QUICK REVISION TEST
INTEGER TYPE QUESTIONS

1 Two balls are projected from a point in two mutually perpendicular vertical planes. Speed of projection of both the balls is 400 m/s. Angle of projection with horizontal is 53^0 for both the balls. After how much time (in seconds), their velocities will be at angle 60^0 from each other ? (Take g = 10 m/s^2)

Answer :8

Solution : Let x – y plane is horizontal plane and z-azis is vertical. If one particle moves in x – z plane, the other moves in y – z plane.

$$\bar{u}_1 = (400\cos 53)\hat{i} + (400\sin 53)\hat{k} = 240\hat{i} + 320\hat{k}$$

$$\bar{u}_2 = (400\cos 53)\hat{j} + (400\sin 53)\hat{k} = 240\hat{j} + 320\hat{k}$$

After time t, $\bar{v}_1 = \bar{u}_1 + \bar{g}t = 240\hat{i} + 320\hat{k} - 10t\hat{k} = 240\hat{i} + (320-10t)\hat{k}$

$$\bar{v}_2 = \bar{u}_2 + \bar{g}t = 240\hat{j} + 320\hat{k} - 10t\hat{k} = 240\hat{j} + (320-10t)\hat{k}$$

$$\cos 60 = \frac{\bar{v}_1.\bar{v}_2}{v_1 v_2} \Rightarrow \frac{1}{2} = \frac{(320-10t)^2}{(240)^2 + (320-10t)^2}$$

$$240^2 + \left(320-10t\right)^2 = 2\left(320-10t\right)^2$$

$$240^2 = \left(320-10t\right)^2 \Rightarrow 320-10t = 240 \Rightarrow 10t = 80 \Rightarrow t = 8\,s$$

2 If the maximum and minimum speeds in the path of a projectile are 20 m/s and 10m/s respectively, what is the ratio of maximum and minimum radii of curvature at points in its path ?

Answer :8

Solution : Initial speed is the maximum speed $\Rightarrow u = 20\,m/s$

Speed at highest point is minimum $\Rightarrow u\cos\theta = 10 \Rightarrow 20\cos\theta = 10 \Rightarrow \theta = 60^0$.

Radius of curvature is minimum at highest point.

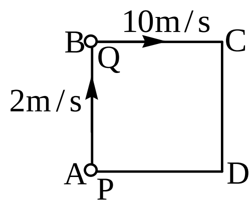
$$a_N = \frac{v^2}{R} \Rightarrow R_{\min} = \frac{(u\cos\theta)^2}{g} = \frac{10^2}{10} = 10$$

R is maximum at the initial point.

$$a_N = \frac{v^2}{R} \Rightarrow g\cos\theta = \frac{u^2}{R_{\max}} \Rightarrow R_{\max} = \frac{u^2}{g\cos\theta} = \frac{20^2}{10\times\frac{1}{2}} = 80$$

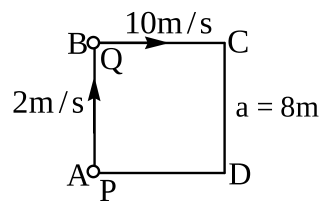
$$\frac{R_{\max}}{R_{\min}} = \frac{80}{10} = 8.$$

3 Two men P & Q are standing at corners A & B of square ABCD of side 8m. They start moving along the track with constant speed 2m/s and 10m/s respectively. The time (in seconds) when they will meet for the first time, is equal to



Answer :3

Solution : They meet when Q displace 8 × 3m



more than P displace $\Rightarrow$ relative displacement = relative velocity × time

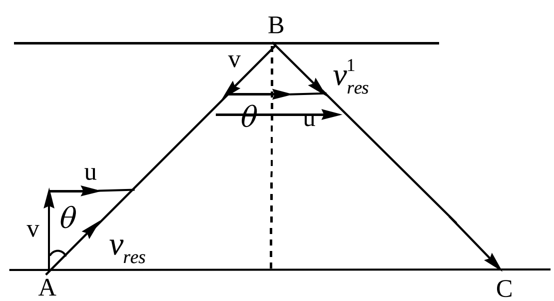
8 × 3 = (10 – 2)t t = 3sec

4 A swimmer crosses a river with minimum possible time 10 seconds. When he reaches the other end, he starts swimming in the direction towards the point from where he started swimming. Keeping the direction fixed, the swimmer crosses the river in 15 s. The ratio of speed of swimmer with respect to water and the speed of river flow is $\frac{2}{\sqrt{x}}$. Find x. (Assume constant speed of river water & swimmer).

Answer :5

Solution : V = velocity of man w.r.t river

U = velocity of river



$$A \to B = \frac{d}{v} \Rightarrow 10 = \frac{d}{v} \Rightarrow d = 10V \to (1)$$

$$B \to C = \frac{d}{\cos\theta} \Rightarrow 15 = \frac{d}{v\cos\theta} \Rightarrow d = 15v\cos\theta \to (2)$$

(1) & (2) $\Rightarrow \cos\theta = 2/3 \Rightarrow \sec\theta = 3/2$

$$\because \tan\theta = \frac{u}{v} \qquad \therefore \sqrt{\sec^2\theta - 1} = \frac{u}{v}$$

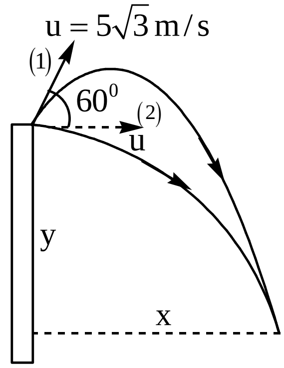
$$\Rightarrow \frac{u}{v} = \sqrt{9/4 - 1} = \frac{\sqrt{5}}{2} \Rightarrow \frac{v}{u} = \frac{2}{\sqrt{5}}$$

5 Two guns situated at a point on the top of a tall hill fire one shot from each gun with the same speed $5\sqrt{3}m/s$ at some interval of time. One gun fires horizontally and the other fires upwards at an angle of 60^0 with the horizontal. If the shots collide in air, what is the time interval between the firings in seconds? (Take g = 10 m/s^2)

Answer :1

Solution : Horizontal velocity of oblique projectile is $5\sqrt{3}\cos 60^0 = \frac{5}{2}\sqrt{3}\, m/s$.

For horizontal projectile, horizontal velocity is $5\sqrt{3}\, ms^{-1}$. To get same horizontal displacement, time of journey for 1st projectile must be more as its horizontal velocity is less. Let t is the time o journey of 1st projectile and Δt is the time interval between firings.



$$x_1 = x_2 \Rightarrow \frac{5}{2}\sqrt{3}(t) = 5\sqrt{3}(t-\Delta t) \Rightarrow \Delta t = \frac{t}{2}$$

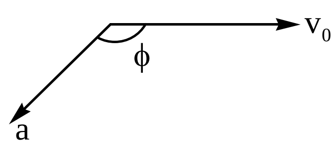
$$y_1 = y_2 \Rightarrow -\left(5\sqrt{3}\sin 60\right)t + \frac{1}{2}gt^2 = \frac{1}{2}g\left(t-\Delta t\right)^2 = \frac{g}{2}\left(t-\frac{t}{2}\right)^2 = \frac{1}{2}\frac{gt^2}{4}$$

$$\frac{15}{2}t = \frac{1}{2}gt^2 \times \frac{3}{4} \Rightarrow t = 2s$$

$$\Delta t = \frac{t}{2} = 1s$$

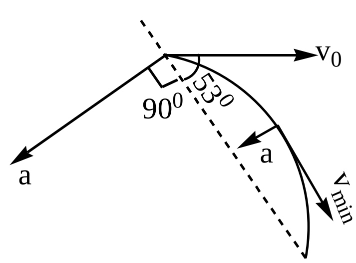
6 Figure shows the velocity and acceleration of a particle at the initial moment of its motion. Acceleration is constant throughout the motion. If the time in seconds when the velocity reaches its minimum value is $\frac{16}{x}$, find x.

$\left(a = 6ms^{-2}, v_0 = 24m/s, \phi = 143^0\right)$



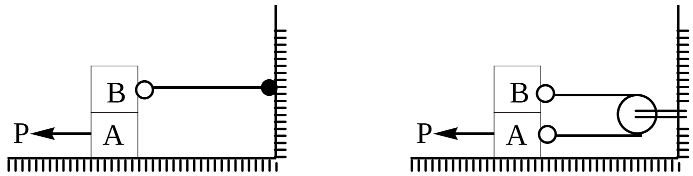
Answer :5

Solution : We can consider it like an oblique projectile from ground with angle of projection $\theta = 53^0$. V is minimum at highest point i.e. after t = time of ascent (T_a)

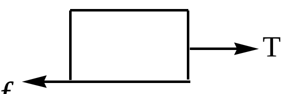


$$t = \frac{v_0 \sin\theta}{a} = \frac{24\times 4/5}{6} = \frac{16}{5} \Rightarrow x = 5$$

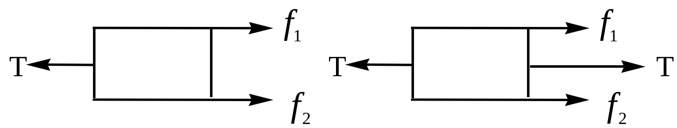
7 Each of the two blocks shown in the figure has a mass m . The coefficient of friction for all surfaces in contact is μ . A horizontal force P is applied to move the bottom block . The value of P , for which acceleration of block A is same in both cases is n times μmg . Then 'n' is equal to _____



Answer :2



Solution : $a_1 = \dfrac{P-(\mu mg - 2\mu mg)}{m}$



$= \left(\dfrac{P-3\mu mg}{m}\right)$

$T - f_1 = ma_2$

$P - T - f_2 - f_1 = ma_2$

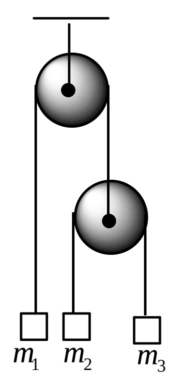
$P - f_2 - f_1 = 2ma_2$

$a_2 = \left(\dfrac{P-4\mu mg}{2m}\right)$

For $2P - 6\mu mg = P - 4\mu mg$

$P = 2\mu mg$

8 In the arrangement shown in figure , pulleys are mass less and frictionless and threads are inextensible. For the block of mass m_1 to remain at rest, $'m_1'$ must be equal to______ kg $(m_2 = 3kg \ \& \ m_3 = 6kg)$



Answer :8

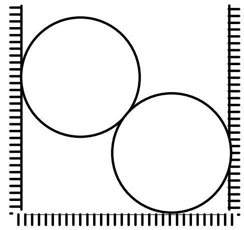
Solution : cve

When m_1 remains at rest, the $T = m_1 g$

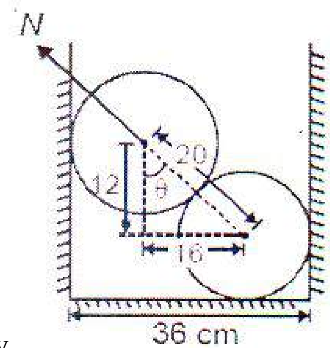
Aslo $T = 2T_o = \dfrac{4m_2m_3}{(m_2+m_3)} g = m_1 g$

$\Rightarrow \dfrac{4}{m_1} = \dfrac{1}{m_2} + \dfrac{1}{m_3}$

9 Two identical spheres of radii 10 cm each and weight 3N each are placed between two rigid vertical walls as shown. The spacing between the walls is 36 cm. Then the force of contact between the two spheres is ________N.(All the surfaces are frictionless)



Answer :5



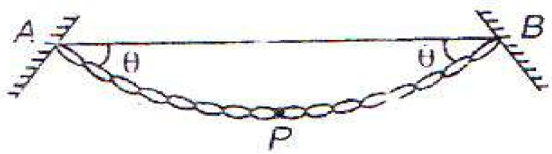
Solution : cv

$$\cos\theta = \frac{12}{20}$$

Also $N\cos\theta = W$

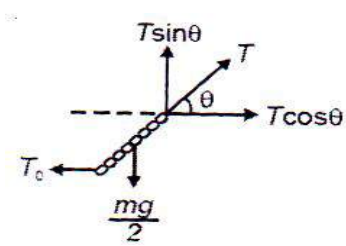
$$N = \frac{W}{\cos\theta} = \frac{W \times 20}{12} = 5N$$

10 A flexible chain of weight 3N hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points of support is θ $(\theta = 37^0)$. The tension at the mid point of the chain in newtons is ________$\left(\sin 37^0 = 0.6\right)$



Answer :2

Solution : At mid point, tension is horizontal, Cosinder the FBD of half part.

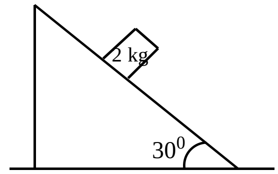


$$T = \sin\theta = \frac{mg}{2}$$

$$T = \cos\theta = T_o$$

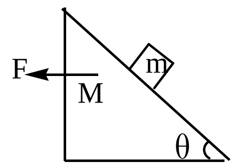
$$\Rightarrow T_O = \frac{mg}{2\tan\theta}$$

11 An inclined plane of angle of inclination $30^0 witha$ block of mass 2 kg kept on the inclined plane is given a horizontal acceleration of $10\sqrt{3}\ ms^{-2}$ towards left. Assuming frictionless contacts. The acceleration of mass 2 kg w.r.t ground is N times $2ms^{-2}$. Then N is equal to ____ (take g=10 m/s^2)



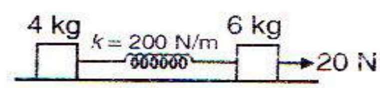
Answer :5
Solution : The force on the wedge is given to be

$F = mg\cot\theta$



In this case, m falls freely $\therefore\ a = 10m/s^2$

12 Two blocks of masses 4 kg and 6 kg are attached by a spring of spring constant k=200 N/m. Both the blocks are moving with same acceleration. Elongation of spring in cm is ______



Answer :4
Solution : For 4 kg block,

$Kx = ma$

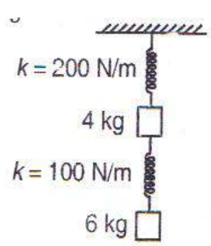
Also $a = \frac{20}{4+6} = 2m/s^2$

$\Rightarrow 200 \times x = 4 \times 2$

$x = \frac{8}{200}$

$\Rightarrow x = 4cm$

13 Two blocks of masses 4 kg and 6 kg are attached by massless springs. They are hanging in vertical position in equilibrium. If lower spring breaks due to excessive force, acceleration of 4 kg block just after breaking is 5 times $x\,ms^{-2}$. Then x is equal to _______



Answer :3
Solution : Just before breaking 4 kg block is in equilibrium

$\Rightarrow kx = 4 + 6 \times 10$

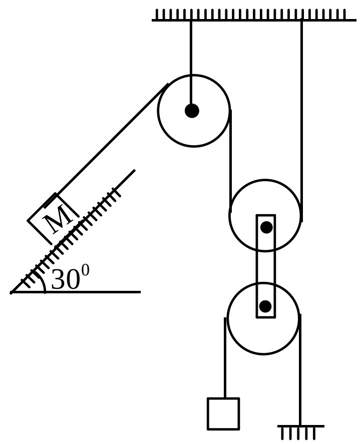
$kx = 100N$

When the lower spring breaks,

$kx - 40 = 4a$

$60 = 4a \Rightarrow a = 15m/s^2$

14 In the arrangement shown, neglect the masses of the ropes and pulley. What must be the value of m in kg to keep the system in equilibrium? There is no friction anywhere.(M=2kg)



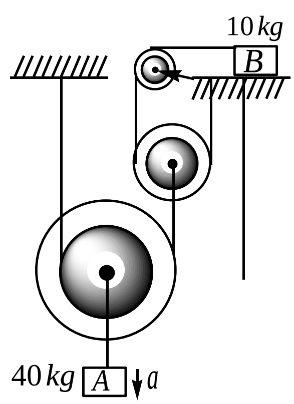
Answer :1

Solution : By drawing F.B.D of both block it can be seen

$Mg\sin 30^0 = mg$

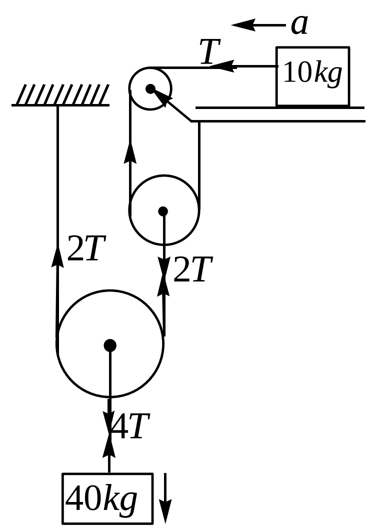
$m = \frac{M}{2}$

15 Figure shows two blocks A and B connected to ideal pulley string system. In this system, when bodies are released, acceleration of block 'A' in ms^{-2} is _____ (Neglect friction and take $g = 10m/s^2$)



Answer :2

Solution : Appling NLM on 40kg block

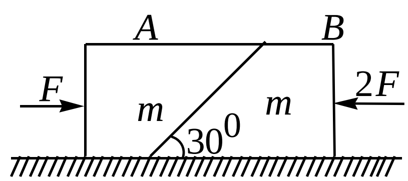


$400 - 4T = 40a$

For 10 kg block $T = 10..(4a)$

Solving $a = 2m/s^2$

16 Two blocks 'A' and 'B' each of mass 'm' are placed on a smooth horizontal surface. Two horizontal forces F and 2F are applied on both the blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. If the normal reaction between the two blocks is N F, then 'N' is equal to __________



Answer :3

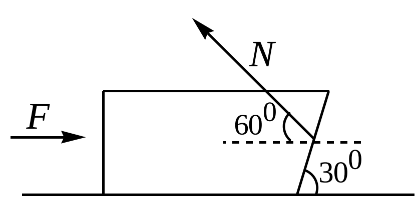
Solution : Acceleration of two mass system is $\alpha - \frac{F}{2m}$ leftward

FBD of block A

$N\cos 60^0 - F = ma - \frac{mF}{2m}$

Solving, $N = 3F$

$N - 3F$



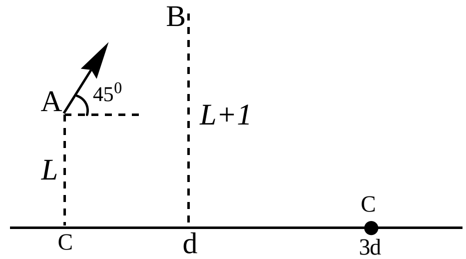
17 The surface of a smooth inclined plane of inclination 30^0 is ABCD. CD is in contact with horizontal ground. A particle is projected parallel to AB from A with a velocity of magnitude u such that it passes through diagonally opposite point C. If AB = BC = 10 m and $g = 10\,ms^{-2}$, its initial speed u in ms^{-1} is ______

Answer :5

Solution : $10 = \frac{1}{2} \times 10 \times \frac{1}{2} \times t^2 \Rightarrow t = 2s$

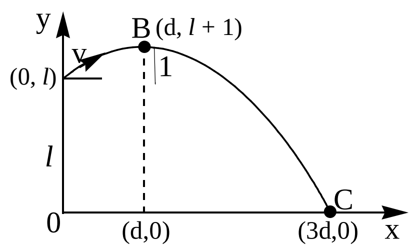
$10 = u \times 2 \Rightarrow u = 5\ m/s$

18 A projectile is launched at time t = 0 from point A which is at height L above the floor with speed v m/s and at an angle $\theta = 45^0$ with horizontal. It passes through a hoop at B which is 1m above the level of A and B is the highest point of the trajectory. The horizontal distance between A and B is d meters. The projectile then falls into a basket, kept at 'C' on the floor at a horizontal distance 3d meters from A. Find L (in m).



Answer :3

Solution : The horizontal and vertical components of the velocity are the same, let it be $u = v\cos 45^0$.



