12

Properties of Fluids

IN THIS CHAPTER ....

The substances which flow are called *fluids*. Fluids include both liquids and gases. The branch of Science which deals with fluids at rest is called fluid statics or hydro statics and the branch of Science which deals with fluids in motion is called *hydrodynamics*. Fluid statics includes hydrostatic pressure, laws of floatation, Pascal's law and Archimedes' principle, while hydrodynamics includes continuity equation and Bernoulli's principle and Torricelli's theorem.

Density and Relative Density

Density and relative density are given below

Density

Fluid is characterised by density ρ at every point which is defined as the ratio of the mass of the fluid contained in an infinitesimal volume element around the point to the volume of element.

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

Density is a positive scalar quantity. In case of a homogeneous isotropic substance, density has no directional properties, so it is a scalar. It has dimensions $[ML^{-3}\ T^0]$. Its SI unit in MKS is kgm^{-3}, while in CGS it is gcc^{-1} with $1\ gcc^{-1} = 10^3\ kgm^{-3}$.

Relative Density

It is defined as the ratio of density of substance to the density of water at 4°C.

$$i.e.\ \text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4^\circ\text{C}}$$

Relative density has no units and no dimensions. It is also known as *specific gravity*.

Important Points Related to Density

- When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom, while that of lowest density at the top and interfaces will be plane.
- If a liquid of mass m_1 and density ρ_1 and other liquid of mass m_2 and density ρ_2 are mixed.

$$m_1 + m_2 = M$$

and $$V = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} \qquad \left[\because V = \frac{m}{\rho}\right]$$

Density of mixed liquids,

$$\rho = \frac{M}{V} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{\Sigma m_i}{\Sigma\left(\frac{m_i}{\rho_i}\right)}$$

If $m_1 = m_2$, then $\rho = \dfrac{2\rho_1 \rho_2}{\rho_1 + \rho_2}$.

- If a liquid of volume V_1 and density ρ_1 and other liquid of volume V_2 and density ρ_2 are mixed.

Then, we have

$$m = \rho_1 V_1 + \rho_2 V_2 \text{ and } V = V_1 + V_2 \qquad \left[\because \rho = \frac{m}{V}\right]$$

$$\rho = \frac{m}{V} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} = \frac{\Sigma \rho_i V_i}{\Sigma V_i}$$

- With increase in pressure due to a decrease in volume, density will increase.

$$\therefore \quad \frac{\rho}{\rho_0} = \frac{m/V}{m/V_0} = \frac{V_0}{V}$$

Example 1. *A rock of volume 10 cm^3 has mass of 100g, the specific gravity of rock is*

(a) 10 (b) 100
(c) 1000 (d) 10000

Sol. (a) Given, $V = 10 \text{ cm}^3 = 10 \times 10^{-6} = 10^{-5} \text{m}^3$

$$m = 100 \text{ g} = 0.1 \text{kg}$$

$$\therefore \text{ Density of rock, } d_r = \frac{\text{Mass}}{\text{Volume}} = \frac{0.1}{10^{-5}} = 10^4 \text{ kg/m}^3$$

$$\therefore \text{ Specific gravity of rock} = \frac{\text{Density of rock}}{\text{Density of water}} = \frac{10^4}{10^3} = 10$$

Example 2. *When equal volumes of two metals are mixed together, the specific gravity of alloys is 4. When equal masses of the same two metals are mixed together, the specific gravity of alloy is 3. What is the specific gravity of each metal?*

(a) 3 *and* 6 (b) 5 *and* 4
(c) 2 *and* 6 (d) 4 *and* 4

Sol. (c) In case of two metals,

$$\rho = \frac{(m_1 + m_2)}{V_1 + V_2}$$

If equal volumes are mixed,

$$V_1 = V_2 = V \text{ and}$$
$$m_1 = V\rho_1 \text{ and } m_2 = V\rho_2$$
$$\rho = \frac{V\rho_1 + V\rho_2}{V + V}$$

i.e. $$\rho = \frac{\rho_1 + \rho_2}{2} = 4$$

$$\Rightarrow \quad \rho_1 + \rho_2 = 8 \qquad \text{...(i)}$$

and when equal masses are mixed, $m_1 = m_2 = m$ and

$$V_1 = \frac{m}{\rho_1}$$

and $$V_2 = \frac{m}{\rho_2}$$

$$\therefore \quad \rho = \frac{m + m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}}$$

$$\therefore \quad \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{2\rho_1\rho_2}{8} = 3$$

$$\Rightarrow \quad \rho_1\rho_2 = 12$$

$$\Rightarrow \quad \rho_1 = \frac{12}{\rho_2} \qquad \text{...(iii)}$$

From Eqs. (i) and (iii), we get

$$\frac{12}{\rho_2} + \rho_2 = 8$$

$$\Rightarrow \quad 12 + \rho_2^2 - 8\rho_2 = 0$$
$$\Rightarrow \quad \rho_2^2 - 8\rho_2 + 12 = 0$$
$$\Rightarrow \quad \rho_2^2 - 6\rho_2 - 2\rho_2 + 12 = 0$$
$$\Rightarrow \rho_2 (\rho_2 - 6) - 2(\rho_2 - 6) = 0$$
$$\Rightarrow \quad (\rho_2 - 2)(\rho_2 - 6) = 0$$
$$\Rightarrow \quad \rho_2 = 2 \text{ or } 6 \qquad \text{...(iv)}$$

Solving Eqs. (iii) and (iv) for ρ_1 and ρ_2, we find specific gravities of metals are 2 and 6.

Pressure due to a Fluid Column

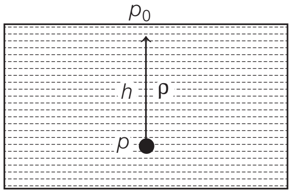
The pressure p is defined as the ratio of the magnitude of the force to the area of the contact, *i.e.*

$$p = \frac{\text{magnitude of force}}{\text{area of the contact}} = \frac{F}{A} \qquad \text{...(i)}$$

For a point at a depth h below the surface of a liquid of density ρ, pressure p is given by

$$p = p_0 + h\rho g \qquad \text{...(ii)}$$

where, p_0 is the *atmospheric pressure.*



The excess pressure above atmospheric pressure is called *gauge pressure* and total pressure is called *absolute pressure*. Thus,

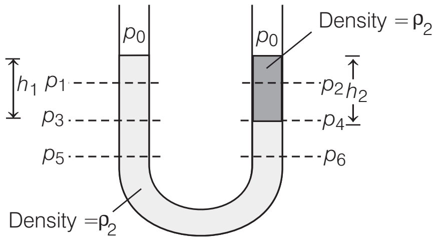
gauge pressure = absolute pressure – atmospheric pressure

i.e. $$h\rho g = p - p_0 \quad \ldots\text{(iii)}$$

Some important points related to fluid column pressure are given below

- Forces acting on a fluid in equilibrium have to be perpendicular to its surface, because it cannot sustain the shear stress. *i.e.* fluid at rest has no tangential force acting on its surface.
- In the same liquid, pressure will be same at all points at the same level.

  For example, in the given figure,



$$p_1 \neq p_2,\ p_3 = p_4 \ \text{ and } \ p_5 = p_6$$

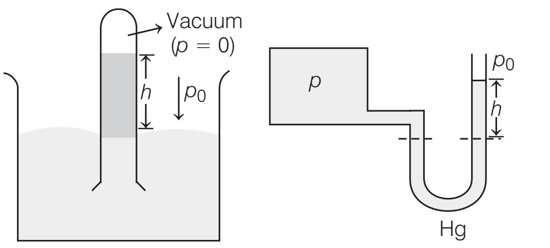
Further, $$p_3 = p_4$$

$$p_0 + \rho_1 g h_1 = p_0 + \rho_2 g h_2$$

or $$\rho_1 h_1 = \rho_2 h_2 \ \text{ or } \ h \propto \frac{1}{\rho}$$

i.e. for a given pressure at a point, the height of liquid column above it is inversly proportional to its density.

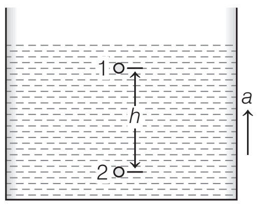
- *Barometer* is used to measure atmospheric pressure, while *manometer* measures pressure difference, *i.e.* gauge pressure.



(a) Barometer $p_0 = h\rho g$ (b) Manometer $p - p_0 = h\rho g$

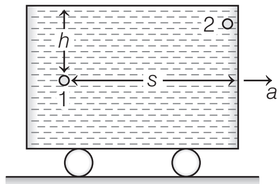
- Pressure at two points within a liquid at vertical separation of h, when the liquid container is accelerating up are related by expression

$$p_2 - p_1 = \rho(g + a)h$$



If container is accelerating down, then $p_2 - p_1 = \rho(g - a)h$ and if container is accelerating down with acceleration $> g$, then the liquid occupies upper part of the vessel.

- Variation of pressure within an accelerating closed container.



Here, all the points lying on a particular line making an angle of $\tan^{-1}(a/g)$ with the horizontal have the same pressure. In the given scenario, point 2 is the least pressure point, if the vessel is completely closed, we can take its pressure to be zero.

$$p_1 = p_2 + \rho g h + \rho s a$$

as $$p_2 = 0$$

So, $$p_1 = \rho g h + \rho s a$$

Pascal's Law and its Applications

Pascal's law states that the increase in pressure at one point of the enclosed liquid in equilibrium at rest is transmitted equally to all other points of liquid provided the gravity effect is neglected.

Some applications of Pascal's law are given as follows

(i) Pascal's law is used in the working of the hydraulic lift which is used to support or lift heavy objects. In hydraulic lift,

$$F_2 = \frac{A_2}{A_1} F_1$$

where, A_1, A_2 = area of cross-section of smaller and larger piston of hydraulic lift, F_1 = force applied on smaller piston.

(ii) Hydraulic lift is a force multiplying device, which is used in dentist's chair, car lifts and jacks, many elevators and hydraulic brakes.

Example 3. *The average depth of Indian Ocean is about 3000 m. Bulk modulus of water is 2.2×10^9 Nm^{-2} and $g = 10$ ms^{-2}, then fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the bottom of the Indian Ocean is*

(a) 1.36% (b) 20.6%
(c) 13.9% (d) 0.52%

Sol. (a) The pressure exerted by a 3000 m column of water on the bottom layer

$$p = h\rho g = 3000 \times 1000 \times 10$$

($\because$ Density of water, $\rho = 1000$ kg/m^3)

$$= 3 \times 10^7 \text{ kg m}^{-1}\text{s}^{-2}$$

$$= 3 \times 10^7 \text{ Nm}^{-2}$$

Fractional compression, $\left(\frac{\Delta V}{V}\right) = \frac{\text{Stress}}{B} = \frac{3 \times 10^7}{2.2 \times 10^9}$

$$= 1.36 \times 10^{-2}$$

$$\frac{\Delta V}{V} = 1.36\%$$

Example 4. *Assuming that the density of atmosphere does not change with altitude. How high would the atmosphere extend? (Given, density of the atmosphere at sea level is 1.29 kg/m^3).*

(a) 2 *km* (b) 4 *km* (c) 8 *km* (d) 16 *km*

Sol. (*c*) Given, $\rho = 1.29$ kg/m^3, $g = 9.8$ ms^{-2}, $p = 1.01 \times 10^5$ Nm^{-2}

$\because$ Pressure $= \rho g h =$ density $\times$ gravity $\times$ height

$\therefore$ $1.01 \times 10^5 = 1.29 \times 9.8 \times h$

$\Rightarrow$ $h = \frac{1.01 \times 10^5}{1.29 \times 9.8} \approx 7989$ m

$\Rightarrow$ $h \approx 8$ km

Note The acceleration due to gravity decreases with height and hence the density of air also decreases. The atmospheric cover extends with decreasing pressure over 100 km. We should also note that the sea level atmospheric pressure is not always 760 mm of Hg. A drop in the Hg level by 10 mm or more is a sign of an approaching storm.

Example 5. *A submarine experience a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa, then $d_2 - d_1$ is approximately (density of water $= 10^3$ kg/m^3 and acceleration due to gravity $= 10$ ms^{-2})* **[JEE Main 2019]**

(a) 500 *m* (b) 400 *m* (c) 600 *m* (d) 300 *m*

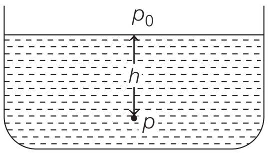
Sol. (*d*) Pressure inside a fluid volume open to atmosphere is

$$p = p_0 + h\rho g$$

where, $p =$ pressure at depth h, $h =$ depth,

$\rho =$ density of fluid

and $g =$ acceleration due to gravity.



In problem given,

when $h = d_1$, pressure $p_1 = 5.05 \times 10^6$ Pa and when $h = d_2$, pressure $p_2 = 8.08 \times 10^6$ Pa

So, we have

$$p_1 = p_0 + d_1 \rho g = 5.05 \times 10^6$$

and $p_2 = p_0 + d_2 \rho g = 8.08 \times 10^6$

$\Rightarrow$ $p_2 - p_1 = (d_2 - d_1)\rho g = 3.03 \times 10^6$

$\Rightarrow$ $d_2 - d_1 = \frac{3.03 \times 10^6}{\rho g}$

Given, $\rho = 10^3$ kgm^{-3}

and $g = 10$ ms^{-2}

$\therefore$ $d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10} = 303$ m ≈ 300 m

Example 6. *At a depth of 1000 m in an ocean, the force acting on the window of area 20 cm $\times$ 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure is [Given, density of sea water $= 1.03 \times 10^3$ kgm^{-3}, $g = 10$ ms^{-2}]*

(a) 3.2×10^8 *N* (b) 4.12×10^5 *N*
(c) 8.3×10^2 *N* (d) 3.1×10^{-5} *N*

Sol. (*b*) Given, $h = 1000$ m, $\rho = 1.03 \times 10^3$ kgm^{-3}, gauge pressure, $p_g = \rho g h$

$$p_g = 1.03 \times 10^3 \times 10 \times 1000$$

$$p_g = 103 \times 10^5 \text{ Pa}$$

The pressure outside the submarine is $p = p_a + \rho g h$ and pressure inside the submarine is p_a. Hence, the net pressure acting on the window is gauge pressure p_g.

Since, area of window is $A = 0.04$ m^2, the force acting on it is

$$F = \rho_g A = 103 \times 10^5 \text{ Pa} \times 0.04 \text{ m}^2$$

$$= 4.12 \times 10^5 \text{ N}$$

Archimedes' Principle and Buoyancy

Whenever a body is immersed in a fluid, the fluid exerts an upward force on the body and this upward force is called *buoyant force*. In fact, any body wholly or partially immersed in a fluid is brought up by a force equal to the weight of the displaced fluid. This result is known as *Archimedes' principle*. Thus, buoyant force $= V_i \rho_l g$, where V_i is the volume of immersed part of body and ρ_l is the density of fluid.

If a body of volume V and density ρ_s is completely immersed in a liquid of density ρ_l, then its observed weight,

$$w_{\text{observed}} = w_{\text{actual}} - \text{upthrust} \qquad \text{(buoyant force)}$$

$$= V\rho_s \cdot g - V\rho_l \cdot g = V(\rho_s - \rho_l)g$$

Let us learn more about buoyant force or buoyancy

(i) It is equal to the weight of liquid displaced by the immersed part of the body.

(ii) The buoyant force acts at the centre of buoyancy which is the centre of gravity of the liquid displaced by the body when immersed in the liquid.

(iii) The line joining the centre of gravity and centre of buoyancy is called *central line*.

Laws of Floatation

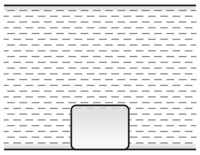
When a body of density ρ_B and volume V is immersed in a liquid of density σ, the forces acting on the body are

(i) The *weight of body* $w = mg = V\rho_B g$ acting vertically downwards through the centre of gravity of the body.

(ii) The *upthrust* $F = V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid, *i.e.* centre of buoyancy.

