CHAPTER 07

Perpetuity, Sinking Funds, Bonds & EMI

In this Chapter...

- Perpetuity
- Sinking Funds
- Bonds
- Equated Monthly Installment (EMI)

Perpetuity

A perpetuity is a type of annuity that lasts forever. In perpetuity, the stream of cash flows continues for an infinite amount of time.

Present Value of Perpetuity

Let us consider an annuity whose periodic payment is ₹ R for infinite periods, interest being $r\%$ per period or $\frac{r}{100} = i$ per period per rupee.

Type I When periodic payment is made at the end of each payment period.

The present value P of the perpetuity is given by

$$P = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots.$$

$$\Rightarrow \quad P = \frac{R}{1+i}\left(\frac{1}{1-\frac{1}{1+i}}\right) = \frac{R}{1+i-1} = \frac{R}{i}$$

$$\Rightarrow \quad P = \frac{R}{i},$$

where R is periodic payments of an annuity.

Hence, the present value of ₹ R payable at the end of each payment period when money is worth i period is $\frac{R}{i}$.

Type II When periodic payment is made at the beginning of each payment period.

The present value P of the perpetuity is given by

$$P = R + \frac{R}{1+i} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \ldots.$$

$$\Rightarrow \quad P = R + \frac{R}{1+i}\left(\frac{1}{1-\frac{1}{1+i}}\right) = R + \frac{R}{i}$$

$$\Rightarrow \quad P = R + \frac{R}{i}$$

Hence, if money is worth i per period, then the present value of perpetuity of ₹ R payble at the beginning of each period is $R + \frac{R}{i}$.

Present Value of a Growing Perpetuity

A growing perpetuity is a perpetuity in which periodic payment do not remain fixed rather these payment keep on growing at the same constant rate of growth.

A growing perpetuity of ₹R , which grows at the rate of g per rupee per period, payable at the end of each payment period.

When money is worth i per period is $P = \frac{R}{i-g}$.

Sinking Fund

A sinking fund is a fund created to accumulate money over the years to discharge a future obligation like replacing an old machine by a new machine, renovation of house, etc.

An annuity whose periodic payment is ₹R payable at the end of each payment period of n period, interest beings $r\%$ per period or $i = \frac{r}{100}$.

Then, the amount of obligation which can be discharged

$$S = \frac{R[(1+i)^n - 1]}{i}$$

Sinking Fund Payment

The periodic payment of ₹ R required to accumulate a sum of ₹ S over n periods with interest charged at the rate of i per period is given by

$$R = \frac{i \times S}{(1+i)^n - 1}$$

Bonds

A bond is a fixed income instrument that represents a loan made by an invester to a borrower.

A bond is characterised by the following terms

(i) **Maturity** Bond maturity is the time when the bond issuer must repay the original bond value to the bond holder.

(ii) **Maturity date** Maturity date is the date on which the bond will mature and the bond issuer will pay the bond holder the face value of the bond.

(iii) **Face value** The face value of a bond is the price that the issuer pays at the time of maturity, also referred to as "Par value".

(iv) **Redemption price** The price at which the issuing company will repurchase the bond from investors before its maturity date.

(v) **Dividend rate** The rate at which a bond yields interest is called the dividend rate or the nominal interest rate.

(vi) **Coupon rate** A bond's coupon rate denotes the annual interest rate paid by the bond issuer to the bond holder.

(vii) **Discount rate or Yield To Maturity (YTM)** The rate of interest used to discount the bond's cash flows is known as discount rate or Yield to Maturity.

Valuation of Bonds

There are mainly four approaches of valuation of bonds

(i) Present value approach

(ii) Relative price approach

(iii) Arbitrage-free pricing approach

(iv) Stochastic-Calculus approach

We will study the one main approaches present value approach.

Present Value Approach

Let a bond of face value ₹ F matures in N yr has a coupon rate of $r\%$ per annum, then each coupon payment $C = F \times \frac{r}{100}$.

If the discount rate is $d\%$ per annum, then

$$i = \frac{d}{100}$$

So, the present vale of bond is given by

$$\text{PV} = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^N} + \frac{F}{(1+i)^N}$$

$$= \frac{\frac{C}{1+i}\left[1 - \frac{1}{(1+i)^N}\right]}{1 - \frac{1}{1+i}} + \frac{F}{(1+i)^N}$$

$$\left[\because \text{ using } S_n = \frac{a(1-r^n)}{1-r}\right]$$

$$\text{PV} = \frac{C[1-(1+i)^{-N}]}{i} + F(1+i)^{-N}$$

(i) If discount rate < coupon rate, then PV > face value.

(ii) If discount rate = coupon rate, then PV = face value

(iii) If discount rate > coupon rate, then PV < face value.

These results show that there is an inverse-relation between yield and bond price.

Calculation of Yield To Maturity (YTM)

It is nearly impossible to calculate the yield to maturity using the formula of present value of a bond. However, approximate value of yield to maturity can be computed using the following formula

Approximate YTM

$$= \frac{\text{Coupon payment} + \frac{\text{Face value} - \text{Present value}}{N}}{\frac{\text{Face value} + \text{Present value}}{2}}$$

i.e. Approximate YTM $= \frac{C + \frac{F - \text{PV}}{N}}{\frac{(F + \text{PV})}{2}}$

Equated Monthly Installment (EMI)

An equated monthly instalment (EMI) is the fixed amount paid by a borrower every month to clear off the loan taken from a lender.

EMI depends on the following factors

(i) Principal amount

(ii) The loan tenure

(iii) The interest rate

EMI can be calculated by two methods

Flat Rate Method

In flat rate method the principal amount remains same throughout the tenure and the interest is charged on it at a constant rate throughout the loan tenure.

Suppose an amount of ₹ P is borrowed at flat rate of r per rupee per month for a period of n months.

Then, Interest $= P \times i \times n$, where $i = \dfrac{r}{100}$

$$\therefore \quad \text{EMI} = \frac{\text{Principal} + \text{Interest}}{n}$$

$$\Rightarrow \quad \text{EMI} = \frac{P + P \times i \times n}{n}$$

$$= P\left(\frac{1 + (i \times n)}{n}\right) = P\left(i + \frac{1}{n}\right)$$

Reducing Balance Method

In this method, EMI is calculated using the following formula

$$\text{EMI} = \frac{P \times i}{1 - (1 + i)^{-n}} \text{ or } \frac{P \times i(1 + i)^n}{(1 + i)^n - 1}$$

where, P = Principal borrowed

i = Rate of interest per month

n = Number of payments.

Amortization of Loans

Amortization refers to the process of paying off a loan through a scheduled, pre-determined sequence of equal payments that includes

(i) interest on the outstanding loan and

(ii) repayment of part of the loan i.e. principal.

A loan is said to be amortized if each installment is used to pay interest and the part of the principal.

Amortization Formulas

While amortizing a loan, one should know the principal outstanding at the beginning of any period, interest and principal amounts in any EMI, total interest paid etc. The formulas for the same are given below.

