of $x^{50} = {}^{1002}c_{50}$

Q.21 n = 12

For T_9 to be the numerically greatest term, $r = \left[\frac{n+1}{1+\left|\frac{x}{a}\right|}\right] =$

$$\left[\frac{n+1}{1+\left|\frac{1}{2}\right|}\right] = 8 \Rightarrow 8 < \frac{2(n+1)}{3} < 9 \Rightarrow 11 < n < 12.5 \Rightarrow n = 12$$

Q.22 [2]

${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$r^2 = 3r$ or $r = 0, 3$

Q.23 [0001]

$2^{60} = (1+7)^{20}$

$= {}^{20}C_0 + {}^{20}C_1.7 + {}^{20}C_2.7^2 + ... + {}^{20}C_{20}.7^{20}$

$\therefore$ The remainder $= {}^{20}C_0 = 1$.

Q.24 [0001]

$3^{400} = (3^4)^{100} = (81)^{100} = (1+80)^{100}$

$= 1 + {}^{100}C_1(80) + {}^{100}C_2(80)^2 + .. + {}^{100}C_{100}(80)^{100}$

= 1 + 8000 + (Last digit in each term is 0)

$\therefore$ Last digit = 1.

Q.25 [0012]

Since, n is even therefore $\left(\frac{n}{2}+1\right)$ th term is the middle terms

$\therefore \; T_{\frac{n}{2}+1} = {}^nC_{n/2}(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2} = 924x^6$

$\Rightarrow x^{n/2} = x^6 \Rightarrow n = 12$

Q.26 [0007]

The general term

$= {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(\frac{-1}{3x}\right)^r = (-1)^r\,{}^9C_r\,\frac{3^{9-2r}}{2^{9-r}}.x^{18-3r}$

The term independent of x, (or the constant term) corresponds to x^{18-3r} being

x^0 or $18 - 3r = 0 \Rightarrow r = 6$

Q.27 [0015]

$S = \sum_{i=0}^{m} {}^{10}C_i\;{}^{20}C_{m-i}$

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + ... + {}^{10}C_{10}x^{10}$...(1)

$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + ... {}^{20}C_{20}x^{20}$...(2)

$\therefore$ S represents coefficient of x^m in (1) × (2)

Coefficient x^m in $(1+x)^{30} = {}^{30}C_m$

$\therefore$ For this to be maximum m = 15

Q.28 [0006]

General term : ${}^{55}C_r\,(y^{1/3})^{55-r}\cdot\left(x^{\frac{1}{10}}\right)^r$

: ${}^{55}C_r \cdot y^{\frac{55-r}{3}} \cdot x^{\frac{r}{10}}$.

Terms free from radical sign, wehn

$r = 0, \quad \frac{55-0}{3} = \frac{55}{3}$ (Not possible)

$r = 10, \quad \frac{55-10}{3} = 15$

$r = 20, \quad \frac{55-20}{3} = \frac{35}{3}$ (Not possible)

$r = 40, \quad \frac{55-40}{3} = 5$

$r = 50, \quad \frac{55-50}{3} = \frac{5}{3}$ (Not possible)

Terms free from radical sign = 2.

EXERCISE-IV

JEE-Main
PREVIOUS YEAR'S

Q.1 (Bonus)

Number of terms = ${}^{n+3-1}C_{3-1} = 28 \Rightarrow n = 6$

Sum of coefficients = $(1-2+4)^6 = 729$

Q.2 (1)

$({}^{21}C_1 + {}^{21}C_2 .... + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 .... {}^{10}C_{10})$

$= \frac{1}{2}\ [({}^{21}C_1 + ... + {}^{21}C_{10}) + ({}^{21}C_{11} + ... {}^{21}C_{20})]$

$-(2^{10} - 1)$

$= \frac{1}{2}\ [2^{21} - 2] - (2^{10} - 1)$

$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

Q.3 (3)

using $(x + a)^5 + (x - a)^5$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3 \cdot a^2 + {}^5C_4 x \cdot a^4]$

$\left(x + \sqrt{x^3 - 1}^{\,5} + (x) - \sqrt{x^3 - 1}^{\,5}\right)$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3(x^3 - 1) + {}^5C_4 x(x^3 - 1)^2]$