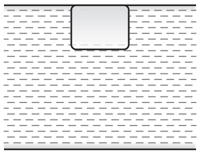
So, the following three cases are possible

***Case* I** The density of body is greater than that of liquid, (*i.e.* $\rho_B > \sigma$). In this case, as weight will be more than upthrust, the body will sink as shown in Fig. (a).



(a) $\rho_B > \sigma$

***Case* II** The density of body is equal to the density of liquid, (*i.e.* $\rho_B = \sigma$). In this case, $w = F$. So, the body will float fully submerged in neutral equilibrium anywhere in the liquid as shown in Fig. (b).

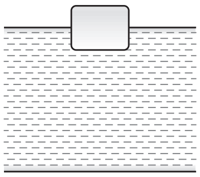


(b) $\rho_B = \sigma$

***Case* III** The density of body is lesser than that of liquid, (*i.e.* $\rho_B < \sigma$). In this case, $w < F$. So, the body will move upwards and in equilibrium will float partially immersed in the liquid such that

$$w = V_{\text{in}}\sigma g$$

[V_{in} is the volume of body in the liquid]



(c) $\rho_B < \sigma$

or $\quad V\rho_B g = V_{\text{in}}\sigma g \quad$ [as, $w = mg = V\rho_B g$]

or $\quad V\rho_B = V_{\text{in}}\sigma \quad$ ...(i)

Some Particular Cases

- If object is immersed in water, then

$$\text{Relative density, RD} = \frac{\text{Weight of body in air}}{\text{Loss in weight in water}}$$

$$\text{RD} = \frac{\text{Weight of body in air}}{\text{Weight in air} - \text{Weight in water}}$$

So, by weighing a body in air and in water, we can determine the relative density of the body.

- The upthrust on a body immersed in a liquid of density ρ_l in a lift moving downwards with acceleration **a** is

$$F = V\rho_l|\mathbf{g} - \mathbf{a}|$$

- The upthrust on a body immersed in a liquid of density ρ_L in a lift moving upwards with acceleration **a** is

$$F = V\rho_l|\mathbf{g} + \mathbf{a}|$$

- If lift is falling freely, then

$$F = 0$$

Example 7. *A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by 1 cm, when a man gets on it. The mass of the man is*

(a) 60 kg (b) 62 kg
(c) 72 kg (d) 128 kg

Sol. (a) Mass of the man = Mass of water displaced

$$= \text{Volume} \times \text{Density}$$

$$= 3 \times 2 \times \frac{1}{100} \times 10^3 \text{ kg}$$

$$= 60 \text{ kg}$$

Example 8. *An ice-berg is floating partially immersed in sea water of density 1.03 gcm^{-3}. The density of ice is 0.92 gcm^{-3}. The fraction of the total volume of the ice-berg above the level of sea water is*

(a) 8.1 % (b) 11 %
(c) 34 % (d) 0.8 %

Sol. (b) Let V be the volume of the ice-berg outside the sea water and V be the total volume of ice-berg. According to question,

Weight of iceberg = Weight of liquid displaced

$$0.92 \times V \times g = 1.03\,(V - V')\,g$$

$$0.92\,V = 1.03\,(V - V')$$

or
$$\frac{V'}{V} = \frac{1.03 - 0.92}{1.03} = \frac{11}{103}$$

$$\frac{V'}{V} \times 100 = \frac{11}{103} \times 100$$

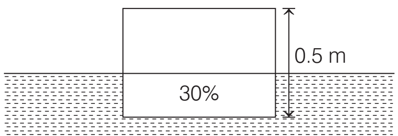
$$= 10.67\% \approx 11\%$$

Example 9. *A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water = 10^3 kg / m^3]*

[JEE Main 2019]

(a) 30.1 kg (b) 46.3 kg
(c) 87.5 kg (d) 65.4 kg

Sol. (c) When only block is floating on water, 30% of its volume is in water as shown below



By Archimedes' principle,

weight of block = weight of displaced water

$\Rightarrow \quad V\rho_b g = (30\%V)\,\rho_w g$

where, $\quad \rho_b$ = density of block,

$\quad \rho_w$ = density of water

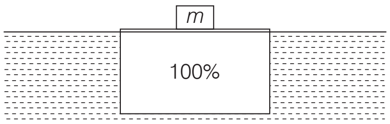
and $\quad V$ = volume of block.

So, $\quad \rho_b = 0.3\,\rho_w$

Now, let a mass m is placed over block to just keep the cube fully immersed in water.

Now, by Archimedes' principle,

weight of water displaced = weight of block + weight of mass m



$$\Rightarrow \quad V\rho_w g = V\rho_b g + mg$$

$$\Rightarrow \quad V(\rho_w - \rho_b) = m \Rightarrow V \times 0.7\rho_w = m \quad [\because \rho_b = 0.3\rho_w]$$

Here, $\quad V = (\text{side})^3 = (0.5)^3 \text{m}^3$

$$\rho_w = 10^3 \text{ kg m}^{-3}$$

Substituting these values in the above relation, we get

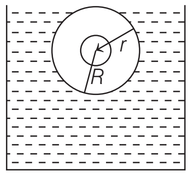
$$m = (0.5)^3 \times 0.7 \times 10^3 = 87.5 \text{ kg}$$

Example 10. *A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ with respect to water, then the value of r is* **[JEE Main 2020]**

(a) $\frac{8}{9}R$ (b) $\frac{1}{3}R$ (c) $\frac{2}{3}R$ (d) $\frac{4}{9}R$

Sol. (a) Relative density or specific gravity,

$$\rho_r = \frac{\text{Density of object}}{\text{Density of water}} = \frac{\rho_o}{\rho_w} = \frac{27}{8}$$



In equilibrium, $\frac{4}{3}\pi R^3 \rho_w g = \frac{4}{3}\pi(R^3 - r^3)\rho_o \cdot g$

$$\Rightarrow \quad R^3 = (R^3 - r^3)\left(\frac{27}{8}\right)$$

$$\Rightarrow \quad 8R^3 = 27R^3 - 27r^3$$

$$\Rightarrow \quad 27r^3 = 27R^3 - 8R^3$$

$$\Rightarrow \quad 27r^3 = 19R^3$$

$$\Rightarrow \quad \frac{r^3}{R^3} = \frac{19}{27} \Rightarrow r = \left(\frac{19}{27}\right)^{1/3} R$$

$$= \frac{2.66}{3}R = 0.88R \Rightarrow r \approx \frac{8}{9}R$$

Example 11. *An air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 g/cm^3 and water offers negligible drag force on the bubble. The mass of the bubble is (Take, $g = 980$ cm/s^2)* **[JEE Main 2020]**

(a) 1.52 g (b) 4.51 g

(c) 4.15 g (d) 3.15 g

Sol. (c) From free body diagram,

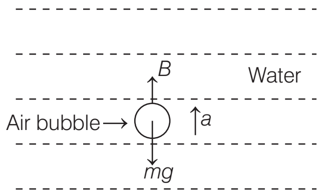
$$B - mg = ma \quad \text{(here, } B = \text{buoyant force)}$$

$$\rho_w Vg - mg = ma \quad \text{(here, } V = \text{volume)}$$

$$\rho_w Vg = mg + ma$$

$$\rho_w Vg = m(g + a)$$

$$\Rightarrow \quad m = \frac{\rho_w Vg}{g + a} \quad \ldots\text{(i)}$$



Given, $\rho_w = 1 \text{ g/cm}^3$, $g = 980 \text{ cm/s}^2$,

$a = 9.8 \text{ cm/s}^2$ and $r = 1$ cm

Substituting all values in Eq. (i), we get

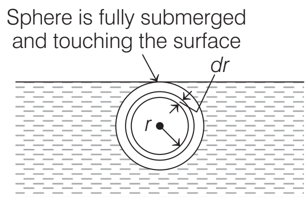
$$m = \frac{(1) \times \frac{4}{3}\pi r^3 \times 980}{980 + 9.8} = \frac{\frac{4}{3} \times \frac{22}{7} \times (1)^3 \times 980}{989.8}$$

$$= \frac{4 \times 22 \times 980}{3 \times 7 \times 989.8} = 4.148 \text{ g} \simeq 4.15 \text{ g}$$

Example 12. *Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is* **[JEE Main 2020]**

(a) $\frac{\rho_0}{3}$ (b) $\frac{2\rho_0}{5}$ (c) $\frac{2\rho_0}{3}$ (d) $\frac{\rho_0}{5}$

Sol. (b) If density of liquid is minimum, then its buoyant force is also minimum. In this case, floating pattern of sphere is as shown below



So, weight of sphere = upthrust or weight of fluid displaced

$$\Rightarrow \quad M_{\text{sphere}} \times g = V_{\text{sphere}} \times \rho_l \times g$$

$$\Rightarrow \quad \int_0^R \rho(r) \cdot 4\pi r^2 \cdot dr = \frac{4}{3}\pi R^3 \cdot \rho_l$$

$$\Rightarrow \quad \int_0^R \rho_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 4\pi r^2 \cdot dr = \frac{4}{3}\pi R^3 \rho_l$$

$$\Rightarrow \quad \rho_0 \times 4\pi \int_0^R \left(r^2 - \frac{r^4}{R^2}\right) dr = \frac{4}{3}\pi R^3 \cdot \rho_l$$

$$\Rightarrow \quad \rho_0 \times 4\pi \left(\frac{R^3}{3} - \frac{R^3}{5}\right) = \frac{4}{3}\pi R^3 \cdot \rho_l$$

$$\Rightarrow \quad \rho_l = \frac{2}{5}\rho_0$$

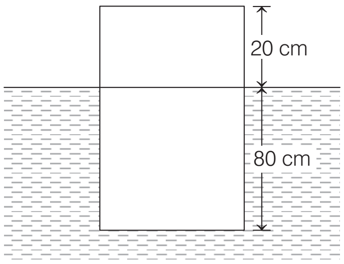
Example 13. *A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0°C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4°C, the height of the cylinder above the water surface becomes 21 cm. The density of water at T = 4 °C, relative to the density at T = 0 °C is close to* **[JEE Main 2020]**

(a) 1.26 (b) 1.03 (c) 1.01 (d) 1.04

Sol. (c) For a floating body, weight of body

= upthrust or buoyant force = weight of water displaced

Initially at 0°C, cylinder floats as shown in the figure



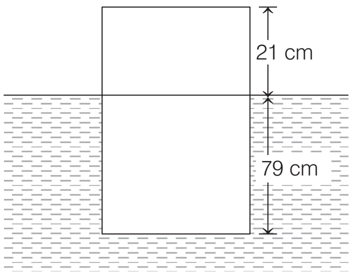
Here, weight of cylinder = weight of water displaced by cylinder of 80 cm length

$\Rightarrow \quad mg = \rho_0 \cdot g \cdot 80\, A \quad \ldots(i)$

where, ρ_0 = density of water at 0°C

and A = area of cross-section of cylinder.

When temperature is raised to 4°C, cylinder floats as shown in the figure.



In above case, weight of cylinder = weight of water displaced by cylinder of 79 cm length

$\Rightarrow \quad mg = \rho_4 \cdot g \cdot 79\, A \quad \ldots(ii)$

From Eqs. (i) and (ii), we have

$$80\, \rho_0 = \rho_4 \cdot 79$$

$$\Rightarrow \quad \frac{\rho_4}{\rho_0} = \frac{80}{79} = 1.01$$

Flow of Liquids

The different types of fluid flow are

Streamline Flow The flow in which each particles of the liquid passing through a point travels along the same path and with the same velocity as the preceeding particles passing through the same point is known as *streamline flow* of a liquid.

Hence, it is a regular flow. The path followed by each element is called *streamline*.

- The tangent drawn at any point of streamline gives the direction of the flow of liquid at that point.
- The streamlines cannot intersect each other.

Turbulent Flow The flow of liquid with a velocity greater than its critical velocity is disordered and called *turbulent flow*. In case of turbulent flow, maximum part of external energy is spent for producing eddies in the liquid and small part of external energy is available for forward flow.

Laminar Flow If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called *laminar flow*. Thus a flow, in which the liquid moves in layers or lamina is called a laminar flow.

Critical Velocity and Reynold's Number

The largest velocity which allows a steady flow is called **critical velocity**.

Value of critical velocity for flow of liquid of density ρ and coefficient of viscosity η flowing through a horizontal tube of diameter D is given by

$$v_c \propto \frac{\eta}{\rho D}$$

Reynold's number is a pure number which determines the nature of flow of liquid through a horizontal tube.

Reynold's number (R_e) is a unitless and dimensionless number given by

$$R_e = \frac{\rho v D}{\eta}$$

If the value of Reynold's number

(i) lies between 0 to 2000, the flow of liquid is streamline or laminar.

(ii) lies between 2000 to 3000, the flow of liquid is unstable and changing from streamline to turbulent flow.

(iii) lies above 3000, the flow of liquid is definitely turbulent.

Example 14. *Water from a pipe is coming at a rate of 100 L/min. If the radius of the pipe is 5 cm, the Reynold's number for the flow is of the order of (density of water = 1000 kg /m^3, coefficient of viscosity of water = 1 mPa-s)* **[JEE Main 2019]**

(a) 10^3 (b) 10^4
(c) 10^2 (d) 10^6

Sol. (b) Reynolds' number for flow of a liquid is given by

$$R_e = \frac{\rho v D}{\eta}$$

where, velocity of flow, $v = \dfrac{\text{volume flow rate}}{\text{area of flow}} = \dfrac{V/t}{A}$

So, $R_e = \dfrac{\rho V D}{\eta A t} = \dfrac{\rho V 2r}{\eta \times \pi r^2 \times t} = \dfrac{2\rho V}{\eta \pi r t}$

Here, ρ = density of water = 1000 kgm^{-3}

$$\frac{V}{t} = \frac{100 \times 10^{-3}}{60}\ \text{m}^3\text{s}^{-1}$$

where, η = viscosity of water $= 1 \times 10^{-3}$ Pa-s

and $\quad r$ = radius of pipe $= 5 \times 10^{-2}$ m

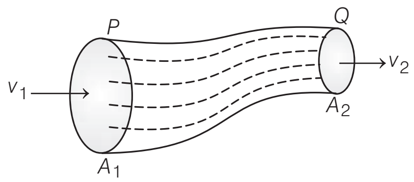
$$R_e = \frac{2 \times 1000 \times 100 \times 10^{-3}}{1 \times 10^{-3} \times 60 \times 3.14 \times 5 \times 10^{-2}}$$

$$= 212.3 \times 10^2 \approx 2.0 \times 10^4$$

So, order of Reynolds' number is of 10^4.

Equation of Continuity

Let us consider streamline flow of an ideal, non-viscous fluid through a tube of varying cross-section. Let at two sections, the cross-section areas be A_1 and A_2 respectively and fluid flow velocities be v_1 and v_2, then according to equation of continuity



$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

where, ρ_1 and ρ_2 are the respective densities of fluid. Equation of continuity is based on the conservation of mass.

If fluid flowing is incompressible, then

$$\rho_1 = \rho_2$$

and equation of continuity is simplified as

$$A_1 v_1 = A_2 v_2$$

Example 15. *Two syringes of different cross-sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1 cm and 3 cm respectively. If the smaller piston is pushed in through 6 cm, how much does the larger piston move out?*

(a) 0.37 cm (b) 0.67 cm
(c) 37 cm (d) 67 cm

Sol. (*b*) Water is considered to be perfectly incompressible. Volume covered by the movement of smaller piston inwards is equal to the volume moved outwards due to longer piston.

$$L_1 A_1 = L_2 A_2$$

$$\Rightarrow \quad L_2 = \frac{A_1}{A_2} L_1$$

$$= \frac{\pi \left(\frac{1}{2} \times 10^{-2}\right)^2}{\pi \left(\frac{3}{2} \times 10^{-2}\right)^2} \times 6 \times 10^{-2}$$

$$= 0.67 \times 10^{-2}\ \text{m} \approx 0.67\ \text{cm}$$

Note Atmospheric pressure is common to both pistons and has been ignored.