Principal outstanding at beginning of kth period

$$= \frac{\text{EMI}\ [(1 + i)^{n - k + 1} - 1]}{i(1 + i)^{n - k + 1}}$$

or $$\frac{\text{EMI}(1 - (1 + i)^{k - (n + 1)})}{i}$$

Interest paid in kth payment $= \dfrac{\text{EMI}[(1 + i)^{n - k + 1} - 1]}{(1 + i)^{n - k + 1}}$

or $$\text{EMI}(1 - (1 + i)^{k - (n + 1)})$$

Principal paid in kth payment

$$= \text{EMI} - \text{Interest paid in } k\text{th period}$$

Total interest paid $= n \times \text{EMI} - P$

Solved Examples

Example 1. What sum of money is needed to invest now, so as to get ₹ 6000 at the beginning of every month forever, if the money is worth 9% per annum compounded monthly?

Sol. Given, $R = ₹\ 6000$,

$$r = \frac{9}{12}\% = 0.75 \text{ per month}$$

and $$i = \frac{0.75}{100} = 0.0075$$

$$\therefore \quad P = R + \frac{R}{i}$$

$$= 6000 + \frac{6000}{0.0075}$$

$$= 6000 + 800000$$

$$= ₹\ 806000$$

Thus, ₹ 806000 are needed to invest to get ₹ 6000 at the beginning of every month forever.

Example 2. Sankar Sharma purchased a number of stocks of Tata power. At the end of first year, he received a payment of ₹ 10000, which grows at a rate of 5% per year and continues forever. If the discount rate is 9%, find the present value of Sankar Sharma's investment.

Sol. Given, $R = ₹\ 10000$, $i = 9\%$ and $g = 5\%$

Let P be the present value of Sankar Sharma's investment.

Then, $$P = \frac{R}{i - g}$$

$$= \frac{10000}{\frac{(9-5)}{100}}$$

$$= \frac{10000 \times 100}{4}$$

$$= ₹\ 250000$$

Example 3. A company anticipates a capital expenditure of ₹ 40000 for a new equipment in 7 yr. How much should be deposited quarterly in a sinking fund carrying 15% per annum compounded quarterly to provide for the purpose?

(given $(1.0375)^{28} = 2.80$)

Sol. Let R be deposited quarterly to accumulate ₹ 40000 in 7 yr.

Given, $S = ₹\ 40000$, $n = 7 \times 4 = 28$ yr

and $$i = \frac{15}{4 \times 100} = 0.0375$$

We know that,

$$R = \frac{i \times S}{(1+i)^n - 1}$$

$$= \frac{0.0375 \times 40000}{(1 + 0.0375)^{28} - 1}$$

$$= \frac{1500}{(1.0375)^{28} - 1} = \frac{1500}{(2.80 - 1)}$$

$$= \frac{1500}{1.80} = 833.33$$

Thus, the required amount is ₹ 833.33

Example 4. A bond that matures in 6 yr has coupon rate of 12% per annum and has a face value of ₹ 15000. Find the fair value of bond, if the yield to maturity is 10%.

Sol. Given, $F = ₹\ 15000$, $r = 12\%$

$N = 6$ yr and $d = 10\%$

$$\therefore \quad i = \frac{10}{100} = 0.1 \qquad \left[\because i = \frac{d}{100}\right]$$

Now, coupon payment, $C = 15000 \times \frac{12}{100}$

$$= ₹\ 1800$$

$$\therefore \quad \text{PV} = \frac{C[1 - (1+i)^{-N}]}{i} + F(1+i)^{-N}$$

$$= \frac{1800[1 - (1 + 0.1)^{-6}]}{0.1} + 1800(1 + 0.1)^{-6}$$

$$= \frac{1800[1 - (1.1)^{-6}]}{0.1} + 15000(1.1)^{-6}$$

$$= \frac{1800(1 - 0.56)}{0.1} + 15000(0.56)$$

$$= 18000\,(0.44) + 8400$$

$$= 7920 + 8400$$

$$= 16320$$

Hence, the fair value of bond is ₹ 16320.

Example 5. A bond of face value of ₹ 1200 has a coupon rate 10% per annum paid semi-annually and matures in 6 yr. If the present value of the bond is ₹ 1560, find the yield to maturity.

Sol. Given, $F = ₹\ 1200$,

$$r = 10\% = \frac{10}{2} = 5\% \text{ per half yearly}$$

$$\therefore \quad C = ₹\left(1200 \times \frac{5}{100}\right) = ₹\ 60$$

$N = 6$ yr $= 12$ half years

Since, PV = ₹ 1560

$$\text{Yield to maturity (YTM)} = \frac{C + \dfrac{(F - \text{PV})}{N}}{\dfrac{(F + \text{PV})}{2}}$$

$$= \frac{60 + \dfrac{(1200 - 1560)}{12}}{\dfrac{1200 + 1560}{2}}$$

$$= \frac{60 + (-30)}{2760 / 2} = \frac{60 - 30}{1380} = 0.0217$$

$\therefore$ Approximate yield to maturity (YTM) = 0.0217
or 2% per half year or 4% per annum.

Example 6. Neha Suman purchased a house from a company for ₹ 600000 and made a down payment of ₹ 200000. She repays the balance in 20 yr by monthly installments at 12% compounded monthly. (given $(1.01)^{-240} = 0.092$).

(i) What are monthly payments?

(ii) What is the total interest payment?

Sol. Given, $P = ₹\,(600000 - 200000) = ₹\,400000$,

$$n = 20 \times 12 = 240$$

and $$i = \frac{12}{12 \times 100} = \frac{12}{1200} = 0.01$$

(i) $\therefore$ The monthly installment (EMI) $= \dfrac{P \times i}{1 - (1 + i)^{-n}}$

$$= \frac{400000 \times 0.01}{1 - (1 + 0.01)^{-240}}$$

$$= \frac{4000}{1 - (1.01)^{-240}}$$

$$= \frac{4000}{1 - 0.092}$$

$$= \frac{4000}{0.908}$$

$$= ₹\,4405.29$$

(ii) $\therefore$ Total interest $= n \times \text{EMI} - P$

$$= 240 \times 4405.29 - 400000$$

$$= 1057269.6 - 400000$$

$$= ₹\,657269.6$$

Chapter Practice

PART 1

Objective Questions

• Multiple Choice Questions

1. The present value of a perpetuity of ₹ 5000 payable at the end of each year, if money is worth 5% compounded annually is

(a) ₹ 20000 (b) ₹ 100000
(c) ₹ 10000 (d) ₹ 25000

2. The rate of interest will the present value of a perpetuity of ₹ 500 payable at the end of every 6 months be ₹ 10000 is

(a) 5% (b) 8%
(c) 10% (d) 4%

3. The present value of a perpetuity of ₹ 3120 payable at the beginning of each year, if money is worth 6% effective is

(a) ₹ 55120 (b) ₹ 56120
(c) ₹ 52120 (d) ₹ 52000

4. Ashwani holds a perpetual bond that generates an annual of ₹ 50000 each year. He believes that the borrower is credit worthy and that an 8% interest rate will be suitable for this bond. Then, present value of this perpetuity is

(a) ₹ 625000 (b) ₹ 400000
(c) ₹ 600000 (d) ₹ 62500

5. Simran purchased a number of stocks of Amazon. At the end of first year, she received a payment of ₹ 8000, which grows at a rate of 3% per year and continues forever. If the discount rate is 7%. Then, the present value of Simran's investment is