$\Rightarrow 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

considering odd degree terms,

$2[x^5 + 5x^7 - 10x^3 + 5x]$

$\therefore$ sum of coefficients of odd terms is 2

Q.4 (2)

$$\frac{2^{403}}{15} = \frac{2^3.2^{400}}{15} = \frac{8.(1+15)^{100}}{15}$$

$$= \frac{8\left({}^{100}C_0 + {}^{100}C_1(15) + {}^{100}C_2(15)^2 + ...\right)}{15}$$

$$\frac{8}{15} + 8\left({}^{100}C_1(15) + {}^{100}C_2(15)^2 + ...\right)$$

Remainder is 8.

Q.5 (2)

$(1-t^6)^3\,(1-t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12})(1 - t)^{-3}$

$\Rightarrow$ coefficient of t^4 in $(1 - t)^{-3}$ is ${}^{3+4-1}C_4 = {}^6C_2 = 15$

Q.6 (4)

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3$$

Now $\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} = \frac{{}^{20}C_{i-1}}{{}^{21}C_1}\ \frac{I}{21}$

Let given sum be S, so

$$S = \sum_{i=1}^{20}\frac{(i)^3}{21^3} = \frac{1}{(21)^3}\left(\frac{20.21}{2}\right)^2 = \frac{100}{21}$$

Given $S = \frac{k}{21} \Rightarrow k = 100$

Q.7 (1)

In the expansion of $\left(1 + x^{\log_2 x}\right)^5$

third term say $T_3 = {}^5C_2\,\left(x^{\log_2 x}\right)^2 = 2560$

$\Rightarrow \left(x^{\log_2 x}\right)^2 = 256$

taking logarithm to the base 2 on both sides

$\Rightarrow 2(\log_2 x)^2 = 8 \Rightarrow (\log_2 x) = \pm 2$

$\Rightarrow x = 4, \frac{1}{4}$

Here $x = \frac{1}{4}$

Q.8 (1)

$$x^2\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$$

Consider constant term

$${}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$$

$\frac{10-r}{2} - 2r = 0$

$10 - 5r = 0$

$r = 2$

$\Rightarrow {}^{10}C_2 \times \lambda^2 = 720 \Rightarrow \lambda = 4$

Q.9 (4)

$$\sum_{r=0}^{25}\frac{\lfloor 50}{\lfloor r \lfloor 50-r} \times \frac{\lfloor 50-r}{\lfloor 25-r \lfloor 25}$$

$$= \sum_{r=0}^{25}\frac{\lfloor 50}{\lfloor r \lfloor 25-r\ \lfloor 25}$$

$$= \frac{\lfloor 50}{\lfloor 25}\sum_{r=0}^{25}\frac{1}{\lfloor r \lfloor 25-r}$$

$$= \frac{50}{\lfloor 25 \lfloor 25}\sum_{r=0}^{25}{}^{25}C_r = {}^{50}C_{25}\left(2^{25}\right).$$

Q.10 (4)

$$\sum_{r=1}^{101}{}^{101}C_r\,S_{r-1}$$

$$= \sum_{r=1}^{101}{}^{101}C_r\,\frac{q^r - 1}{q - 1}$$

$$=\frac{1}{q-1}\left(\sum_{r=1}^{101}{}^{101}C_r q^r-\sum_{r=1}^{101}{}^{101}C_r\right)$$

$$=\frac{1}{q-1}((1+q)^{101}-1-2^{101}+1)$$

$$=\frac{\alpha}{2^{100}}\left(\frac{(1+q)^{101}-2^{101}}{q-1}\right)$$

$\Rightarrow\ \alpha=2^{100}$

Q.11 (3)

$(10+x)^{50}+(10-x)^{50}$

$a_0=(10^{50})(2)$

$a_2={}^{50}C_2(10)^{48}(2)$

$$\frac{a_2}{a_0}=\frac{{}^{50}C_2(10)^{48}(2)}{10^{52}(2)}=12.25$$

Q.12 (2)

${}^{20}C_r.\,{}^{20}C_0+{}^{20}C_{r-1}.\,{}^{20}C_1+............+{}^{20}C_0.\,{}^{20}C_r=$

Selecting r student from 20 boys and 20 girls = ${}^{40}C_r$

${}^{40}C_r$ will be maximum if $r=20$.