Example 16. *In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to the second piston of a radius 15 cm. If the mass of the car to be lifted is 1350 kg, then the pressure necessary to accomplish this task is*

(a) 1.9×10^5 Pa (b) 3×10^6 Pa
(c) 6.5×10^3 Pa (d) 0.23×10^3 Pa

Sol. (*a*) Since, pressure is transmitted undiminished throughout the fluid

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi (5 \times 10^{-2}\ \text{m})^2}{\pi (15 \times 10^{-2}\ \text{m})^2} (1350\ \text{N} \times 9.8\ \text{ms}^{-2})$$

$$= 1470\ \text{N} \approx 1.5 \times 10^3\ \text{N}$$

The air pressure that will produce this force is

$$p = \frac{F_1}{A_1} = \frac{1.5 \times 10^3\ \text{N}}{\pi (5 \times 10^{-2})^2} = 1.9 \times 10^5\ \text{Pa}$$

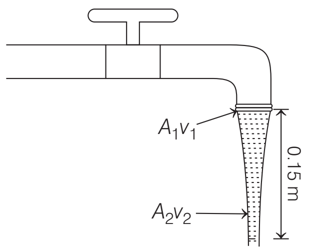
Note This is almost double the atmospheric pressure.

Example 17. *Water from a tap emerges vertically downwards with an initial speed of $1.0\,ms^{-1}$. The cross-sectional area of the tap is $10^{-4}\,m^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be (Take, $g = 10\,ms^{-2}$)*

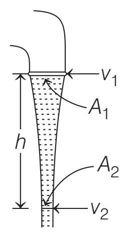
[JEE Main 2019]

(a) $2 \times 10^{-5} m^2$ (b) $1 \times 10^{-5} m^2$
(c) $5 \times 10^{-4} m^2$ (d) $5 \times 10^{-5} m^2$

Sol. (*d*) Given situation is as shown in the figure below



From equation of continuity, $A \propto \frac{1}{v}$



where, A = area of flow

and $\quad v$ = velocity of flow.

$\therefore$ Increase in speed of flow causes a decrease in area of flow.

Here given that height of fall, $h = 0.15$ m

Area, $A = 10^{-4}\text{m}^2$

Initial speed, $v = 1\ \text{ms}^{-1}$

Velocity of water stream below h height is

$$v_2 = \sqrt{v_1^2 + 2gh} \qquad [\because v^2 - u^2 = 2gh]$$

Substituting the given values, we get

$$= \sqrt{1^2 + 2 \times 10 \times 0.15}$$

$$= \sqrt{4} = 2\ \text{ms}^{-1}$$

Now, from the equation of continuity, we have

$$A_1v_1 = A_2v_2 \text{ or } A_2 = \frac{A_1v_1}{v_2}$$

$$\therefore \quad A_2 = \frac{10^{-4} \times 1}{2} = 0.5 \times 10^{-4}$$
$$= 5 \times 10^{-5} \text{ m}^2$$

Energy of a Flowing Liquid

There are three types of energies in a flowing liquid

Pressure Energy If p is the pressure on the area A of a fluid and the liquid moves through a distance l due to this pressure, then

pressure energy of liquid = work done

= force × displacement = pAl

The volume of the liquid is Al.

Hence, pressure energy per unit volume of liquid

$$= \frac{pAl}{Al} = p$$

Kinetic Energy If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy is $\frac{1}{2}mv^2$.

$\therefore$ Kinetic energy per unit volume of liquid

$$= \frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$$

Here, ρ is the density of liquid.

Potential Energy If a liquid of mass m is at a height h from the reference line ($h = 0$), then its potential energy is mgh.

$\therefore$ Potential energy per unit volume of the liquid

$$= \left(\frac{m}{V}\right)gh = \rho gh$$

Bernoulli's Principle

If an ideal liquid is flowing in streamlined flow, then total energy, *i.e.* sum of pressure energy, kinetic energy and potential energy per unit volume of the liquid remains constant at every cross-section of the tube.

Mathematically, $p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

It can be expressed as, $\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$

where, $\frac{p}{\rho g}$ = pressure head, $\frac{v^2}{2g}$ = velocity head

and h = gravitational head or potential head.

For horizontal flow of liquid, $p + \frac{1}{2}\rho v^2 = \text{constant}$

where, p is called *static pressure* and $\frac{1}{2}\rho v^2$ is called *dynamic pressure*.

Therefore in horizontal flow of liquid, if p increases, v decreases and *vice-versa*.

This theorem is applicable to ideal liquid, *i.e.* a liquid which is non-viscous, incompressible and irrotational.

Applications of Bernoulli's Theorem

- The action of carburetor, paintgun, scent sprayer, atomiser and insect sprayer is based on Bernoulli's theorem.
- The action of Bunsen's burner, gas burner, oil stove and exhaust pump is also based on Bernoulli's theorem.
- Motion of a spinning ball (Magnus effect) is based on Bernoulli's theorem.
- Blowing of roofs by wind storms, attraction between two closely parallel moving boats, fluttering of a flag etc are also based on Bernoulli's theorem.
- Bernoulli's theorem helps in explaining blood flow in artery.
- Working of an aeroplane is based on Bernoulli's theorem.

Example 18. *A fluid is flowing through a horizontal pipe of varying cross-section with speed v ms^{-1} at a point where the pressure is p Pa. At another point, where pressure is $\frac{p}{2}$ Pa its speed is V ms^{-1}. If the density of the fluid is ρ kg m^{-3} and the flow is streamline, then V is equal to* **[JEE Main 2020]**

(a) $\sqrt{\frac{p}{\rho} + v}$ (b) $\sqrt{\frac{2p}{\rho} + v^2}$

(c) $\sqrt{\frac{p}{2\rho} + v^2}$ (d) $\sqrt{\frac{p}{\rho} + v^2}$

Sol. (*d*) Since, pipe is horizontal. Applying Bernoulli's equation at two different cross-section,

$$p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

For same level,

$$p + \frac{1}{2}\rho v_1^2 = \frac{p}{2} + \frac{1}{2}\rho v_2^2$$

Given, $v_1 = v$ m/s and $v_2 = V$ m/s

$$\frac{p}{2} = \frac{1}{2}\rho(V^2 - v^2) \Rightarrow V^2 - v^2 = \frac{p}{\rho}$$

$$\Rightarrow \quad V = \sqrt{\frac{p}{\rho} + v^2}$$

Example 19. *At what speed, the velocity head of water is equal to pressure head of 40 cm of Hg?*

(a) 10.32 ms^{-1} (b) 2.8 ms^{-1}

(c) 5.5 ms^{-1} (d) 8.4 ms^{-1}

Sol. (*a*) From Bernoulli's equation,

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

Given that, velocity head = pressure head

$$\frac{v^2}{2g} = \frac{p}{\rho g}$$

$$\Rightarrow \quad v^2 = \frac{2p}{\rho}$$

Given, $p = 40$ cm of Hg $= 40 \times 10^{-2} \times 9.8 \times 13.6 \times 10^3$

$$v^2 = \frac{2 \times 13.6 \times 10^3 \times 40 \times 10^{-2} \times 9.8}{10^3}$$

$$\Rightarrow \quad v = 10.32 \text{ ms}^{-1}$$

Example 20. *Water is flowing with a speed of 2 ms^{-1} in a horizontal pipe with cross-sectional area decreasing from $2 \times 10^{-2} m^2$ to 0.01 m^2 at pressure 4×10^4 Pa. What will be the pressure at small cross-section?*

(a) 2×10^4 Pa (b) 3.4×10^4 Pa
(c) 2.4×10^4 Pa (d) 4×10^4 Pa

Sol. (b) Here, $v_1 = 2 \text{ ms}^{-1}$, $A_1 = 2 \times 10^{-2} \text{m}^2$,

$$p_1 = 4 \times 10^4 \text{ Pa}, A_2 = 0.01 \text{m}^2, p_2 = ?$$

As, $A_1 v_1 = A_2 v_2$

or $$v_2 = \frac{A_1 v_1}{A_2}$$

$$= \frac{2 \times 10^{-2} \times 2}{0.01} = 4 \text{ ms}^{-1}$$

Now, $$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or $$p_2 = p_1 + \frac{1}{2}\rho (v_1^2 - v_2^2)$$

or $$p_2 = 4 \times 10^4 + \frac{1}{2} \times 10^3 (2^2 - 4^2)$$

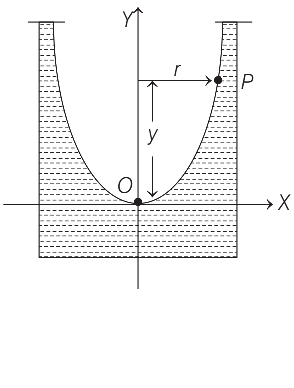
$$= 4 \times 10^4 - 6 \times 10^3$$

$$= 3.4 \times 10^4 \text{ Pa}$$

Example 21. *A long cylindrical vessel is half-filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rps, then the difference in the heights between the centre and the sides (in cm) will be* **[JEE Main 2019]**

(a) 0.1 (b) 1.2 (c) 0.4 (d) 2.0

Sol. (d) When liquid filled vessel is rotated the liquid profile becomes a paraboloid due to centripetal force, as shown in the figure below



Pressure at any point P due to rotation is given as,

$$P_r = \frac{1}{2}\rho v^2 = \frac{1}{2}\rho \cdot (\omega t)^2 = \frac{1}{2}\rho \omega^2 r^2$$

Gauge pressure at depth y is $p_G = -\rho g y$

If p_0 is atmospheric pressure, then total pressure at point P is

$$p = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

For any point on surface of rotating fluid, $p = p_0$

Hence, for any surface point.

$$p_0 = p_0 + \frac{1}{2}\rho r^2 \omega^2 - \rho g y$$

or $$\frac{1}{2}\rho r^2 \omega^2 = \rho g y$$

or $$y = \frac{r^2 \omega^2}{2g} \quad \dots \text{(i)}$$

In the given case,

Angular speed, $\omega = 2$ rps $= 2 \times 2\pi = 4\pi$ rad s^{-1}

Radius of vessel, $r = 5$ cm $= 0.05$ m and $g = 10 \text{ ms}^{-2}$

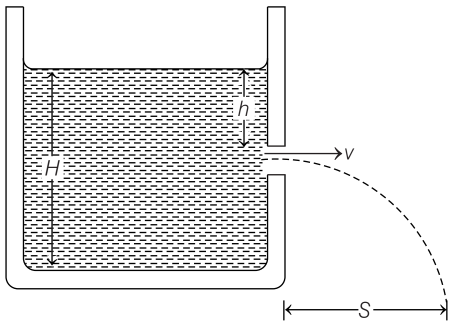
Hence, substituting these values in Eq. (i), we get

$$y = \frac{\omega^2 r^2}{2g} = \frac{(4\pi)^2 (0.05)^2}{2 \times 10}$$

$$= 0.02 \text{ m} = 2 \text{ cm}$$

Torricelli's Theorem

Velocity of efflux (the velocity with which the liquid flows out of a orifice or narrow hole) is equal to the velocity acquired by a freely falling body through the same vertical distance equal to the depth of orifice below the free surface of liquid.



Velocity of efflux, $v = \sqrt{2gh}$

where, h = depth of orifice below the free surface of liquid.

Time taken by the liquid to reach the base-level

$$t = \sqrt{\frac{2(H-h)}{g}}$$

Horizontal range, $S = \sqrt{4h(H-h)}$

where, H = height of liquid column.

Horizontal range is maximum, equal to height of the liquid column H, when orifice is at half of the height of liquid column.

If the hole is at the bottom of the tank, then time required to make the tank empty is $t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$

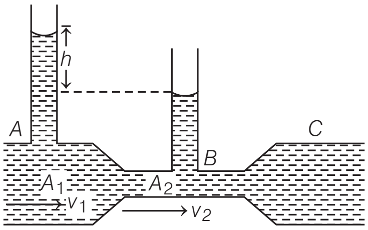
where, A is area of the container and A_0 is area of orifice.

Volume of liquid coming out from the orifice per second

$$= VA_0 = A_0\sqrt{2gh}$$

Venturimeter

It is a device which is used to measure the rate of flow of fluids through pipes. Its working is based on Bernoulli's theorem. In the arrangement shown, the rate of flow of fluid V is given by



$$V = A_1A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$$

$$V = A_1A_2\sqrt{\frac{2(p_1 - p_2)}{\rho(A_1^2 - A_2^2)}}$$

Example 22. *The diameter of a pipe at two points, where a venturimeter is connected is 8 cm and 5 cm and the difference of levels in it is 4 cm. The volume of water flowing through the pipe per second is*

(a) 1889 ccs^{-1} (b) 1520 ccs^{-1}
(b) 1321 ccs^{-1} (d) 1125 ccs^{-1}

Sol. (a) Here, $r_1 = 8/2 = 4.0$ cm,

$r_2 = 5/2 = 2.5$ cm, $h = 4$ cm

Now, $A_1 = \pi r_1^2 = \pi(4)^2 = 16\pi \text{cm}^2$

and $A_2 = \pi r_2^2 = \pi(2.5)^2 = 6.25\pi \text{ cm}^2$

Here, $\rho = \rho_m$

So, the rate of flow of water in venturimeter is given by

$$V = A_1A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$$

$$= 6.25\pi \times 16\pi\sqrt{\frac{2 \times 980 \times 4}{(16\pi)^2 - (6.25\pi)^2}}$$

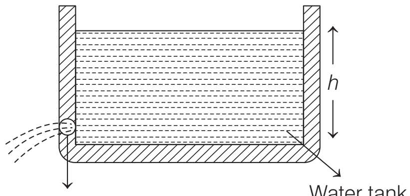
$$= \frac{100\pi^2 \times 28\sqrt{10}}{\sqrt{(16\pi - 6.25\pi)(16\pi + 6.25\pi)}}$$

$$= 1889 \text{ ccs}^{-1}$$

Example 23. *The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to* **[JEE Main 2019]**

(a) 4.8 *m* (b) 6.0 *m* (c) 2.9 *m* (d) 9.6 *m*

Sol. (a) For the given condition, a water tank is open to air and its water level maintained.



Suppose the depth of the centre of the opening from level of water in tank is h and the radius of opening is r.

According to question, the water per minute through a circular opening

Flow rate $(Q) = 0.74 \text{ m}^3/\text{min} = \frac{0.74}{60} \text{ m}^3/\text{s}$

r = radius of circular opening = 2 cm

Here, the area of circular opening $= \pi(r^2)$

$$a = \pi \times (2 \times 10^{-2})^2 = 4\pi \times 10^{-4} \text{ m}^2$$

Now, flow rate through an area is given by

Q = Area of circular opening $\times$ Velocity of water

$Q = a \times v = \pi(r^2) \times v$

$$\Rightarrow \quad \frac{0.74}{60} = (4\pi \times 10^{-4}) \times v \qquad \text{...(i)}$$

According to Torricelli's law (velocity of efflux)

$$v = \sqrt{2gh}$$

Putting the value of v in Eq. (i), we get

$$\sqrt{2gh} = \frac{0.74 \times 10^4}{60 \times 4\pi}$$

$$\Rightarrow \quad h = \left(\frac{0.74 \times 10^4}{60 \times 4\pi}\right)^2 \times \frac{1}{2g}$$

$$h \approx 4.8 \text{ m}$$

Surface Tension

The property of a liquid at rest by virtue of which its free surface behaves like a stretched membrane under tension and tries to occupy as small area as possible is called **surface tension**.

$$\text{Surface tension, } S = \frac{\text{Force}}{\text{Length}} = \frac{F}{L}$$

The SI unit of surface tension is N/m or J/m². It is a scalar quantity and its dimensional formula is $[MT^{-2}]$.

Surface tension of a liquid depends only on the nature of liquid and is independent of the surface area of film or length of the line considered. Small liquid drops are spherical due to the property of surface tension.

Surface tension of a liquid decreases with an increase in temperature.