(a) ₹ 32000 (b) ₹ 20000
(c) ₹ 200000 (d) ₹ 40000

6. A person has set up a sinking fund in order to have ₹ 100000 after 10 yr for his children's college education. How much amount should be set aside bi-annually into an account paying 5% per annum compounded half-yearly? (given $(1.025)^{20} = 1.6386$)

(a) ₹ 3914.81 (b) ₹ 2914.81
(c) ₹ 3614.81 (d) ₹ 3814.81

7. A machine costing ₹ 200000 has effective life of 7 yr and its scrap value is ₹ 30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost ₹ 300000 after 7 yr (given $(1.05)^7 = 1.4071$)

(a) ₹ 32169.53 (b) ₹ 34169.53
(c) ₹ 33161.38 (d) ₹ 36169.53

8. A company intends to create a sinking fund to replace at the end of 20th year assets costing ₹ 50000. Calculate the amount to be retained out of profit every year, if the interest rate is 5% (given $(1.05)^{20} = 2.6532$)

(a) ₹ 15122.18 (b) ₹ 15322.18
(c) ₹ 15422.18 (d) ₹ 15022.18

9. A bond has a face value of ₹ 1000, matures in 4 yr. Coupon rate is 4% per annum. The bond makes annual payments. If the yield to maturity is 4%, then the fair value of bond is (given $(1.04)^{-4} = 0.8551$)

(a) ₹ 1500 (b) ₹ 1000
(c) ₹ 1200 (d) ₹ 1600

10. A bond of face value of ₹ 1000 matures in 5 yr. Interest is paid semi-annually and bond is priced to yield 8% per annum. If the present value of bond is ₹ 800, then the annual coupon rate is (given $(1.04)^{-10} = 0.6761$)

(a) 3.06% (b) 4%
(c) 4.2% (d) None of these

11. A company ABC Ltd has issued a bond having a face value of ₹ 10000 paying annual dividend at 8%. The bond will be redeemed at par at the end of 10 yr. The purchase price of this bond, if the investor wishes a yield rate of 8% is (given $(1.08)^{-10} = 0.4632$)

(a) ₹ 8000 (b) ₹ 12000
(c) ₹ 10000 (d) ₹ 9000

12. A sinking fund is created for the redemption of debentures of ₹ 100000 at the end of 25 yr. How much money should be provided out of profits each year for the sinking fund, if the investment can earn interest 4% per annum (given $(1.04)^{25} = 2.6658$)

(a) ₹ 2401.25 (b) ₹ 2501.25
(c) ₹ 2500 (d) ₹ 2700

13. A bond of face value ₹ 1000 has a coupon rate of 10% per annum paid semi-annually and mature in 4 yr, if the present value of the bond is ₹ 1140, then the yield to maturity is

(a) 5% (b) 6% (c) 4% (d) 8%

14. A bond of face value ₹ 1000 has a coupon rate of 6% per annum with interest paid semi-annually and matures in 5 yr. If the bond is priced to yield 8% per annum, then the value of the bond is (given $(1.04)^{-10} = 0.6761$)

(a) ₹ 919.03 (b) ₹ 802.03
(c) ₹ 719.03 (d) ₹ 1019.03

15. A bond of face value of ₹ 500 mature in 3 yr. Interest is paid half-yearly and bond is price to yield 8% annually. If the present value of bond is ₹ 450, then annual coupon rate is

(a) less than 10% (b) equal to 10%
(c) more than 10% (d) None of these

16. Manish takes a loan of ₹ 300000 at an interest of 10% compounded annually for a period of 3 yr, then EMI by using flat rate method is

(a) ₹ 1083.33 (b) ₹ 1073.33
(c) ₹ 1093.33 (d) ₹ 1063.33

17. Mr. Ahuja borrowed ₹ 100000 from a bank to purchase a car and decided to repay the loan by equal monthly installments in 10 yr. If bank charges interest at 9% per annum compounded monthly, then the value of EMI is (given $(1.0075)^{120} = 2.4514$)

(a) ₹ 1250
(b) ₹ 1266.74
(c) ₹ 1300
(d) None of the above

18. A person buys a house for which he agrees to pay ₹ 5000 at the end of each month for 8 yr. If money is worth 12% converted monthly. Then, the cash price of house is (given $(1.01)^{-96} = 0.3847$)

(a) ₹ 307650 (b) ₹ 407650
(c) ₹ 507650 (d) None of these

19. Mr. Malik borrowed ₹ 500000 from a bank to purchase a house and decided to repay the loan by equal monthly payment in 10 yr. If bank charges interest at 7.5% per annum compounded monthly, then EMI is (given $(1.00625)^{120} = 2.1121$)

(a) ₹ 5935 (b) ₹ 6380
(c) ₹ 7340 (d) ₹ 8520

• Case Based MCQs

20. Aakriti invest in bond of face value of ₹ 1000 has a coupon rate of 8% per annum with interest paid semi-annually and mature in 5 yr.

Based on above information, answer the following questions.

(i) If discount rate is 6% per annum, then present value of bond is
(a) greater than 1000
(b) less than 1000
(c) equal to 1000
(d) None of the above

(ii) If discount rate is 8% per annum, then present value of bond is
(a) ₹ 1000 (b) ₹ 900
(c) ₹ 1200 (d) ₹ 1100

(iii) If discount rate is 10% per annum, then present value of bond is
(a) greater than 1000 (b) less than 1000
(c) equal to 1000 (d) None of these

(iv) If the present value of bond is ₹ 1000, then discount rate is
(a) 6% (b) 8%
(c) 10% (d) 5%

(v) If the present value of bond is ₹ 1100, then discount rate is
(a) less than 8% (b) greater than 8%
(c) equal to 8% (d) None of these

21. In year 2018, Mr. Verma took a loan of ₹ 250000 at the interest of 6% per annum compounded monthly is to be amortized by equal payment at the end of each of 5 yr.

[given $(1.005)^{60} = 1.3489$ and $(1.005)^{21} = 1.1104$]

Based on the above information, answer the following questions

(i) The number of payment is
(a) 10 (b) 12
(c) 30 (d) 60

(ii) The EMI is calculated using formula

(a) $\text{EMI} = \frac{Pi(1+i)^{-n}}{(1+i)^{-n}-1}$ (b) $\text{EMI} = \frac{P(1+i)^{-n}}{i}$

(c) $\text{EMI} = \frac{Pi(1+i)^{n}}{(1+i)^{n}-1}$ (d) $\text{EMI} = \frac{Pi(1+i)^{n}}{i}$

(iii) The size of each monthly payment is

(a) ₹ 4832.69 (b) ₹ 4932.69

(c) ₹ 5000 (d) ₹ 4700

(iv) The interest paid in 40th payment is

(a) ₹ 500 (b) ₹ 600

(c) ₹ 480.48 (d) ₹490.48

(v) The total interest paid by Mr. Verma is

(a) ₹ 40000 (b) ₹ 50000

(c) ₹ 39961.40 (d) ₹ 40961.40

PART 2

Subjective Questions

• Short Answer Type Questions

1. Find the present value of a sequence of payments of ₹ 2000 made at the end of every 6 months and continuing forever. If money is worth 5% per annum compounded semi-annually.