Q.13 (1)

5^{th} term will be the middle term.

$$t_{4+1}={}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4=5670$$

$$={}^8C_4.x^8=5670$$

$$=\frac{8\times7\times6\times5}{4\times3\times2}x^8=5670$$

$$=x^8=\frac{567}{7}=81\ =x^8-81=0$$

$\Rightarrow$ Real value of $x=\pm\sqrt{3}$

Q.14 (3)

$$\frac{5^{th}\text{ term from begining}}{5^{th}\text{ term from end}}=\frac{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^4 2^{6/3}}{{}^{10}C_4(2)^{4/3}\left(\frac{1}{2(3^{1/3})}\right)^6}$$

$$=\frac{2^2 2^{-2}3^{-4/3}}{2^4 2^{(4/3)-6}3^{-2}}=3^{2/3}.2^{8/3}=4.(36)^{1/3}$$

Q.15 (4)

General term $T_{r+1}={}^{60}C_r,7^{\frac{60-r}{5}}3^{\frac{r}{10}}$

$\therefore$ for rational term, $r=0, 10, 20, 30, 40, 50, 60$

$\Rightarrow$ number of rational terms = 7

$\therefore$ number of irrational terms = 54

Q.16 (2)

$2.{}^nC_5={}^nC_4+{}^nC_6$

$$2.\frac{\lfloor n}{\lfloor 5\lfloor n-5}=\frac{\lfloor n}{\lfloor 4\lfloor n-4}+\frac{\lfloor n}{\lfloor 6\lfloor n-6}$$

$$\frac{2}{5}\cdot\frac{1}{n-5}=\frac{1}{(n-4)(n-5)}+\frac{1}{30}$$

n = 14 satisfying equation

Q.17 **(2)**

$2.{}^{20}C_0+5.{}^{20}C_1+8.{}^{20}C_2+11.{}^{20}C_3+.......+62.{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r.\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r\left(\frac{20}{r}\right){}^{19}C_{r-1}+2.2^{20}=60.2^{19}+2.2^{20}=2^{25}$$

Q.18 **(4)**

$$\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$$

$=2[{}^6C_0x^6+{}^6C_2x^4(x^3-1)+{}^6C_4x^2(x^3-1)^2+{}^6C_6(x^3-1)^3]$

$=2[{}^6C_0x^6+{}^6C_2x^7-{}^6C_2x^4+{}^6C_4x^8+{}^6C_4x^2-2{}^6C_4x^5+(x^9-1$
$-3x^6+3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x

$=2[1-15+15+15-1-3]=24$

Q.19 (Bonous)

$$200={}^6C_3\left(x^{\frac{1}{1+\log_{10}x}}\right)^{\frac{3}{2}}\times x^{\frac{1}{4}}$$

$$\Rightarrow 10=x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$$

$$\Rightarrow 1=\left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$$

where $t = \log_{10} x$

$\Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow x = 10 (\text{As } x > 1)$

Q.20 (2)

$$T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 . \left(x^{\log_8 x}\right)^3$$

$$20 \times 8^7 = \frac{160}{x^3} . x^{3\log_8 x}$$

$$8^6 = x^{\log_2 x} - 3$$

$$2^{18} = x^{\log_2 x - 3}$$

$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow (t-6)(t+3) = 0$

$\Rightarrow t = 6, -3$

$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$

$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

Q.21 (4)

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \quad \Rightarrow \quad \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$15r = 2n - 2r + 2$

$\boxed{17r = 2n + 2}$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14} \quad \Rightarrow \quad \frac{r+1}{n-r} = \frac{3}{14}$$

$14r + 14 = 3n - 3r$

$3n - 17r = 14$

$$\frac{2n - 17r = -2}{n = 16}$$

$17r = 34, r = 2$

${}^{16}C_1, {}^{16}C_2, {}^{16}C_3$

$$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$$

$$\frac{680 + 16}{3} = \frac{696}{3} = 232$$

Q.22 (2)

$$T_r = \sum_{r=0}^{n} {}^nC_r x^{2n-2r} . x^{-3r}$$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$

for $r = 15 . n = 38$

smallest value of n is 38.