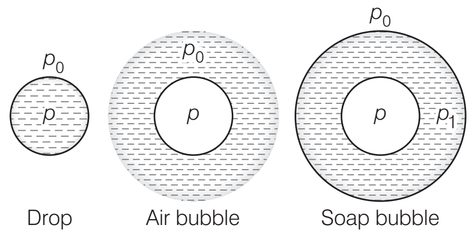
Applications of Surface Tension

- Oil spreads over the water surface because the surface tension of oil is smaller than the water.
- The surface tension of points and all lubricating oils is low.
- The stromy waves at the sea are calmed by pouring oil on the sea water.
- The surface tension of antiseptics like dettol is low and hence they spread faster.
- The surface tension of soap solution is low, therefore it can spread over large area.

Surface Tension of Drops and Bubbles

Due to the property of surface tension, a drop or bubble tends to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So, in equilibrium, the pressure inside a bubble or drop is greater than outside and difference of pressure between two sides of the liquid surface is called **excess pressure**.

Excess pressure in different cases is given below

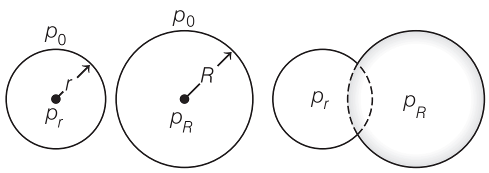


Excess pressure inside a drop, $\Delta p = \dfrac{2S}{r}$

Excess pressure inside an air bubble, $\Delta p = \dfrac{2S}{r}$

Excess pressure inside a soap bubble, $\Delta p = \dfrac{4S}{r}$

- When two soap bubbles of same material having different radii r and $R(>r)$ are combined to form a double bubble, then



$$p_r - p_0 = \frac{4S}{r}$$

and $$p_R - p_0 = \frac{4S}{R}$$

$\therefore$ $$p_r - p_R = 4S\left(\frac{1}{r} - \frac{1}{R}\right)$$

So, the radius of the common surface is given as

$$R_0 = \frac{rR}{R-r}.$$

- If two plates are placed in contact with a thin film of liquid in between them to pull them apart, a large force is needed.

Excess pressure in this case is $\dfrac{2S}{d}$, where d is the separation between the plates. Force required to separate two plates, each of area A is given by

$$F = \frac{2\,A \times S}{d}$$

Surface Energy

If we increase the free surface area of a liquid, then work has to be done against the force of surface tension. This work done is stored in liquid surface as potential energy.

This additional potential energy per unit area of free surface of liquid is called surface energy.

Surface energy $(E) = S \times \Delta A$

where, S = surface tension and ΔA = increase in surface area.

(i) **Work Done in Blowing a Liquid Drop** If a liquid drop is blown up from a radius r_1 to r_2, then work done for that is

$$W = S \cdot 4\pi(r_2^2 - r_1^2)$$

(ii) **Work Done in Blowing a Soap Bubble** As a soap bubble has two free surfaces, hence work done in blowing a soap bubble so as to increase its radius from r_1 to r_2 is given by

$$W = S \cdot 8\pi(r_2^2 - r_1^2)$$

(iii) **Work Done in Splitting a Bigger Drop into *n* Smaller Droplets**

If a liquid drop of radius R is split up into n smaller droplets, all of same size, then radius of each droplet

$$r = R \cdot (n)^{-1/3}$$

Work done, $W = 4\pi S(nr^2 - R^2) = 4\pi SR^2\,(n^{1/3} - 1)$

(iv) **Coalescance of Drops** If n small liquid drops of radius r each combine together so as to form a single bigger drop of radius $R = n^{1/3} \cdot r$, then in the process energy is released. Release of energy is given by $\Delta U = S \cdot 4\pi(nr^2 - R^2) = 4\pi Sr^2 n(1 - n^{-1/3})$

Example 24. *A rectangular film of liquid is extended from 5 cm × 3 cm to 6 cm × 5 cm. If the work done is 3.0×10^{-4} J. The surface tension of liquid is*

(a) $0.05\ Nm^{-1}$ (b) $0.1\ Nm^{-1}$
(c) $0.2\ Nm^{-1}$ (d) $2\ Nm^{-1}$

Sol. (a) Increase in area,

$$\Delta A = 2(6 \times 5 - 5 \times 3) = 2 \times (30 - 15)\ \text{cm}^2$$
$$= 30 \times 10^{-4}\ m^2$$

As, work done, W = surface tension × increase in area

$$3.0 \times 10^{-4} = S \times 2 \times 30 \times 10^{-4} \quad \text{or } S = 0.05\ \text{Nm}^{-1}$$

Example 25. *Surface tension of a detergent solution is $2.8\times10^{-2}\ Nm^{-1}$. What is the work done in blowing a bubble of 2 cm diameter?*

(a) $4\times10^{-6}\ J$ (b) $70.3\times10^{-6}\ J$
(c) $50.8\times10^{-6}\ J$ (d) $60.8\times10^{-6}\ J$

Sol. (b) Given that, $S = 2.8\times10^{-2}\,\text{Nm}^{-1}$

and $R = \frac{2}{2} = 1\,\text{cm} = 0.01\,\text{m}$

As soap bubble has two free surfaces,

$\therefore$ Work done, $W = S\cdot 8\pi R^2 = 2.8\times10^{-2}\times8\times3.14\times(0.01)^2$

$= 70.3\times10^{-6}\ \text{J}$

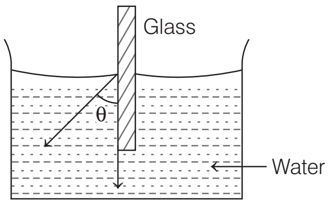
Angle of Contact

The angle subtended between the tangents drawn at liquid surface and at solid surface inside the liquid at the point of contact is called angle of contact (θ).

If liquid molecules is in contact with solid (*i.e.* wall of capillary tube), then forces acting on liquid molecules are

(i) Force of cohesion F_c (acts at an angle 45° to the vertical)

(ii) Force of adhesion F_a (acts outwards at right angle to the wall of the tube)



Angle of contact depends upon the nature of the liquid and solid in contact and the medium which exists above the free surface of the liquid.

When wax is coated on a glass capillary tube, it becomes water-proof. The angle of contact increases and becomes obtuse. Water does not rise in it. Rather it falls in the tube by virtue of obtuse angle of contact.

- If θ is acute angle, *i.e.* $\theta < 90°$, then liquid meniscus will be concave upwards.
- If θ is 90°, then liquid meniscus will be plane.
- If θ is obtuse, *i.e.* $\theta > 90°$, then liquid meniscus will be convex upwards.
- If angle of contact is acute angle, *i.e.* $\theta < 90°$, then liquid will wet the solid surface.
- If angle of contact is obtuse angle, *i.e.* $\theta > 90°$, then liquid will not wet the solid surface.

Angle of contact increases with increase in temperature of liquid. Angle of contact decreases on adding soluble impurity to a liquid.

Angle of contact for pure water and glass is zero.

For ordinary water and glass, it is 8°.

For mercury and glass, it is 138°.

For pure water and silver, it is 90°.

For alcohol and clean glass $\theta = 0°$.

Angle of Contact, Meniscus and Shape of liquid surface

Property	Angle of Contact < 90°	Angle of Contact = 90°	Angle of Contact > 90°
Substances	Water and glass	Water and silver	Mercury and glass
Angle of contact	Almost zero, acute angle	Right angle = 90°	Obtuse angle = 138°
Meniscus shape	Concave	Plane	Convex
Capillary action	Liquid rises	No effect	Liquid falls
Sticking to solid	Stick/wets	Does not wet	Does not wet
Relation between cohesive force (F_c) and adhesive force (F_a)	$F_a > \frac{F_c}{\sqrt{2}}$ $F_a > F_c$	$F_a = \frac{F_c}{\sqrt{2}}$	$F_a < \frac{F_c}{\sqrt{2}}$ $F_c > F_a$
Shape of liquid surface	Almost round	Spreads on surface	Flat on interface

Capillarity

The phenomenon of rise or fall of liquid column in a capillary tube is called capillarity.

Ascent of a liquid column in a capillary tube is given by

$$h = \frac{2S\cos\theta}{r\rho g} - \frac{r}{3}$$

If capillary is very narrow, then $h = \dfrac{2S\cos\theta}{r\rho g}$

where, r = radius of capillary tube, ρ = density of the liquid,

θ = angle of contact and S = surface tension of liquid.

- If $\theta < 90°$, $\cos\theta$ is positive, so h is positive, *i.e.* liquid rises in a capillary tube.
- If $\theta > 90°$, $\cos\theta$ is negative, so h is negative, *i.e.* liquid falls in a capillary tube.
- Rise of liquid in a capillary tube does not violate law of conservation of energy.

Some Practical Examples of Capillarity

(i) The kerosene oil in a lantern and the melted wax in a candle, rise in the capillaries formed in the cotton wick and burns.

(ii) Coffee powder is easily soluble in water because water immediately wets the fine granules of coffee by the action of capillarity.

(iii) The water given to the fields rises in the innumerable capillaries formed in the stems of plants and trees and reaches the leaves.

Example 26. *The lower end of a capillary tube of diameter 2 mm is dipped 8 cm below the surface of water in a beaker. The surface tension of water at temperature of the experiment is $7.3\times10^{-2}\ Nm^{-1}$, 1 atmospheric pressure*

$= 1.01\times 10^5$ *Pa, density of water* $= 1000$ kg/m^3, $g = 9.8$ ms^{-2}, *then the pressure inside the bubble is*

(a) 2.13×10^3 *Pa* (b) 1.02×10^5 *Pa*

(c) 5×10^{-5} *Pa* (d) 7.3×10^{-3} *Pa*

Sol. (*b*) The excess pressure in a bubble of gas in a liquid is given by $\frac{2S}{r}$, where S is the surface tension of the liquid gas interface. The radius of the bubble is r.

The pressure outside the bubble p_0 equals atmospheric pressure plus the pressure due to 8 cm of water column. That is

$$p_0 = (1.01\times 10^5 \text{ Pa} + 0.08 \text{ m}\times 1000 \text{ kg m}^{-3}\times 9.80 \text{ ms}^{-2})$$

$$p_0 = 1.01784\times 10^5 \text{ Pa}$$

The pressure inside the bubble is

$$p_i = p_0 + \frac{2S}{r} = 1.01784\times 10^5 + \frac{(2\times 7.3\times 10^{-2})}{10^{-3}}$$

$$= (1.01784 + 0.00146)\times 10^5$$

$$= 1.02\times 10^5 \text{ Pa}$$

Note This is a 100% increase in pressure from surface level. At a depth of 1 km, the increase in pressure is 100 atm. Submarines are designed to withstand such enormous pressures.

Example 27. *The lower end of a capillary tube is dipped into water and it is seen that water rises through 7.5 cm in the capillary. Given surface tension of water is* 7.5×10^{-2} Nm^{-1} *and angle of contact between water and glass capillary tube is zero. What will be the diameter of the capillary tube? (Given,* $g = 10$ ms^{-2}*)*

(a) 0.2 *mm* (b) 0.3 *mm* (c) 0.4 *mm* (d) 0.5 *mm*

Sol. (c) Given, $h = 7.5$ cm $= 7.5\times 10^{-2}$ m, $S = 7.5\times 10^{-2}$ Nm^{-1},

$$\theta = 0°,\ 2r = ?$$

As, $$h = \frac{2S\cos\theta}{r\rho g}$$

$$2r = \frac{4S\cos\theta}{h\rho g}$$

$$= \frac{4\times 7.5\times 10^{-2}\times\cos 0°}{7.5\times 10^{-2}\times 10^3\times 10}$$

$$\Rightarrow \quad = 4\times 10^{-4}$$

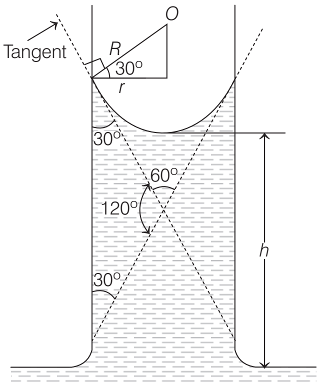
$$= 0.4 \text{ mm}$$

Example 28. *A capillary tube made of glass of radius 0.15 mm is dipped vertically in a beaker filled with methylene iodide, which rises to height h in the tube. It is observed that the two tangents drawn from liquid- glass interface (from opposite sides of the capillary) make an angle of* $60°$ *with one another. Then, h is close to (Given, surface tension* $= 0.05$ Nm^{-1}*, density* $= 667$ $kg\ m^{-3}$ *and* $g = 10$ ms^{-2}*)* ***[JEE Main 2020]***

(a) 0.049 *m* (b) 0.087 *m* (c) 0.137 *m* (d) 0.172 *m*

Sol. (*b*) It is given that tangents drawn from point of contact of meniscus makes $60°$ angle with each other.

We have following situation, as shown in figure



A normal drawn on the tangent, will pass through the centre O of meniscus. Let r be the radius of capillary tube and R be the radius of meniscus.

From geometry of figure, $\frac{\text{radius of capillary}}{\text{radius of meniscus}} = \frac{r}{R} = \cos 30°$

$$\Rightarrow \quad \frac{r}{R} = \frac{\sqrt{3}}{2}$$

or $$R = \frac{2r}{\sqrt{3}} = \frac{2\times 0.15\times 10^{-3}}{\sqrt{3}}$$

$$\Rightarrow \quad R = \sqrt{3}\times 10^{-4} \text{ m}$$

Now, by ascent formula,

$$h = \frac{2S}{\rho g R} = \frac{2\times 0.05}{667\times 10\times\sqrt{3}\times 10^{-4}} = 0.087 \text{ m}$$

Viscosity

The property of a fluid due to which it opposes the relative motion between its different layers is called *viscosity* (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called *viscous force*.

According to Newton, the frictional force F (or viscous force) between two layers depends upon the following factors

(i) Force F is directly proportional to the area (A) of the layers in contact, *i.e.* $F \propto A$

(ii) Force F is directly proportional to the velocity gradient $\left(\frac{dv}{dy}\right)$ between the layers, $\frac{dv}{dy}$ is also called *strain rate*.

Combining these two, we have

$$F \propto A\frac{dv}{dy} \text{ or } F = -\eta A\frac{dv}{dy}$$

where, η is a constant called *coefficient of viscosity*.

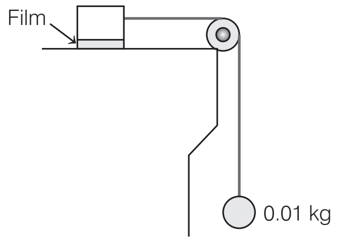
The negative sign shows that viscous force on a liquid layer acts in a direction opposite to the relative velocity of flow of fluid.

Its unit is poise or dyne cm^{-2} s in CGS system and poiseuille or deca poiseuille or Newton-s-m^{-2} in SI system. It is a scalar quantity.

$$1 \text{ poiseuille} = 1 \text{ deca poiseuille} = 10 \text{ poise}$$

Note In case of a steady flow of a liquid of viscosity η in a capillary tube of length L and radius i under a pressure difference p across it, the velocity of flow at a distance x from the axis is given by $v = \frac{p}{4\eta L}(r^2 - x^2)$

Example 29. *A metal block of area* 0.10 m^2 *is connected to a* 0.010 *kg mass via a string that passes over an ideal pully (considered massless and frictionless). A liquid with a film thickness of* 0.30 *mm is placed between the block and the table. When released the block moves to the right with a constant speed of* 0.085 ms^{-1}. *The coefficient of viscosity of the liquid is*



(a) 45.2×10^2 *Pa-s* (b) 13.4×10^{-4} *Pa-s*
(c) 3.45×10^{-3} *Pa-s* (d) 1.42×10^{-2} *Pa-s*

Sol. (c) The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m. Thus, the shear force

$$F = T = mg = 0.01 \text{ kg} \times 9.8 \text{ ms}^{-2} = 9.8 \times 10^{-2} \text{N}$$

$$\text{Shear stress on the fluid} = \frac{F}{A} = \frac{9.8 \times 10^{-2}}{0.10}$$

$$\text{Strain rate} = \frac{v}{t} = \frac{0.085}{0.030}$$

$$\eta = \frac{\text{Stress}}{\text{Strain rate}}$$

$$\eta = \frac{(9.8 \times 10^{-2} \text{ N})\,(0.30 \times 10^{-3} \text{ m})}{(0.085 \text{ ms}^{-1})\,(0.10 \text{ m}^2)}$$

$$\eta = 3.45 \times 10^{-3} \text{ Pa-s}$$

Poiseuille's Formula

In case of steady flow of a liquid of viscosity η in a capillary tube of length L and radius R under a pressure difference p across it, the volume of liquid flowing per second is given by

$$\frac{dQ}{dt} = \frac{\pi p R^4}{8\eta L}$$

This is called *Poiseuille's formula*.