2. At what rate of interest will the present value of ₹ 300 payable at the end of each quarter be ₹ 24000?

3. At 6% converted quarterly, find the present value of a perpetuity of ₹ 45000 payable at the end of each quarter.

4. What sum of money is needed to invest now, as to get ₹ 10000 at the beginning of every month forever, if the money is worth 8% per annum compounded quarterly?

5. The present value of a perpetual income of ₹ R at the end of each 6 months is ₹ 42000. Find the value of R, if money is worth 6% compounded semi-annually.

6. A company intends to create a sinking fund to replace at the end of 20th year assets costing ₹ 500000. Calculate the amount to be retained out of profits every year, if the interest rate is 5% per annum. (given $(1.05)^{20} = 2.6532$)

7. A firm anticipates a capital expenditure of ₹ 80000 for a new equipment in 5 yr. How much should be deposited quarterly in sinking fund carrying 12% per annum compounded quarterly to provide for the purchase? (given $(1.03)^{20} = 1.8061$)

8. A company establish a sinking fund to provide for payment of ₹ 250000 debt, maturing in 4 yr. Contributions to the fund are made at the end of every year. Find the amount of each annual deposit, if the interest is 18% per annum.
(given $(1.18)^{4} = 1.9387$).

9. A bond has face value of ₹ 10000 and maturity period of 10 yr. The nominal interest rate is 6% per annum. What should be the price of the bond to yield an effective interest of 8%?
(given $(1.08)^{-10} = 0.4631$)

10. A bond of face value of ₹ 1000 matures in 5 yr. Interest is paid semi-annually and bond is priced to yield 8% per annum. If the present value of bond is ₹ 1100, find the annual coupon rate.
(given $(1.04)^{-10} = 0.6761$)

11. Face value of bond is ₹ 1000, coupon rate 4.25% per annum paid semi-annually and matures in 10 yr. If present value of bond is ₹ 918.23. What is the yield to maturity.

12. A coupon bond has a ₹ 1000 face value and provides 10.5% semi-annual coupon for 14 yr. The discount rate is 9% per annum. What is the value of the coupon bond. (given $(1.045)^{-28} = 0.2915$)

13. Mr. Taneja purchase a house of ₹ 4500000 with a down payment of ₹ 500000 and balance in EMI for 25 yr. If bank charges 6% per annum compounded monthly. Calculate the EMI.
(given $(1.005)^{300} = 4.4650$).

14. A person buys a house for which be agrees to pay ₹ 25000 at the end of each month for 8 yr. If money is worth 12% converted monthly, what is the cash price of house? (given $(1.01)^{-96} = 0.3847$)

15. Mr. Kailash has taken a personal loan of ₹ 200000 for 2 yr at an interest rate of 20% per annum, which is to be paid back in equal monthly installments. How much monthly installment Mr. Kailash will pay? $\left(\text{given}\left(\frac{61}{60}\right)^{-24} = 0.6725\right)$.

• Long Answer Type Questions

16. A machine costs a company ₹ 52000 and its effective life is estimated to be 25 yr. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of ₹ 2500 only. The price of the new model is estimated to be 25% more than the price of present one. Find what amount should be set aside at the end of each year out of the profit for the sinking fund, if it accumulates at 3.5% compound annually? (given $(1.035)^{25} = 2.3632$)

17. A machine is bought for ₹ 320000. Its effective life is 8 yr, after which its salvage value would be ₹ 25000. It is decided to create a sinking fund to replace this machine at the end of its effective life by making half yearly payments that will earn an interest of 8% per annum compounded half yearly. If it is known that the cost of machine increases by 5% per annum. Calculate the amount of each payment to the sinking fund.
[given $(1.04)^{16} = 1.8730$ and $(1.05)^{8} = 1.4774$]

18. Find the purchase price of a ₹ 20000 bond, redeemable at the end of 10 yr at 110 and paying annual dividends at 4%, if the yield rate is to be 5% effective. (given $(1.05)^{-10} = 0.6139$)

19. Rajesh purchased a house from a company for ₹ 2500000 and made a down payment of ₹ 500000. He repays the balance in 25 yr by monthly instalments at the rate of 9% per annum compounded monthly
(i) What are the monthly payment?
(ii) What is the total interest payment?
(given $(1.0075)^{-300} = 0.1062$)

20. A loan of ₹ 400000 at the interest rate of 6.75% per annum compounded monthly is to be amortized by equal payment at the end of each month for 10 yr, find
(i) the size of each monthly payment
(ii) the principal outstanding at the beginning of 61st month.
[given $(1.005625)^{120} = 1.9603$ and $(1.005625)^{60} = 1.4001$]

SOLUTIONS

Objective Questions

1. (b) We know that,

$$P = \frac{R}{i}$$

Here, $R =$ ₹ 5000 and $i = 5\%$

$$\therefore \quad P = \frac{5000}{5\%} = \frac{5000 \times 100}{5} = 100000$$

$\therefore$ Present value = ₹ 100000

2. (c) Here, $P =$ ₹ 10000 and $R =$ ₹ 500

Let rate of interest be $r\%$ per annum

$$\therefore \quad i = \left(\frac{r}{2}\right)\% \text{ (compounded semi-annually)}$$

$$\therefore \quad i = \frac{R}{P}$$

$$\Rightarrow \quad \frac{r}{200} = \frac{500}{10000}$$

$$\Rightarrow \quad r = 10\%$$

3. (a) Here, $R =$ ₹ 3120 and $i = \frac{6}{100}$

$$\therefore \quad P = R + \frac{R}{i} = 3120 + \frac{3120 \times 100}{6}$$

$$= 3120 + 52000 = ₹\ 55120$$

4. (a) Here, $R =$ ₹ 50000 and $i = 8\%$

$$\therefore \quad P = \frac{R}{i} = \frac{50000 \times 100}{8} = 625000$$

$\therefore$ Present value = ₹ 625000

5. (c) Here, $R =$ ₹ 8000, $i = 7\%$ and growth of rate $(g) = 3\%$

$$\therefore \quad P = \frac{R}{i-g} = \frac{8000}{(7-3)\%} = \frac{8000 \times 100}{4} = 200000$$

$\therefore$ Present value = ₹ 200000

6. (a) Let ₹ R be set a side bi-annually for 10 yr in order to have ₹ 100000 after 10 yr.

Then, $S =$ ₹ 100000, $n = 10 \times 2 = 20$ yr

and $\quad i = \frac{5}{200} = 0.025$

$$\therefore \quad R = \frac{i \times S}{(1+i)^n - 1} = \frac{0.025 \times 100000}{(1.025)^{20} - 1}$$

$$= \frac{2500}{1.6386 - 1}$$

$$= \frac{2500}{0.6386} = ₹\ 3914.81$$

7. (c) Cost of new machine = ₹ 300000

Scrap value of old machine = ₹ 30000

Hence, the money required for new machine after 7 yr

$$= 300000 - 30000$$

$$= ₹\ 270000$$

Here, $S =$ ₹ 270000, $n = 7$ yr and $i = 5\% = \frac{5}{100} = 0.05$

$$\therefore \quad R = \frac{i \times S}{(1+i)^n - 1} = \frac{0.05 \times 270000}{(1.05)^7 - 1}$$

$$= \frac{13500}{1.4071 - 1} = \frac{13500}{0.4071} = ₹\ 33161.38$$

8. (*a*) Here, $S = ₹\ 50000$, $i = 5\% = 0.05$ and $n = 20$ yr

$$\therefore \quad R = \frac{i \times S}{(1+i)^n - 1} = \frac{0.05 \times 50000}{(1.05)^{20} - 1}$$

$$= \frac{25000}{2.6532 - 1} = \frac{25000}{1.6532}$$

$$= ₹\ 15122.18$$

9. (*b*) Here, $F = ₹\ 1000$, $r = 4\%$, $N = 4$ yr and $d = 4\%$

$$\Rightarrow \quad i = \frac{4}{100} = 0.04 \qquad \left[\because i = \frac{d}{100}\right]$$