From A to B: $1 = \dfrac{u^2}{2g} \Rightarrow u^2 = 2g$

At B: $d = ut_2 \Rightarrow t_1 = d/u$

$$1 = ut_1 - \frac{g}{2}t_1^2 = u\frac{d}{u} - \frac{g}{2}\frac{d^2}{u^2}$$

$$\Rightarrow 1 = d - \frac{g}{2}\frac{d^2}{u^2} \Rightarrow 1 = d - \frac{gd^2}{4g}$$

$$\Rightarrow 4 = 4d - d^2 \Rightarrow d^2 - 4d + 4 = 0$$

$$\Rightarrow d = 2m$$

At C: $3d = ut_2 \Rightarrow t_2 = \dfrac{3d}{u}$

$$-\ell = ut_2 - \frac{1}{2}gt_2^2 = u \cdot \frac{3d}{4} - \frac{g}{2}\frac{9d^2}{4^2} = 3d - \frac{9gd^2}{4g}$$

$$= 3d - \frac{9d^2}{4} = 3 \times 2 - \frac{9}{4} \times 4 = 6 - 9 = -3$$

$$\Rightarrow \ell = 3m$$

19 In a car race, car A takes 4 s less than car B at the finish and passes the finishing point with a velocity v m/s more than the car B. Assuming that the cars start from rest and travel with constant accelerations $a_1 = 4ms^{-2}$ and $a_2 = 1ms^{-2}$ respectively, find the velocity v in m/s

Answer :8

Solution : $t_1 = t_2 - t, v_1 = v_2 = v, S = \dfrac{1}{2}a_1t_1^2,\ S = \dfrac{1}{2}a_2t_2^2$

$v_1 = a_1t_1,\ v_2 = a_2t_2 \Rightarrow v_2 + v = a_1t_1$

$\Rightarrow a_2t_2 + v = a_1t_1 = a_1t_2 \Rightarrow t_2 = \dfrac{v + a_1t}{a_1 - a_2}$

$\sqrt{\dfrac{a_2}{a_1}} = \dfrac{t_1}{t_2} = 1 - \dfrac{t}{t_2} \Rightarrow \sqrt{\dfrac{a_2}{a_1}} = 1 - \dfrac{t(a_1 - a_2)}{(v + a_1t)}$

$\dfrac{\sqrt{a_2}}{\sqrt{a_1}} = \dfrac{v + a_2t}{v + a_1t} \Rightarrow \sqrt{a_1}v + a_1\sqrt{a_2}t = v\sqrt{a_1} + a_2\sqrt{a_1}t$

$\Rightarrow v = \left(\sqrt{a_1a_2}\right)t = 8\ ms^{-1}$

$v_1 = u\cos\theta$

20 The velocity of a projectile when it is at the greatest height is $\sqrt{2/5}$ times its velocity when it is at half of its greatest height. Then its angle of projection is $10 \times n^0$ where n =

Answer :6

Solution : At half of the greatest height y = h/2, $a_y = -g,\ u_y = u\sin\theta$

$v_y = \dfrac{u\sin\theta}{\sqrt{2}}$

$v_2 = \sqrt{v_x^2 + v_y^2}$

$\dfrac{v_1}{v_2} = \sqrt{\dfrac{2}{5}}$

$\tan\theta = \sqrt{3}$

$\theta = 60^0$

21 A particle is projected up from the bottom of a hollow wedge of inclination 30^0, with a velocity $40\sqrt{2}\,ms^{-1}$ at 45^0 to horizontal. (Take point of projection as origin, horizontal direction as x-axis and vertical direction as y-axis). There is a hole to the surface of wedge at (240 m, 60 m). This particle passes through that hole. If the base length of wedge is 280 m, the speed with which this particle hits the vertical face of the wedge is $10x\,ms^{-1}$. Then x = ____ (Take g = $10\,ms^{-2}$)

Answer :5

Solution : The vertical face is at a horizontal distance of 280m.

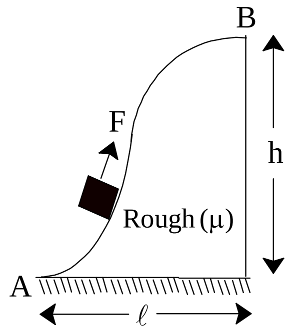
$280 = 40 \times t \Rightarrow t = 7s$

We have to find magnitude of velocity at 7s.

$v_x = 40ms^{-1}, v_y = 40 - 10 \times 7 = -30m/s$

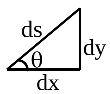
$|\overline{v}| = \sqrt{40^2 + 30^2} = 50 = 10 \times 5 \Rightarrow x = 5$

22 A body of mass m is slowly hauled up the hill by a force F which at each point is directed along a tangent to the trajectory. The work performed by this force is nmgh in moving the body from A to B if the height of the surface is h, the length of its base is 'l' and the coefficient of friction between the body and the surface is given by $\mu = \tan\alpha$ where α is the angle between the normal force applied by the surface and the vertical at every point. Find the value of n.



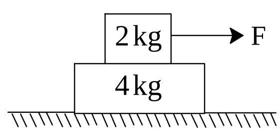
Answer :2

Solution : $dw_f = -\left(\mu mg\cos\theta\right)ds$



$$= -\left(\tan\theta\right)\left(mg\cos\theta\right)ds$$
$$= -(mg\sin\theta)ds$$
$$= -mg\ dy$$
$$w_f = \int dw_f = -mg\int dy = -mgh$$
$$w_F + w_N + w_g + w_f = 0$$
$$\Rightarrow w_F + O - mgh - mgh = 0$$
$$\Rightarrow w_F = mgh + mgh = 2mgh$$

23 A plank of mass 4kg is placed on a smooth horizontal surface. A block of mass 2 kg is placed on the plank and is being acted upon a horizontal force F = 0.5 t where F is in newton and t is in s. If the coefficient of friction between the block and the plank is 0.10, the work done by friction on the system between t = 0 and t = 6s in joules is _________ (Take $g = 10ms^{-2}$)



Answer :0

Solution : $f_\ell = 2N$ and they move together upto 6 sec.

$\therefore$ The frictional force between the block is static upto t = 6 sec.

Hence $w_f = 0$

24 A body of mass 4 kg is moving at speed 1 m/s on circular path of radius 1m. Speed of particle is continuously increasing at the rate of 3 m/s^2. Force acting on particle at the instant when speed 2 m/s is n x 10N. Find the value of n.

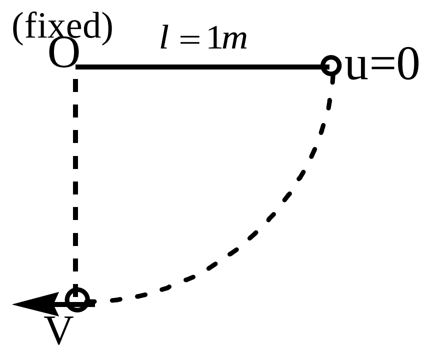
Answer :2

Solution : $a_t = 3m/s^2$

$$a = a_t + a_r$$
$$a = \sqrt{3^2 + \frac{4^2}{1}} = \sqrt{9+16} = 5m/s^2$$

F = 20 N

25 A particle of mass 2kg is tied to a light inextensible string of length 1m at one end. Other end of string is fixed at ‘O’. If the particle is released from rest with the string horizontal, the acceleration $(in\, ms^2)$ of the particle when the string becomes vertical is $10\times n$. find n. $g = 10\, ms^{-2}$.



Answer :2

Solution : $mg\ell = \frac{1}{2}mv^2 \Rightarrow V^2 = 2g\ell$

At the bottom point $a = a_{normal} = \frac{V^2}{\ell} = 2g$

$\therefore n = 2$

26

A particle of mass 0.5 kg travels along x-axis with velocity of magnitude $V = a\,x^{3/2}$ where $a = 5\,m^{-\frac{1}{2}}.s^{-1}$ and x is the position. If the work done by the net force during its displacement from x=0 to x=2 m is 10n joules, find n.

Answer :5

Solution : at x=0, V = a(0)=0

at x=2, $V = (5)2^{3/2} = 10\sqrt{2}\, m/s$

$$W = \Delta KE = \frac{1}{2}mV^2 - 0 = \frac{1}{2}\times\frac{1}{2}\times\left(10\sqrt{2}\right)^2 = 50 = 10\times 5\, J$$

27 A gun is fired from a moving platform and the ranges of the shots are observed to be $2x_0$ and x_0 when platform is moving forward or backward respectively with velocity V. If the elevation of the gun is θ with horizontal then $\tan\theta = \frac{gx_0}{2kv^2}$, where k = ____

Answer :6

Solution : Velocity components of body

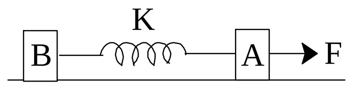
Platform	Horizontal	Vertical	Range
Rest	u_x	u_y	
Forward motion (v)	$u_x + v = u_{x1}$	u_y	$2x_0 = \frac{2(u_{x1})(u_y)}{g}$ -------(1)
Backward motion (v)	$u_x - v = u_{x2}$	u_y	$x_0 = \frac{2(u_{x2})(u_y)}{g}$ ------(2)

Angle of projection $\tan\varphi = \frac{u_y}{u_x}$ --------(3)

On solving (1), (2) & (3)

$\tan\varphi = \frac{g(x_0)}{12v^2} \Rightarrow k = 6$

28 Two blocks A and B connected by an ideal spring of spring constant $K = 100\frac{N}{m}$ are moving on a smooth horizontal plane due to the action of a horizontal force F. Mass of A is 5 kg, mass of B is 2 kg and F = 35 N. The extension of the spring at an instant when both A and B move with constant acceleration is ______ cm.



Answer :1

Solution : When A and B have constant acceleration

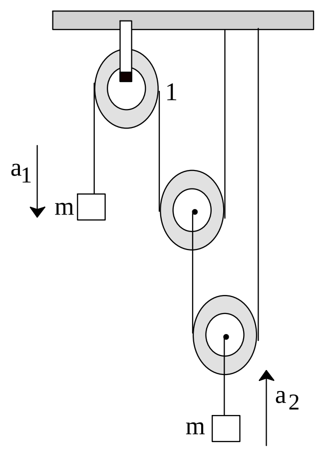
(spring has maximum extension)

And $a_A = a_B = a$

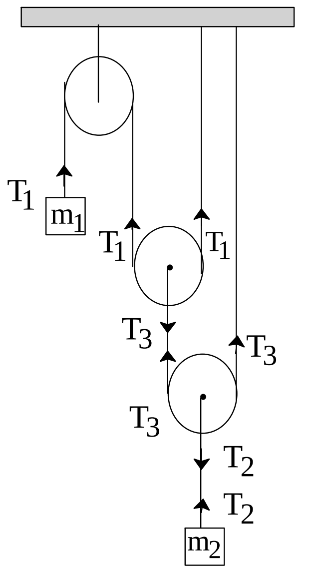
$\therefore a = \frac{F}{m_A + m_B} = 5\,m/s^2$

$\Rightarrow kx = m_B a \Rightarrow x = 1\ cm$

29 Two blocks of masses m_1 and m_2 are connected by massless threads. The pulleys are massless and smooth. If a_1 is the magnitude value of acceleration of m_1 and a_2 is the magnitude value of acceleration of m_2, find the ratio $\frac{a_1}{a_2}$.



Answer :4



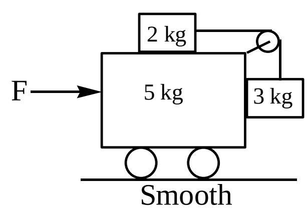
Solution :

Here $T_3 = 2T$; $T_2 = 2T_3 = 4T_1$

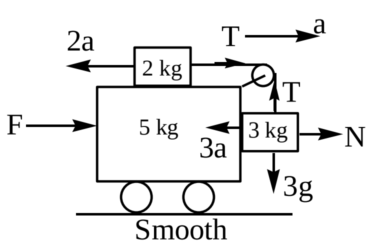
<u>Constraint equation:</u>

$\Sigma \vec{T}.\vec{V} = 0 \Rightarrow \dfrac{a_1}{a_2} = 4$

30 In the figure shown, if all the surfaces are smooth, then the horizontal force, F required to keep the 2kg and 3kg blocks stationary is $\dfrac{30g}{K} N$. Then value of K is ________



Answer :2



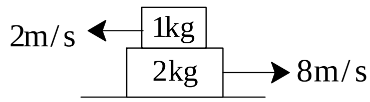
Solution :

T = 2a = 3g

$a = \dfrac{3g}{2}$

$$F = (5+3+2)a = 10 \times \frac{3g}{2} = 15g = \frac{30g}{K} \Rightarrow K = 2$$

31 Coefficient of friction between two blocks shown in figure is $\mu = 0.4$. The blocks are given velocities of 2 m/s and 8 m/s in the directions shown in figure. The time when relative motion between them will stop is 5t/3 sec, then t is



Answer :1

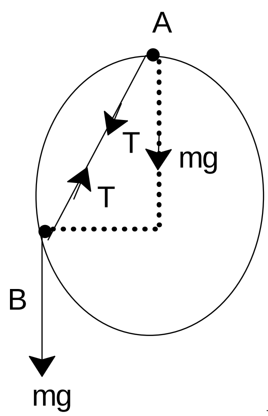
Solution : Relative motion between them stops when $V_1 = V_2$ at an instant $t = t_0$

Here $a_1 = \mu g$; $\quad a_2 = \mu\left(\frac{m_1}{m_2}\right)g$

$\therefore v_1 = v_2$

$\Rightarrow -2 + \mu g t_0 = 8 - \mu\left(\frac{1}{2}\right)g t_0 \Rightarrow t_0 = \frac{5}{3}\text{sec}$

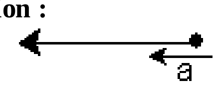
32 Object A and B each of mass ‘m’ are connected by light inextensible cord. They are constrained to move on a friction less ring in a vertical plane as shown in figure. The objects are released from rest at the positions shown. The tension in the cord just after



relax will be $= \frac{mg}{\sqrt{k}}$ then value of K is

Answer :2

Solution :



For A

T cos 45 = ma ----------------- (1)

For B mg – T sin 45° = ma

$$\frac{T\cos 45}{mg - T\sin 45^\circ} = 1$$

$$\frac{T}{\sqrt{2}} = mg - \frac{T}{\sqrt{2}}$$

$$\frac{2T}{\sqrt{2}} = mg$$

$$\sqrt{2}T = mg$$

$$T = \frac{mg}{\sqrt{2}}$$

33 A train is moving along a straight track with a uniform acceleration. A boy standing in the train throws a ball forward with a speed of 10 m/s relative to the train at angle 60^0 to the horizontal. The boy moves forward by 1.15 m inside the train to catch the ball at the initial height. Then the acceleration of the train is __________ m/s^2.

Answer :5

Solution : $x = (u\cos\theta)t - \frac{1}{2}gt^2$, where $t = \frac{2u\sin\theta}{g} = \sqrt{3}\,s$

$\Rightarrow 1.15 = 10x\frac{1}{2}x\sqrt{3} - \frac{1}{2}ax3 = 5x1.73 - \frac{3}{2}a = 8.65 - \frac{3}{2}a$

$\Rightarrow \frac{3}{2}a = 7.5 \;\; or \;\; a = 5m/s^2$

34 Two solid cylinders, each of mass m and radius r, are placed touching along their lengths on a rough horizontal surface of coefficient of friction μ . A third cylinder of same length, made of same material, but of radius 2r is placed lengthwise over them so that the system just remains at rest. There is no friction between the cylinders. Find the value of $9\sqrt{2}\mu$

Answer :3

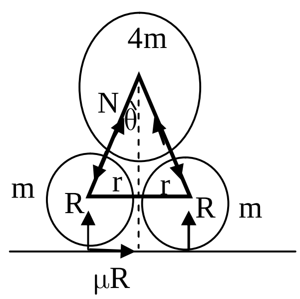
Solution : 2R = 6 mg

$2N\cos\theta = 4mg$

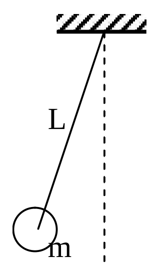
$N\sin\theta = \mu R$

Solving $\mu = \frac{1}{3\sqrt{2}}$

$\Rightarrow 9\sqrt{2}\mu = 9\sqrt{2}x\frac{1}{3\sqrt{2}} = 3$



35 A ball of mass m = 0.5 kg is attached to the end of a string of length L = 0.5 m. The other end of the string is fixed. The ball is made to rotate on a horizontal circular path about the vertical axis through the fixed end of the string. The maximum tension that the string can bear is 324 N. The maximum possible angular velocity of the ball is n^2 rad/s, where n = _________

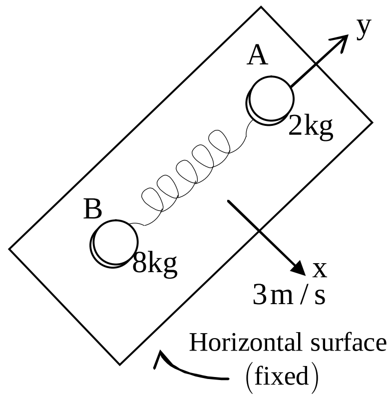


Answer :6

Solution : $T = m\omega^2 l \Rightarrow \omega^2 = \dfrac{T}{ml} = \dfrac{324}{\frac{1}{2}x\frac{1}{2}} = 324x4$

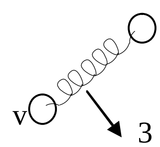
$\Rightarrow \omega = 18\times2 = 36 \Rightarrow$ n = 6.

36 A mass less spring of force constant 1000 N/m is compressed through a distance of 20 cm between two discs of masses 2 kg and 8 kg on a smooth horizontal surface. The discs are not attached to the spring. The system is given an initial velocity of 3 m/s perpendicular to the length of the spring. Find the velocity (in m/s) of the 2 kg disc relative to ground when the spring regains its natural length.



Answer :5

Solution : $\frac{1}{2}\mu\ x(5v)^2 = \frac{1}{2}x1000x(0.2)^2$

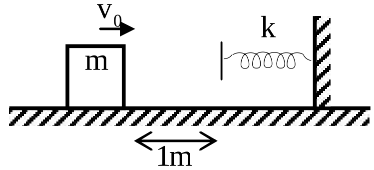


Where $\mu = \dfrac{16}{10}$

$\Rightarrow v = 1\ m/s$

$\Rightarrow v_A = \sqrt{4^2+3^2} = 5\ m/s$

37 A block of mass m = 2kg is moving with velocity v_0 towards a massless unstretched spring of force constant k = 12 N/m. Coefficient of friction between the block and the floor is $\mu = 0.2$. The block, after pressing the spring, just stops there without returning. Find v_0^2 in m^2s^{-2}.



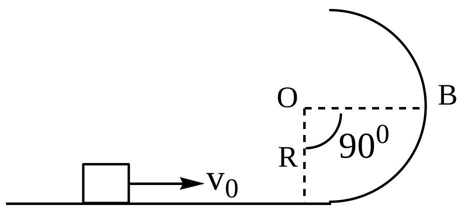
Answer :6

Solution : $\mu mg = kx$

$\frac{1}{2}mv_0^2 = \mu mg(x+1) + \frac{1}{2}kx^2$

$$\Rightarrow \frac{1}{2}mv_0^2 = \frac{3}{2}kx^2 + \mu mg \Rightarrow v_0^2 = \frac{3\mu^2 mg^2}{k} + 2\mu g = 2+4=6$$

38 A body is projected with velocity v_0 to move along a vertical circular track of radius R as shown. It presses the surface at B with a force of $\frac{6}{5}mg$, where m is the mass of the body. Neglecting friction, the value of initial velocity v_0 is $n\sqrt{\frac{gR}{5}}$ where n = ___________



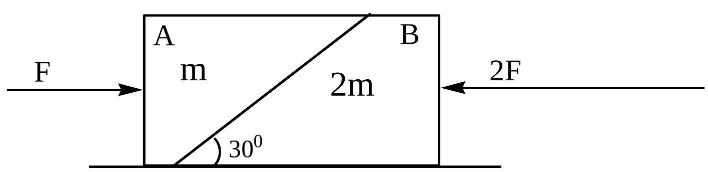
Answer :4

Solution : $\frac{6}{5}mg = \frac{mv^2}{R} \Rightarrow v^2 = \frac{6gR}{5} = v_0^2 - 2gR$

$$\Rightarrow v_0^2 = \frac{6gR}{5} + 2gR = \frac{16}{5}gR$$

$$\Rightarrow v_0 = 4\sqrt{\frac{gR}{5}}$$

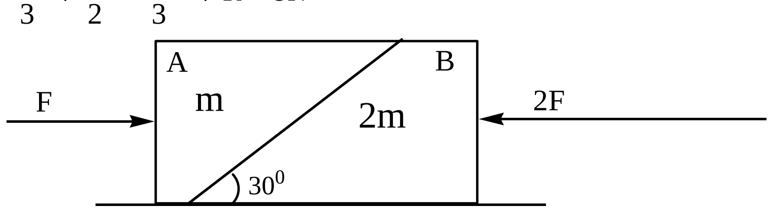
39 Two blocks A and B of mass m and 2m are placed on a smooth horizontal surface. Two horizontal forces F and 2F are applied on blocks A and B respectively, where F = 3N. The block A does not slide on block B. Then the normal reaction acting between the blocks is _______ N.