Q.23 (1)

Coefficient of $x^2 = {}^{15}C_2 \times 9 - 3a({}^{15}C_1) + b = 0$

$\Rightarrow -45a + b + {}^{15}C_2 + 9 = 0$ ....(i)

Also, $-27 \times {}^{15}C_3 + 9a \times {}^{15}C_2 - 3b \times {}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ....(ii)

(i) + (ii)

$-24\, a + 672 = 0$

$\Rightarrow a = 28$

So, $b = 315$

Q.24 **(2)**

$$(1+x)(1-x)^{10}\left(1+x+x^2\right)^9$$

$$\left(1-x^2\right)\left(1-x^3\right)^9$$

$${}^9C_6 = 84$$

Q.25 **(4)**

$$\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81} . x^8 \left(2x^2 - \frac{3}{x^2}\right)^6$$

its general term

$$\frac{1}{60}\, {}^6C_r 2^{6-r} (-3)^r x^{12-4r} - \frac{1}{81}\, {}^6C_r 2^{6-r} (-3)^r 12^{20-4r}$$

for term independent of x, r for I[st] expression is 3 and r for second expression is 5

$\therefore$ term independent of $x = -36$

Q.26 **[30]**

Let $(1 - x + x^2 \ldots\ldots)(1 + x + x^2 + \ldots\ldots) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots$

Put $x = 1$

$1(2n + 1) = a_0 + a_1 + a_2 + \ldots\ldots + a_{2n}$ ......(1)

Put x = – 1

$(2n + 1)\ 1 = a_0 - a_1 + a_2 + \ldots\ldots + a_{2n}$ ……(2)

From (1) + (2),

$4n + 2 = 2\ (a_0 + a_2 + \ldots\ldots) = 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30.$

Q.27 (3)

$$\frac{{}^{36}C_{r+1}}{{}^{35}C_r} = \frac{6}{k^3 - 3}$$

$$k^2 - 3 = \frac{r+1}{6} \qquad \Rightarrow k^2 = 3 + \frac{r+1}{6}$$

r can be 5, 35

for r = 5, k = ±2

r = 35, k = ±3

Hence number of order pair = 4

Q.28 (2)

$$\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$$

$$\frac{(1+x)^{10}\left[(1+x)^{11}-x^{11}\right]}{(1+x)^{11}\times\frac{1}{(1+x)}} = (1+x)^{11} - x^{11}$$

Coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_4 = 330$

Q.29 **(3)**

We know ${}^nC_r$ is max at middle term

$a = {}^{19}C_p = {}^{19}C_{10} = {}^{19}C_9$

$b = {}^{20}C_q = {}^{20}C_{10}$

$c = {}^{21}C_6 = {}^{21}C_{10} = {}^{21}C_{11}$

$$\frac{a}{{}^{19}C_9} = \frac{b}{\frac{20}{10}\cdot{}^{19}C_9} = \frac{c}{\frac{21}{11}\cdot\frac{20}{10}\cdot{}^{19}C_9}$$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{42/11}$$

$$\frac{a}{11} = \frac{b}{22} = \frac{c}{42}.$$

Q.30 **[615]**

General term $= \dfrac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

for coefficient of $x^4 \Rightarrow \beta + 2\gamma = 4$

$\gamma = 0, \beta = 4, \alpha = 6 \quad \Rightarrow \dfrac{10!}{6!4!0!} = 210$

$\gamma = 1, \beta = 2, \alpha = 7 \quad \Rightarrow \dfrac{10!}{7!2!1!} = 360$

$\gamma = 2, \beta = 0, \alpha = 8 \quad \Rightarrow \dfrac{10!}{8!0!2!} = 45$

Total = 615

Q.31 (4)

$$T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$$

for r = 8 term is free from ‘x’

$$T_9 = {}^{16}C_8\ \frac{1}{\sin^8\theta\cos^8\theta}$$

$$T_9 = {}^{16}C_8\ \frac{2^8}{(\sin 2\theta)^8}$$

in $\theta \in \left[\frac{\pi}{8},\frac{\pi}{4}\right]$, $L_1 = {}^{16}C_8\, 2^8$ $\because$ {Min value of L_1 at $\theta = \pi/4$}