So, Coupon payment, $C = 1000 \times \frac{4}{100} = ₹\ 40$

$$\text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$$

$$= \frac{40(1-(1.04)^{-4})}{0.04} + 1000(1.04)^{-4}$$

$$= \frac{40(1-0.8551)}{0.04} + 1000(0.8551)$$

$$= \frac{40(0.1449)}{0.04} + 855.10$$

$$= 144.90 + 855.10 = ₹\ 1000$$

10. (*a*) Let the annual coupon rate is $r\%$.

Here, $F = ₹\ 1000$, then $C = 1000 \times \frac{r}{200} = 5r$

$$d = 8\% \text{ or } i = \frac{8}{200} = 0.04 \text{ (semi-annually)}$$

$$N = 5 \times 2 = 10 \text{ yr and PV} = ₹\ 800$$

$$800 = \frac{5r(1-(1.04)^{-10})}{0.04} + 1000(1.04)^{-10}$$

$$800 = \frac{5r(1-0.6761)}{0.04} + 1000(0.6761)$$

$$\Rightarrow \quad \frac{5r(0.3239)}{0.04} = 800 - 676.10 = 123.90$$

$$r = \frac{123.90 \times 0.04}{5 \times (0.3239)} = 3.06\%$$

11. (*c*) Here, $F = ₹\ 10000$, $N = 10$ yr, $i = 8\% = 0.08$

and $\quad C = \frac{8}{100} \times 10000 = ₹\ 800$

$$\text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$$

$$= \frac{800(1-(1.08)^{-10})}{0.08} + 10000(1.08)^{-10}$$

$$= \frac{800(1-0.4632)}{0.08} + 10000(0.4632)$$

$$= 10000(0.5368) + 4632$$

$$= 5368 + 4632 = ₹\ 10000$$

12. (*a*) Here, $S = ₹\ 100000$, $n = 25$ yr and $i = 4\% = 0.04$

$$\therefore \quad R = \frac{i \times S}{(1+i)^n - 1} = \frac{0.04 \times 100000}{(1.04)^{25} - 1} = \frac{4000}{2.6658 - 1}$$

$$= \frac{4000}{1.6658}$$

$$= ₹\ 2401.25$$

13. (*b*) Here, $F = ₹\ 1000$, $r = 10\%$

$$r = \frac{10}{2} = 5\% = 0.05 \text{ (semi-annually)}$$

and $\quad C = 1000 \times \frac{5}{100} = ₹\ 50$

$N = 8$ yr

PV = ₹ 1140

Approximate YTM

$$= \frac{C + \left(\frac{F - \text{PV}}{N}\right)}{\frac{F + \text{PV}}{2}} = \frac{50 + \left(\frac{1000 - 1140}{8}\right)}{\frac{1000 + 1140}{2}}$$

$$= \frac{50 - 17.5}{1070} = 0.0304$$

Approximate YTM = 3% half yearly = 6% per annum

14. (*a*) Here, $F = ₹\ 1000$, $r = \frac{6}{2} = 3\%$ (semi- annually)

$$N = 5 \times 2 = 10 \text{ yr and } d = \frac{8}{2} = 4\%$$

or $\quad i = \frac{4}{100} = 0.04 \qquad \left[\because i = \frac{d}{100}\right]$

So, $\quad C = 1000 \times \frac{3}{100} = ₹\ 30$

$$\therefore \quad \text{PV} = \frac{C(1-(1+i)^{-N}}{i} + F(1+i)^{-N}$$

$$= \frac{30(1-(1.04)^{-10})}{0.04} + 1000 \times (1.04)^{-10}$$

$$= \frac{30(1-0.6761)}{0.04} + 1000 \times 0.6761$$

$$= 242.93 + 676.10 = ₹\ 919.03$$

15. (*a*) Let annual coupon rate is $r\%$

Here, $F = ₹\ 500$, then $C = 500 \times \frac{r}{200} = \frac{5}{2}r$

$$i = \frac{8}{2} = 4\% = 0.04,\ N = 3 \times 2 = 6 \text{ yr and PV} = ₹\ 450$$

$$\therefore \quad \text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$$

$$450 = \frac{5r}{2}\frac{(1-(1.04)^{-6})}{0.04} + 500(1.04)^{-6}$$

$$450 = \frac{5r}{0.08}(1 - 0.7903) + 500(0.7903)$$

$$\frac{500}{8}(0.2097)r = 450 - 395.15 = 54.85$$

$$r = \frac{54.85 \times 8}{500 \times 0.2097} = 4.18$$

Hence, coupon rate is 4.18%, which is less than 10%.

16. (*a*) We have, P = Principal = ₹ 30000

$$i = \frac{10}{1200} = \frac{1}{120} \text{ and } n = 12 \times 3 = 36$$

$$\text{EMI} = P\left(i + \frac{1}{n}\right) = 30000\left(\frac{1}{120} + \frac{1}{36}\right)$$

$$= 250 + 833.33 = ₹\ 1083.33$$

17. (*b*) Given, $P = ₹\ 100000$, $i = \dfrac{9}{12 \times 100} = 0.0075$

and $n = 12 \times 10 = 120$

So, $\text{EMI} = \dfrac{P \times i \times (1+i)^n}{(1+i)^n - 1}$

$= \dfrac{100000 \times 0.0075 \times (1.0075)^{120}}{(1.0075)^{120} - 1}$

$= \dfrac{750 \times 2.4514}{2.4514 - 1} = ₹\ 1266.74$

18. (*a*) Here, EMI = ₹ 5000, $n = 12 \times 8 = 96$ and $i = \dfrac{12}{1200} = 0.01$

$\therefore \quad \text{EMI} = \dfrac{P \times i}{1 - (1+i)^{-n}}$

$5000 = \dfrac{P(0.01)}{1 - (1.01)^{-96}}$

$\Rightarrow \quad P = \dfrac{5000(1 - 0.3847)}{0.01} = 50000 \times 0.6153$

$= ₹\ 307650$

19. (*a*) Here, $P = ₹\ 500000$, $i = \dfrac{7.5}{1200} = 0.00625$

and $n = 12 \times 10 = 120$

$\text{EMI} = \dfrac{P \times i(1+i)^n}{(1+i)^n - 1}$

$\text{EMI} = \dfrac{500000 \times 0.00625 \times (1.00625)^{120}}{(1.00625)^{120} - 1}$

$= \dfrac{3125 \times 2.1121}{2.1121 - 1} = \dfrac{6600.3125}{1.1121} = ₹\ 5934.99$

$= ₹\ 5935$ (approx.)