Poiseuille's equation can also be written as

$$Q = \frac{p_1 - p_2}{\left(\frac{8\eta L}{\pi R^4}\right)} = \frac{\Delta p}{X}$$

Here, $$X = \frac{8\eta L}{\pi R^4}$$

Example 30. *Water is flowing through a horizontal tube 8 cm in diameter and 4 km in length at the rate of 20 L/s. Assuming only viscous resistance. The pressure required to maintain the flow in terms of mercury column. (Coefficient of viscosity of water is* 0.001 *Pa-s) is*

(a) 69.68 *cm* (b) 59.68 *cm* (c) 49.68 *cm* (d) 39.68 *cm*

Sol. (b) Here, $2r = 8 \text{ cm} = 0.08 \text{ m}$

or $$r = 0.04 \text{ m};\ l = 4 \text{ km} = 4000 \text{ m};$$

$$V = 20 \text{ L/s} = 20 \times 10^{-3} \text{ m}^3\text{s}^{-1},$$

$$\eta = 0.001 \text{Pa-s},\ p = ?$$

As, $$V = \frac{\pi p r^4}{8\eta l}$$

or $$p = \frac{8V\eta l}{\pi r^4} = \frac{8 \times (20 \times 10^{-3}) \times 0.001 \times 4000}{\left(\frac{22}{7}\right) \times (0.04)^4}$$

$$= 7.954 \times 10^4 \text{ Pa}$$

$\therefore$ Height of mercury column for pressure difference p will be

$$h = \frac{p}{\rho g} = \frac{7.954 \times 10^4}{(13.6 \times 10^3) \times 9.8}$$

$$= 0.5968 \text{ m} = 59.68 \text{ cm}$$

Stoke's Law and Terminal Velocity

When a small spherical body falls in a long liquid column, then after sometime it falls with a constant velocity, called *terminal velocity*.

When a small spherical body falls in a liquid column with terminal velocity, then viscous force acting on it is

$$F = 6\pi\eta rv$$

where, r = radius of the body, v = terminal velocity and η = coefficient of viscosity.

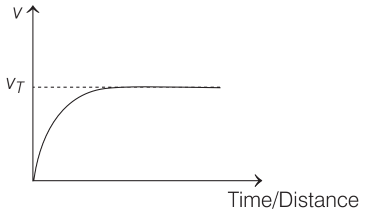
This is called *Stoke's law*.

Terminal velocity, $$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where, ρ = density of body,
σ = density of liquid,
η = coefficient of viscosity of liquid
and g = acceleration due to gravity.

(i) If $\rho > \sigma$, the body falls downwards.
(ii) If $\rho < \sigma$, the body moves upwards with the constant velocity.
(iii) If $\sigma << \rho$, $v = \frac{2r^2\rho g}{9\eta}$.

- Terminal velocity depends on the radius of the sphere in such a way that, if radius becomes n times, then terminal velocity will become n^2 times.
- Terminal velocity-Time/distance graph



Importance of Stoke's Law

- This law is used in the determination of electronic charge by Millikan in his oil drop experiment.
- This law helps a man coming down with the help of a parachute.
- This law accounts for the formation of clouds.

Example 31. *A rain drop of radius* 0.3 *mm has a terminal velocity in air* $1\ ms^{-1}$. *The viscosity of air is* 18×10^{-5} *poise. Find the viscous force on the rain drops.*

(a) $2.05\times10^{-7}\ N$ (b) $1.018\times10^{-7}\ N$
(c) $1.05\times10^{-7}\ N$ (d) $2.058\times10^{-7}\ N$

Sol. (*b*) Here, $r = 0.3\ \text{mm} = 0.3\times10^{-3}$ m; $v = 1\text{ms}^{-1}$

$$\eta = 18\times10^{-5}\ \text{poise} = 18\times10^{-6}\ \text{decapoise}$$

Viscous force, $F = 6\pi\eta rv$

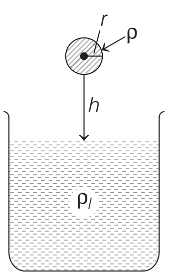
$$= 6\times\frac{22}{7}\times(18\times10^{-6})\times(0.3\times10^{-3})\times1$$

$$= 1.018\times10^{-7}\ \text{N}$$

Example 32. *In an experiment to verify Stoke's law, a small spherical ball of radius r and density* ρ *falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to (Ignore viscosity of air)* **[JEE Main 2020]**

(a) r^4 (b) r (c) r^3 (d) r^2

Sol. (a) Let ρ_l be the density of water.



Velocity of the ball after free fall, $v = \sqrt{2gh}$ ...(i)

After falling through distance h, the velocity of the ball will be equal to terminal velocity.

Terminal velocity, $v = \dfrac{2r^2g(\rho_l - \rho)}{9\eta}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2g}{\eta}(\rho_l - \rho)$$

$$\Rightarrow \quad 2gh = \frac{4}{81}\frac{r^4g^2}{\eta^2}(\rho_l - \rho)^2$$

$$\Rightarrow \quad h = \frac{2}{81}\frac{r^4g^2}{\eta^2}(\rho_l - \rho)^2 \Rightarrow h \propto r^4$$

Hence, correct option is (a).

Example 33. *The terminal velocity of a copper ball of radius 2 mm falling through a tank of oil at* $20°C$ *is* $6.5\ cms^{-1}$. *The viscosity of the oil at* $20°C$ *is [Given, density of oil is* $1.5\times10^3\ kgm^{-3}$, *density of copper is* $8.9\times10^3\ kgm^{-3}$*]*

(a) $3.3\times10^{-1}\ kgm^{-1}s^{-1}$ (b) $6.3\times10^{-2}\ kgm^{-1}s^{-1}$
(c) $9.2\times10^{-3}\ kgm^{-1}s^{-1}$ (d) $9.9\times10^{-1}\ kgm^{-1}s^{-1}$

Sol. (*d*) Given, $v_t = 6.5\times10^{-2}\ \text{ms}^{-1}$, $a = 2\times10^{-3}$ m, $g = 9.8\ \text{ms}^{-2}$, $\rho = 8.9\times10^3\ \text{kgm}^{-3}$

$$\sigma = 1.5\times10^3\ \text{kgm}^{-3}$$

$$\therefore \quad \eta = \frac{2a^2(\rho-\sigma)g}{9v_t}$$

$$\eta = \frac{2}{9}\times\frac{(2\times10^{-3})^2\times9.8}{6.5\times10^{-2}}\times7.4\times10^3\ \text{kgm}^{-3}$$

$$\eta = 9.9\times10^{-1}\ \text{kgm}^{-1}\text{s}^{-1}$$

Example 34. *Eight spherical rain drops of equal size are falling vertically through air with a terminal velocity of* $0.10\ ms^{-1}$. *What should be the velocity, if these drops were to combine to form one large spherical drop?*

(a) $0.1\ ms^{-1}$ (b) $0.2\ ms^{-1}$
(c) $0.3\ ms^{-1}$ (d) $0.4\ ms^{-1}$

Sol. (*d*) Let r be the radius of each of the small rain drop and R be the radius of big rain drop formed.

As, volume of big drop = 8 × volume of each small drop

$$\therefore \quad \frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3$$

$$R = 2r$$

Let terminal velocity of small drop be v_1 and of big drop be v_2.

As, terminal velocity,

$$v = \frac{2r^2(\rho-\sigma)g}{9\eta} \quad \text{or} \quad v \propto r^2$$

$$\therefore \quad \frac{v_2}{v_1} = \frac{R^2}{r^2}$$

$$\text{or} \quad v_2 = v_1\frac{R^2}{r^2} = 0.1\left(\frac{2r}{r}\right)^2 = 0.1\times4 = 0.4\ \text{ms}^{-1}$$

Practice Exercise

ROUND I Topically Divided Problems

Density and Relative Density of Substance

1. A beaker containing water is balanced on the pan of a common balance. A solid of specific gravity 1 and mass 5 g is tied to the arm of the balance and immersed in water contained in the beaker. The scale pan with the beaker

(a) goes down
(b) goes up
(c) remains unchanged
(d) None of the above

2. A U-tube contains 10 cm of water and 12.5 cm of methylated spirit separated by mercury. If the 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

(a) 0.221 cm (b) 2.22 cm
(c) 0.02 cm (d) None of these

3. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. The specific gravity of spirit would be

(a) 0.70 (b) 0.80
(c) 0.90 (d) 0.60

4. The density ρ of water of bulk modulus B at a depth y in the ocean is related to the density at surface ρ_0 by the relation

(a) $\rho = \rho_0\left(1 - \frac{\rho_0 gy}{B}\right)$ (b) $\rho = \rho_0\left(1 + \frac{\rho_0 gy}{B}\right)$

(c) $\rho = \rho_0\left(1 + \frac{B}{\rho_0 hgy}\right)$ (d) $\rho = \rho_0\left(1 - \frac{B}{\rho_0 gy}\right)$

5. An ice block floats in a liquid whose density is less than water. A part of block is outside the liquid. When whole of ice has melted, the liquid level will

(a) rise
(b) go down
(c) remain same
(d) first rise then go down

Pressure due to Fluid Column and Pascal's Law

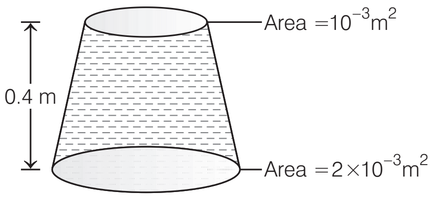
6. Density of ice is ρ and that of water is σ. What will be the decrease in volume when a mass M of ice melts?

(a) $\frac{M}{\sigma - \rho}$ (b) $\frac{\sigma - \rho}{M}$ (c) $M\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$ (d) $\frac{1}{M}\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$

7. A 50 kg girl wearing high heel shoes balances on a single heel. If the heel is circular with a diameter 1.0 cm. What is the pressure exerted on the horizontal floor?

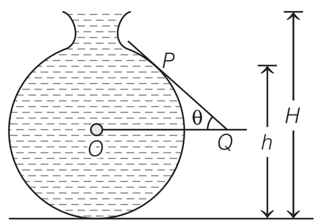
(a) 6.9×10^6 Pa (b) 6.2×10^6 Pa
(c) 9.6×10^6 Pa (d) 9.0×10^6 Pa

8. A uniform tapering vessel shown in figure is filled with liquid of density 900 kgm^{-3}. The force that acts on the base of the vessel due to liquid is (Take, $g = 10\,\text{ms}^{-2}$)



(a) 3.6 N (b) 7.2 N
(c) 9.0 N (d) 12.0 N

9. Figure shows the vertical cross-section of a vessel filled with a liquid of density ρ. The normal thrust per unit area on the walls of the vessel at point P, as shown will be



(a) $h\rho g$ (b) $H\rho g$
(c) $(H - h)\rho g$ (d) $(H - h)\rho g \cos\theta$

10. Torricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg/m^3. Determine the height of the wine column for normal atmospheric pressure.
(a) 9.5 m (b) 5.5 m
(c) 10.5 m (d) 11.5 m

11. A tank 5m high is half filled with water and then is filled to the top with oil of density 0.85 gcm^{-3}. The pressure at the bottom of the tank, due to these liquids is
(a) 1.85 g dyne cm^{-2} (b) 89.25 g dyne cm^{-2}
(c) 462.5 g dyne cm^{-2} (d) 500 g dyne cm^{-2}

12. A cylindrical vessel is filled with equal amounts of weight of mercury and water. The overall height of the two layers is 29.2 cm, specific gravity of mercury is 13.6. Then the pressure of the liquid at the bottom of the vessel is
(a) 29.2 cm of water (b) 29.2/13.6 cm of mercury
(c) 4 cm of mercury (d) 15.6 cm of mercury

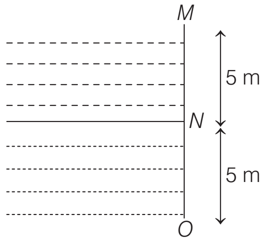
13. An aquarium tank is in the shape of a cube with one side a 4m tall glass wall. When the tank is half filled and the water is 2 m deep, the water exerts a force F on the wall. What force does the water exerts on the wall when the tank is full and the water is 4 m deep?
(a) 1/2 F (b) F
(c) 2 F (d) 4 F

14. The surface area of air bubble increases four times when it rises from bottom to top of a water tank where the temperature is uniform. If the atmospheric pressure is 10 m of water, the depth of the water in the tank is
(a) 30 m (b) 40 m
(c) 70 m (d) 80 m

15. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be (Assume that, atmospheric pressure $= 1 \times 10^5$ Pa, density of water $= 10^3$ kg m^{-3}, $g = 10\,\text{ms}^{-2}$) **[JEE Main 2021]**
(a) $\frac{200}{3}\%$ (b) $\frac{200}{5}\%$ (c) $\frac{5}{200}\%$ (d) $\frac{3}{200}\%$

16. Two cubes each weighting 22 g exactly are taken. One is of iron ($d = 8 \times 10^3$ kgm^{-3}) and the other is of marble ($D = 3 \times 10^3$ kgm^{-3}). They are immersed in alcohol and then weighted again
(a) iron cube weights less (b) iron cube weights more
(c) both have equal weight (d) nothing can be said

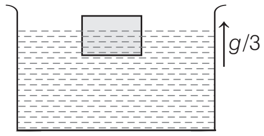
17. Two liquids of densities ρ_1 and $\rho_2 (\rho_2 = 2\rho_1)$ are filled up behind a square wall of side 10 m as shown in figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (assume that the liquids are not mixing)



[JEE Main 2020]
(a) 1/2 (b) 2/3 (c) 1/4 (d) 1/3

Archimedes' Principle and Laws of Floatation

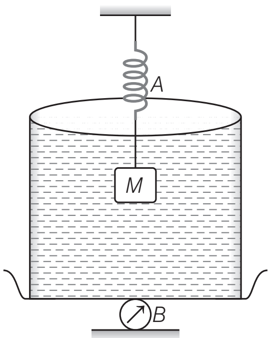
18. A cubic block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upward with acceleration of $g/3$, then the fraction of volume immersed in the liquid will be



(a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

19. The total weight of a piece of wood is 6 kg. In the floating state in water, its $\frac{1}{3}$ part remains inside the water. On this floating solid, what maximum weight is to be put such that the whole of the piece of wood is to be drowned in the water?
(a) 12 kg (b) 10 kg (c) 14 kg (d) 15 kg

20. The spring balance A reads 2 kg with a block of mass m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in a beaker as shown in figure.



(a) The balance A will read more than 2 kg
(b) The balance B will read less than 5 kg

(c) The balance A will read less than 2 kg and B will read more than 5 kg

(d) The balance A will read more than 2 kg and B will read less than 5 kg

21. A cylinder of mass m and density ρ hanging from a string is lowered into a vessel of cross-sectional area A containing a liquid of density $\sigma\,(<\rho)$ until it is fully immersed. The increase in pressure at the bottom of the vessel is

(a) Zero (b) $\frac{mg}{A}$

(c) $\frac{mg\rho}{\sigma A}$ (d) $\frac{m\sigma g}{\rho A}$

22. A rectangular plate 2m × 3m is immersed in water in such a way that its greatest and least depth are 6 m and 4 m respectively, from the water surface. The total thrust on the plate is

(a) 294×10^3 N (b) 294 N

(c) 100×10^3 N (d) 400×10^3 N

23. A body of density ρ is dropped from rest at a height h into a lake of density σ, where $\sigma > \rho$. Neglecting all dissipative forces, calculate the maximum depth to which the body sinks before returning to float on the surface.