In $\theta \in \left[\frac{\pi}{16},\frac{\pi}{8}\right]$, $L_2 = {}^{16}C_8\ \dfrac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8.2^8.2^4$ {$\because$ min value of L_2 at $\theta = \pi/8$]

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8.2^8 2^4}{{}^{16}C_8.2^8} = 16$$

Q.32 [51]

$$\sum_{r=0}^{25}(4r+1)^{25}C_r = 4\sum_{r=0}^{25} r^{25}C_r + \sum_{r=0}^{25}{}^{25}C_r$$

$$= 4\sum_{r=1}^{25} r \times \frac{25}{r}\,{}^{24}C_{r-1} + 2^{25} = 100\sum_{r=1}^{25}{}^{24}C_{r-1} + 2^{25}$$

$= 100\ .2^{24} + 2^{25} = 2^{25}\ (50 + 1) = 51.2^{25}$

So k = 51

Q.33 [118]

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} : 2 : 5 : 12$

$$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2} \text{ and } \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{5}{2} \text{ and } \frac{n-r}{r+1} = \frac{12}{5}$$

$\Rightarrow 2n - 7r + 2 = 0$ and $5n - 17r - 12 = 0$

Solving ; x = 118, r = 34

Q.34 (3)

General term of

$$\left(\alpha x^{1/9}+\beta x^{1/6}\right)^{10} = {}^{10}C_r\left(\alpha x^{1/9}\right)^{10-r}\left(\beta x^{-1/6}\right)^r$$

For term independent of 'x' r = 4

$\therefore$ Term independent of x = ${}^{10}C_r\,\alpha^6\beta^4$

Also $\alpha^3+\beta^2 = 4$

By AM - GM inequality

$$\frac{\alpha^3+\beta^2}{2}\ge\left(\alpha^3\beta^2\right)^{1/2}$$

$\Rightarrow (2)^2\ge\alpha^3\beta^2 \qquad \Rightarrow \alpha^6\beta^4\le 16$

$\therefore\ 10k = {}^{10}C_4\cdot 16 \qquad \Rightarrow k = 336$

Q.35 (4)

$$\text{General term} = T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

$$= {}^9C_r\,(-1)^r.\ \frac{3^{9-2r}}{2^{9-r}}\ x^{18-3r}$$

If term is independent of x then r = 6.

$$\therefore\ k = {}^9C_6\,.\ \frac{3^{-3}}{2^3} = \frac{7}{18}$$

$\therefore\ 18\,K = 7$

Q.36 (3)

$$\left(3^{1/2}+5^{1/8}\right)^n$$

Let $T_{r+1} = {}^nC_r\,(3)^{\frac{n-r}{2}}\,(5)^{\frac{r}{8}}$

So, r must be 0, 8, 16, 24 ......

Now $n = t_{33} = 0 + 32\times 8 = 256$

$\Rightarrow n = 256$

Q.37 (2)

$$\because\ (r+1).\,{}^rP_{r-1} = (r+1)\,.\,\frac{\lfloor r}{\lfloor 1} = \lfloor r+1$$

So $(2.\,{}^1P_0 - 3.\,{}^2P_1 + ...51 \text{ terms}) +$

$(\lfloor 1 - \lfloor 2 + \lfloor 3 - ....$ upto 51 terms)

$= [\lfloor 2 - \lfloor 3 + \lfloor 4 - .... + \lfloor 52] + [\lfloor 1 - \lfloor 2 + \lfloor 3 - .... + \lfloor 51]$

$= [\lfloor 52 + \lfloor 1 = \lfloor 52 + 1$

Q.38 (2)

Consider the three consecutive coefficients as ${}^{n+5}C_r,\ {}^{n+5}C_{r+1},\ {}^{n+5}C_{r+2}$

$$\because\ \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{1}{2}$$

$$\Rightarrow\ \frac{r+1}{n+5-r} = \frac{1}{2}\Rightarrow 3r = n+3 \qquad .....(i)$$

$$\text{and}\ \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \frac{5}{7}$$

$$\Rightarrow\ \frac{r+2}{n+4-r} = \frac{5}{7}\Rightarrow 12r = 5n+6 \qquad ....(ii)$$

From (i) and (ii) n = 6

Largest coefficient in the expansion is ${}^{11}C_6 = 462$

Q.39 (1)

$$\text{Here}\ \sum_{r=0}^{20}{}^{50-r}C_6$$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + ...\ {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$

$\Rightarrow ({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$\Rightarrow ({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{32}C_7 + {}^{32}C_6) + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

....