20. (i) (*a*) Discount rate < Coupon rate

$\Rightarrow$ Present value > Face value

$\Rightarrow$ Present value > 1000

(ii) (*a*) Discount rate = Coupon rate

$\Rightarrow$ Present value = Face value

$\Rightarrow$ Present value =1000

(iii) (*b*) Discount rate > Coupon rate

$\Rightarrow$ Present value < Face value

$\Rightarrow$ Present value < 1000

(iv) (*b*) Present value =1000

Here, Present value = Face value

$\therefore$ Discount rate = Coupon rate =8%

(v) (*a*) Present value =1100

Here, Present value > Face value

$\therefore$ Discount rate < Coupon rate

$\therefore$ Discount rate is less than 8%.

21. (i) (*d*) Here, Time = 5 yr

$\therefore$ Total number of payment = $5 \times 12 = 60$

(ii) (*c*) $\text{EMI} = \dfrac{P \times i(1+i)^n}{(1+i)^n - 1}$

(iii) (*a*) Given, $P = ₹\ 250000$,

$i = \dfrac{6}{1200} = 0.005$ and $n = 60$

$\text{EMI} = \dfrac{250000 \times 0.005 (1.005)^{60}}{(1.005)^{60} - 1}$ [from (ii)]

$= \dfrac{250000 \times 0.005 \times 1.3489}{1.3489 - 1} = ₹\ 4832.69$

(iv) (*c*) We know that, interest paid in *k*th payment

$= \dfrac{\text{EMI}\,[(1+i)^{n-k+1} - 1]}{(1+i)^{n-k+1}}$

So, interest paid in 40th payment

$= \dfrac{\text{EMI}[(1+i)^{60-40+1} - 1]}{(1+i)^{60-40+1}}$

$= \dfrac{4832.69 \times ((1.005)^{21} - 1)}{(1.005)^{21}}$

$= \dfrac{4832.69 \times (1.1104 - 1)}{1.1104}$

$= \dfrac{4832.69 \times 0.1104}{1.1104} = ₹\ 480.48$

(v) (*c*) Total interest paid by Mr. Verma

$\therefore$ Total interest = $n \times \text{EMI} - P$

$= 60 \times (4832.69) - 250000$

$= 289961.40 - 250000$

$= ₹\ 39961.40$

Subjective Questions

1. The given annuity is perpetuity of first type in which

$R = ₹\ 2000$ and $r = \dfrac{5}{2}\%$ (semi-annually)

So, $i = \dfrac{5}{2}\% = \dfrac{5}{200}$

We know that,

$P = \dfrac{R}{i} = \dfrac{2000}{5/200} = \dfrac{2000 \times 200}{5} = ₹\ 80000$

Hence, the present value is ₹ 80000.

2. Let the rate of interest be $r\%$ per annum.

Then, $i = \dfrac{r}{400}$ (quarterly)

Here, $R = ₹\ 300$ and $P = ₹\ 24000$

We know that, $P = \dfrac{R}{i} \Rightarrow i = \dfrac{R}{P}$

$\Rightarrow \quad \dfrac{r}{400} = \dfrac{300}{24000}$

$\Rightarrow \quad r = \dfrac{300 \times 400}{24000} = 5\%$

Hence, rate of interest is 5% per annum.

3. Let P be the present value of perpetuity.

Given, $R = ₹\ 45000$ and $i = \dfrac{6}{4}\% = \dfrac{6}{400}$

$\therefore P = \dfrac{R}{i} = \dfrac{45000}{\frac{6}{400}} = \dfrac{45000 \times 400}{6} = ₹\ 3000000$

Hence, the present value is ₹ 3000000.

4. Given, $R = ₹\ 10000$ and $r = \frac{8}{4}\% = 2\% = 0.02$

So, $i = 0.02$

$\therefore \quad P = R + \frac{R}{i} = 10000 + \frac{10000}{0.02}$

$= 10000 + 500000$

$= ₹\ 510000$

Hence, ₹ 510000 are needed to invest now to get ₹ 10000 at the beginning of every quarter forever.

5. Given, $P = ₹\ 42000$ and $i = \frac{6}{200} = 0.03$

$\therefore \quad R = P \times i = 42000 \times 0.03 = ₹\ 1260$

Hence, $R = ₹\ 1260$

6. Given, $S = ₹\ 500000$ and $i = \frac{5}{100} = 0.05$ and $n = 20$ yr

We know that,

$$R = \frac{i \times S}{(1+i)^n - 1}$$

$$= \frac{(0.05)\,(500000)}{(1+0.05)^{20} - 1} = \frac{(0.05)(500000)}{(1.05)^2 - 1}$$

$$= \frac{25000}{2.6532 - 1} = \frac{25000}{1.6532}$$

$= ₹\ 15122.18$

Thus, ₹ 15122.18 are retained out of profits every year for 20 yr to accumulate ₹ 500000.

7. Given, $S = ₹\ 80000$, $i = \frac{12}{400} = 0.03$ and $n = 5 \times 4 = 20$ yr

$\therefore \quad R = \frac{i \times S}{(1+i)^n - 1} = \frac{(0.03)(80000)}{(1+0.03)^{20} - 1}$

$= \frac{2400}{(1.03)^{20} - 1}$

$= \frac{2400}{1.8061 - 1} = \frac{2400}{0.8061}$

$= ₹\ 2977.29$

Thus, ₹ 2977.29 deposited quarterly in a sinking fund.

8. Here, $S = ₹\ 250000$, $n = 4$ yr and $i = \frac{18}{100} = 0.18$

Now, $R = \frac{i \times S}{(1+i)^n - 1} = \frac{(0.18) \times (250000)}{(1+0.18)^4 - 1}$

$= \frac{45000}{(1.18)^4 - 1} = \frac{45000}{1.9387 - 1} = \frac{45000}{0.9387}$

$= ₹\ 47938.64$

Hence, the required amount is ₹ 47938.64.

9. We have, F = Face value of bond = ₹ 10000

N = Number of period = 10

$C = \frac{8}{100} = 0.08$

C = Coupon payment = Annual dividend $\times$ 10000

$= \frac{6}{100} \times 10000 = ₹\ 600$

F = Maturity value = Face value = ₹ 10000

Let PV be the price of the bond.

Then, $\text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$

$$\text{PV} = 600\,\frac{(1-(1.08)^{-10})}{0.08} + 10000(1.08)^{-10}$$

$$= 600\,\frac{(1-0.4631)}{0.08} + 10000(0.4631)$$

$$= \frac{600 \times 0.5369}{0.08} + 4631$$

$$= \frac{322.14}{0.08} + 4631 = 4026.75 + 4631 = ₹\ 8657.75$$

Hence, the price of bond is ₹ 8657.75.

10. Let the annual coupon rate is 6%

Given, $F = ₹\ 1000$, then $C = 1000 \times \frac{r}{200} = ₹\ 5r$

$d = 8\%$ per annum or 4% per half year

$i = \frac{4}{100} = 0.04$

$N = 5 \times 2 = 10$ yr and PV = ₹ 1100

$\therefore \quad \text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$

$$1100 = \frac{5r(1-(1.04)^{-10})}{0.04} + 1000(1.04)^{-10}$$

$$1100 = \frac{5r(1-0.6761)}{0.04} + 1000(0.6761)$$

$$1100 = \frac{5r \times 0.3239}{0.04} + 676.10$$

$$r = \frac{0.04\,(1100 - 676.10)}{5 \times 0.3239} = \frac{0.04 \times 423.90}{5 \times 0.3239}$$

$r = 10.47$

Hence, coupon rate is 10.47%.