(a) $\frac{h}{\sigma-\rho}$ (b) $\frac{h\rho}{\sigma}$

(c) $\frac{h\rho}{\sigma-\rho}$ (d) $\frac{h\sigma}{\sigma-\rho}$

24. A hemispherical bowl just floats without sinking in a liquid of density 1.2×10^3 kgm^{-3}. If outer diameter and the density of the bowl are 1 m and 2×10^4 kgm^{-3} respectively, then the inner diameter of the bowl will be

(a) 0.94 m (b) 0.96 m

(c) 0.98 m (d) 0.99 m

25. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now, the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is **[JEE Main 2019]**

(a) zero (b) 5.0 mm

(c) 4.0 mm (d) 3.0 mm

26. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is **[JEE Main 2019]**

(a) 0.6 (b) 0.8 (c) 0.7 (d) 0.5

27. A cubical block of wooden edge l and a density ρ floats in water of density 2ρ. The lower surface of cube just touches the free end of a massless spring of force constant k fixed at the bottom of the vessel. The weight w put over the block so that it is completely immersed in water without wetting the weight is

(a) $a\,(l\rho g + k)$ (b) $a\,(l^2\rho g + k)$

(c) $a\left(\frac{l\rho g}{2} + 2k\right)$ (d) $l\left(l^2\rho g + \frac{k}{2}\right)$

28. Two cylinders of same cross-section and length L but made of two material of densities ρ_1 and ρ_2 (in CGS units) are cemented together to form a cylinder of length $2L$. If the combination floats in water with a length $L/2$ above the surface of water and $\rho_1 < \rho_2$, then

(a) $\rho_1 > 1$ (b) $\rho_1 < 3/4$

(c) $\rho_1 > 1/2$ (d) $\rho_1 > 3/4$

29. The density of ice is 0.9 gcc^{-1} and that of sea water is 1.1 gcc^{-1}. An ice berg of volume V is floating in sea water. The fraction of ice berg above water level is

(a) 1/11 (b) 2/11 (c) 3/11 (d) 4/11

30. A hollow cylinder of mass m made heavy at its bottom is floating vertically in water. It is tilted from its vertical position through an angle θ and then released. The restoring force acting on it is

(a) $mg\cos\theta$ (b) $mg\sin\theta$

(c) $mg\left[\frac{1}{\cos\theta} - 1\right]$ (d) $mg\left[\frac{1}{\cos\theta} + 1\right]$

Flow of Liquids

31. Fig. (i) and Fig. (ii) refer to the steady flow of a (non-viscous) liquid. Which of the figures is/are incorrect?

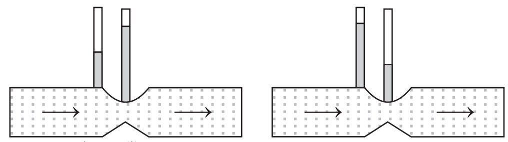
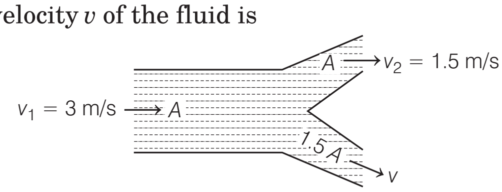


Fig. (i) Fig. (ii)

(a) Fig. (i) (b) Fig. (ii)

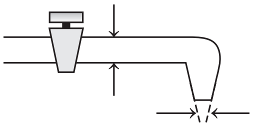
(c) both (i) and (ii) (d) None of these

32. An incompressible liquid flows through a horizontal tube as shown in the figure. Then, the velocity v of the fluid is



(a) 3 m/s (b) 1.5 m/s (c) 1.0 m/s (d) 2.25 m/s

33. Water flowing out of the mouth of a tap and falling vertically in streamline flow forms a tapering column, *i.e.* the area of cross-section of the liquid column decreases as it moves down. Which of the following is the most accurate explanation for this?



(a) Falling water tries to reach a terminal velocity and hence, reduces the area of cross-section to balance upward and downward forces
(b) As the water moves down, its speed increases and hence, its pressure decreases. It is then compressed by atmosphere
(c) The surface tension causes the exposed surface area of the liquid to decrease continuously
(d) The mass of water flowing out per second through any cross-section must remain constant. As the water is almost incompressible, so the volume of water flowing out per second must remain constant. As this is equal to velocity × area, the area decreases as velocity increases.

34. Two water pipes P and Q having diameter 2×10^{-2} m and 4×10^{-2} m respectively are joined in series with the main supply line of water. The velocity of water flowing in pipe P is

(a) 4 times that of Q (b) 2 times that of Q
(c) (1/2) times that of Q (d) (1/4) times that of Q

35. A fluid flows through a horizontal pipe having two different cross-sections of area A and $2A$. If the pressure at the thin cross-section is p and fluid velocity is v, the velocity and pressure at the thicker cross-section is (take the density of fluid as ρ)

(a) $\frac{v}{2}, p+\frac{1}{2}\rho v^2$ (b) $\frac{v}{4}, p+\frac{3}{8}\rho v^2$
(c) $\frac{v}{2}, p+\frac{3}{8}\rho v^2$ (d) $v, p+\frac{3}{4}\rho v^2$

36. If two ping pong balls are suspended near each other and a fast stream of air is produce within the space of the balls, the balls

(a) come nearer to each other
(b) move away from each other
(c) remain in their original positions
(d) move far away

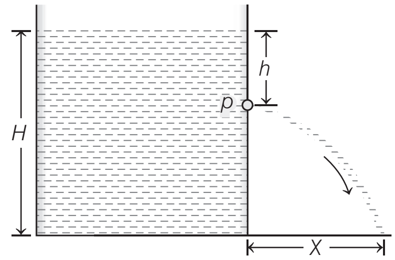
37. Two identical cylindrical vessels are kept on the ground and each contains the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is **[JEE Main 2020]**

(a) $gdS\,(x_2^2 + x_1^2)$ (b) $\frac{3}{4}\,gdS\,(x_2 - x_1)^2$
(c) $gdS\,(x_2 + x_1)^2$ (d) $\frac{1}{4}\,gdS\,(x_2 - x_1)^2$

38. Air is streaming past a horizontal air plane wing such that its speed is 120 ms^{-1} over the upper surface and 90 ms^{-1} at the lower surface. If the density of air is 1.3 kgm^{-3}, what will be the gross lift on the wing? If the wing is 10 m long and has an average width of 2 m,

(a) 81.9 N (b) 8.19 kN (c) 81.9 kN (d) 819 kN

39. A tank is filled with water upto a height H. Water is allowed to come out of a hole P in one of the walls at a depth h below the surface of water (see figure). Express the horizontal distance X in terms of H and h.



(a) $X = \sqrt{h\,(H-h)}$ (b) $X = \sqrt{\frac{h}{2}(H-h)}$
(c) $X = 2\sqrt{h\,(H-h)}$ (d) $X = 4\sqrt{(H-h)}$

40. A cylindrical drum, open at the top, contains 15 L of water. It drains out through a small opening at the bottom. 5 L of water comes out in time t_1, the next 5 L in further time t_2 and the last 5 L in further time t_3. Then

(a) $t_1 < t_2 < t_3$ (b) $t_1 > t_2 > t_3$
(c) $t_1 = t_2 = t_3$ (d) $t_2 > t_1 = t_3$

41. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are 70 m/s and 63 m/s respectively. What is the lift on the wing, if its area is 2.5 m^2? Take the density of air to be 1.3 kg/m^3.

(a) 5.1×10^2 N (b) 6.1×10^2 N
(c) 1.6×10^3 N (d) 1.5×10^3 N

Surface Tension and Surface Energy

42. A thin metal disc of radius r float on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of water w and surface tension of water is T, then the weight of metal disc is

(a) $2\pi rT + w$ (b) $2\pi rT\cos\theta - w$
(c) $2\pi rT\cos\theta + w$ (d) $w - 2\pi rT\cos\theta$

43. A ring is cut from a platinum tube 8.5 cm internal diameter and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so, that it comes in contact with the water in glass vessel. If an extra 3.47 g-wt is required to pull it away from water, surface tension of water is

(a) 62.96 dyne cm^{-1} (b) 70.80 dyne cm^{-1}
(c) 65.35 dyne cm^{-1} (d) 60.00 dyne cm^{-1}

44. What is the radius of the biggest aluminium coin of thickness, t and density ρ, which will still be able to float on the water surface of surface tension S?

(a) $\frac{4S}{3\rho gt}$ (b) $\frac{3S}{4\rho gt}$
(c) $\frac{2S}{\rho gt}$ (d) $\frac{S}{\rho gt}$

45. 8000 identical water drops are combined to form a big drop then the ratio to the final surface energy to the initial surface energy, if all the drops together is

(a) 1 : 10 (b) 1 : 15
(c) 1 : 20 (d) 1 : 25

46. A mercury drop of radius 1 cm is broken into 10^6 droplets of equal size. The work done is
(Take, $S = 35 \times 10^{-2}\ \text{Nm}^{-1}$)

(a) 4.35×10^{-2} J (b) 4.35×10^{-3} J
(c) 4.35×10^{-6} J (d) 4.35×10^{-8} J

47. What change in surface energy will be noticed when a drop of radius R splits up into 1000 droplets of radius r, surface tension S ?

(a) $4\pi R^2 S$ (b) $7\pi R^2 S$
(c) $16\pi R^2 S$ (d) $36\pi R^2 S$

48. Let W be the work done, when a bubble of volume V is formed from a given solution. How much work is required to be done to form a bubble of volume 2 V?

(a) W (b) $2W$
(c) $2^{1/3}\ W$ (d) $4^{1/3}\ W$

49. What is the ratio of surface energy of 1 small drop and 1 large drop if 1000 drops combine to form 1 large drop?

(a) 100 : 1 (b) 1000 : 1
(c) 10 : 1 (d) 1 : 100

50. Two spherical soap bubbles of radii a and b in vacuum coalesce under isothermal conditions. The resulting bubble has a radius given by

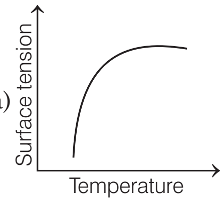
(a) $\frac{(a+b)}{2}$

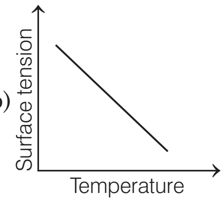
(b) $\frac{ab}{a+b}$

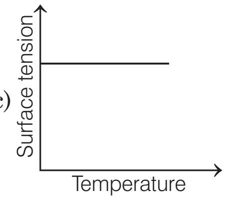
(c) $\sqrt{a^2+b^2}$

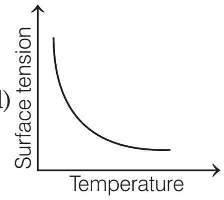
(d) $a+b$

51. Which graph represent the variation of surface tension with temperature over small temperature ranges for water?

(a) 

(b) 

(c) 

(d) 

52. The amount of work done in blowing a soap bubble such that its diameter increases from d to D is
(Here, S = surface tension of solution)

(a) $\pi(D^2-d^2)S$ (b) $2\pi(D^2-d^2)S$
(c) $4\pi(D^2-d^2)S$ (d) $8\pi(D^2-d^2)S$

Excess of Pressure, Shape of Meniscus and Capillarity

53. Pressure inside two soap bubbles are 1.01 atm and 1.02 atm, respectively. The ratio of their volume is **[JEE Main 2020]**

(a) 4 : 1 (b) 0.8 : 1 (c) 8 : 1 (d) 2 : 1

54. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles with glass are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio (r_1 / r_2), is then close to **[JEE Main 2019]**

(a) 3/5 (b) 2/3 (c) 2/5 (d) 4/5

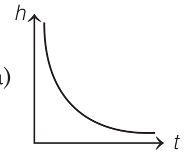
55. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80\ \text{m/s}^2$ and the simplified relation $T = \frac{rhg}{2} \times 10^3 \text{N/m}$, the possible error in surface tension is closest to **[JEE Main 2017]**

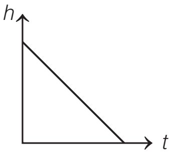
(a) 1.5% (b) 2.4% (c) 10% (d) 0.15%

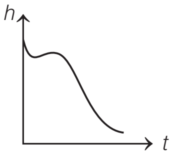
56. When two soap bubbles of radius r_1 and r_2 $(r_2 > r_1)$ coalesce, the radius of curvature of common surface is

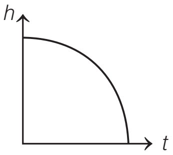
(a) $(r_2 - r_1)$ (b) $(r_2 + r_1)$ (c) $\frac{r_2 - r_1}{r_1 r_2}$ (d) $\frac{r_2 r_1}{r_2 - r_1}$

57. Water in a vessel of uniform cross-section escapes through a narrow tube at the base of the vessel. Which graph given below represents the variation of the height h of the liquid with time t?

(a) 

(b) 

(c) 

(d) 

58. A capillary tube of radius R and length L is connected in series with another tube of radius $R/2$ and length $L/4$. If the pressure difference across the two tubes taken together is p, then the ratio of pressure difference across the first tube to that across the second tube is

(a) 1 : 4 (b) 1 : 1
(c) 4 : 1 (d) 2 : 1

59. The rate of steady volume flow of water through a capillary tube of length l and radius r under a pressure difference of p, is V. This tube is connected with another tube of the same length but half the radius in series. Then the rate of steady volume flow through them is (The pressure difference across the combination is p)

(a) $\frac{V}{16}$ (b) $\frac{V}{17}$
(c) $\frac{16V}{17}$ (d) $\frac{17V}{16}$

60. The rate of flow of liquid through a capillary tube of radius r is V, when the pressure difference across the two ends of the capillary is p. If pressure is increased by $3p$ and radius is reduced to $r/2$, then the rate of flow becomes

(a) $V/9$ (b) $3V/8$
(c) $V/4$ (d) $V/3$

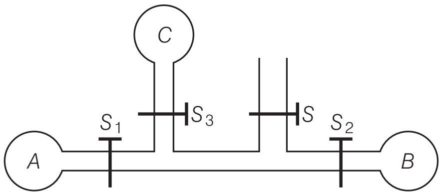
61. When two soap bubbles of radii a and $b\,(b > a)$ coalesce, the radius of curvature of common surface is **[JEE Main 2021]**

(a) $\frac{ab}{b-a}$ (b) $\frac{a+b}{ab}$
(c) $\frac{b-a}{ab}$ (d) $\frac{ab}{a+b}$

62. The angle of contact at the interface of water-glass is 0° Ethylalcohol-glass is 0°, Mercury-glass is 140° and Methyliodide-glass is 30°. A glass capillary is put in a trough containing one of these four liquids. It is observed that the meniscus is convex. The liquid in the trough is **[NCERT Exemplar]**

(a) water (b) ethylalcohol
(c) mercury (d) methyliodide

63. The diagram shows three soap bubbles A, B and C prepared by blowing the capillary tube fitted with stop cocks S, S_1, S_2 and S_3. With stop cock S closed and stop cocks S_1, S_2 and S_3 opened



(a) B will start collapsing with volumes of A and C increasing
(b) C will start collapsing with volume of A and B increasing
(c) volume of A, B and C will become equal in equilibrium
(d) C and A will both start collapsing with volume of B increasing

64. Water rises in a capillary tube to a height h. It will rise to a height more than h

(a) on the surface of sun
(b) in a lift moving down with an acceleration
(c) at the poles
(d) in a lift moving up with an acceleration

65. Water rises to a height of 16.3 cm in a capillary of height 18 cm above the water level. If the tube is cut at a height of 12 cm in the capillary tube, then

(a) water will come as a fountain from the capillary tube
(b) water will stay at a height of 12 cm in the capillary tube
(c) the height of water in the capillary tube will be 10.3 cm
(d) water height flow down the sides of the capillary tube

66. Two capillary tubes of radii 0.2 cm and 0.4 cm are dipped in the same liquid. The ratio of heights through which liquid will rise in the tubes is

(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

67. By inserting a capillary tube upto a depth l in water, the water rises to a height h. If the lower end of the capillary tube is closed inside water and the capillary is taken out and closed end opened, to what height the water will remain in the tube, when $l > h$?