....

....

$= {}^{51}C_7 - {}^{30}C_7$

Q.40 (120)

$$(1+x+x^2+x^3)^6 = \left(\frac{1-x^4}{1-x}\right)^6$$

Coefficient of x4 in $\left(\frac{1-x^4}{1-x}\right)^6$ = coefficient of x^4

in $(1-6x^4)(1-x)^{-6}$

= coweefficient of x^4 in

$(1-6x^4)\,[1 + {}^6C_1x + {}^7C_2x^2 + ...]$

$= {}^9C_4 - 6.1 = 126 - 6 = 120$

Q.41 [13]

$$T_{r+1} = {}^{22}C_r\cdot (x^m)^{22-r}.\left(\frac{1}{x^2}\right)^r$$

$T_{r+1} = {}^{22}C_r\cdot\ x^{22m-mr-2r}$

$\because\ 22m - mr - 2r = 1$ amd ${}^{22}C_r = 1540$

$\because\ {}^{22}C_3 = 1540 \Rightarrow r = 3$ or 19

Now, for r = 3; 22m – 3m – 6 = 1

$\Rightarrow 19m = 7 \quad \Rightarrow m = \frac{7}{19}$ (not acceptable)

for r = 19; 22m – 19m = 39

$\Rightarrow m = 13$

Q.42 (2)

General term $= T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\cdot\left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r\,(-k)^r \cdot x^{\frac{10-r}{2}-2r}$

$= {}^{10}C_r\,(-k)^r \cdot x^{\frac{10-5r}{2}}$

If it is constant term then r = 2

$\because\ {}^{10}C_2\,(-k)^r = 405$

$\Rightarrow k^2 = \frac{405\times 2}{10\times 9} = \frac{81}{9} = 9$

$|k| = 3$

JEE-ADVANCED
PREVIOUS YEAR'S

Q.1 **(D)**

$B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10}(A_r)^2 = {}^{20}B_{10}({}^{30}C_{20} - 1) - {}^{30}C_{10}$

$({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

Q.2 **[6]**

${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5 : 10 : 14$

$\Rightarrow \frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = \frac{10}{5} \quad \& \quad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{14}{10}$

$\Rightarrow \frac{(n+5)-r+1}{r} = 2 \quad \& \quad \frac{(n+5)-(r+1)+1}{r+1} = \frac{7}{5}$

$\Rightarrow \frac{n+6}{r} = 3 \quad \& \quad \frac{n+6}{r+1} = \frac{12}{5}$

$\Rightarrow 3r = \frac{12}{5}(r+1) \quad \Rightarrow r = 4$

$\therefore\ n + 6 = 12 \quad \Rightarrow n = 6$

Q.3 **(C)**

Coefficent of x^{11}

$\equiv \frac{(1+x^2)^4(1+x^3)^7(1+x^4)^{12}(1-x^2)^4}{(1-x^2)^4}$

Coefficent of $x^{11} \equiv (1-x^8)^4(1+x^4)^8(1+x^3)^7(1-x^2)^{-4}$

$= (1-4x^8)(1+x^4)^8(7x^3+35x^9)(1-x^2)^{-4}$

$= (7x^3+35x^9-28x^{11})(1+x^4)^8(1-x^2)^{-4}$

Coefficent of $x^8 = (7x+35x^6-28x^8)(1+8x^4+28x^8)(1-x^2)^{-4}$

$= (7+35x^6-28x^8+56x^4+196x^8)(1-x^2)^{-4}$

Coefficent of $t^4 \equiv (7+56t^2+35t^3+168t^4)(1-t)^{-4}$

$= 7\cdot{}^7C_3 + 56\cdot{}^5C_3 + 35\cdot{}^4C_3 + 168$

$= 245 + 700 + 168 = 1113.$

Alterantive :