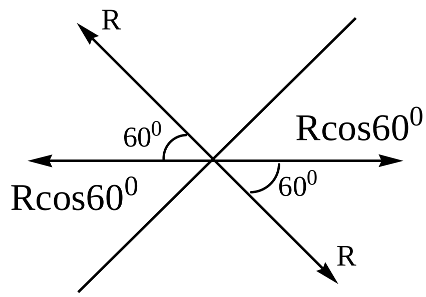


Answer :8

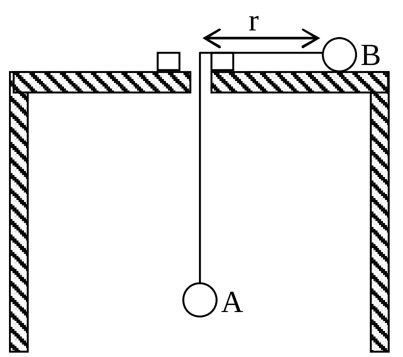
Solution : $a = \frac{F}{3m}$

$$R\cos 60° - F = ma = \frac{F}{3} \Rightarrow \frac{R}{2} = \frac{4F}{3} \Rightarrow R = 8N$$





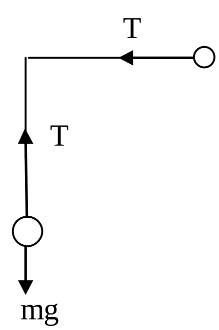
40 Two identical balls A and B are attached to the ends of a thread passed through a narrow hole on a smooth horizontal table. The distance of ball B from the hole is r = 20 cm. As ball B revolves with angular velocity ω about a vertical axis passing through the hole on the table, ball A neither rises nor falls. Then ω = ________ rad/sec.



Answer :7

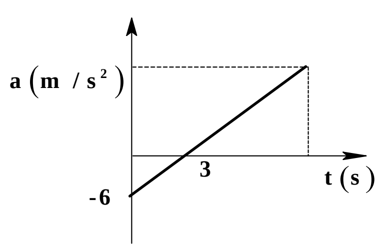
Solution : $T = m\omega^2 r = mg$

$$\Rightarrow \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.8}{0.2}} = \sqrt{49} = 7 rad/s$$



41 The acceleration of a particle vary with respect to time and is given by a = (2t - 6), where t is in seconds. Find the time (in seconds) at which velocity of particle in negative direction is maximum, if its initial speed is zero.

Answer :3

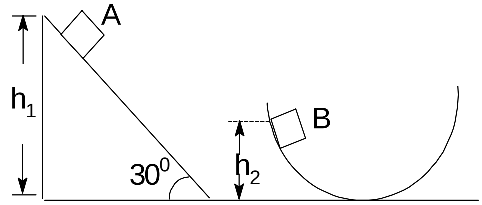


Solution :

Acceleration – time graph of the particle is shown.

Maximum velocity in negative direction will be at t = 3 sec, as acceleration becomes positive after 3 sec.

42 Two small blocks A and B are released from rest on a fixed inclined plane of angle 30^0 and a circular track of radius R from different heights h_1 and h_2 respectively as shown in figure. The mass of each block is m. If F_1 and F_2 are the magnitudes of respective resultant forces experienced by two blocks at the bottom-most points of the tracks and $F_1 = F_2$, then find the value of h_2 (in m) for R = 8 m.



Answer :2

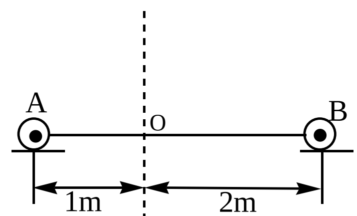
Solution : $F_1 = ma_1 = mg\sin 30^0 = mg/2$

$F_2 = ma_2 = m(v^2/R)$

$= m\left(\frac{2gh_2}{R}\right) = 2mg\left(\frac{h_2}{R}\right)$

For $h_2 = R/4, F_2 = mg/2 = F_1$ So, $h_2 = \frac{R}{4} = \frac{8}{4} = 2m$

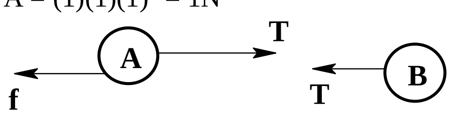
43 A massless string of 3 m length joins two small spheres A and B of mass 1 kg and 2 kg respectively. The spheres are placed on the horizontal surfaces at the same level. The string is horizontal and is rotated at 1 rad/s about a vertical axis passing through point O. The surface on which sphere B is placed is smooth and A is kept on rough surface. Find the value of frictional force between sphere A and the surface (in N) acting parallel to the string.



Answer :3

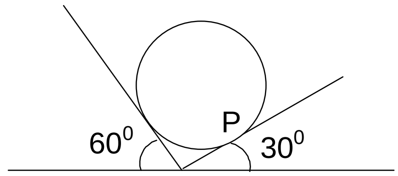
Solution : Centripetal force required for sphere B = $mrw^2 = (2)(2)(1)^2 = 4N$

Centripetal force required for sphere A = $(1)(1)(1)^2 = 1N$

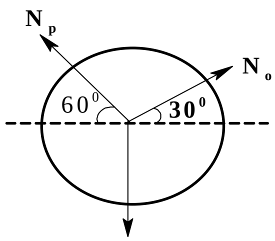


Since, there is no frictional force acting on B and T = 4N, is providing sphere A an extra force of 3N, which will be balanced by the friction, f = 3N

44 A sphere of mass $\left(\frac{1}{\sqrt{3}}\right)$ kg is placed on two smooth inclined planes of angles 30^0 and 60^0 with horizontal, as shown. Find the normal reaction at point P (in N). ($g = 10\ ms^{-2}$)



Answer :5



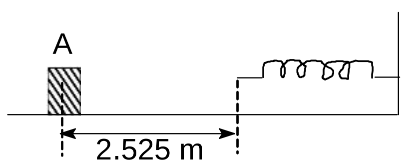
Solution :

N_O, N_P and mg will pass through centre

$$\frac{N_P}{\sin\left(90^0+30^0\right)}=\frac{N_o}{\sin\left(90^0+60^0\right)}=\frac{(10/\sqrt{3})}{\sin 90^0}$$

$$N_P=10\cos 30^0=\left(\frac{10}{\sqrt{3}}\right)\frac{\sqrt{3}}{2}=5N$$

45 A block A of mass 1 kg which lies on a rough horizontal surface has a velocity v_0 directed towards a relaxed spring. After travelling a distance of 2.525m, it strikes the spring. After compressing the spring, the block comes to rest and remains there permanently. Find the maximum value of v_0 in m/s. (g = 10 ms^{-2})



Answer :5

Solution : Spring is compressed by x. Block stops and will not return bock if kx = μmg .

$x=\frac{\mu mg}{k}=\frac{(0.4)(1)(10)}{10}=0.4m$. Work done against friction $\mu mg\left(x+2.525\right)=\frac{1}{2}mv_0^2-\frac{1}{2}kx^2$

(0.4)(1)(10)(0.4 +2.525) = $\frac{1}{2}(1)v_0^2-\frac{1}{2}(10)(0.16)$

23.40 = $v_0^2-1.6 \Rightarrow v=5m/s$

46 The coefficient of friction between a rough horizontal floor and a box of weight 1000 N kept on it, is $\frac{3}{k}$. If a minimum force of 600 N is required to start the box moving, then k =

Answer :4

Solution : F$_{\text{minimum}}$ = mg $\sin\theta \Rightarrow \sin\theta=\frac{600}{1000}=\frac{3}{5}$

$$\mu=\tan\theta=\frac{3}{4}=\frac{3}{k}\Rightarrow k=4$$

47 A pump motor delivers water at a certain rate. The power of motor is to be increased to obtain twice as much water from the same pipe and in same time. The power of motor has to be increased to how many times (in an integer)?

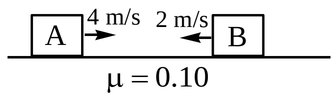
Answer :8

Solution : Mass flowing out per per second, m = Avp Rate of increase of kinetic energy

$$=\frac{1}{2}mv^2 = \frac{1}{2}A\rho v^3 = \frac{P'}{P} = \frac{(A\rho v'^3)}{(A\rho v^3)} = \frac{v'^3}{v^3}$$

Now, $\frac{m'}{m} = \frac{A\rho v'}{A\rho v} = \frac{v'}{v}$ As m' = 2m, so v' = 2v and thus $\frac{P'}{P}$ =$(2)^3$ = 8

48 Blocks A and B each of mass 1 kg are moving with 4m/s and 2m/s respectively as shown. The coefficient of friction for all the surfaces is 0.10. Find the distance (in m) by which centre of mass will travel before the center of mass coming to rest. Assume that blocks do not collide before v_{cm} becomes zero.



Answer :3

Solution : The block B will stop in 2 seconds. The block A will stop in 4 seconds. From 0 to 2 seconds equal force of friction are acting on the blocks in opposite directions and thus system will remain conserved as net force is zero.

At t = 0.

$$V_{CM} = \frac{m_1 + V_1 + m_2V_2}{m_1 + m_2} = \frac{(1)(4)+(1)(-2)}{2} = 1\text{m /sec}$$

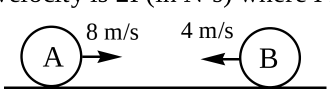
Up to t = 2 second, V_{CM} is constant = $d_1 = V_{CM}$ T = (1)(2) = 2m

After t > 2 second, retardation of centre of mass

a = $\frac{\mu g}{2} = \frac{0.1(10)}{2} = \frac{1}{2} m/s^2$ = $d_2 = \frac{V^2{}_{CM}}{2a} = \frac{(1)^2}{2(1/2)} = 1m$

Total distance = 2 + 1 = 3 m

49 Two spheres A and B of masses 2 kg and 1 kg respectively are moving with 8 m/s and 4 m/s on a smooth horizontal surface. Let head on collision takes place between them. During collision, they exert impulse of magnitude J on each other. The minimum value of J (in N - s) for which sphere A will change its direction of velocity is 2I (in N-s) where I is an integer. Find the value of I.



Answer :9

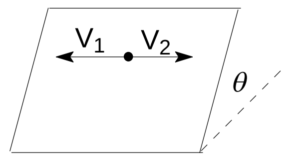
Solution : Initial momentum of A is 2 × 8 = 16 kg. m/s. To reverse the direction of velocity of A, impulse on it J must be greater than 16 N.s

2I > 16 $\Rightarrow$ I > 8

If I = 8, A will just stop only.

Next allowed integer is 9.

50 Two particles are projected horizontally in opposite directions from a point on a smooth inclined plane of inclination $\theta = 60^0$ with the horizontal as shown in figure. Find the separation between the particles on the inclined plane when their velocity vectors become perpendicular to each other. $v_1 = 1m/s, v_2 = 3m/s.$ Express your answer in the form of k/10 m. Then, find the value of k.



Answer :8

Solution : $\vec{v_1} = \vec{v_1}\ \hat{i} - g\sin\theta t\ \hat{j}$

$\vec{v_2} = -\vec{v_2}\ \hat{i} - g\sin\theta t\ \hat{j}$

$\vec{v_1}.\vec{v_2} = 0$

Or $-v_1 v_2 + g^2 \sin^2\theta t^2 = 0$

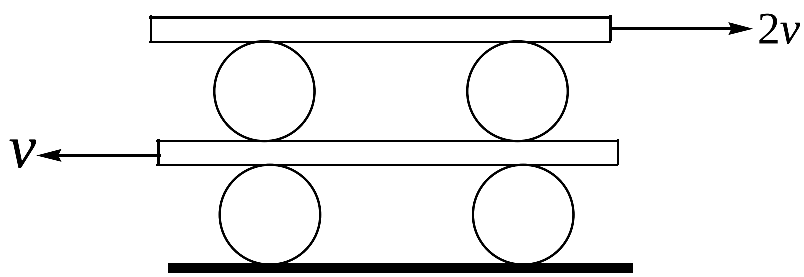
Or $v_2 v_2 = g^2 \sin^2\theta \times t^2$

Or $t = \dfrac{\sqrt{v_1 v_2}}{g\sin\theta}$

The particles line will be parallel to x – axis. Separation between the particles will be

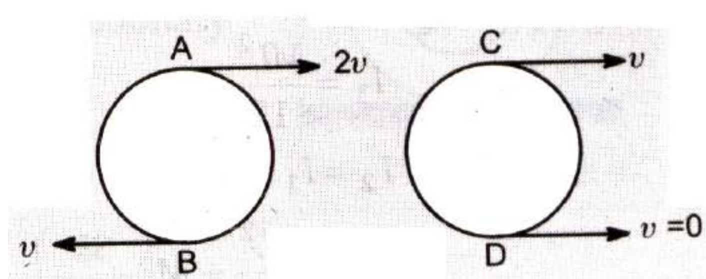
$= x_1 + x_2 = \dfrac{(v_1 + v_2)\sqrt{v_1 v_2}}{g\sin\theta}$

51 A system of uniform cylinders and plates is shown. All the cylinders are identical and there is no slipping at any contact. Velocity of lower and upper plates is 'v' and $2v$ respectively as shown. Then the ratio of angular speeds of the upper cylinders to lower cylinders is :



Answer :3

Solution : In the absence of slipping, velocities of contact points of upper cylinders and lower cylinders are respectively.

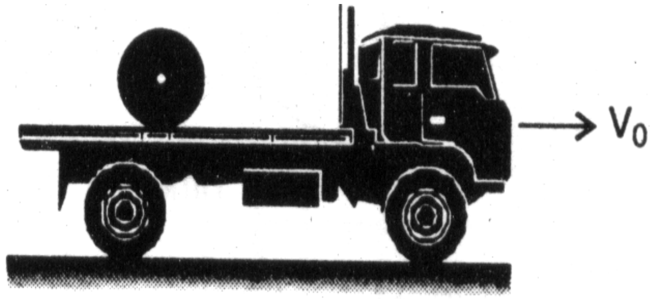


$\omega_{up} = \dfrac{3v}{2R} = \dfrac{v_{AB}}{AB};$

$\omega_{lower} = \dfrac{v_{CD}}{CD} = \dfrac{v}{2R};$

$\dfrac{\omega_{up}}{\omega_{lower}} = 3$

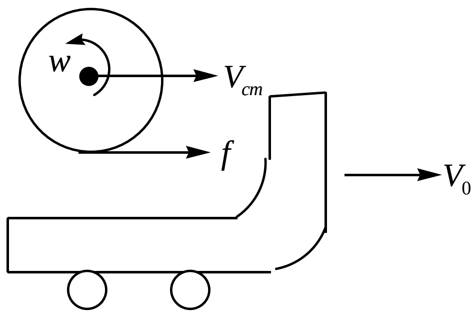
52 A solid sphere of mass 'M' and radius 'R' is initially at rest. Solid sphere is gradually lowered onto a truck moving with constant velocity V_0.



The final speed of sphere's centre of mass in ground frame when eventually pure rolling sets in (in multiples of $\frac{V_0}{7}$?

Answer :2

Solution : $f\Delta t \times R = \frac{2}{5}MR^2\omega$ ___________(1)



$f\Delta t = M\,V_{cm}$ ________(2)

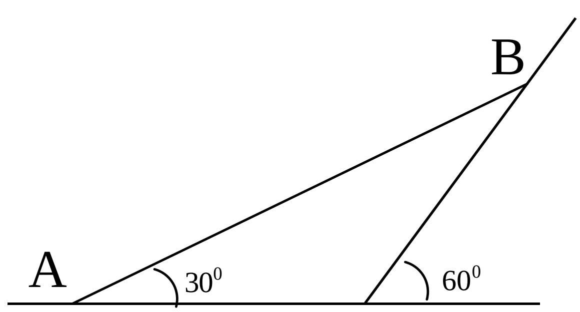
From (1), (2) $V_{cm} = \frac{2}{5}R\omega$

$V_0 = V_{cm} + Rw$ (Condition for pure rolling)

$\Rightarrow V_{CM} = V_0 - Rw = V_0 - \frac{5}{2}V_{cm}$

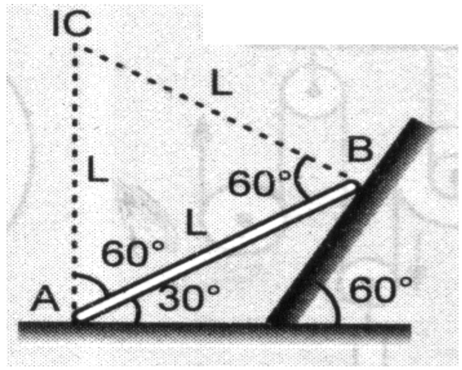
$V_{cm}\left(1+\frac{5}{2}\right) = V_0 \Rightarrow V_{cm} = \frac{2}{7}V_0$

53 In the figure shown, ends A and B of rod of length L slide on smooth horizontal ground and smooth inclined wall. Instantaneous speed of end A of the rod is 'v' to the left. The angular velocity of the rod, in multiples of $\frac{v}{L}$ is



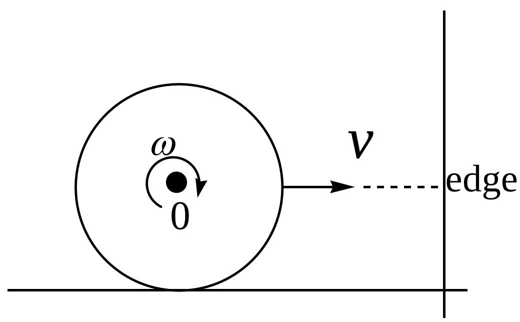
Answer :1

Solution : Draw normal at A and B to locate IC.



$$\omega = \frac{V}{L}$$

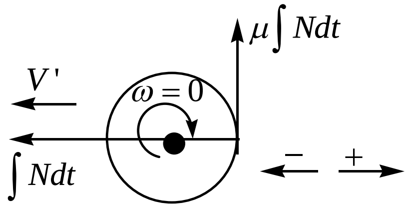
54 A uniform solid sphere of radius 'r' is rolling on a smooth horizontal surface with velocity 'v' and angular velocity $\omega (v = \omega r)$. The sphere collides with a sharp edge on the wall as shown. The coefficient of friction between the sphere and the edge is $\mu = \frac{1}{5}$. Just after the collision the angular velocity of the sphere becomes zero. The linear velocity of the sphere just after the collision in multiples of $\frac{v}{5}$ is :



Answer :5

Solution : $-\int Ndt = -mV' - (mV)$ ……….. (1)

$\mu R \int Ndt = \frac{2}{5} mR^2 \left(\frac{V}{R} \right)$ ……….. (2)

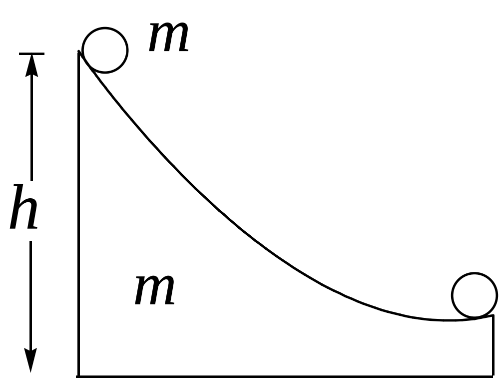


From eqns. (1) and (2), we get

$\int Ndt = 2mV$

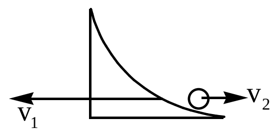
And $V' = V$

55 A hollow sphere is released from the top of a wedge, friction is sufficient for pure rolling of sphere on the wedge. There is no friction between the wedge and the ground. At the instant it leaves the wedge horizontally, velocity of centre of mass of the sphere w.r.t ground is $\sqrt{\frac{n}{7} gh}$. The value of 'n' is :



Answer :3

Solution : $mv_2 = mv_1 \Rightarrow v_1 = v_2$ (say v)