(a) zero (b) $l + h$ (c) $2h$ (d) h

68. Water rises to a height of 10 cm in a capillary tube and mercury falls to a depth of 3.42 cm in the same capillary tube. If the ratio of density of mercury and water is 13.5, then the ratio of surface tension of water and mercury is
(a) 1 : 0.15
(b) 1 : 3
(c) 1 : 6.5
(d) 1.5 : 1

Stoke's Law, Terminal Velocity and Variation of Viscosity

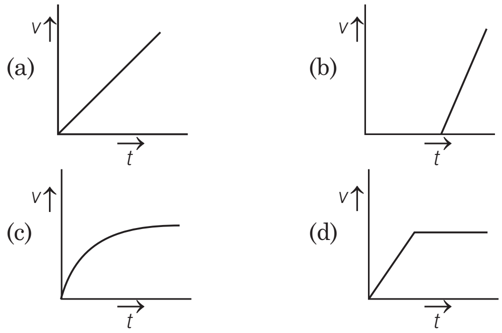
69. The relative velocity of two parallel layers of water is 8 cms^{-1}. If the perpendicular distance between the layers is 0.1 cm, then velocity gradient will be
(a) 40 s^{-1}
(b) 50 s^{-1}
(c) 60 s^{-1}
(d) 80 s^{-1}

70. A small spherical ball of steel falls through a viscous medium with terminal velocity v. If a ball of twice the radius of the first one but of the same mass is dropped through the same method, it will fall with a terminal velocity (neglect buoyancy)
(a) $\frac{v}{2}$ (b) $\frac{v}{\sqrt{2}}$
(c) v (d) $2v$

71. The terminal velocity v of a spherical ball of lead of radius R falling through a viscous liquid varies with R such that
(a) $\frac{v}{R} = \text{constant}$ (b) $vR = \text{constant}$
(c) $v = \text{constant}$ (d) $\frac{v}{R^2} = \text{constant}$

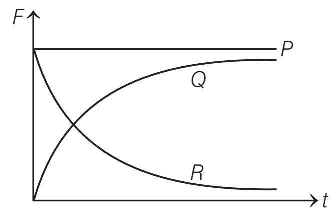
72. A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2 when falling through the same fluid, the ratio (v_1 / v_2) equals **[JEE Main 2019]**
(a) 9 (b) 1/27
(c) 1/9 (d) 27

73. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in figure, indicate the one that represents the velocity (v) of the pebble as a function of time (t).



74. A rain drop of radius 1.5 mm, experiences a drag force $F = (2 \times 10^{-5}\, v)$ N, while falling through air from a height 2 km, with a velocity v. The terminal velocity of the rain drop will be nearly (use $g = 10\ ms^{-2}$)
(a) 200 ms^{-1} (b) 80 ms^{-1}
(c) 7 ms^{-1} (d) 3 ms^{-1}

75. A spherical ball is dropped in a long column of viscous liquid. Which of the following graphs represent the variation of
(i) gravitational force with time
(ii) viscous force with time
(iii) net force acting on the ball with time?



(a) Q, R, P (b) R, Q, P
(c) P, Q, R (d) R, P, Q

76. A small iron sphere is dropped from a great height. It attains its terminal velocity after fallen 32 m. Then, it covers the rest of the path with terminal velocity only. The work done by air friction during the first 32 m of fall is W_1. The work done by air friction during the subsequent 32 m fall is W_2, then
(a) $W_1 > W_2$ (b) $W_1 < W_2$
(c) $W_1 = W_2$ (d) $W_2 = 32\, W_1$

77. A marble of mass x and diameter $2r$ is gently released a tall cylinder containing honey. If the marble displaces mass $y\ (< x)$ of the liquid, then the terminal velocity is proportional to
(a) $(x + y)$ (b) $(x - y)$
(c) $\frac{x+y}{r}$ (d) $\frac{(x-y)}{r}$

ROUND II Mixed Bag

Only One Correct Option

1. Wax is coated on the inner wall of a capillary tube and the tube is then dipped in water. Then, compared to the unwaxed capillary, the angle of contact θ and the height h upto which water rises change. These changes are **[JEE Main 2013]**

(a) θ increases and h also increases
(b) θ decreases and h also decreases
(c) θ increases and h decreases
(d) θ decreases and h increases

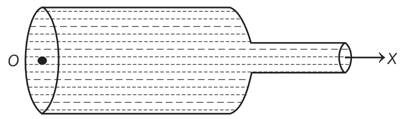
2. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa-s, respectively. (Take, density of water = 10^3 kg/m^3) **[JEE Main 2021]**

(a) Unsteady flow to steady flow
(b) Remains steady flow
(c) Remains turbulent flow
(d) Steady flow to unsteady flow

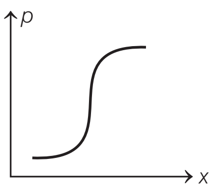
3. A film of water is found between two straight parallel wires of length 10 cm each separated by 0.2 cm. If their separation is increased by 1 mm, while still maintaining their parallelism, how much work will have to be done? (surface tension of water is 7.2×10^{-2} Nm^{-1})

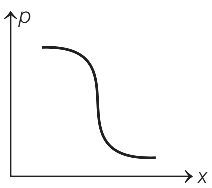
(a) 7.22×10^{-6} J (b) 1.44×10^{-5} J
(c) 2.88×10^{-8} J (d) 5.76×10^{-5} J

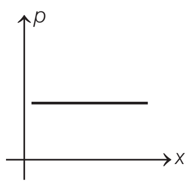
4. A non-viscous liquid is flowing through a frictionless duct, with cross-section varying as shown in figure.

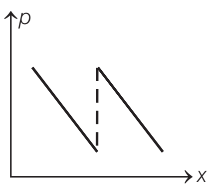


Which of the following graph represents the variation of pressure p along the axis of tube?

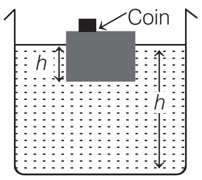
(a)


(b)


(c)


(d)


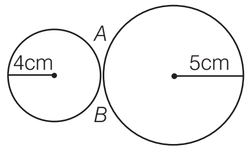
5. A wooden block with a coin placed on its top.floats in water as shown in figure



The distance l and h are shown in the figure. After some time the coin falls into the water. Then, **[NCERT Exemplar]**

(a) l decreases (b) h increases
(c) l increases (d) None of these

6. Two soap bubbles of radii r_1 and r_2 equal to 4 cm and 5 cm respectively are touching each other over a common surface AB (shown in figure). Its radius will be

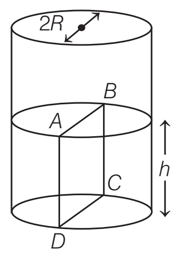


(a) 4 cm (b) 4.5 cm (c) 5 cm (d) 20 cm

7. A spring balance reads w_1 when a ball of mass m is suspended from it. A weighing machine reads w_2 when a beaker of liquid is kept on the pan of balance. When the ball is immersed in liquid, the spring balance reads w_3 and the weighing machine reads w_4. The two balances are now so arranged that the suspended mass is inside the liquid in a beaker. Then,

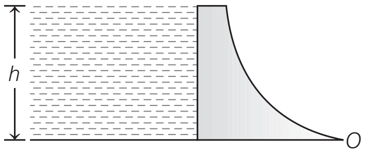
(a) $w_3 > w_1$ (b) $w_4 > w_2$
(c) $w_3 < w_1$ and $w_4 > w_2$ (d) $w_3 > w_1$ and $w_4 < w_2$

8. Water is filled up to a height h in beaker of radius R as shown in the figure. The density of water ρ the surface tension of water is T and the atmosphere pressure is p_0. Consider a vertical section $ABCD$ of the water column through n diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude.



(a) $|2p_0Rh + \pi r^2\rho gh - 2RT|$ (b) $|2p_0Rh + R\rho gh^2 - 2RT|$
(c) $|p_0\pi R^2 + R\rho gh^2 - 2RT|$ (d) $|p_0\pi R^2 + R\rho gh^2 + 2RT|$

9. Water of density ρ at a depth h behind the vertical face of dam whose cross-sectional length is λ and cross-sectional area A. It exerts a horizontal resultant force on the dam tending to slide it along its foundation and a torque tending to overturn the dam about the point O.



The height at which the resultant force would have to act to the same torque is

(a) $\frac{h}{6}$ (b) $\frac{h}{3}$

(c) $\frac{h}{2}$ (d) $\frac{2h}{3}$

10. Water flows through a vertical tube of variable cross-section. The area of cross-section at A and B are 6 mm^2 and 3 mm^2, respectively. If 12 cc of water enters per second through A, find the pressure difference $p_A - p_B$ ($g = 10\,ms^{-2}$) The separation between cross-section at A and B is 100 cm.

(a) 1.6×10^5 dyne cm^{-2}

(b) 2.29×10^5 dyne cm^{-2}

(c) 5.9×10^4 dyne cm^{-2}

(d) 3.9×10^5 dyne cm^{-2}

11. A canister has a small hole at its bottom. Water penetrates into the canister when its base is at a depth of 40 cm from the surface of water. If surface tension of water is 73.5 dyne/cm, find the radius of the hole.

(a) 375 mm

(b) 3.75 mm

(c) 0.0375 mm

(d) zero

12. A piece of gold weights 50 g in air and 45 g in water. If there is a cavity inside the piece of gold, then find its volume [Density of gold = 19.3 g/cc].

(a) 2.4 cm^3 (b) 2.4 m^3

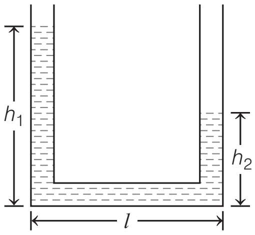
(c) 4 .2 m^3 (d) 4.2 mm^3

13. The bottom of a cylindrical vessel has a circular hole of radius r and at depth h below the water level. If the diameter of the vessel is D, then find the speed with which the water level in the vessel drops.

(a) $\frac{4r^2}{D^2}\sqrt{2gh}$ (b) $\frac{4D^2}{r^2}$

(c) $\frac{4D^2}{r^2}\sqrt{2gh}$ (d) None of these

14. A liquid of density ρ is filled in a U-tube is accelerated with an acceleration a so that the height of liquid in its two vertical arms are h_1 and h_2 as shown in the figure. If l is the length of horizontal arm of the tube, the acceleration a is



(a) $\frac{g(h_1 - h_2)}{2l}$ towards right

(b) $\frac{g(h_1 - h_2)}{2l}$ towards left

(c) $\frac{g(h_1 - h_2)}{l}$ towards right

(d) $\frac{g(h_1 - h_2)}{l}$ towards left

15. A trough contains mercury to a depth of 3.6 cm. If same amount of mercury is poured in it, then height of mercury in the trough will be

(a) 3.6 cm (b) 7.2 cm

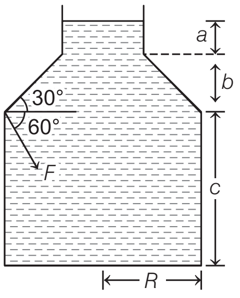
(c) 6 cm (d) None of these

16. A vessel whose bottom has round holes with diameter of 1 mm is filled with water. Assuming that surface tension acts only at holes, then the maximum height to which the water can be filled in vessel without leakage is (Surface tension of water is $75 \times 10^{-3}\,Nm^{-1}$ and $g = 10\,ms^{-2}$)

(a) 3 cm (b) 0.3 cm

(c) 3 mm (d) 3 m

17. A jar shown in figure is filled with a liquid of density ρ. The jar is placed in vacuum. Cross-section of the jar is circular and base is having a radius R. The force exerted by the liquid column on the base of the jar is



(a) $\rho g(a + b + c)\pi R^2$

(b) less than $\rho g(a + b + c)\pi R^2$

(c) greater than $\rho g(a + b + c)\pi R^2$

(d) $2\rho g(a + b + c)\pi R^2$

18. Water flows into a large tank with flat bottom at the rate of $10^{-4}\ m^3s^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is **[JEE Main 2019]**

(a) 4 cm (b) 2.9 cm (c) 5.1 cm (d) 1.7 cm

19. A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take note that the surface tension applies an upward force on the droplet) **[JEE Main 2020]**

(a) $r = \sqrt{\dfrac{3T}{(2d-\rho)g}}$ (b) $r = \sqrt{\dfrac{T}{(d-\rho)g}}$

(c) $r = \sqrt{\dfrac{T}{(d+\rho)g}}$ (d) $r = \sqrt{\dfrac{2T}{3(d+\rho)g}}$

20. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is **[JEE Main 2013]**

(a) $\dfrac{Mg}{k}$ (b) $\dfrac{Mg}{k}\left(1-\dfrac{LA\sigma}{M}\right)$

(c) $\dfrac{Mg}{k}\left(1-\dfrac{LA\sigma}{2M}\right)$ (d) $\dfrac{Mg}{k}\left(1+\dfrac{LA\sigma}{M}\right)$

21. The glycerine of density $1.25 \times 10^3\ kmg^{-3}$ is flowing through a conical tube with end radii 0.1 m and 0.04 m respectively. The pressure difference across the ends is 10 Nm^{-2}. The rate of flow of glycerine through the tube is

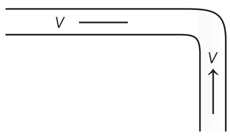
(a) $6.4 \times 10^{-2}\ m^3s^{-1}$ (b) $6.4 \times 10^{-4}\ m^3s^{-1}$

(c) $12.8 \times 10^{-2}\ m^3s^{-1}$ (d) $12.8 \times 10^{3}\ m^3s^{-1}$

22. A wooden ball of density ρ is immersed in water of density ρ_0 to depth h and then released. The height H above the surface of water upto which the ball jump out of water is

(a) zero (b) h (c) $\dfrac{\rho_0 h}{\rho}$ (d) $\left(\dfrac{\rho_0}{\rho}-1\right)h$

23. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upwards through the hydrant and then does 90° turn to emerge horizontally at speed v. The pipe and nozzle have uniform cross-section throughout. The force exerted by water on the corner of the hydrant is



(a) zero (b) $\rho v L$

(c) $2\rho v L$ (d) $\sqrt{2}\rho v L$

24. A plane of mass 3250 g is in level flight at a constant speed and each wing has an area of 25 m^2. During flight the speed of the air is 216 kmh^{-1} over the lower wing surface and 252 kmh^{-1} over the upper wing surface of each wing of aeroplane. Take density of air = 1 kgm^{-3} and $g = 10\ ms^{-2}$.

If a plane is in level flight with a speed of 360 kmh^{-1} then the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface is

(a) 13 % (b) 9 %

(c) 6.5 % (d) 4.5 %

25. Two capillaries of radii r_1 and r_2, lengths l_1 and l_2 respectively are in series. A liquid of viscosity η is flowing through the combination under a pressure difference p. What is the rate of volume flow of liquid?

(a) $\dfrac{\pi p}{8\eta}\left(\dfrac{l_4}{r_1^4}+\dfrac{l_4}{r_2^4}\right)^{-1}$ (b) $\dfrac{8\pi p}{\eta}\left(\dfrac{l_1}{r_1^4}+\dfrac{l_2}{r_2^4}\right)$

(c) $\dfrac{\pi p}{8\eta}\left(\dfrac{r_1^4}{l_1}+\dfrac{r_2^4}{l_2}\right)^{-1}$ (d) $\dfrac{\pi p}{8\eta}\left(\dfrac{l_1}{r_1^4}+\dfrac{l_2}{r_2^4}\right)^{-1}$

26. A metal ball immersed in alcohol weighs W_1 at 0°C and W_2 at 59°C. The coefficient of cubical expansion of the metal is less than that of alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

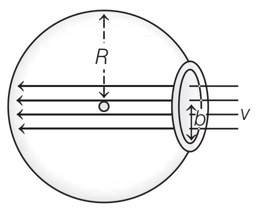
(a) $W_1 > W_2$

(b) $W_1 < W_2$

(c) $W_1 = W_2$

(d) $W_1 = 2W_2$

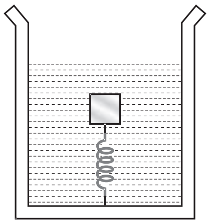
27. A soap film is made by dipping a circular frame of radius b in soap solution. A bubble is formed by blowing air with speed v in the form of cylinder. The radius of the bubble formed $R >> b$ so that the air is incident normally on the surface of bubble. Air stops after striking surface of soap bubble. Density of air is ρ. The radius R of the bubble when the soap bubble separates from the ring is (surface tension of liquid is S).