$2x + 3y + 4z = 11$

$(x, y, z) = (0, 1, 2)\ {}^4C_0 \times {}^7C_1 \times {}^{12}C_2$

$(1, 3, 0)\quad {}^4C_1 \times {}^7C_3$

$(2, 1, 1)\ {}^4C_2 \times {}^7C_1 \times {}^{12}C_1$

$(4, 1, 0)\quad {}^7C_1$

coefficient of x^{11} $= 66 \times 7 + 35 \times 4 + 42 \times 12 + 7$

= 1113. **Ans.**

Q.4 **[8]**

9 = (0, 9) (1, 8), (2, 7), (3, 6), (4, 5) # 5 cases

9 = (1,2,6), (1,3,5), (2, 3, 4) # 3 cases

total = 8

Q.5 **[5]**

coefficient of x^2 in $(1+x)^2 + (1+x)^3 + ... + (1+x)^{49} + (1+mx)^{50}$

$= (3n+1)\ {}^{51}C_3$

$\Rightarrow$ coefficient of x^2 in

$\frac{(1+x)^2\left((1+x)^{48}-1\right)}{1+x-1} + (1+mx)^{50} = (3n+1)\ {}^{51}C_3$

$\Rightarrow {}^{50}C_3 + {}^{50}C_2 \times m^2 = = (3n+1)\ {}^{51}C_3$

$\Rightarrow 16 + m^2 = (3n+1)\ 17$

$\Rightarrow n = 5$

Q.6 **[646]**

$X = \sum_{r=0}^{n} r.\left({}^nC_r\right)^2 ; n = 10$

$X = n.\sum_{r=0}^{n} {}^nC_r.{}^{n-1}C_{r-1}$

$X = n.\sum_{r=1}^{n} {}^nC_{n-r}\cdot{}^{n-1}C_{r-1}$

$X = n.^{2n-1}C_{n-1}; n = 10$

$X = 10.^{19}C_9$

$\frac{X}{1430} = \frac{1}{143}.^{19}C_9$

$= 646$

Q.7 [6.20]

Suppose

$$\begin{vmatrix} \frac{n(n+1)}{2} & n(n-1).2^{n-2} + n.2^{n-1} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$\frac{n(n+1)}{2} . 4^n - n^2(n-1).2^{2n-3} - n^2\, 2^{2n-2} = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$$

$n^2 - 3n - 4 = 0$

$n = 4$

Now $\sum_{k=0}^{4} \frac{^4C_k}{k+1} = \sum_{k=0}^{4} \frac{k+1}{5} . ^5C_{k=1} \frac{1}{k+1}$

$= \frac{1}{5} . [^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5] = \frac{1}{5} [2^5 - 1]$

$= \frac{31}{5} = 6.20$

Q.8 (A,B,D)

Solving

$f(m, n, p) = \sum_{i=0}^{p} {}^mC_i \, {}^{n+i}C_p . {}^{p+n}C_{p-i}$

${}^mC_i . {}^{n+i}C_p . {}^{p+n}C_{p-i}$

${}^mC_i . \frac{(n+i)!}{p!(n-p+i)!} \times \frac{(n+p)!}{(p-i)!(n+i)!}$

${}^mC_i \times \frac{(n+p)!}{p!} \times \frac{1}{(n-p+i)!(p-i)!}$

${}^mC_i \times \frac{(n+p)!}{p!n!} \times \frac{n!}{(n-p+i)!(p-i)!}$

${}^mC_i . {}^{n+p}C_p . {}^nC_{p-i}$ $\{{}^mC_i . {}^nC_{p-i} = {}^{m+n}C_p\}$

$f(m,n,p) = {}^{n+p}C_p . {}^{m+n}C_p$

$\frac{f(m,n,p)}{{}^{n+p}C_p} = {}^{m+n}C_p$

Now

$g(m, n) = \sum_{p=0}^{m+n} \frac{f(m,n,p)}{{}^{n+p}C_p}$

$g(m, n) = \sum_{p=0}^{m+n} {}^{m+n}C_p$

$g(m, n) = 2^{m+n}$

(A) $g(m,n) = q(n, m)$

(B) $g(m,n+1) = 2^{m+n+1}$

$g(m+n,n) = 2^{m+1+n}$

(D) $g(2m, 2n) = 2^{2m+2n}$

$= (2^{m+n})^2$

$= (g(m,n))^2$