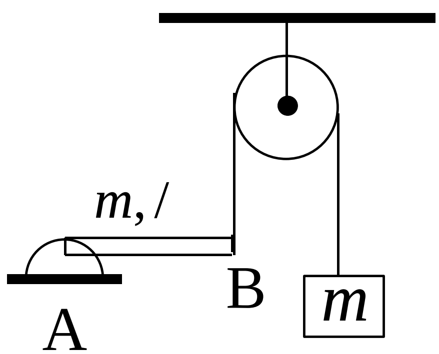


Constraint equation is $v_2 + v_1 = R\omega \Rightarrow 2v = R\omega \Rightarrow \omega = \frac{2v}{R}$

$$mgh = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega^2 = mv^2 + \frac{1}{2}\times\frac{2}{3}mR^2 \times \frac{4v^2}{R^2}$$

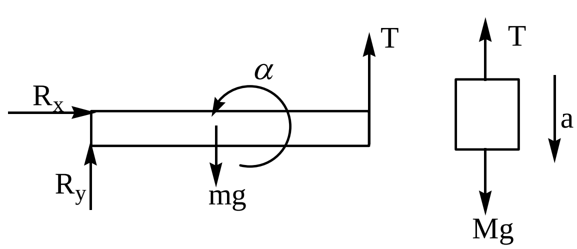
$$\Rightarrow v = \sqrt{\frac{3}{7}gh} \Rightarrow n = 3$$

56 Uniform rod AB is hinged at end A in horizontal position as shown in the fig. The other end is connected to a block through a massless string as shown. The pulley is smooth and massless. Masses of block and rod are same and equal to 'm'. Then acceleration of block just after release from this position in multiples of $\frac{g}{8}$ is :



Answer :3

Solution : mg – T = ma



$$Tl - \frac{mgl}{2} = \frac{ml^2}{3}\alpha$$

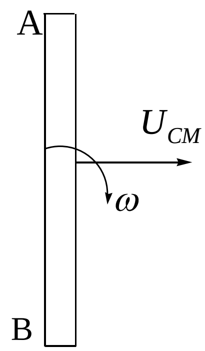
Also $a = l\alpha$

Acc. Of end B of rod is $l\alpha$

Thus $a = \frac{3g}{8}$; $T = \frac{5mg}{8}$

$$\alpha = \frac{3g}{8l}$$

57 A rod of length 'l' is traveling on a smooth horizontal surface with velocity u_{CM} and rotating with an angular velocity ω such that $u_{CM} = \frac{\omega\ell}{2}$. The distance covered by the point B in multiples of 'ℓ' when the rod completes one full rotation is :

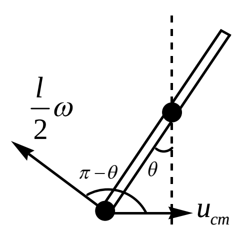


Answer :4

Solution : $u = \sqrt{\left(\frac{\omega l}{2}\right)^2 + u_{CM}^2 + 2u_{CM}\frac{\omega l}{2}\cos(\pi-\theta)}$

$$\frac{ds}{dt} = \frac{\omega l}{2}\sqrt{2+2\cos(\pi-\theta)}$$

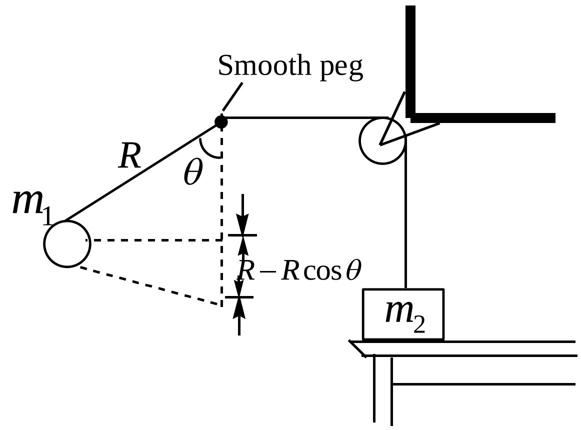
$$\frac{ds}{d\theta} = \frac{l}{2}\sqrt{2(1-\cos\theta)}$$



$$\int ds = \frac{l}{2}\int_0^{2\pi}\sqrt{2}\left(1-\cos\theta\right)^{1/2} d\theta$$

$$s = \frac{l}{2}\times 2\int_0^{2\pi}\sin\frac{\theta}{2}d\theta = l\frac{\left[-\cos\frac{\theta}{2}\right]_0^{2\pi}}{\left(\frac{1}{2}\right)} = 2l\left[-\cos\pi+\cos 0\right] = 4l$$

58 Two blocks are connected by a massless string that passes over a frictionless peg as shown in Fig. One end of the string is attached to a mass m1 = 3 kg, i.e., a distance R = 1.2 m from the peg. The other end of the string is connected to a block of mass m2 = 6 kg resting on a table. From what angle θ (in multiples of 10^0) measured from the vertical must the 3 kg block be released in order to just lift the 6 kg block off the table?



Answer :6

Solution : $m_2 g = m_1 g + \dfrac{m_1 v^2}{R}$

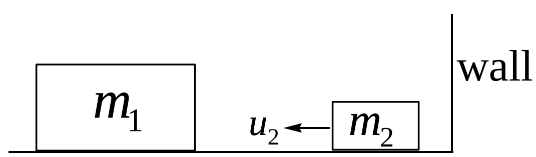
$v = \sqrt{2gR(1-\cos\theta)}$

59 A train has to negotiate a curve of radius 400 m. The distance between the rails is 1m. The height the outer rail should be raised with respect to inner rail for a speed of 36 km/hr is p/2 cm. Find the value of ‘p’ (take g = 10 m/s^2)

Answer :5

Solution : $\tan\theta = \dfrac{h}{l} = \dfrac{v^2}{rg}$

60 A block of mass m_1 = 150 kg is at rest on a very long frictionless table, one end which is terminated in a wall. Another block of mass m_2 is placed between the first block and the wall, and set in motion towards m_1 with constant speed u_2. Assume that all collisions are perfectly elastic. Find the value of m_2 (in multiples of 10 kg)for which both the blocks move with the same velocity after m_2 collides once with m_1 and once with the wall. The wall has effectively infinite mass.



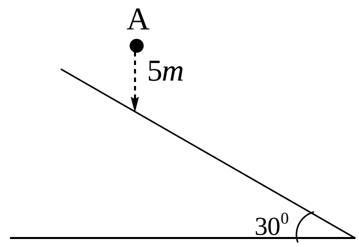
Answer :5

Solution : $v_1 = \dfrac{2m_2}{m_1+m_2}u_2,\ v_2 = \dfrac{(m_1-m_2)}{m_1+m_2}u_2$

$v_1 = v_2 \Rightarrow 2m_2 = m_1 - m_2$

$3m_2 = m_1 \ \therefore m_2 = 50\ kg$

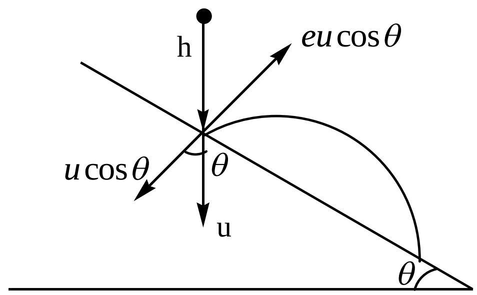
61 A ball is released from position A and travels 5m before striking the smooth fixed inclined plane as shown. If the coefficient of restitution in the impact is $e = \dfrac{1}{2}$, the time taken by the ball to strike the plane again is (in sec) (g = 10 m/s^2)



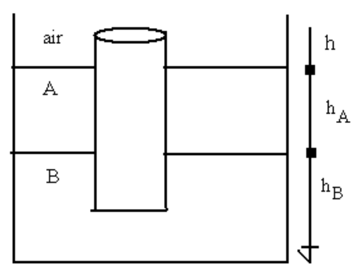
Answer :1

Solution : $T = \dfrac{2eu\cos\theta}{g\cos\theta}, \qquad u = \sqrt{2gh} = 10\,m/s$

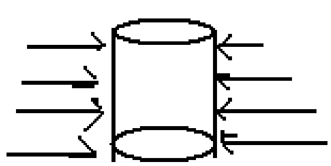
$\therefore T = 1s$



62 A uniform solid cylinder of density 0.8 g/cc floats in equilibrium in a combination of two non- mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cc and 1.2 g/cc respectively. The height of liquid A is $h_A = 1.2cm$. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8cm$. The total force exerted by liquid A on the cylinder is (in N)

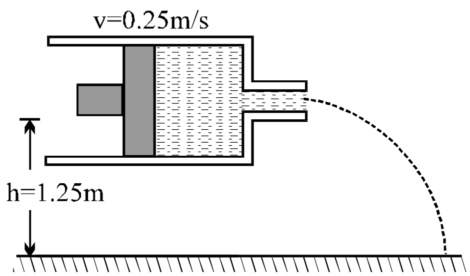


Answer :0
Solution :



The total force exerted by liquid A is Zero. This can be easily explained by pressure profile

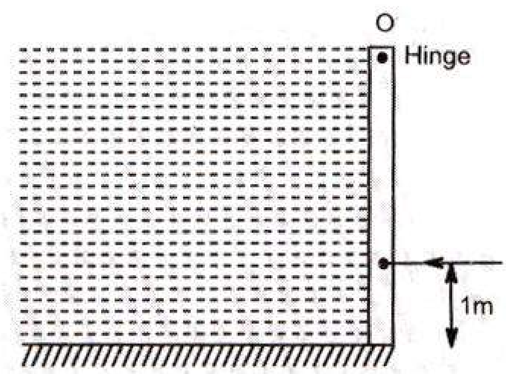
63 A syringe of diameter D = 8mm and having a nozzle of diameter d = 2mm is placed horizontally at a height of 1.25 m as shown in the figure. An incompressible and non-viscous liquid is filled in syringe and the piston is moved at a speed of v = 0.25 m/s. Find the range of liquid jet on the ground (in meters). (g = 10m/ s^2).



Answer :2

Solution : $A_1v_1 = A_2v_2, R = \sqrt{\frac{2h}{g}}.v_2$

64 A square gate of size $4m \times 4m$ is hinged at topmost point. A liquid of density ρ fills the space left of it. The force which acting at a height 1m from lowest point can hold the gate stationary is $\frac{2^N}{9}\rho g$. The value of 'N' is



Answer :8

Solution : Force acting on any elementary strip at a distance y from O

$dF = (\rho gy)(ady)$

Torque about O

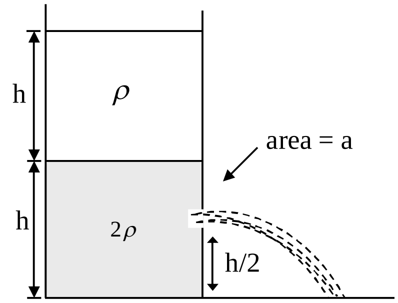
$d\tau = y(dF)$

$d\tau = y(dF)$

$d\tau = (\rho gy^2 a)dy$

Net torque $\tau = \int_0^a (\rho gy^2)dy = \rho ga \left.\frac{y^2}{3}\right|_0^a$

65 A fixed cylindrical tank having large cross-section area is filled with two liquids of densities a ρ and 2ρ and in equal volumes as shown in the figure. A small hole of area of cross-section $a = \sqrt{6}cm^2$ is made at height h/2 from the bottom. Find the area of cross- section of stream of liquid in cm^2 just before it hits the ground



Answer :2
Solution : Applying Bernoulli's equation at cross-section 1 & 2

$$P_{atm} + \rho gh + 0 = P_2 + 0 + 0 \Rightarrow P_2 = P_{atm} + \rho gh.....(1)$$

Again applying Bernoulli's equation at section 2 & 3

$$P_2 + 0 + 2\rho g \times \frac{h}{2} = P_{atm} + \frac{1}{2}2\rho V^2 \quad .....(2)$$

$$\Rightarrow V = \sqrt{2gh}$$

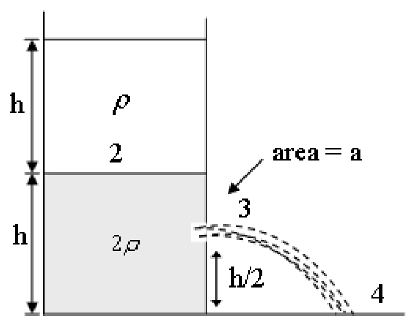
This is required velocity of efflux
Applying continuity equation between 3 & 4 cross-section.

$$aV = a_1 V_1$$

This is required velocity of efflux
Applying Bernoulli's equation between 3 & 4

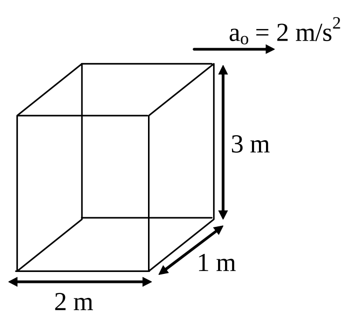
$$P_{atm} + \frac{1}{2}(2\rho)V^2 + 2\rho g \times \frac{h}{2} = P_{atm} + \frac{1}{2}(2\rho)V_1^2 + 0$$

$$\rho V^2 + \rho gh = \rho V_1^2 \Rightarrow V_1^2 = 3gh$$

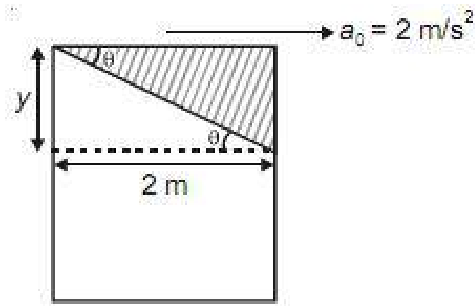


$$a_1 = \frac{aV}{V_1} = \frac{\sqrt{2gh}}{\sqrt{3gh}} = 2cm^2$$

66 An open tank having dimensions 1 m ´ 2 m ´ 3m completely filled with water is kept on a horizontal surface. The mass of water that spills out is 100x kg, when the tank is slowly accelerated horizontally at the rate of 2 m/ s^2 . Then find the value of x.



Answer :4



Solution :

$\tan\theta = a_o / g \Rightarrow \tan\theta = 2/10 \Rightarrow \tan\theta = 0.2$

$\tan\theta = y/2$ \ y = 0.4 m

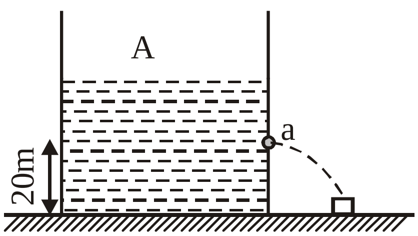
Þ volume of shaded region

Þ V = (1/2) ´ 0.4 ´ 2 ´ 1 = 0.4 m^3

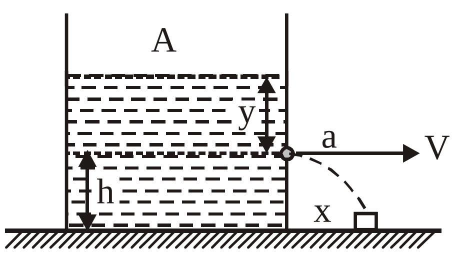
mass = V ´ d = 0.4 ´ 1000 = 400 kg

Þ x = 4

67 Water is filled in a uniform container of area of cross section A. A hole of cross section area a (<< A) is made to the wall of container at a height of 20 m above the base. Water streams out and hits a small block placed at some distance from container. With what speed (in ms^{-1}) should the block be moved such that water stream always hits the block before the level of water in the vessel reaches 20 m. (Given $\frac{a}{A} = \frac{1}{20}$). (Take g = $10\,ms^{-2}$)



Answer :1



Solution :

Velocity of efflux $v = \sqrt{2gy}$

Range $x = \sqrt{2gy} \times \sqrt{\frac{2h}{g}}$

The velocity of the block must be $(\frac{dx}{dt})$.

$\therefore \quad V_b = \frac{dx}{dt} = \sqrt{\frac{2h}{g}} \times \sqrt{2g} \times \frac{1}{2\sqrt{y}} \frac{dy}{dt}$

$V_b = \frac{\sqrt{h}}{\sqrt{y}} . \frac{dy}{dt}$ ....(i)

Using equation of continuity

$$\frac{Ady}{dt} = a\sqrt{2gy} \text{ ………..(ii)}$$

equation (i) and (ii)

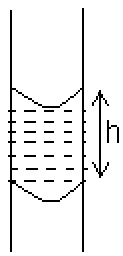
$$V_b = \sqrt{\frac{h}{y}} \times \frac{a}{A}\sqrt{2gy}$$

$$V_b = \sqrt{2gh} \times \frac{a}{A} = 20 \times \frac{1}{20} = 1 \text{ ms}^{-1}.$$

68 A long glass capillary tube of radius r is placed horizontally and filled with water (angle of contact for water – glass = 0^o). If the tube is made vertical, then the length of water column that remains in the capillary is $\frac{\eta T}{r\rho g}$ where 'T' is the surface tension of water, ρ is density of water. Find η

Answer :4

Solution :



$$P_0 - \frac{2T}{R} + \rho gh - P_0 = \frac{2T}{R} Þ\, h = \frac{4T}{R\rho g}$$

69 A ball A moving with momentum $2\hat{i} + 6\hat{j}$ collides with another identical moving ball B with momentum $-4\hat{j}$ and momentum of ball B after collision is $2\hat{j}$. The coefficient of restitution in the collision is $\frac{x}{15}$. Find the value of x.

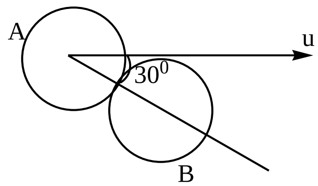
Answer :3

Solution : Since the momentum exchange in $\hat{j}$ - direction $\Rightarrow$ line of impulse is in $\hat{j}$-direction

$$\left(2\hat{i} + 6\hat{j}\right) - 4\hat{j} = \overrightarrow{P_A} + 2\hat{j} \Rightarrow \overrightarrow{P_A} = 2\hat{i}$$

$$\therefore e = \frac{\frac{2}{m} - 0}{\frac{6}{m} + \frac{4}{m}} = \frac{2}{10} = \frac{1}{5}$$

70 A moving sphere A of mass 'm' experience a perfectly elastic collision with a stationary sphere B of same mass 'm' as shown in the fig. At the instant of collision the velocity vector of A makes an angle of 30^0 with the line joining the centers of A and B. After collision the spheres fly apart then the angle between their velocity vectors is $K \times 10^0$. Then the value of 'K' is



Answer :9

Solution : After oblique elastic collision the balls are moving mutually perpendicular directions hence K = 9

71 The number of possible overtones of air column in a closed pipe of length 83.2 cms and diameter 6 cms whose frequencies lie below 1000 Hz will be, given velocity of sound in air =340 m/sec.(Consider end correction ; end correction equal to 0.3 times of diameter)

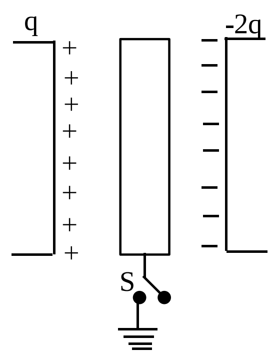
Answer :4

Solution : $\frac{\lambda}{4} = l + 0.3d = 83.2 + 0.3 \times 6 = 85$

$\lambda = 340cm$

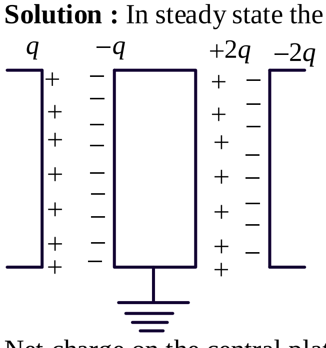
$n = \frac{V}{\lambda} = 100Hz$ (possible frequencies are 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz. No. of over times “4”).

72 The metal plate on the left in the figure carries a charge $+q$. The metal plate on the right has a charge of $-2q$. The magnitude of that charge will flow through S when it is closed (if the central plate is initially neutral) is xq where x = ______



Answer :1

Solution : In steady state the following charges will appear on different faces of the plates.



Net charge on the central plate is $+q$. Thus, $+q$ charge will flow through the switch.

73 An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the each. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed (in km/sec) with which it hits the surface of the earth ($g = 10\,m/s^2$ and R = 6400 km).