11. Given, $F = ₹\ 1000$, $r = 4.25\%$

and $i = \frac{4.25}{2}\% = \frac{4.25}{200}$ (semi-annually)

So, $C = ₹\ 1000 \times \frac{4.25}{200} = ₹\ 21.25$

$N = 10 \times 2 = 20$ yr and PV = ₹ 918.23

$$\text{Approximate Yield to Maturity (YTM)} = \frac{C + \frac{(F - \text{PV})}{N}}{\frac{F + \text{PV}}{2}}$$

$$= \frac{21.25 + \frac{1000 - 918.23}{20}}{\frac{1000 + 918.23}{2}}$$

$$= \frac{21.25 + 4.0885}{959.115} = \frac{25.3385}{959.115} = 0.0264$$

Approximate YTM = 2.64% semi-annually = 5.28% per annum

Hence, yield to maturity is 5.28 per annum.

12. Given, $F = ₹\ 1000$, $C = 1000 \times \frac{10.5}{200} = ₹\ 52.5$ and $d = 9\%$

So, $i = \frac{9}{200} = 0.045$ (semi-annually)

and $N = 14 \times 2 = 28$ yr

$$\text{PV} = \frac{C(1-(1+i)^{-N})}{i} + F(1+i)^{-N}$$

$$= 52.5\,\frac{(1-(1.045)^{-28})}{0.045} + 1000(1.045)^{-28}$$

$$= 52.5\,\frac{(1-0.2915)}{0.045} + 1000\,(0.2915)$$

$$= \frac{52.5 \times 0.7085}{0.045} + 291.50$$

$$= 826.58 + 291.50 = ₹\ 1118.08$$

Hence, the value of coupon bond is ₹ 1118.08.

13. Cost of house = ₹ 4500000

Down payment = ₹ 500000

$\therefore$ Balance = ₹ (4500000 − 500000) = ₹ 4000000

So, P = ₹ 4000000, $i = \frac{6}{1200} = 0.005$

and $n = 25 \times 12 = 300$

$$\therefore \text{EMI} = \frac{P \times i(1+i)^n}{(1+i)^n - 1}$$

$$= \frac{4000000 \times 0.005 \times (1.005)^{300}}{(1.005)^{300} - 1}$$

$$= \frac{20000 \times 4.4650}{4.4650 - 1} = \frac{89300}{3.4650} = ₹\ 25772$$

Hence, EMI of Mr. Taneja is ₹ 25772.

14. Given, EMI = ₹ 25000, $n = 12 \times 8 = 96$ and $i = \frac{12}{1200} = 0.01$

Let P be the cash price of house.

Then, $$P = \frac{\text{EMI}(1-(1+i)^{-n})}{i}$$

$$= \frac{25000\,(1-(1.01)^{-96})}{0.01}$$

$$= \frac{25000(1-0.3847)}{0.01}$$

$$= \frac{25000 \times 0.6153}{0.01} = ₹\ 1538250$$

Hence, the cash price of house is ₹ 1538250.

15. Given, P = ₹ 200000 $n = 12 \times 2 = 24$ and $i = \frac{20}{1200} = \frac{1}{60}$

$$\therefore \quad \text{EMI} = \frac{P \times i}{1-(1+i)^{-n}} = \frac{200000 \times \frac{1}{60}}{1-\left(1+\frac{1}{60}\right)^{-24}}$$

$$= \frac{200000}{60\left(1-\left(\frac{61}{60}\right)^{-24}\right)} = \frac{200000}{60(1-0.6725)}$$

$$= \frac{200000}{60 \times 0.3275} = ₹\ 10178$$

Hence, EMI of Mr. Kailash is ₹ 10178.

16. Let ₹ R be the amount set aside each year.

Since, the cost of new machine is 25% more than the cost of present.

Cost of machine = ₹ 52000

Cost of new machine after increasing the 25% of the cost of machine,

i.e. 52000 + 25% of 52000

$$= 52000\left(1+\frac{25}{100}\right)$$

$$= 52000 \times \frac{5}{4} = ₹\ 65000$$

Scrap value of the present machine = ₹ 2500

So, net amount required at the end of 25 yr to purchase the new model = ₹ (65000 − 2500) = ₹ 62500

We know that,

$$R = \frac{i \times S}{(1+i)^n - 1}$$

Here, S = ₹ 62500, n = 25 yr and $i = \frac{3.5}{100} = 0.035$

$$\Rightarrow \quad R = \frac{(0.035) \times (62500)}{(1+0.035)^{25} - 1}$$

$$= \frac{2187.5}{(1.035)^{25} - 1}$$

$$= \frac{2187.5}{2.3632 - 1}$$

$$= \frac{2187.5}{1.3632} = ₹\ 1604.68$$

Thus, ₹ 1604.68 are set aside each year out of the profit to purchase the new model of the machine.

17. Let each semi-annually deposit in the sinking fund of ₹ R. Since, the cost of new machine is increases by 5% per annum the cost of present.

Cost of machine at present = ₹ 320000

Cost of machine after increasing 5% per annum after 8 yr

$$= 320000\left(1+\frac{5}{100}\right)^8 = 32000\,(1.05)^8$$

$$= 320000 \times 1.4774 = ₹\ 472768$$

Salvage value of present machine = ₹ 25000

So, net amount required at the end of 8 yr to purchase the new model is ₹ (472768 − 25000) = ₹ 447768

We know that, $R = \frac{i \times S}{(1+i)^n - 1}$

Here, S = ₹ 447768, $n = 8 \times 2 = 16$ yr and $i = \frac{8}{200} = 0.04$

$$\therefore \quad R = \frac{(0.04) \times (447768)}{(1+0.04)^{16} - 1}$$

$$= \frac{17910.72}{(1.04)^{16} - 1}$$

$$= \frac{17910.72}{1.8730 - 1}$$

$$= \frac{17910.72}{0.8730} = ₹\ 20516.28$$

Thus, ₹ 20516.28 deposited half yearly out of the profit to purchase the new model of the machine.

18. We have,

n = Number of period = 10, i = yield rate = $\frac{5}{100} = 0.05$

R = Annual dividend = 4% of face value = ₹ $\left(\frac{4}{100} \times 20000\right)$

= ₹ 800

The bond is redeemed at 110.

Therefore, the redemption price of the bond is 110% of its face value.