Answer :8

Solution : Given $v_0 = v_e / 2$

$$\left(\frac{GM}{R+h}\right)^{1/2} = \frac{1}{2}\left(\frac{2GM}{R}\right)^{1/2}$$

On solving, $h = R.$

From law of conservation of energy,

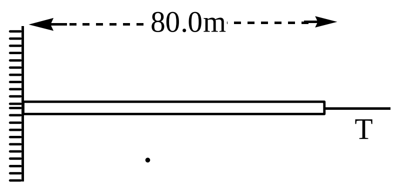
$$-\frac{GMm}{(R+h)} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

or $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$

or $v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$= \left[(10)\left(6.4\times10^6\right)\right]^{1/2} = 8\ km/s$$

74 A rope is held horizontally as shown in the figure. The rope is under a uniform tension $T = 196$ newton. The mass per unit length of the rope is $\frac{49}{2025}\ kg/m$. At the free end of the rope, a vertical jerk is given with a frequency of $2.0\ Hz$, generating a transverse wave in the rope. Neglecting the fact that the weight of the rope may cause the rope to have a curved shape rather than a straight one, The number of wave cycles in the rope's length is $\frac{16}{x}$, where x = _____



Answer :9

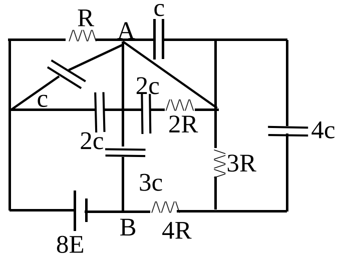
Solution : $V = \sqrt{\frac{196\times2025}{49}} = 90\ ms^{-1}$

$$\lambda = \frac{V}{f} = \frac{90}{2} = 45\,m$$

$\therefore$ No. of wave cycles $= \frac{80}{45} = \frac{16}{9} = 1.77$, An: x = 9

75 Six capacitors are arranged as shown in the figure. The circuit is in steady state The current through the battery is steady state is 2x amp where x = _____

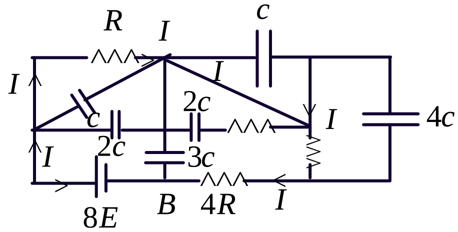
$C = 1\mu F; R = 1\Omega; E = 10\,Volt$



Answer :5

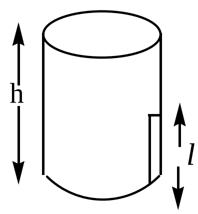
Solution : $8RI = 8E \Rightarrow E \backslash R = 10\,Amp$; An: x = 2

$(V_A - V_B) - 8E + IR = 0$



76 A cylinder of height $h = 1$ m has a narrow vertical slit along its wall running up to a length $l = 40$ cm from the bottom. The width of the slit $b = 1$mm. The cylinder is filled with water with the silt closed. The force experienced by the vessel immediately after the silt is opened, is nearly $(x + 0.4)$ N where x = ______

$(g = 10m/s^2)$



Answer :6

Solution : Consider an element of the slit of length dx at a depth x from the top.

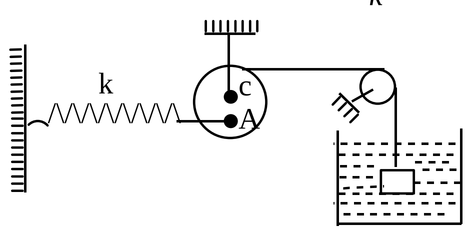
Velocity of the efflux at $x = \sqrt{2gx}$

Force on elementary area = rate of change of momentum $= (bdx\,\rho v)v = b\rho dx(2gx)$

$\therefore$ F (total force experienced) $= 2b\rho d\int_{h-l}^{h} xdx = b\rho gl(2h - l)$

Or , $F = 6.27\,N$ An; x = 6

77 A cubical block of volume v and density 3ρ is placed inside a liquid of density ρ and attached to a spring of spring constant k as shown in the figure. Assuming ideal spring and pulley and spring is attached at A which is at $R/2$ from centre. The compression in the spring at equilibrium is $\frac{xpvg}{k}$, where x = _____



Answer :4

Solution : Tension in the string $T = mg - F_B = 3\rho vg - \rho vg = 2\rho vg$

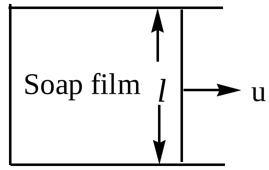
Balancing torque

$kxR \backslash 2 = TR$

$\Rightarrow x = 2T/k = 4\rho Vg/k$ An; x = 4

78 A soap film is created in a small wire frame as shown in the figure. The sliding wire of mass m is given a velocity

u to the right and assume that u is small enough so that film doesn't break. Plane of the film is horizontal and the surface tension is T. Then time to regain the original position of wire is $\frac{xum}{T\ell}$, where x = ___



Answer :1

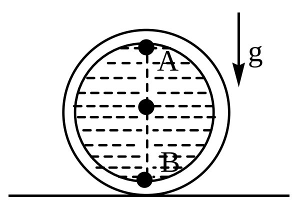
Solution : $a = \frac{2Tl}{m}$

$v = u + at \Rightarrow 0 = u - at$

$t = \frac{u}{a} = \frac{um}{2Tl}$

$\Rightarrow \quad Total\ time\ T = 2t = \frac{um}{Tl}$; here x = 1

79 A non-viscous incompressible liquid of mass m is filled fully inside a thin uniform spherical shell of mass m and radius R performing pure rolling on a rough horizontal surface. There are two points A and B inside the liquid on the vertical diameter separated by 2R as shown in the figure. Pressure difference between B and A is $p_B - p_A$. At the given instant velocity of centre of mass of the given system is v_0 and kinetic energy of this system is K. There is no slipping of sphere on surface. If the value of $\frac{K}{p_B - p_A}$ is of the form $\frac{X\pi v_0^2 R^2}{9g}$, then find the value of X.



No slipping

Answer :8

Solution : Liquid will be in pure translation.

80 A heavy uniform rope of length L is suspended from a ceiling. The rope is given a sudden sideways jerk at the bottom. A particle of mass 3kg is dropped from the ceiling at the instant at which jerk is given at the bottom of rope. The kinetic energy of the particle when the particle and wave are at the same level is $(\sqrt{x})\, gL$. Find the value of x. Find the value of x.

Answer :4

Solution : $v = \sqrt{\frac{T}{\mu}} = \sqrt{gx} = \frac{dx}{dt}$

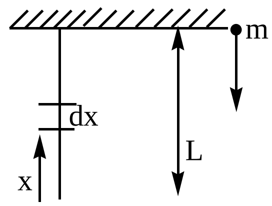
$x = \frac{gt^2}{4}$

$L - x = \frac{1}{2}gt^2 \Rightarrow L = \frac{3}{4}gt^2$

$t = \sqrt{\frac{4L}{3g}}$

Velocity of the particle $\upsilon = gt = g\sqrt{\frac{4L}{3g}}$

KE of the particle $= \frac{1}{2}m\upsilon^2 = \frac{2mg\ell}{3} = \frac{2(3)g\ell}{3} = 2g\ell$



81 A small ball of radius r is falling in a viscous liquid under gravity. Find the power of radius r on which the rate of heat produce depends after the drop attains terminal velocity.

Answer :5

Solution : From Stoke's law,

$F = 6\pi\eta\upsilon$

Rate of heat produced $Q = \bar{F}.\bar{v} = 6\pi\eta r\upsilon^2$

Terminal velocity $\upsilon = \frac{2}{9}\frac{r^2(\rho - \rho_0)g}{\eta}$

$\therefore \quad Q = 6\pi\eta r(\frac{2}{9}\frac{r^2(\rho - \rho_0)g}{\eta})^2$

$= \frac{4}{3}\frac{\pi r^5(\rho - \rho_0)^2 g^2}{\eta}$

$\therefore \quad Q \propto r^5$

The power is 5.

82 A block of mass m produces an extension of 9 cm in an elastic spring of length 60 cm when it is hung by it, and the system is in equilibrium. The spring is cut in two parts of 40 cm and 20 cm lengths. The same block hangs in equilibrium with the help of these two parts. Find the extension (in cm) in this case.

Answer :2

Solution : $mg = (K)(9)$ or K=mg/9

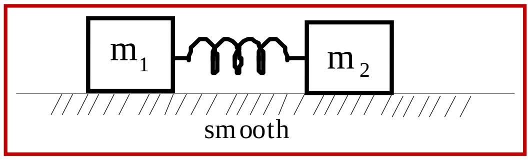
The spring constant is inversely proportional to length. So the spring constant for the spring of 40 cm length is $\frac{3}{2}K$. The spring constant for the spring of 20 cm length is 3K. The spring constant for the combination is

$\frac{3}{2}K + 3K = \frac{9}{2}K$

Now $mg = \frac{9}{2}Kx_0$ or $9K = \frac{9}{2}Kx_0$

$x_0 = 2\,cm$

83 Two blocks with masses $m_1 = 1kg$ and $m_2 = 2kg$ are connected by a spring of force constant K=24 N/m. The left block is imparted an initial velocity of 12 cm/s. The amplitude of the oscillations is _______cm



Answer :2

Solution : Initial velocity of the centre of mass

$$\upsilon_{cm} = \frac{m_1\upsilon_0}{m_1 + m_2}$$

$$E_{translattional} = \frac{1}{2}(m_1 + m_2)\upsilon_{cm}^2$$

Energy imparted to $'m_1', E = \frac{1}{2}m_1\upsilon_0^2$

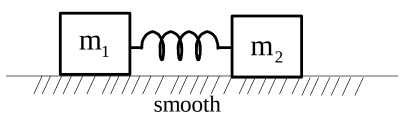
$$E_{vibrational} = E - E_{translational}$$

$$= E - \frac{E\,m_1}{m_1 + m_2} = \frac{E\,m_2}{m_1 + m_2} \Rightarrow \frac{1}{2}ka^2 = \frac{1}{2}\frac{m_1 m_2 \upsilon_0^2}{m_1 + m_2}$$

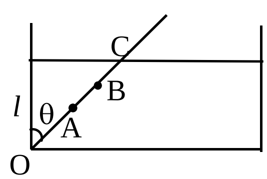
Amplitude $= a = \upsilon_0\sqrt{\frac{m_1 m_2}{(m_1 + m_2)k}}$

$$= (12cm/s)\sqrt{\frac{2}{3\times 24}}$$

= 2 cm

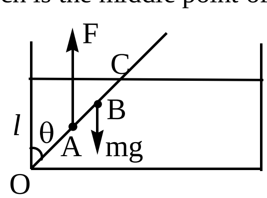


84 A wooden plank of length 1 m and uniform cross section is hinged at one end to the bottom of a tank as shown. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. The angle θ that the plank makes with the vertical in the equilibrium position (exclude $\cos\theta = 0$) is $\theta = \frac{\pi}{x}$. Find the value of x.



Answer :4

Solution : The forces acting on the plank are shown in the figure. The height of water level is l=0.5 m. The length of the plank is 1.0 m=2l. The weight of the plank acts through the centre B of the plank. We have OB=l. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.



(i) We have $OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$

Let the mass per unit length of the plank be ρ.

Its weight $mg = 2\ell\rho g$

The mass of the part OC of the plank $= \left(\frac{\ell}{\cos\theta}\right)\rho$

The mass of water displaced $= \frac{1}{0.5}\frac{\ell}{\cos\theta}\rho = \frac{2\ell\rho}{\cos\theta}$

Therefore, the buoyant force $F = \frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium the torque of mg about O should balance the torque of F about O.

So, $mg(OB)\sin\theta = F(OA)\sin\theta$

$$(2\ell\rho)\ell = \left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}} or\, \theta = 45^0 = \frac{\pi}{4}$$

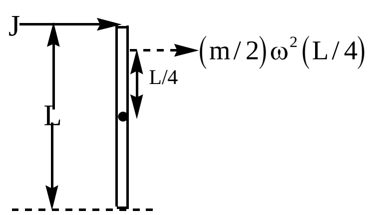
85 A uniform rod mass m= 5.0 kg, length L=90 cm rests on a smooth horizontal surface. One of the ends of the rod is struck with the impulse J=3.0 N.s in a horizontal direction perpendicular to the rod. As a result the rod obtains the momentum p=3.0 N.s. Find the force with which one half of the rod will exert on the other in the process of motion.

Answer :9

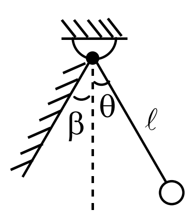
Solution : $J \times \frac{L}{2} = \left(\frac{mL^2}{12}\right)\omega \Rightarrow \omega = \frac{6J}{mL}$

Rod will rotate about its c.m., one half exerts centrifugal force on the other half, therefore

$$F = \frac{m\omega^2}{2} \times \frac{L}{4} = \frac{9J^2}{2mL} = 9N$$

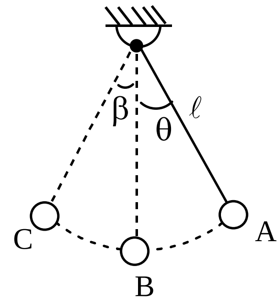


86 The bob is released from $\theta = 4^0$, so that it collides with the inclined wall of inclination $\beta = 2^0$. If the length of the string is $\ell = 10\,cm$. The time after which the bob collides with the wall is $\frac{4\pi}{10X}s$, then value of X is



Answer :3

Solution : The total time period motion is



$T = (t_{AB} + t_{BC})$ .... (i)

Where $t_{AB} = \pi\sqrt{\frac{\ell}{g}}$ .... (ii)

And t_{BC} can given as $\beta = \theta\sin(\omega t_{BC})$ or $t_{BC} = \frac{1}{\omega}\sin^{-1}\frac{\beta}{\theta} = \frac{1}{\sqrt{\frac{g}{\ell}}}\sin^{-1}\frac{\beta}{\theta} = \sqrt{\frac{\ell}{g}}\sin^{-1}\frac{\beta}{\theta}$ ...... (iii)

From equations (i), (ii) and (iii), $T = \sqrt{\frac{\ell}{g}}[\pi + 2\sin^{-1}(\frac{\beta}{\theta})] = \sqrt{\frac{10}{100\times10}}[\pi + 2\sin^{-1}(\frac{2^0}{4^0})]$

$= \frac{1}{10}[\pi + \frac{2\pi}{6}] = \frac{4\pi}{30}s$.

87 For two satellites at distances R and 7R above the earth's surface, the ratio of their total energies are ________

Answer :4

Solution : Distance of the two satellites from the centre of the earth are $r_1 = 2R$ and $r_2 = 8R$ respectively

R=earth's radius, Their potential energies are

$V_1 = \frac{GmM}{r_1}$ and $V_2 = \frac{GmM}{r_2}$

Their ratio is $\frac{V_1}{V_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

The kinetic energy of a satellite can be obtained from relation

$\frac{mV^2}{r} = \frac{GmM}{r^2}$

or $K = \frac{1}{2}mv^2 = \frac{GmM}{2r}$

Thus $K_1 = \frac{GmM}{2r_1}$ and $K_2 = \frac{GmM}{2r_2}$

The ratio of their kinetic energies is

$\frac{K_1}{K_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

Their total energies are

$E_1 = -\frac{GmM}{r_1} + \frac{GmM}{2r_1} = -\frac{GmM}{2r_1}$

and $E_2 = -\frac{GmM}{r_2} + \frac{GmM}{2r_1} = -\frac{GmM}{2r_2}$

Their ratio is $\frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

88 Two balls A and B are thrown vertically upwards from the same location on the surface of the earth with velocities $2\sqrt{\frac{gR}{3}}$ and $\sqrt{\frac{2gR}{3}}$ respectively, where R is the radius of the earth and g is the acceleration due to gravity on the surface of the earth. The ratio of the maximum height attained by A to that attained by B above the earth's surface is

Answer :4

Solution : I h is maximum height attained, then we have

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

Which gives $V^2 = \frac{2ghR}{(R+h)}$ $\left(\therefore g = \frac{GM}{R^2}\right)$

For ball A, we have $\frac{4gR}{3} = \frac{2gh_A R}{(R+h_A)} \Rightarrow h_A = 4R$

For ball B, we have $\frac{2gR}{3} = \frac{2gh_B R}{(R+h_B)} \Rightarrow h_B = \frac{R}{2}$

$\therefore \frac{H_\Delta}{h_B} = 8$.

89 The electric potential V (in volt) varies with x (in metre) according to the relation $V = 5 + 4x^2$. The magnitude of force experienced by a negative charge of 2×10^{-6} C located at x = 0.5 m is $z\times 10^{-6}$ N, then the value of z is ________

Answer :8

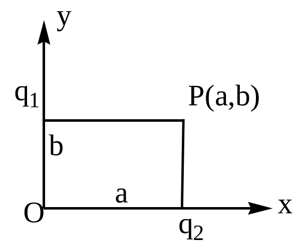
Solution : Electric field $E - \frac{dV}{dx} = -\frac{d}{dx}\left(5x - 4x^2\right) = -8x$

Force on charge $(-q) = -qE = +8qx$

At x=0, 5m, force $-8\times 2\times 10^{-6}\times 1.5 = 8\times 10^{-6}\,N$

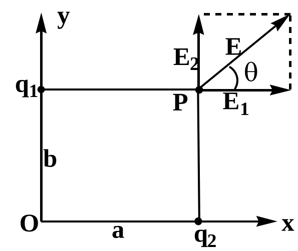
$\therefore z = 8$

90 Two point charge $q_1 = 2\mu C$ and $q_2 = 1\mu C$ are placed at distances b = 1cm and a = 2cm from the origin on y and x–axes as shown in figure. The electric field vector at point P(a,b) will subtend an angle θ with the x–axis given by $\tan\theta =$ _________



Answer :2

Solution : The electric field, E_1 at (a,b) due to q_1 has a magnitude $E_1 = \frac{1}{4\pi \in_0}.\frac{q_1}{a^2}$



And is directed along +x- axis. The electric field E_2 at (a,b) due to q_2

Has a magnitude $E_2 = \frac{1}{4\pi \in_0}.\frac{q_2}{b^2}$ and is directed along +y-axis. The angle θ subtended by the resultant field E with the x-axis is given by $\tan\theta = \frac{E_2}{E_1} = \frac{q_2}{q_1}.\frac{a^2}{b^2} = \frac{1}{2}x\left(\frac{2}{1}\right)^2 = 2$

91 A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring. The flux of the electric field through the surface of the sphere is $\frac{Q}{n\varepsilon_0}$. The value of n is

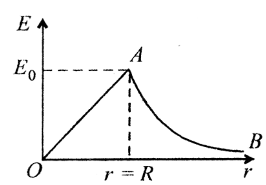
____

Answer :3

Solution : Due to a charged non-conducting sphere,

(i) Intensity inside sphere = E_1

$$E_1 = \frac{1}{4\pi \in_0}\frac{Q}{R^3}r$$



$\therefore E_1 \alpha r$, for r < R, as (OA)in figure.

Lotion (A) is correct.

(ii) Intensity outside sphere = E_2

$E_2 = \frac{1}{4\pi \in_0}\frac{Q}{r^2}$ when R < r < ∞

$\therefore\ E_2 \alpha \frac{1}{r^2}$, as (AB) in the figure.

$\therefore E_2$ decreases as r increases, Option (C) is correct.

(iii) Option (B) is incorrect in view of (A) and (C) above.

(iv) Intensity at surface = E_0

$\therefore E_0 = \frac{1}{4\pi \in_0} \frac{Q}{R^2}$, at point A in the figure.

E_0 is continuous at r = R. Option (D) is incorrect.

92 A mass of 6×10^{24} kg is to be compressed in a sphere in such a way that the escape velocity from its surface is 3×10^{8} m/s. The radius of the sphere is ________ mm(nearly)

Answer :9

Solution : Kinetic energy $\frac{1}{2}mv^2 = \frac{GmM}{2r}$ or $KE\alpha \frac{1}{r}$. Thus choice (A) is correct.

Angular momentum = mvr = $mx\sqrt{\frac{GM}{r}}xr = m\sqrt{GMr}$, which is proportional to $\sqrt{r}$. Hence choice (B) is wrong.

Linear momentum mv = $m\sqrt{\frac{GM}{r}}$, which is proportional to $\frac{1}{\sqrt{r}}$. Hence choice (C) is correct.

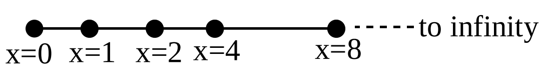
The freuqeuncy of revolution is $v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{GM}{r^3}}.i.e.\ v\alpha \frac{1}{r^{3/2}}$.