Thus, C = Redemption price = ₹ $\left(20000 \times \frac{110}{100}\right)$ = ₹ 22000

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C(1+i)^{-n}$$

$$\Rightarrow V = ₹\left[800\left\{\frac{1-(1+0.05)^{-10}}{0.05}\right\} + 22000(1+0.05)^{-10}\right]$$

$$= ₹\,[16000\{1-(1.05)^{-10}\} + 22000(1.05)^{-10}]$$

$$= ₹\,[16000(1-0.6139) + 22000(0.6139)]$$

$$= ₹\,[16000(0.3861) + 13505.80]$$

$$= ₹\,(6177.60 + 13505.80)$$

$$= ₹\,19683.40$$

∴ The purchase price of bond is ₹ 19683.40

19. Cost of house = ₹ 2500000

Down payment = ₹ 500000

∴ Principal amount = ₹ (2500000 − 500000) = ₹ 2000000,

$$n = 25 \times 12 = 300$$

and $$i = \frac{9}{1200} = 0.0075$$

(i) We know that,

$$\text{EMI} = \frac{P \times i}{1-(1+i)^{-n}}$$

$$= \frac{2000000 \times 0.0075}{1-(1+0.0075)^{-300}} = \frac{15000}{1-(1.0075)^{-300}}$$

$$= \frac{15000}{1-(0.1062)}$$

$$= \frac{15000}{0.8938} = ₹\ 16782.27$$

∴ Monthly payment = ₹ 16782.27

(ii) We have, EMI = ₹ 16782.27

n = 300 and P = 2000000

∴ Total interest = $n \times$EMI − P

= 300 × 16782.27 − 2000000

= 5034681 − 2000000

= ₹ 3034681

∴ Total interest = ₹ 3034681.

20. Given, P = ₹ 400000, $n = 12 \times 10 = 120$

and $$i = \frac{6.75}{1200} = 0.005625$$

(i) Size of monthly payment

$$\text{EMI} = \frac{P \times i(1+i)^n}{(1+i)^n - 1}$$

$$= \frac{400000 \times 0.005625\,(1+0.005625)^{120}}{(1+0.005625)^{120} - 1}$$

$$= \frac{2250 \times 1.9603}{1.9603 - 1}$$

$$= \frac{4410.675}{0.9603} = ₹\ 4593$$

∴ Monthly payment = ₹ 4593

(ii) We know that,

Principal outstanding at beginning of kth period

$$= \frac{\text{EMI}[(1+i)^{n-k+1} - 1]}{i(1+i)^{n-k+1}}$$

So, principal outstanding at beginning of 61st month

$$= \frac{\text{EMI}\,((1+i)^{120-61+1} - 1)}{i(1+i)^{120-61+1}}$$

$$= \frac{\text{EMI}\,((1+i)^{60} - 1)}{i(1+i)^{60}}$$

$$= \frac{4593\,((1.005625)^{60} - 1)}{0.005625(1.005625)^{60}}$$

$$= \frac{4593\,(1.4001 - 1)}{0.005625(1.4001)}$$

$$= \frac{4593 \times 0.4001}{0.005625 \times 1.4001} = ₹\ 233336.89$$

Principal outstanding at beginning of 61st Month is ₹ 233336.89

Chapter Test

Multiple Choice Questions

1. A fund which is created to accumulate money over the years to discharge a future obligation is called ..... .
(a) Perpetuity (b) Sinking fund
(c) Bonds (d) EMI

2. The present value of a perpetual of ₹ R at the end of each of the 6 months is ₹ 144000 at 6% compounded semi-annually, then the value of R is
(a) ₹ 4000 (b) ₹ 4200
(c) ₹ 4320 (d) ₹ 4230

3. If the discount rate is greater than coupon rate, then the present value is
(a) greater than face value (b) less than face value
(c) equal to face value (d) None of these

4. A bond of the face value ₹ 1000 in 7 yr, coupon rate is 8% paid semi-annually. If the discount rate is 8% per annum, then the present value of the bond is
(a) ₹ 950 (b) ₹ 1048
(c) ₹ 1000 (d) ₹ 1150

5. The present value of a perpetuity of ₹ 500 payable at the end of each quarter, if money is worth 8% per annum is
(a) ₹ 25000 (b) ₹ 20000
(c) ₹ 30000 (d) None of these

6. The present value of a perpetuity of ₹ 7800 payable at the beginning of each year. If money is worth 6% effective is
(a) ₹ 138800 (b) ₹ 137800
(c) ₹ 136800 (d) ₹ 135800

7. At what rate of interest will the present value of a perpetuity of ₹ 500 payable at the end of each quarter be ₹ 40000?
(a) 1.25% per annum (b) 2.5% per annum
(c) 5% per annum (d) 6% per annum

Case Based MCQs

8. In year 2020, Mr. Jain amortizes a loan of ₹ 1500000 for renovation of his house by 8 yr mortgage at rate of 12% per annum compounded monthly.

Based on the above information, answer the following questions.

(i) The equated monthly installment paid by Mr. Jain was
(a) ₹ 24378.35 (b) ₹ 30379.10
(c) ₹ 35691.10 (d) None of these

(ii) The interest paid by Mr. Jain in 50th month
(a) ₹ 11000 (b) ₹ 9105.31
(c) ₹ 10055.30 (d) ₹ 12135.40

(iii) The principal outstanding at the beginning of 50th month
(a) ₹ 110541 (b) ₹ 10341
(c) ₹ 910531 (d) ₹ 1213540

(iv) The principal paid by Mr. Jain in 50th month
(a) ₹ 91053 (b) ₹ 16753
(c) ₹ 14273 (d) ₹ 15273

(v) Total interest paid by Mr. Jain
(a) ₹ 840321 (b) ₹ 905310
(c) ₹ 105316 (d) None of these

[given $(1.01)^{-96} = 0.3847$ and $(1.01)^{-47} = 0.6265$]

Short Answer Type Questions

9. Find the present value of a perpetuity of ₹ 1200 payable at the end of each year, if money worth is 4% per annum.

10. What sum of money is needed to invest now, so as to get ₹ 6000 at the beginning of every month, of the money is worth 8% per annum compounded monthly.

11. A firm anticipates a capital expenditure of ₹ 10000 for a new equipment in 5 yr. How much should be deposited quarterly in a sinking fund earning 10% per annum compounded quarterly to provide for the purchase? (given $(1.025)^{20} = 1.6836$).

12. A bond that matures in 5 yr has coupon rate of 10% per annum and has a face value of ₹ 10000. Find the fair value of bond, if yield to maturity is 8%.
(given $(1.08)^{-5} = 0.6808$)

13. Mrs. Sharma took a housing loan of ₹ 800000 to be paid in 10 yr by equal monthly installments. The interest charged is 10.5% per annum compounded monthly. Find her monthly installment.
(given $(1.00875)^{-120} = 0.3515$)

Long Answer Type Questions

14. A machine costs a company ₹ 52000 and its effective life is estimated to be 12 yr. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of ₹ 5000 only. The price of new model is estimated to be 25% higher than the price of the present one. Find what amount should be set aside at the end of each year, out of the profit, for the sinking fund, if it accumulates at 10% effective. (given $(1.1)^{12} = 3.1384$)

15. A person amortizes a loan of ₹ 150000 for a new home by obtaining a 10 yr mortgage at the rate of 9% per annum compounded monthly. Find
(i) the monthly payment (ii) the total interest paid
(given $(1.0075)^{-120} = 0.4079$)

Answers

1. *(b)* ***2.*** *(c)* ***3.*** *(b)* ***4.*** *(c)* ***5.*** *(a)* ***6.*** *(b)* ***7.*** *(c)*
8. *(i) (a) (ii) (b) (iii) (c) (iv) (d) (v) (a)* ***9.*** *₹ 30000* ***10.*** *₹ 906000*
11. *₹ 365.71* ***12.*** *₹ 10798* ***13.*** *₹ 10794.14*
14. *₹ 2805.85* ***15.*** *(i) ₹ 1900 (ii) ₹ 78000*