Hence the correct choices are (A), (C) and (D)

93 A particle of mass 1kg is placed at a distance 4m from the centre and on the axis of a uniform ring of mass of 5kg and radius 3m. The work required against mutual gravitational attraction to increase the distance of the particle from 4m to $3\sqrt{3}\,m$ is $\frac{G}{6x}J$. The value of x is ______ (G = universal gravitational constant)

Answer :1

Solution : The potential at x = 0 due to set of infinite number of charges placed on the x- axis as shown in figure, is



$$V = \frac{1}{4\pi \in_0}[\frac{q}{1} + \frac{q}{2} + \frac{q}{4} + \frac{q}{8} + ...+\infty]$$

$$= \frac{q}{4\pi \in_0}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....to\,\infty\right) == \frac{q}{4\pi \in_0}\left(\frac{1-\left(\frac{1}{2}\right)^{\infty}}{1-\frac{1}{2}}\right)$$

$$= \frac{q}{4\pi \in_0} x \frac{(1-0)}{\left(\frac{1}{2}\right)} = \frac{q}{2\pi \in_0}$$

The charges are placed along the same straight line, the electric filed at x = 0 will be directed along the x – axis and its magnitude is given by

$$E = \frac{1}{4\pi \in_0}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + ...to\,\infty\right] = \frac{q}{4\pi \in_0}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....to\,\infty\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{1-\left(\frac{1}{4}\right)^{\infty}}{1-\frac{1}{4}}\right] = \frac{q}{4\pi \in_0} x \frac{(1-0)}{\left(\frac{3}{4}\right)} = \frac{q}{3\pi \in_0}$$

94 Two identical small charged spheres are suspended by strings of equal lengths. The strings make an angle of 30^0 with each other in air. When suspended in a liquid of density 0.8 g/cc, the angle remains same. If the density of the material of the sphere is 1.6 g/cc, the dielectric constant of the liquid is _________

Answer :2
Solution : If the consecutive charges have opposite sign, the potential at x = 0 is given by

$$V = \frac{1}{4\pi \in_0}\left[\frac{q}{1} - \frac{q}{2} + \frac{q}{4} - \frac{q}{8} + \frac{q}{16} - \frac{q}{32} ..... \ to \infty\right]$$

$$= \frac{q}{4\pi \in_0}\left[\left(1 + \frac{1}{4} + \frac{1}{16} + ... \ to \infty\right) - \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ... \ to \ \infty\right)\right]$$

$$= \frac{q}{4\pi \in_0}\left[\left(\frac{1}{1-\frac{1}{4}}\right) - \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right)\right] = \frac{q}{4\pi \in_0}\left[\frac{4}{3} - \frac{1}{2} x \frac{4}{3}\right] = \frac{q}{6\pi \in_0}$$

$$E = \frac{1}{4\pi \in_0}\left[\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{8^2} + \frac{q}{16^2} - \frac{q}{32^2} + ... \ to \infty\right]$$

$$= \frac{q}{4\pi \in_0}\left[\left(1 + \frac{1}{16} + \frac{1}{256} + ... \ to \infty\right) - \left(\frac{1}{4} + \frac{1}{64} + \frac{1}{1024} + ... \ to \infty\right)\right]$$

$$= \frac{q}{4\pi \in_0}\left[\left(\frac{1}{1-\frac{1}{16}}\right) - \frac{1}{4}\left(\frac{1}{1-\frac{1}{16}}\right)\right] = \frac{q}{4\pi \in_0}\left[\frac{16}{15} - \frac{1}{4} x \frac{16}{15}\right] = \frac{q}{4\pi \in_0}$$

95 A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where 'k' and 'a' are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at r = R, find the value of a.

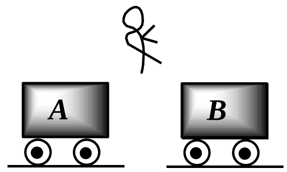
Answer :2
Solution : Electric flux through a plane surface of area S is given by where θ is the angle which the normal to the surface makes with the direction of the electric filed.
Now, S = 10cm x 10cm = 100 cm^2 =100 x 10^{-4} m^2 and $E = 3x10^3 \ NC^{-1}$ along the positive x- direction. $\theta = 0^0$
Hence $\phi = 3x10^3 x 100 x 10^{-4} x \cos 0^0 = 30NC^{-1}m^2$.

96 In a uniform electric field, the potential is 10V at the origin of coordinates, and 8V at each of the points (1,0,0), (0,1,0) and (0,0,1). The potential at the point (1,1,1) will be ________(in volts)

Answer :4

Solution : In this case $\theta = 90^0 - 30^0 = 60^0$, hence $\phi = 15NC^{-1}m^2$.

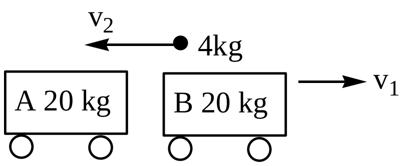
97 A child of mass 4 kg jumps from cart B to cart A and then immediately back to cart B. The mass of each cart is 20 kg and they are initially at rest. In both the cases the child jumps at 6 m/s relative to the cart. If the cart moves along the same line with negligible friction with final velocities of V_B and V_A, respectively, find the ratio of $6v_B$ and $5v_A$.



Answer :1

Solution : All the velocities shown in diagrams are w.r.t. ground.

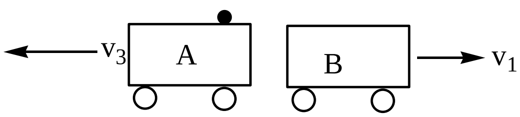
After first jump :



$20v_1 = 4v_2$ and $v_1 + v_2 = 6(given)$

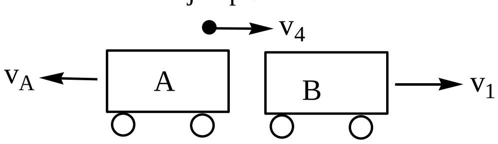
Solve to get $b_1 = m/s, v_2 = 5m/s$

When child arrives on A:



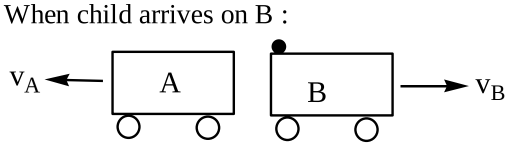
$(20+4)v_3 = 4v_2 \Rightarrow v_3 = 5/6\,m/s$

After the second jump :



$v_4 + v_A = 6, 24v_3 = 20v_A - 4v_4$ Solve to get $v_A = \dfrac{11}{6}m/s, v_4 = \dfrac{25}{6}m/s$

When child arrives on B :

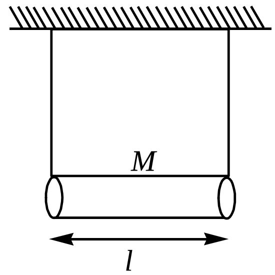


$24v_B = 4v_4 + 20v_1$

$24v_B = 4\left(\dfrac{25}{6}\right) + 20\times 1 \Rightarrow v_B = \dfrac{55}{36}m/s$

Now $\dfrac{6v_B}{5v_A} = \dfrac{6\times 55\times 6}{36\times 5\times 11} = 1$

98 A uniform rod of length 1 m and mass 2 kg is suspended. Calculate tension $T(in\,N)$ in the string at the instant when the right string snaps $(g = 10\,m/s^2)$.

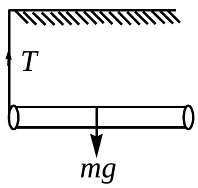


Answer :5

Solution : $Mg - T = ma_y$

$T(\frac{L}{2}) = \frac{ML^2}{12}\alpha$ and $a_y = \frac{L}{2}\alpha$ on solving, we get

$T = \frac{Mg}{4} = 5N$



99 A stone of mass *m*, tied to the end of a string, is whirled around in a horizontal circle (neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then the tension in the string is given by $T = A/r^n$, where A is a constant , r is the instantaneous radius of the circle and *n* is _____

Answer :3

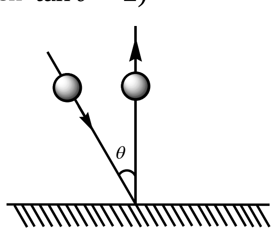
Solution : For circular motion of the stone . $\frac{mv^2}{r} = T[as\, g = 0](i)$

And as A.M is constant, $mvr = K, v = \frac{K}{mr}(ii)$ i.e.,

Eliminating v between Eqs. (i) and (ii) , we get $\frac{m}{r}\left[\frac{K}{mr}\right]^2 = T, T = \frac{K^2}{m}r^{-3}T = Ar^{-3}$ with $A = \left(\frac{K^2}{m}\right)(iii)$

Comparing Eqs. (iii) with $T = A/r^n$, we find n = 3.

100 A ball of mass 1 kg moving with a velocity of 5 m/s collides elastically with rough ground at an angle θ with the vertical as shown in Fig. What can be the minimum coefficient of friction if ball rebounds vertically after collision? (given $\tan\theta = 2$)



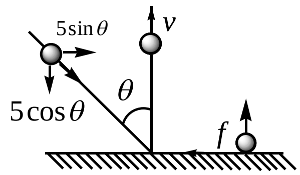
Answer :1

Solution : From impulse momentum theorem $\int N\, dt = m(v + 5\cos\theta)$

$\int f\, dt = m5\sin\theta$

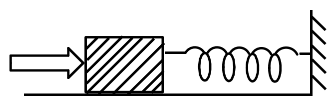
$\mu\int N\, dt = m5\sin\theta$

$\Rightarrow \mu m(v + 5\cos\theta) = m5\sin\theta$

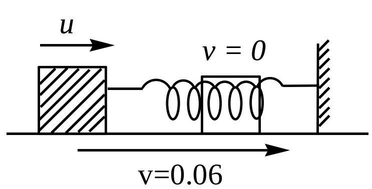


According to Newton's law of restitution, $v = e5\cos\theta$ Solve to get $\mu = 1$

101 A blocks of mass 0.18 kg is attached to a spring of force constant $2Nm^{-1}$. The coefficient of friction between the block and the floor is 0.1 Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in ms^{-1} is $V = N/10.$ Then N is



Answer :4



Solution :

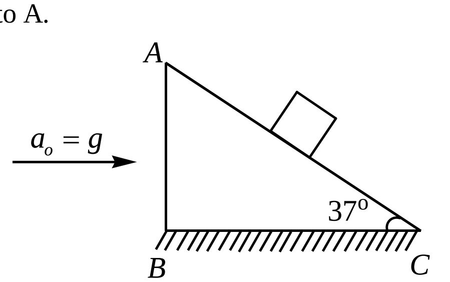
$$\frac{1}{2}mu^2 = \mu mg \times 0.06 + \frac{1}{2}kx^2$$

$$\frac{1}{2}\times 0.18u^2 = 0.1\times 1.18\times 10\times 0.06 + \frac{1}{2}\times 2\times (0.06)^2$$

$$u = 0.4ms^{-1}$$

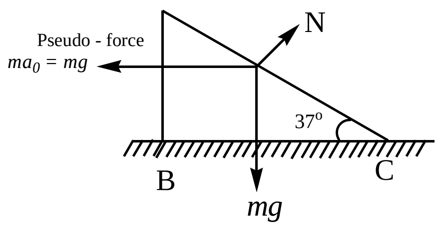
$$0.4 = \frac{N}{10} \Rightarrow N = 4$$

102 A block is placed on an inclined plane moving towards right horizontally with an acceleration $a_0 = g.$ The length of the plane AC = 1m. Friction is absent everywhere. Find the time taken (in seconds) by the block to reach from C to A.



Answer :1

Solution : (1) Drawing free – body diagram of block with respect to plane. Acceleration of the block up the plane is

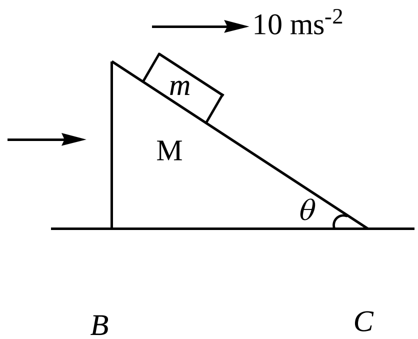


$$a = \frac{mg\cos 37^o - mg\sin 37^o}{m} = g\left(\frac{4}{5} - \frac{3}{5}\right) = 2ms^{-2}$$

Applying, $s = \frac{1}{2}at^2$ we get

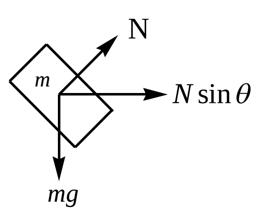
$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{2}} = 1\sec$$

103 In the figure shown all the surface are frictionless, and mass of the block is m = 100 g. The block and the wedge are held initially at rest. Now the wedge is given a horizontal acceleration of $10\,ms^{-2}$ by applying a force on the wedge, so that the block does not slip on the wedge. Then find the work done in joules by the normal force in ground frame on the block in 1 s.



Answer :5

Solution : (5) If block does not slip then $a, = g\tan\theta \Rightarrow 10 = 10\tan\theta \Rightarrow \theta = 45^o$



$$N\sin\theta = ma \Rightarrow N\times\frac{1}{\sqrt{2}} = 0.1\times 10 \Rightarrow N = \sqrt{2}N$$

$$S = \frac{1}{2}\times at^2 = \frac{1}{2}\times 10(1)^2 = 5m$$

$$W_N = NS\cos\theta = \sqrt{2}\times 5\times 1/\sqrt{2} = 5J$$

Alternatively : $W_N = \frac{1}{2}mv^2 = \frac{1}{2}(0.1)(at)^2 = 5J$

104 A binary star consists of two stars A (mass 2.2Ms) and B (mass 11 Ms). Where Ms is the mass of the sun. They are separated by distance d and are rotating about their centre of mass,which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is.

Answer :6

Solution : (6) $\frac{L_{total}}{L_B} = \frac{m_1 r_1^2}{m_2 r_2^2} + 1$

105 A steel wire of length 1 metre, mass 0.1 kg and uniform cross sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20^0C. If transverse waves are setup by plucking the string in the middle, the frequency of the fundamental mode of vibration. [Young's modulus of steel = 2×10^{11} N/m^2, coefficient of linear expansion of steel =$1.21\times10^{-5/0}$C] is $\frac{22}{x} Hz$ find "x"

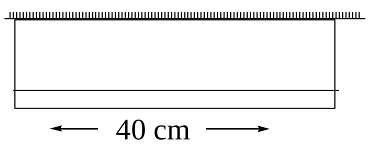
Answer :2

Solution : $y = \frac{Tl}{A\Delta l}$ and $al = l\alpha\Delta\theta$

$\therefore T = yA\alpha\Delta\theta = 48.4N$

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times1}\sqrt{\frac{48.4}{0.1}} = \frac{22}{2} \qquad \therefore x = 2$$

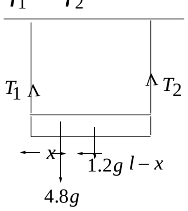
106 A uniform horizontal rod of length 40 cm and mass 1.2 kg is supported by two identical wires as shown in figure. Where should a mass of 4.8 kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? from left in cm (Take g = 10 m/s^2)



Answer :5

Solution : $T_1 + T_2 = 6g$

$f_1 = f_2$



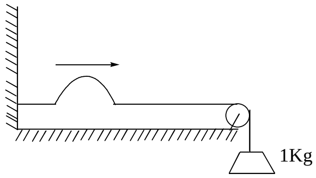
$$\frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{2}{2l}\sqrt{\frac{T_2}{\mu}}$$

$\therefore T_1 = 4T_2$

$(4.8g)x + (1.2g)l/2 = \frac{T}{2}l$

$\therefore x = 5cm$

107 Figure shows a string of linear mass density 1.0 g/cm on which a wave pulse is travelling. The time taken by the pulse in travelling through a distance of 50 cm on the string is $x\times10^{-2}$ s find x Take g = 10 m/s^2



Answer :5

Solution : $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{10}{0.1}}=10m/s$

Time $t=\frac{s}{v}=\frac{1/2}{10}=\frac{1}{20}=5\times10^{-2}\sec$

108 Two identical piano wires have a fundamental frequency of 600 vib/sec. When kept under the same tension. What percentage (nearly) increase in the tension of one of wire will lead to the occurrence of six beats per second.

Answer :2

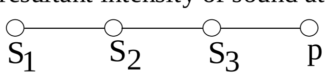
Solution : $f_1=\frac{1}{2l}\sqrt{\frac{T_1}{\mu}}=600$; $f_2=\frac{1}{2l}\sqrt{\frac{T_2}{\mu}}=60.6$

Solving $\frac{T_2}{T_1}=\left(\frac{101}{100}\right)^2$

$$\left(\frac{T_2-T_1}{T_1}\right)100=\left[\frac{(101)^2}{(100)^2}-1\right]100$$

= 2%

109 Three sources of sound S_1, S_2 and S_3 producing equal intensity at P are placed in a straight line with $S_1S_2=S_2S_3$ (figure). At a point P, far away from the sources, the wave coming from S_2 is 120^0 ahead in phase of that from S_1. Also, the wave coming from S_3 is 120^0 ahead of that from S_2. What would be the resultant intensity of sound at P?



Answer :0

Solution : $\bar{A}_1=A\uparrow$

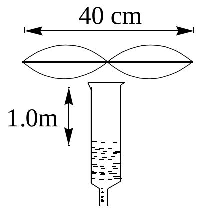
$\bar{A}_2=-A\cos60^0\uparrow+A\sin60^0\hat{j}$

$\bar{A}_3=-A\cos60^0\uparrow-A\sin60^0\hat{j}$

$\bar{A}=\bar{A}_1+\bar{A}_2+\bar{A}_3=0$

$\therefore$ Resultant intensity = 0

110 Consider the situation shown in figure. The wire which has a mass of 4.00 g oscillates in its second harmonic and sets the air column in the tube into vibrations in its fundamental mode. Assuming that the speed of sound in air is 340 m/s. The tension in the wire is $\left(11+\frac{x}{10}\right)$ Find x



Answer :6

Solution : $\frac{2}{2l}\sqrt{\frac{T}{\mu}}=\frac{v}{4l}$

$\frac{1}{0.4}\sqrt{\frac{T}{10^{-2}}}=\frac{340}{4}\Rightarrow T=11.6=11+\frac{x}{10}$

111 The equation of a longitudinal stationary wave in a metal rod is given by, y=0.002 sin $\frac{\pi x}{3}$ sin 1000 π t where x & y are in cm and t is in seconds. Maximum change in pressure (the maximum tensile stress) at the point x=2 cm, [if young's modulus of the material is $\frac{3}{8\pi}$ dynes/Cm^2] is $\frac{1}{p}\times10^{-3}$ dy/Cm^2. Find p

Answer :8

Solution : Maximum change in pressure

Maximum tensile stress = YAK

$=\frac{3}{8\pi}\times10^{-3}\times\frac{\pi}{3}$

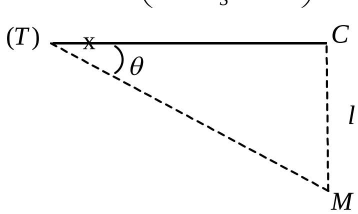
$=\frac{1}{8}\times10^{-3}\,dyne/cm^2$

112 A road passes at some distance from a standing man. A truck is coming on the road with some acceleration. The truck driver blows a whistle of frequency 500 Hz when the line joining the truck and the man makes an angle θ with the road. The man hears a note having a frequency of 600 Hz when the truck is closest to him. Also the speed of truck has got doubled during this time. The value of 'q' is $\frac{\pi}{x}$ find x.

Answer :3

Solution : Let Velocity of truck be V_S

Then $600=\left(\frac{V}{V-V_S\cos\theta}\right)500$



As $2V_S=V_S+at\Rightarrow at=V_S$

As $Vt=\frac{l}{\sin\theta};V=l\,Velocity\,of\,\,sound$

$$l\cot\theta = V_S t + \frac{1}{2}at^2$$

$$\frac{l\cos\theta}{\sin\theta} = \frac{3at^2}{2}[\therefore V_S = at]$$

$$\frac{p\cos\theta}{\sin\theta} = \frac{3a}{2}\left[\frac{1}{V\sin\theta}\right]^2$$

$$\frac{al}{V^2} = \frac{2}{3}\sin\theta\cos\theta$$

from (1)

$$\frac{6}{5} = \frac{V}{V - V_V\cos\theta} \qquad \Rightarrow \frac{6}{5} = \frac{V}{V - at\cos\theta} \qquad \Rightarrow \frac{6}{5} = \frac{V}{V - \frac{al}{V\sin\theta}}\cos\theta$$

$$\frac{6}{5} = \frac{V}{V - \frac{\cos\theta}{\sin\theta}}\left(\frac{2}{3}\sin\theta\cos\theta\right)V \Rightarrow \therefore \cos\theta = 60^0$$

113 A source of sonic oscillations with frequency $v_0 = 1700$ Hz and a receiver are located on the same normal to a wall. Both the source and the receiver are stationary, and the wall recedes from the source with velocity u = 6.0 cm/s. The beat frequency registered by the receiver is $\frac{K}{10}HZ$ find K. The velocity of sound v = 340 m/s

Answer :6

Solution : Let cl be The velocity of wall

$V_O \rightarrow$ velocity of sound then

beat frequency $\Delta f = \frac{2\mu f_O}{\mu + V_O}{}^{\simeq 0.589}_{\simeq 0.6H_2}$

114 A man standing in front of a mountain beats a drum at regular intervals. The drumming rate is gradually increased and be finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves near to the mountain by 90m and finds that the echo is again not heard when the drumming rate becomes 60 per minute. The distance (in metre) between the mountain and the initial position of the man is $90x$ m find $'x'$ value ?

Answer :3

Solution : Initially time interval beats $b/n] = \frac{60}{40} = 1.5\sec$

$S \rightarrow$ Initial distance and

$V \rightarrow$ speed of the sound ; $V \rightarrow t = 1.5 = \frac{25}{V}$

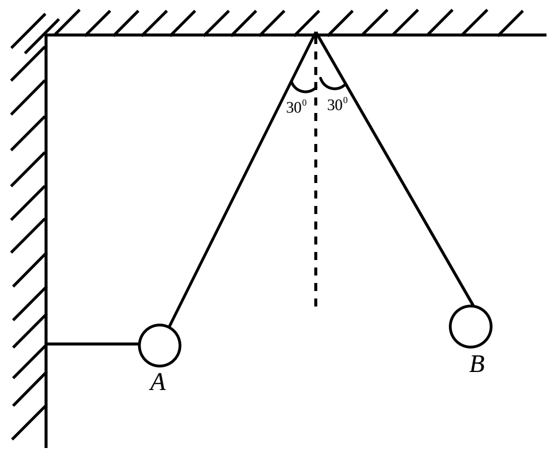
When man approaches 90m towards the wall.

$$\frac{2(S-90)}{V} = 1\ [\therefore dru\min g\ rate = 60/\min]\ \ 2\Delta\ \ (2)$$

Solve (1) & (2)

$S = 270m = 90\times 3; N = 3$

115 A ball is held at rest in position A by two light cords. The horizontal cord is now cut and the ball swings to the position B. Then the ratio of the tension in the cord in position B to that in position A originally is $\frac{3}{x}$, find x



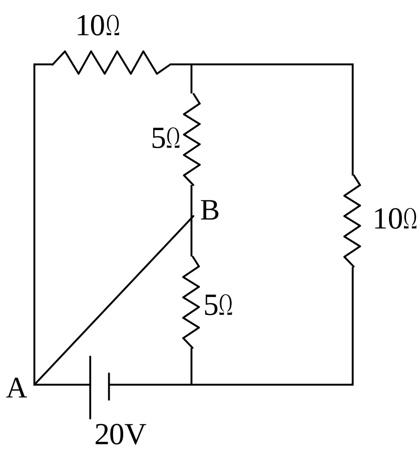
value ……….

Answer :3

Solution : . Tension in string at two position is same

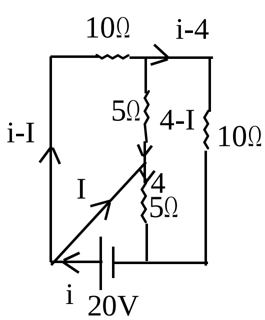
$\therefore x = 3$

116 Find the current in amperes in the branch AB in the circuit shown in the figure.



Answer :5

Solution : 5Ω is in parallel to cell. Current through it should be 4A.



$$\left.\begin{array}{l} 10(i-4) = 5(4-I)+20 \\ 10(i-I)+5(4-I) = 0 \end{array}\right\} \rightarrow I = 5A$$

117 A series R-C combination is connected to an AC voltage of angular frequency $\omega = 600\ rad.s^{-1}$. The impedance

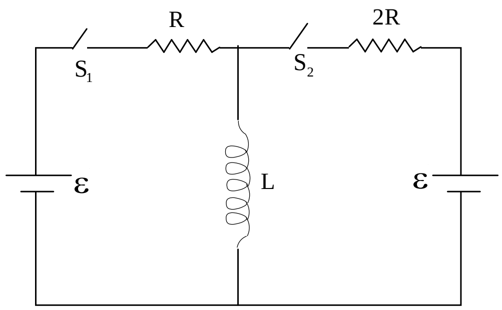
of the circuit is $\sqrt{1.25}\ R.$ Then the time constant of the circuit is $\frac{10}{n}$ millisecond, where n = ________

Answer :3

Solution : $Z^2 = X_C^2 + R^2 = \frac{5}{4}R^2 \Rightarrow X_C^2 = \frac{R^2}{4}$

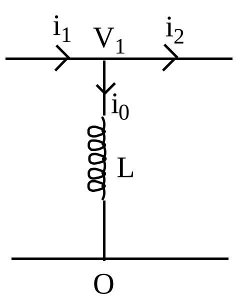
$\Rightarrow \frac{1}{\omega C} = \frac{R}{2} \Rightarrow \tau = CR = \frac{2}{\omega} = \frac{10}{3} ms$

118 In the circuit shown, switch S_2 remains closed for a long time, with S_1 open. It is given that $R = 10\Omega$, $L = 1\ mH$ and $\varepsilon = 3V$. Now switch S_1 is also closed. Immediately afterwards, if the magnitude of rate of change of current (in As^{-1}) in the inductor is 1000 x, find x.



Answer :2

Solution : Steady current L (with S_2 closed): $i_0 = \frac{\varepsilon}{2R}$



Current is same in L just before and just after closing S_1 .

$i_1 + i_2 = i_0 \Rightarrow \frac{\varepsilon - V_1}{R} + \frac{\varepsilon - V_1}{2R} = \frac{\varepsilon}{2R}$

$\Rightarrow \frac{3}{2}V_1 = \varepsilon$

$\frac{Ldi_0}{dt} = V_1 \Rightarrow \frac{di_0}{dt} = \frac{2}{3} \times \varepsilon \times \frac{1}{L} = \frac{2}{3} \times \frac{3}{10^{-3}} = 2000 \Rightarrow x = 2$

119 Two spheres of relative densities 0.8 and 1.2 respectively, each having equal volume $250\ cm^3$, are connected by

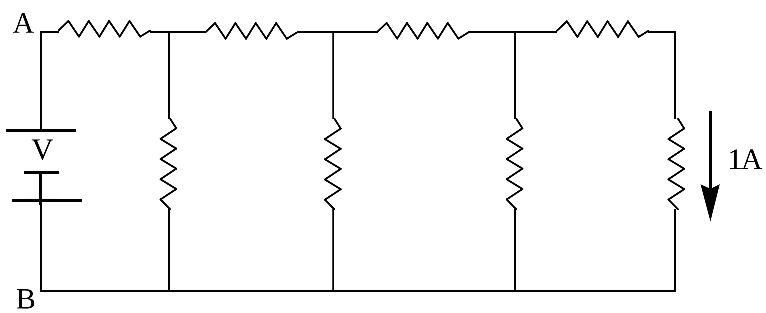
a string and the combination is immersed in a liquid of relative density 1.0. As the system freely floats in the liquid, the tension in the string is $\frac{n}{10}N$, where n = _____ $(g = 10\ m/s^2)$.

Answer :5

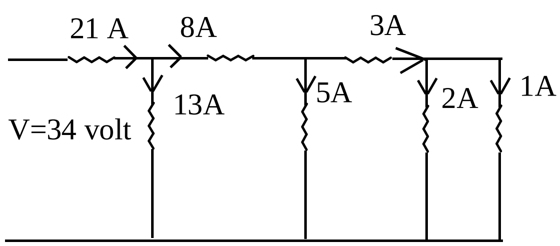
Solution : $(250\times10^{-6}\times800g+T)\downarrow=(250\times10^{-6}\times1200g-T)\downarrow=$

$\Rightarrow 2T=250\times10^{-6}\times400g\Rightarrow T=0.5N$

120 Each resistor in the finite network shown in the figure is $1\Omega.$ A current of 1 A flows through the last branch. The potential difference across the terminals A and B is $V=(10a+b)$ volt with a,b as integers , where a + b = ______



Answer :7



Solution :

a = 3, b = 4 $\Rightarrow$ a + b = 7

121 From the top of a tower of height 80m, a body is projected up with velocity 50m/s at an angle of inclination of 37^0 with horizontal. If mass of the body is m=0.01 kg and acceleration due to gravity is $10\,\text{ms}^{-2}$, then find power (in watts) supplied by gravitational force on the body at an instant seventh second after the projection .

Answer :4

Solution : $\vec{u}=40\hat{i}+30\hat{j}\;\; and\; \vec{a}=-10\hat{j}\;\; velocity\;\; after\; 7\sec=40\hat{i}+(30-70)\hat{j}=40\hat{i}-40\hat{j}$

$=-mg\hat{j}=-0.01\times10\hat{j}=0.1\hat{j}$

$\therefore power=F.v=4W$

122 An object of mass 0.2kg executes SHM along x-axis about origin with a frequency of $\frac{25}{\pi}Hz.$ At the position x = 0.04m, the object has K.E of 0.5J and P.E 0.4J. If the amplitude of osillations is A cm, find A.

Answer :6

Solution : $v=\frac{1}{2\pi}\sqrt{\frac{k}{m}}\Rightarrow\frac{25}{\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{0.2}}\Rightarrow k=500N\backslash m$

$\Rightarrow\frac{1}{2}KA^2=0.5+0.4\Rightarrow A=\sqrt{\frac{2\times0.9}{500}}=6cm$

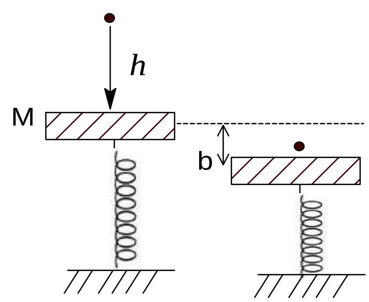
123 A block of mass 25kg rests on a horizontal floor $(\mu = 0.2)$ .It is attached by a 5m long light, straight horizontal rope to a peg fixed on floor. The block is pushed along the ground with an initial velocity of 10 ms-1 so that it moves in a circle around the peg. Find the time (in seconds) when tension in the rope just becomes zero. $(g = 10\,ms^{-2})$

Answer :5

Solution : $\frac{dv}{dt} - a_1 = -\mu g \Rightarrow \int_{10}^{v} dv = \int_{0}^{t} -2dt \Rightarrow v = 10 - 2t$

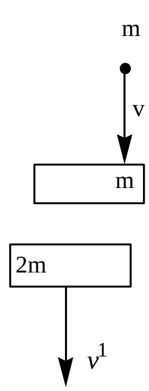
The tension in the rope will become zero, when centripetal acceleration becomes zero I,e., when speed becomes zero i.e., $10 - 2t = 0 \Rightarrow t = 5\sec$

124 A mass M is in static equilibrium on a massless vertical spring as shown in fig. A ball of mass 'm' dropped from certain height sticks to the mass 'M' after colliding with it. Combined mass executes SHM. During oscillations, it rises to a height ‘a’ above the equilibrium level of M and a depth b below it. What is the height (in meters) above the initial level of M from which the mass 'm' was dropped?(given M=m,b=3m, m=1kg, k=10N/m, g=10m/s2)



Answer :3

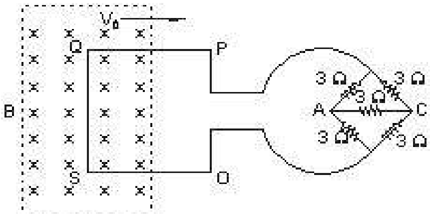
Solution : $v = \sqrt{2gh}$ After collision $v^1 = \frac{v}{2}$



$$w_s + w_g = \Delta K \Rightarrow -\frac{1}{2}k\left[\left(\frac{mg}{k}+b\right)^2 - \left(\frac{mg}{k}\right)^2\right] + 2mb = 0 - \frac{1}{2}\times 2mv'^2$$

$$\Rightarrow -\frac{1}{2}k(15) + 6mg = -\frac{mv^2}{4} \Rightarrow h = 3m$$

125 A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 weber/ m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohm. The resistances of the lead wires *OS* and *PQ* are negligible. What should be the speed of the loop (in cm/s) so as to have a steady current of 1 mA in the loop?



Answer :2

Solution : Given network forms a balanced Whetstones bridge. The net resistance of the circuit is therefore $3\Omega + 1\Omega = 4\Omega$. Emf of the circuit is $Bv_0 l$. Therefore, current in the circuit would be $i = \dfrac{Bv_0 l}{R}$

or $v_0 = \dfrac{iR}{Bl} = \dfrac{(1\times10^{-3})(4)}{2\times0.1}$

= 0.02 m/s = 2 cm/s

126 An LCR series circuit with $10\,\Omega$ resistance is connected to an AC source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by $60°$. When only the inductance is removed, the current leads the voltage by $60°$. Calculate the power (in kW) dissipated in the LCR circuit.

Answer :4

Solution : When the capacitance is removed, the circuit becomes LR with

$$\tan\phi = \frac{X_L}{R},\ i.e.\ X_L = R\tan\phi = 10\sqrt{3}$$

and when inductance is removed, the circuit becomes CR with

$$\tan\phi = \frac{X_C}{R},\ i.e.\ X_C = R\tan\phi = 10\sqrt{3}\,\Omega$$

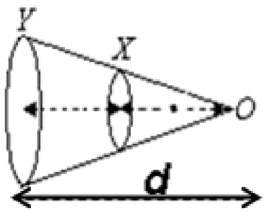
as here $X_L = X_C$, so the circuit is series resonance and hence as

$$X = X_L - X_C = 0,\ i.e. Z = R,\ so\ I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{200}{10} = 20A$$

and $P_{av} = V_{rms/ms}\cos\phi = 200\times20\times1 = 4kW$

127 Two circular coils *X* and *Y* have equal number of turns and carry equal currents in the same sense and subtend same solid angle at point *O*. If the smaller coil *X* is midway between *O* and *Y*, then if we represent the magnetic induction due to bigger coil *Y* at *O* as B_Y and that due to smaller coil *X* at *O* as

B_X , then find $\frac{B_x}{B_y}$

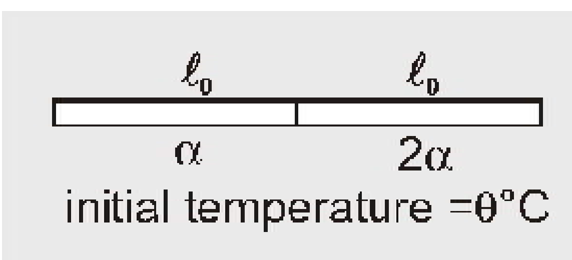


Answer :2
Solution : Conceptual

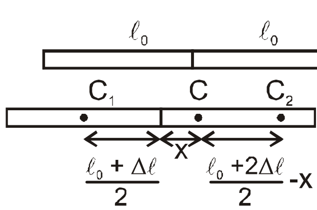
128 A current I, is flowing through a circular loop of radius 'R' made up of thin copper wire. When a uniform magnetic field 'B' is produced perpendicular to the plane of the loop, the tension developed in the loop is found to be $\frac{BIR}{x}$. Find the value of $'x'$.

Answer :1
Solution : Conceptual

129 Two uniform rods of equal lengths (l_0) and equal masses have coefficient of linear expansion $\alpha \; and \; 2\alpha$ are placed in contact on a smooth horizontal surface as shown. The temperature of system is $\theta^0 C$. Now the temperature is increased by $\Delta\theta^0 C$. The junction of the rods will shift from its initial position by $\frac{\ell_0 \alpha \Delta\theta}{x}$. The value of 'x' is



Answer :4
Solution : The extension in left rod $= \Delta l = l_0 \alpha\Delta\theta$



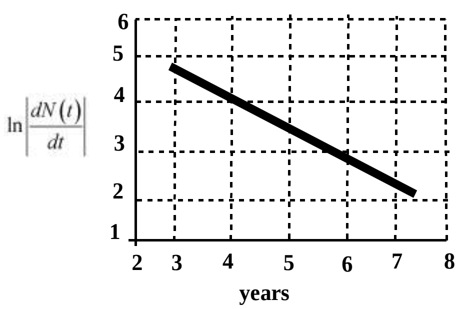
The extension in right rod $= 2\Delta l = 2l_0 \alpha\Delta\theta$

Since the centre of mass of two rod system will not shift, the junction will shift by x as shown. Taking moment of mass about original centre of mass C,

$$m \times \left(\frac{\ell_0 + \Delta\ell}{2} + x \right) = m\left(\frac{\ell_0 + 2\Delta\ell}{2} - x \right)$$

Solving we get x $= \frac{\Delta\ell}{4} = \frac{\ell_0 \, \alpha \, \Delta\theta}{4}$

130 To determine the half life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time 't'. If the number of radioactive nuclei of this element decreases by a factor of P after 4.16 years, the value of 'P' is



Answer :8

Solution : $\frac{dN}{dt} = \lambda N, \ln\frac{dN}{dt} = \ln\lambda + \ln N$

$\lambda = 1/2$ put $N = N_0 e^{-\lambda t}$

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{P} = N_0 e^{-\frac{1}{2}4.16}$$

$$P = e^{+2.08} = 8$$

131 A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is

Answer :4

Solution : For adiabatic process

$$T_1 V_1^{\lambda - 1} = T_2 V_2^{\lambda - 1} \qquad (\lambda = 7/5)$$

Hence, we have

$$T_1.V_1^{\gamma-1} = \alpha T_1.\left(\frac{V}{32}\right)^{\gamma-1}$$

Solving $\alpha = 4$

132 Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour?

Answer :3

Solution : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{3}{25}+\frac{1}{u_1}=\frac{2}{20}$$

$$\Rightarrow u_1 = 50m$$

Now, $\frac{7}{50}+\frac{1}{u_2}=\frac{2}{20}$

$$\Rightarrow u_2 = -25m$$

Speed of object $=\frac{u_1-u_2}{t}$

$$=\frac{-50-(-25)}{3}=\frac{-5}{6}m/s$$

$$=\frac{5}{6}\times\frac{18}{5}=3kmph$$

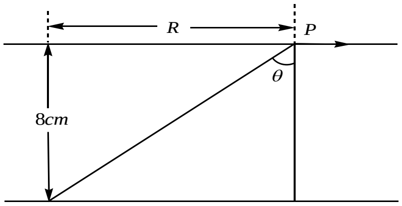
Hence $v=\frac{25\times 18}{30\times 5}$

$$=3kmph$$

133 A large glass slab $(\mu = 5/3)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

Answer :6

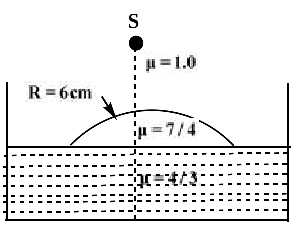
Solution : As shown, at 'P' TIR will occur



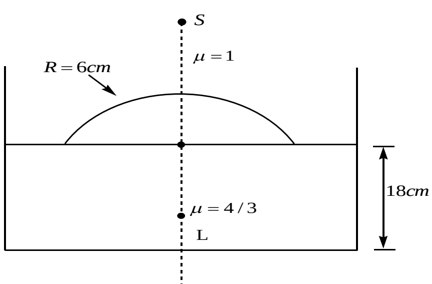
Thus, $\sin\theta = 3/5 \, i.e., \theta = 37^0$

Now R= $8 \times \tan 37^0$ = 6cm

134 Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature R = 6 cm as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Answer :2
Solution : Refraction at curved surface



For refraction at air-oil interface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$ we have,

$$\frac{7/4}{v_1} - \frac{1}{-24} = \frac{(7/4) - 1}{6}$$

$$v_1 = 21cm$$

Now refraction at plane surface (oil-water surface)

$$\frac{4/3}{v_2}-\frac{7/4}{21}=\frac{4/3-7/4}{\infty}$$

Hence $v_2 = 16cm$

Therefore,

$x = 18-16 = 2$

$\therefore$ Answer is 2 cm

135 A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A\times10^{z}$ (where 1 < A < 10). The value of 'Z' is

Answer :7

Solution : $E_{photon}=\frac{hc}{\lambda}=\frac{1242eV-nm}{200nm}=6.21eV$

$$V=\frac{1}{4\pi\varepsilon_0}.\frac{ne}{R}$$

Emission will stop when potential reached by sphere is capable to restrict fastest electron to escape. Hence

$$(eV)=\left(\frac{hc}{\lambda}-\phi\right)=6.21-4.7=1.51eV$$

$$\therefore \frac{n\times(1.6\times10^{-19})^2\times9\times10^9}{10^{-2}}=1.51\times(1.6\times10^{-19})$$

$$n=\frac{15.1}{1.6\times9}\times10^7=A\times10^Z$$

$\therefore$ Z= 7

136 The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second. Whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is
Answer :1
Solution : $A=\lambda N$

$$N=\frac{10^{10}}{\lambda}=10^{10}\times10^9=10^{19}$$

$M = Nm = 10^{19} \times 10^{-25} = 10^{-6} kg = 1mg$

137 The focal length of a thin biconvex lens is 20cm. When an object is moved from a distance of 25 cm in front of it to 50cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Answer :6

Solution : In first case $u = -25; v = ?; m = ?$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Solving $v = +100$ and $m_{25} = -4$

In second case

$u = -50$

$m_{50} = -2/3$

Thus, $\frac{m_{25}}{m_{50}} = +6$

138 Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500nm and in that of B is at 1500nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Answer :9

Solution : By Wien's, displacement law,

$\lambda_{\max}.T = \text{constant.}$

$\lambda_{\max}.T = \lambda_{\max 2}.T_2$

Thus, $(500 \times T_1) = (1500)(T_2)$

Hence, $T_1 = 3T_2$

Also $E_A = \sigma(4\pi)(6 \times 10^{-2})^2 (T_1^4)$

$E_B = \sigma(4\pi)(18 \times 10^{-2})^2 (T_2)^4$

Thus, $\frac{E_A}{E_B} = \left(\frac{1}{3}\right)^2 \times 3^4 = 9$

139 A piece of ice of specific (heat capacity = $2100 Jkg^{-1} {}^0C^{-1}$ and latent heat $= 3.36\times10^5 J/kg^{-1}$) of mass 'm' grams is at $-5^0 C$ at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice water mixture is in equilibrium, it is found that 1gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Answer :8
Solution : Heat lost= Heat gained

Mass=m grams, Thus, we have,

$[m(2100)(5)+1\times(336\times10^5)]\times10^{-3} = 420$

Solving, we get m = 8 gram

140 Monochromatic lights of wavelength 400 nm and 560 nm are incident simultaneously and normally on a double-slit apparatus whose slit separation is 0.1 mm and screen distance is 1m. Distance between areas of total darkness will be 4z mm. Find the value of z.

Answer: 7
Solution : At the area of total darkness, minima will occur for both the wavelengths.

$$\therefore \frac{(2n+1)}{2}\lambda_1 = \frac{(2m+1)}{2}\lambda_2$$

$$\Rightarrow (2n+1)\lambda_1 = (2m+1)\lambda_2$$

Or $\frac{(2n+1)}{(2m+1)} = \frac{560}{400} = \frac{7}{5}$

Or 10n = 14m + 2

By inspection: For m = 2, n = 3. For m = 7, n= 10. The distance between them will be the distance between such points, i.e.,

$\Delta s = \frac{D\lambda_1}{d}\left\{\frac{(2n_2+1)-(2n_1+1)}{2}\right\}$ Putting $n_2 = 10$ and $n_1 = 3$ and on solving, we get $\Delta s = 28 mm.$

141 In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same setup, the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of intensity of light at the mid-point of the screen in the first case to that in the second case is

Answer :2

Solution : In the first case, $I = I_0 + I_0 \cos 0^0$ or $I = 4I_0$

In the second case, $I' = I_0 + I_0 = 2I_0$ $\quad \therefore \frac{I}{I'} = \frac{4I_0}{2I_0} = \frac{2}{1}$

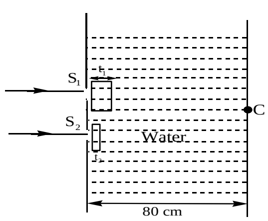
142 The wave front of a light beam is given by the equation $x + 2y + 3z = c$ (where c is arbitrary constant), then the angle made by the direction of motion of light with the $y - axis$ is $\cos^{-1}\left(\frac{\theta}{\sqrt{14}}\right)$ find θ.

Answer :2

Solution : Here, direction of light is given by normal vector $\vec{n} = \vec{t} + 2\hat{j} + 3\hat{k}$

$\therefore$ Angle made by the $\vec{n}$ with y-axis is given by $\cos\beta = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{14}}$

143 A screen is at a distance D = 80 cm from a diaphragam having two narrow slits S_1 and S_2 which are d = 2 mm apart. Slit S_1 is covered by a transparent sheet of thickness $t_1 = 2.5\,\mu m$ and S_2 by another sheet of thickness $t_2 = 1.25\,\mu m$ as shown in the figure. Both sheets are made of same material having refractive index $\mu = 1.40$. Water is filled in space between diaphragm and screen. A monochromatic light beam of wavelength $\lambda = 5000\overset{0}{A}$ is incident normally on the diaphragm. Assuming intensity of beam to be uniform, calculate ratio of intensity of centre of screen 'C' to intensity of individual slit. $(\mu_w = 4/3)$.



Answer :3

Solution : Path difference at C,

$$\Delta x = t_1(\mu - 1) - t_2(\mu - 1) \;= \mu(t_1 - t_2) - (t_1 - t_2)$$

$$= (t_1 - t_2)(\mu - 1) = (2.5 - 1.25)\left(\frac{14\times 3}{4\times 10} - 1\right)$$

$$= 1.25\times\frac{2}{40} = \frac{2.5}{40} \Rightarrow \Delta x = \frac{1}{16}\mu m$$

$$\phi = \frac{2\pi}{\lambda_W}.\Delta x = \frac{2\pi\times 4}{5000\times 3\times 10^{-10}}.\frac{1}{16}\times 10^{-6} = \frac{\pi}{3}$$

I at C, $I_c = 2I_0\left(1+\cos\frac{\pi}{3}\right) = 3I_0$

Required ratio $= \frac{I_C}{I_0} = 3$

144 A monochromatic beam of light of wavelength $5000\overset{0}{A}$ is used in Young's double slit experiment. If one of the slits is covered by a transparent sheet of thickness $1.4\times10^{-5}\,m$, having refractive index of its medium 1.25, then the number of fringes shifted is

Answer :7
Solution : Number of fringes is

$$t\frac{(\mu-1)D/d}{D\lambda/d} = \frac{(\mu-1)t}{\lambda} = 7$$

145 Assuming that about $200MeV$ of energy is released per fission of $_{92}U^{235}$ nuclei, the mass of U^{235} consumed per day in a fission reactor of power 1 megawatt will be approximately in grams is

Answer :1
Solution : Power P of fission reactor,

$P = 10^6 W = 10^6\, Js^{-1}$

Time = $t = 1day = 24\times36\times10^2\, s$

Energy produced, $U = Pt \text{ or } U = 10^6\times24\times36\times10^2 = 24\times36\times10^8\, J$

Energy released per fission of U^{235}

$= 200MeV = 32\times10^{-12}\, J$

Number of U^{235} atoms used

$$= \frac{24\times36\times10^8}{32\times10^{-12}} = 27\times10^{20}$$

mass of 6×10^{23} atoms of $U^{235} = 235g$

mass of 27×10^{20} atoms of $U^{235} = \left(\frac{235}{6\times10^{23}}\right)\left(27\times10^{20}\right) = 1.058g = 1g$

146 There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After n half-lives of A, rates of disintegration of both are equal. The minimum value of n is

Answer: 1

Solution: Let $\lambda_A = \lambda$ *and* $\lambda_B = 2\lambda$.$\Rightarrow$ Half life of A is $T_A = 2T_B$

Initially rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$. After one half-life of A, rate of disintegration of A will become $\frac{\lambda N_0}{2}$. (Half-life of B = one – half the half-life of A). So, after one half-life of A or two half – lives of B $\left(-\frac{dN}{dt}\right) = \left(-\frac{dN}{dt}\right)_B$ $\therefore n = 1$

147 The ratio of molecular mass of two radioactive substances is $\frac{3}{2}$ and the ratio of their decay constants is $\frac{4}{3}$. Then, the ratio of their initial activity per mole will be $\frac{K}{3}$. Find K

Answer :4

Solution : Activity , $R = \lambda N$. Number of nuclei (N) per mole are equal for both the substances.

$\therefore R \propto \lambda$

or $\frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$

148 At any instant, the ratio of the amounts of two radioactive substances is $2:1$. If their half-lives be respectively $12h$ and $16h$, then after two days, what will be the ratio of the substances?

Answer :1

Solution : Let $\frac{M_1}{M_2}(mass\ ratio) = \frac{2}{1}$

$2days = 2 \times 24h = 48h$

For first substance, 4 half-life periods and for second substance 3 half-life periods are passed; the masses are reduced to

$M_1' = M_1 \times \left(\frac{1}{2}\right)^4$

$M_2' = M_2 \times \left(\frac{1}{2}\right)^3$

$\therefore \frac{M_1'}{M_2'} = \frac{M_1}{M_2} \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2} = \frac{1}{1}$

149 A steel rod of length l m is heated from 25^0C to 75^0C keeping its length constant. The longitudinal strain developed in the rod is (Given: Coefficient of linear expansion of steel $=12\times10^{-6}/^{0}C$) $-n\times10^{-4}$ then n is

Answer :6

Solution : Strain developed :

$$\varepsilon=\alpha\ \Delta T=(12\times10^{-6})(50)=6\times10^{-4}$$

Strain will be negative, as the rod is in a compressed state.

150 Two light waves are given by, $E_1=2\ \sin(100\pi t-kx+30^0)$ and $E_2=3\ \cos(200\pi t-k'x+60^0)$. The ratio intensity of first wave to that of second wave is $\frac{4}{k}$ then k is

Answer :9

Solution : $I\alpha A^2\quad \therefore \frac{I_1}{I_2}=\frac{2^2}{3^2}=4/9$

151 In a hydrogen atom, the electron is in n^{th} excited state. It comes down to first excited state by emitting 10 different wavelengths. The value of n is.

Answer :6

Solution : $10=\frac{(n-1)(n-2)}{2},\ n=6$

152 The wavelengths of $K_\alpha\ X-$ rays of two metal 'A' and 'B' are $\frac{4}{1875R}$ and $\frac{1}{675R}$ respectively, where 'R' is rydebrg constant. The number of elements lying between 'A' and 'B' according to their atomic numbers is

Answer :4

Solution : Using $\frac{1}{\lambda}=R(z-1)^2\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right]$

For α particle ; $n_1=2, n_2=1$

For metal A; $\frac{1875R}{4}=R(Z_1-1)^2\left(\frac{3}{4}\right)$ $\qquad\Rightarrow\ Z_1=26$

For metal B; 675R $=R(Z_2-1)^2\left(\frac{3}{4}\right)$ $\qquad\Rightarrow\ z_2=31$

Therefore, 4 elements lie between A and B.

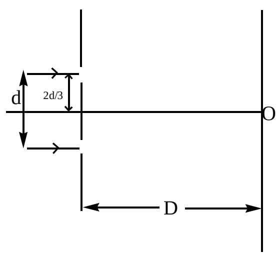
153 An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron increases k times then k is

Answer :8

Solution : $i = \frac{q}{T}$

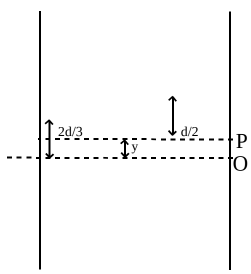
Now $T^2 \alpha r^3 \alpha n^6 \quad \Rightarrow \quad i \alpha \frac{1}{n^3}$

154 In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the nearest white spot on the screen from O is: [assumed $d << D, \lambda << d$] $\frac{d}{m}$ than 'm' is



Answer :6

Solution : The nearest white spot will be at P, the central maxima.



$\therefore \quad y = \frac{2d}{3} - \frac{d}{2} = \frac{d}{6}$

155 An interference is observed due to two coherent sources S_1 placed at origin and S_2 placed at $(0, 3\lambda, 0)$. Here λ is the wavelength of the sources. A detector D is moved along the positive x-axis. Find the integral value of the x-coordinate (excluding x = 0 and $x = \infty$) where maximum intensity is observed.

Answer :4

Solution : At x = 0, path difference is 3λ. Hence, third order maxima will be obtained. At $x = \infty$, path difference is zero. Hence, zero order maxima is obtained. In between, first and second order maximas will be obtained.

First order maxima :

$S_2P - S_1P = \lambda$ or $\sqrt{x^2 + 9\lambda^2} - x = \lambda$ or $\sqrt{x^2 + 9\lambda^2} = x + \lambda$. Squaring both sides, we get

$x^2 + p\lambda^2 = x^2 + \lambda^2 + 2x\lambda$. Solving this, we get $x = 4\lambda$

Second order maxima : $S_2P - S_1P = 2\lambda$

Or $\sqrt{x^2 + 9\lambda^2} - x = 2\lambda$ or $\sqrt{x^2 + 9\lambda^2} = (x + 2\lambda)$

Squaring both sides, we get $x^2 + 9\lambda^2 = x^2 + 4\lambda^2 + 4x\lambda$

Solving, we get $x = \frac{5}{4}\lambda = 1.25\lambda$

Hence, the desired x coordinates are,

$x = 1.25\lambda$ and $x = 4\lambda$.

156 Two kilograms of ice at -20^0C is mixed with 5kg of water at 20^0C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that specific heats of water and ice are $1 kcal/kg/^0C$ and $0.5 kcal/kg/^0C$ while the latent heat of fusion of ice is $80 kcal/kg$

Answer :6

Solution : Heat required to convert 5 kg of water at 20^0C to 5 kg of water at 0^0C

$mC_u\Delta T = 5\times1\times20 = 100 kcal$

Heat released by 2kg ice at -20^0C to convert 2 kg of ice at 0^0C is

$mC_{ice}\Delta T = 2\times0.5\times20 = 20 kcal$

Amount of ice that will convert into water at 0^0C for giving another 80 Kcal of heat can be found as follows;

$Q = mL \Rightarrow 80 = m\times80 \rightarrow m = 1kg$

Therefore, the amount of water at 0^0C is

5kg+1kg=6kg

Thus, at equilibrium we have, 6 kg water at 0^0C +1kg ice at 0^0C

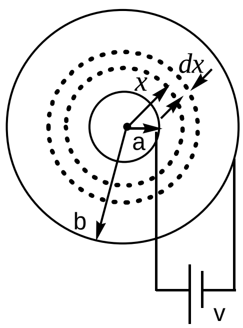
157 A metal sphere of radius $'a'$ is surrounded by a concentric metal spherical shell of radius b $(b > a)$. The space between the spheres is filled with material whose electrical conductivity σ varies with the electric field strength E according to the rotation $\sigma = KE$, where K is a constant. A potential difference V is maintained between inner sphere and outer shell. If the current (in ampere) between the inner sphere and outer shell is x, find $\frac{x}{4}$.

[Here $K = \frac{1}{4}\times10^{-2}\ \Omega^{-1}V^{-1}, V = 20 volt$, b = ea (e = exponential)

Answer :4

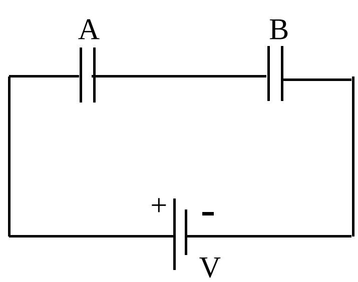
Solution : For the element, $i=\dfrac{dV}{\dfrac{1}{\sigma}\cdot\dfrac{dx}{4\pi x^2}}=\dfrac{Edx}{\dfrac{1}{KE}\cdot\dfrac{dx}{4\pi x^2}} \Rightarrow i=KE^2.4\pi x^2$

$$\Rightarrow \sqrt{\frac{i}{4\pi K}}=Ex=\frac{dv}{dx}.x \ \Rightarrow \sqrt{\frac{i}{4\pi K}}\int_a^b \frac{dx}{x}=\int_o^V dV$$



$$\Rightarrow\ i=\frac{4\pi KV^2}{\left[\ln\left(\frac{b}{a}\right)\right]^2}=\frac{4\pi\times\frac{1}{\pi}\times10^{-2}\times400}{\ln e}=16A$$

158 Two identical parallel plate capacitors A and B are connected in series through a battery of potential difference V. Area of each plate is $'a'$ and initially plates of capacitors are separated by a distance $'d'$. Now separation between plates of capacitor B starts increasing at constant rate v. The rate by which work is done on the battery when separation between



plates of capacitor B is 2d is $\dfrac{a\in_o vV^2}{nd^2}$. Find the value of n.

Answer :9

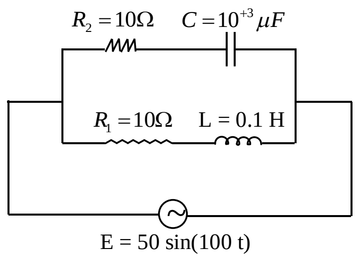
Solution : Let at any instant separation between plates of capacitor be x, then

$$C_e=\frac{\frac{a\in_\circ}{d},\frac{a\in_\circ}{x}}{\frac{a\in_\circ}{d}+\frac{a\in_\circ}{x}}=\frac{a\in_\circ(xd)}{xd(d+x)}=\frac{a\in_\circ}{(d+x)}$$ and $$Q=C_eV=\frac{a\in_\circ}{(d+x)}V$$

$$\frac{dQ}{dt} = -\frac{a \in_\circ V}{(d+x)^2} \cdot \frac{dx}{dt} = -\frac{a \in_o V \vartheta}{(d+x)^2}$$

Rate of work done on the battery $= -\left(\frac{dQ}{dt}\right) V = \frac{a \in_o \vartheta V^2}{9d^2}$ $\therefore n = 9$

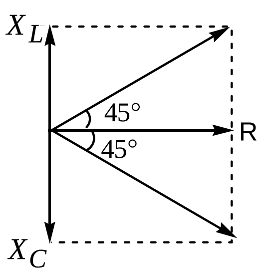
159 For the circuit shown in the figure, find the peak current (in ampere) through the source



Answer :5

Solution : Peak current through $R_1, i_1 = \frac{50}{10\sqrt{2}} = \frac{5}{\sqrt{2}} A$

Peak current through $R_2, i_2 = \frac{50}{10\sqrt{2}} = \frac{5}{\sqrt{2}} A$



$R_1 = R_2 = X_L = X_C$

place difference between i_1 *and* i_2 is $\frac{\pi}{2}$

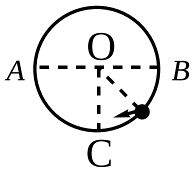
$\therefore$ peak current through the source is ,

$$i = \sqrt{i_1^2 + i_2^2} = \sqrt{\frac{25}{2} + \frac{25}{2}} = 5A$$

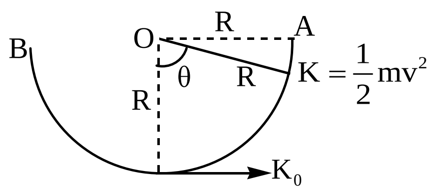
160 A particle of mass m oscillates about C between A,B inside a smooth spherical shell of radius R and center O.

At any instant kinetic energy of the particle is K. If the magnitude of force applied by particle on the shell at

this instant is $\frac{nK}{R}$ the value of n is ______



Answer :3



Solution :

$$mgR = K + mgR(1-\cos\theta)$$

$$mgR\cos\theta = K$$

$$mg\cos\theta = \frac{K}{R}$$

$$N = \frac{mv^2}{R} + mg\cos\theta$$

$$= \frac{2K}{R} + mg\cos\theta = \frac{2K}{R} + \frac{K}{R}$$

$$N = \frac{3K}{R}$$