(a) $\dfrac{S}{\rho v^2}$ (b) $\dfrac{4S}{\rho v^2}$

(c) $\dfrac{Sb}{\rho v}$ (d) $\dfrac{4Sb}{\rho v^2}$

28. A block is submerged in vessel filled with water by a spring attached to the bottom of the vessel. In equilibrium, the spring is compressed. The vessel now moves downward with an acceleration a ($< g$). The spring length



(a) will become zero
(b) will decrease but not zero
(c) will increase
(d) may increase or decrease or remain constant

29. A glass tube 80 cm long and open at both ends is half immersed in mercury. Then the top of the tube is closed and it is taken out of the mercury. A column of mercury 20 cm long then remains in the tube. The atmospheric pressure (in cm of Hg) is

(a) 90 (b) 75
(c) 60 (d) 45

30. There is a hole of area A at the bottom of a cylindrical vessel. Water is filled upto a height h and water flows out in t sec. If water is filled to a height $4h$, then it will flow out in time

(a) $2\ t$ (b) $4\ t$
(c) $16\ t$ (d) $7/4\ t$

31. Two soap bubbles A and B are kept in closed chamber where the air is maintained at pressure $8\ \text{N/m}^2$.The radius of bubbles A and B are 2 cm and 4 cm respectively surface tension of the soap water used to make bubbles is 0.04 N/m. Find the ratio n_B / n_A, where n_A and n_B are the number of moles of air in bubbles A and B respectively
[Neglect the effect of gravity]

(a) 2 (b) 9
(c) 8 (d) 6

32. What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20° C) is 2.50×10^{-2} N/m? If an air bubble of the same dimension were formed at a depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is 1.01×10^5 Pa.)

(a) 7.06×10^5 Pa
(b) 2.06×10^5 Pa
(c) 1.06×10^5 Pa
(d) 1.86×10^5 Pa

33. A uniform rod of density ρ is placed in a wide tank containing a liquid σ ($\sigma > \rho$). The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position, the rod makes an angle θ with the horizontal. Then, $\sin\theta$ is equal to

(a) $\frac{1}{2}\sqrt{\frac{\sigma}{\rho}}$ (b) $\frac{1}{2}\frac{\sigma}{\rho}$
(c) $\sqrt{\frac{\rho}{\sigma}}$ (d) $\sqrt{\frac{\rho}{\sigma}}$

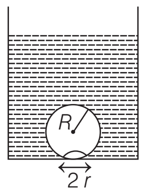
34. If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is **[JEE Main 2019]**

(a) $2M$ (b) $4M$
(c) $\frac{M}{2}$ (d) M

35. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected 25% losses all of its momentum and, 25% comes back with the same speed. The resultant pressure on the mesh will be **[JEE Main 2019]**

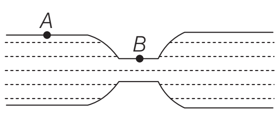
(a) ρv^2 (b) $\frac{1}{2}\rho v^2$
(c) $\frac{1}{4}\rho v^2$ (d) $\frac{3}{4}\rho v^2$

36. On heating water, bubbles beings formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ) **[JEE Main 2014]**



(a) $R^2\sqrt{\frac{2\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$
(c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

37. Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B, where the area of cross- section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube.
(Take, density of water = 1000 kgm^{-3}) **[JEE Main 2020]**



(a) 3020 cm^3/s (b) 2420 cm^3/s
(c) 2720 cm^3/s (d) 1810 cm^3/s

38. An alloy of Zn and Cu (*i.e.* brass) weighs 16.8 g in air and 14.7 g in water. If relative density of Cu and Zn are 8.9 and 7.1 respectively, then the amount of Cu and Zn in the alloy, respectively are

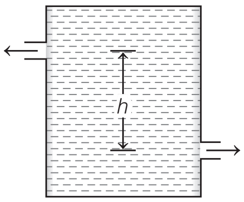
(a) 2g, 4g

(b) 4g, 2g

(c) 9.345g, 7.455 g

(d) 0, 3g

39. There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in height between the two holes is h. As the liquid comes out of the two holes, the tank will experience a net horizontal force proportional to



(a) $h^{1/2}$ (b) $h^{3/2}$

(c) h (d) h^2

40. A streamline body with relative density ρ_1 falls into air from a height h_1 on the surface of a liquid of relative density ρ_2, where $\rho_2 > \rho_1$. The time of immersion of the body into the liquid will be

(a) $\sqrt{\frac{2h_1}{g}}$ (b) $\sqrt{\frac{2h}{g}} \times \frac{\rho_1}{\rho_2}$

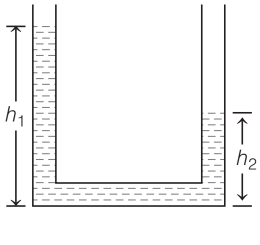
(c) $\sqrt{\frac{2h_1}{g} \times \frac{\rho_1}{\rho_2}}$ (d) $\sqrt{\frac{2h_1}{g}} \times \frac{\rho_1}{(\rho_2 - \rho_1)}$

41. Calculate the force of attraction between two parallel plates separated by a distance 0.2 mm after a water drop of mass 80 mg is introduced between them. The wetting is assumed to be complete. (surface tension of water is 0.07 Nm^{-1})

(a) 0.14 N (b) 0.28 N

(c) 0.42 N (d) 0.56 N

42. The U-tube has a uniform cross-section as shown in figure. A liquid is filled in the two arms upto heights h_1 and h_2 and then the liquid is allowed to move. Neglect viscosity and surface tension. When the level equalize in the two arms, the liquid will



(a) be at rest

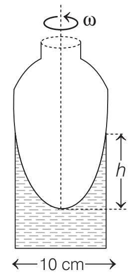
(b) be moving with an acceleration of $g\left(\frac{h_1 - h_2}{h_1 + h_2 + 2}\right)$

(c) be moving with a velocity of $(h_1 - h_2)\sqrt{\frac{g}{2(h_1 + h_2 + h)}}$

(d) exert a net force to the right on the cube

43. A cylindrical vessel containing a liquid is rotated about its axis, so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height h (in cm) of liquid at the centre of vessel and at the side will be

[JEE Main 2020I]

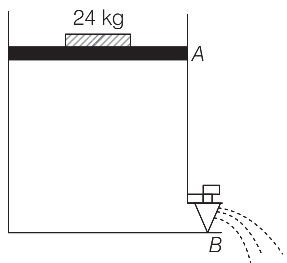


(a) $\frac{25\,\omega^2}{2g}$ (b) $\frac{5\,\omega^2}{2g}$ (c) $\frac{2\,\omega^2}{25g}$ (d) $\frac{2\,\omega^2}{5g}$

Numerical Value Questions

44. Consider a water tank as shown in the figure. Its cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-sectional area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}. The value of v, to the nearest integer, is …… . (Take, g = 10 ms^{-2})

[JEE Main 2021]



45. If a solid floats with (1/4)th its volume above the surface of water, then the density (in kgm^{-3}) of the solid will be ……… .

46. The largest average velocity (in ms^{-1}) of blood flow in artery of radius 2×10^{-3}, if the flow must remain laminar will be ……… . (Take, viscosity of blood $= 2.084 \times 10^{-3}$ Pa-s and density of blood $= 1.06 \times 10^3$ kgm^{-3})

47. A wire ring of 30.0 mm radius resting flat on the surface of the liquid is raised. If the pull required is 3.03 gf force before the film breaks, so the surface tension (in dyne cm^{-1}) of the liquid will be ……… .

48. A square plate of side 10 cm moves parallel to another plate with a velocity of 10 cm s^{-1}; both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, then their separation distance is found to be $x \times 10^{-2}$ m, then the value of x is ……… .

49. The density of ice is 917 kg m^{-3}. Therefore, the fraction of the volume of a piece of ice while floating above the fresh water will be ……… .

50. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it.If the contact angle between the liquid and glass is close to 0°, the surface tension of the liquid (in mN m^{-1}) to the nearest integer is (Take, $\rho_{(\text{liquid})} = 900\ \text{kgm}^{-3}$, $g = 10\ \text{ms}^{-2}$) ....

[JEE Main 2020]

51. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})} = 6 \times 10^{-6}\ °C^{-1}$ and $\gamma_{(\text{mercury})} = 1.5 \times 10^{-4}\ °C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ....

[JEE Main 2020]

Answers

Round I

1. (c)	2. (a)	3. (b)	4. (b)	5. (b)	6. (c)	7. (b)	8. (b)	9. (c)	10. (c)
11. (c)	12. (c)	13. (d)	14. (c)	15. (a)	16. (b)	17. (c)	18. (a)	19. (a)	20. (c)
21. (d)	22. (a)	23. (c)	24. (c)	25. (d)	26. (a)	27. (d)	28. (b)	29. (b)	30. (c)
31. (a)	32. (c)	33. (d)	34. (a)	35. (c)	36. (a)	37. (d)	38. (c)	39. (c)	40. (a)
41. (d)	42. (c)	43. (a)	44. (c)	45. (c)	46. (a)	47. (d)	48. (d)	49. (d)	50. (c)
51. (b)	52. (b)	53. (c)	54. (c)	55. (a)	56. (d)	57. (a)	58. (a)	59. (b)	60. (c)
61. (a)	62. (c)	63. (b)	64. (b)	65. (b)	66. (b)	67. (c)	68. (c)	69. (d)	70. (a)
71. (d)	72. (a)	73. (c)	74. (c)	75. (c)	76. (b)	77. (c)			

Round II

1. (c)	2. (d)	3. (b)	4. (b)	5. (a)	6. (d)	7. (b)	8. (b)	9. (b)	10. (a)
11. (c)	12. (a)	13. (a)	14. (c)	15. (b)	16. (a)	17. (c)	18. (c)	19. (a)	20. (c)
21. (b)	22. (d)	23. (d)	24. (c)	25. (d)	26. (b)	27. (b)	28. (c)	29. (c)	30. (a)
31. (d)	32. (c)	33. (a)	34. (a)	35. (d)	36. (a)	37. (c)	38. (c)	39. (c)	40. (d)
41. (b)	42. (c)	43. (a)	44. 3	45. 750	46. 0.98	47. 78.76	48. 5	49. 0.083	50. 101
51. 20									

Solutions

Round I

1. Effective weight of solid of specific gravity 1 when immersed in water, will be zero. Hence, the scale pan with the beaker remains unchanged.

2. When 15.0 cm of water is poured in each arm, then

height of water column $(h_1) = 10 + 15 = 25$ cm

Height of spirit column $(h_2) = 12.5 + 15 = 27.5$ cm

Density of water $(\rho_w) = 1$ g/cm^3

Density of spirit $(\rho_s) = 0.80$ g/cm^3

Density of mercury $(\rho_m) = 13.6$ g/cm^3

Let in equilibrium, the difference in the level of mercury in both arms be h cm.

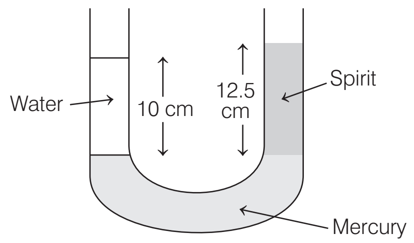
$$\therefore \quad h\rho_m g = h_1\rho_w g - h_2\rho_s g$$

$$\text{or} \quad h = \frac{h_1\rho_w - h_2\rho_s}{\rho_m} = \frac{25 \times 1 - 27.5 \times 0.80}{13.6}$$

$$= 0.221 \text{ cm}$$

Therefore, mercury will rise in the arm containing spirit by 0.221 cm.

3.



Height of water column $(h_1) = 10.0$ cm

Density of water $(\rho_1) = 1$ g/cm^3

Height of spirit column $(h_2) = 12.5$ cm

Density of spirit $(\rho_2) = ?$

The mercury column in both arms of the U-tube are at same level, therefore pressure in both arms will be same.

$\therefore$ Pressure exerted by water column = Pressure exerted by sprit column

$$\therefore \quad p_1 = p_2$$

$$h_1\rho_1 g = h_2\rho_2 g$$

$$\text{or} \quad \rho_2 = \frac{h_1\rho_1}{h_2} = \frac{10 \times 1}{12.5} = 0.80 \text{ g/cm}^3$$

$$\text{Specific gravity of spirit} = \frac{\text{Density of spirit}}{\text{Density of water}}$$

$$= \frac{0.80}{1} = 0.80$$

4. As, Bulk modulus, $B = -V_0 \dfrac{\Delta p}{\Delta V}$

$$\Rightarrow \quad \Delta V = -V_0 \frac{\Delta p}{B}$$

$$\Rightarrow \quad V = V_0\left(1 - \frac{\Delta p}{B}\right)$$

$$\therefore \text{ Density,} \quad \rho = \rho_0\left(1 - \frac{\Delta p}{B}\right)^{-1} = p_0\left(1 + \frac{\Delta p}{B}\right)$$

where, $\Delta p = p - p_0 = h\rho_0 g$

Pressure difference between depth and surface of ocean.

$$\therefore \quad \rho = \rho_0\left(1 + \frac{\rho_0 g y}{B}\right) \quad (\text{As, } h = y)$$

5. Ice is lighter than water. When ice melts, the volume occupied by water is less than that of ice. Due to which the level of water goes down.

6. Volume of ice $= \dfrac{M}{\rho}$, volume of water $= \dfrac{M}{\sigma}$

$$\text{Change in volume} = \frac{M}{\rho} - \frac{M}{\sigma} = M\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$$

7. Given, mass of girl $(m) = 50$ kg

Diameter of circular heel $(2r) = 1.0$ cm

$\therefore$ Radius $(r) = 0.5$ cm $= 5 \times 10^{-3}$ m

Area of circular heel $(A) = \pi r^2$

$$= 3.14 \times (5 \times 10^{-3})^2 \text{ m}^2$$

$$= 78.50 \times 10^{-6} \text{m}^2$$

$\therefore$ Pressure exerted on the horizontal floor,

$$p = \frac{F}{A} = \frac{mg}{A}$$

$$= \frac{50 \times 9.8}{78.50 \times 10^{-6}}$$

$$= 6.24 \times 10^6 \text{ Pa}$$

$$\simeq 6.2 \times 10^6 \text{ Pa}$$

8. Force on the base of the vessel

$$= \text{Pressure} \times \text{Area of the base}$$

$$= h\rho g \times A$$

$$= 0.4 \times 900 \times 10 \times 2 \times 10^{-3}$$

$$= 7.2 \text{ N}$$

9. Depth of point P below the free surface of water in the vessel $= (H - h)$. Since, the liquid exerts equal pressure in all direction at one level, hence the pressure at $p = (H - h)\rho g$.

10. Atmospheric pressure $(p) = 1.013 \times 10^5$ Pa

Density of French wine $(\rho) = 984$ kg/m^3

Let h be the height of the wine column for normal atmospheric pressure.

For normal atmospheric pressure $(p) = h\rho g$

$$\therefore \quad h = \frac{p}{\rho g} = \frac{1.013 \times 10^5}{984 \times 9.8} = 10.5 \text{ m}$$

11. Pressure at the bottom $p = (h_1 d_1 + h_2 d_2) g$

$$= [250 \times 1 + 250 \times 0.85]\, g$$
$$= 250\, [1.85]\, g$$
$$= 462.5\, g \text{ dyne/cm}^2$$

12. Let A be the area of cross-section of the cylindrical vessel and x cm be the height of mercury in vessel. The height of water in the vessel $= (29.2 - x)$ cm.

As per question,

$$Ax \times 13.6 = (29.2 - x) \times 1$$

or $\quad x = 2$ cm

$\therefore$ Height of water column $= (29.2 - 2) = 27.2$ cm

$\therefore$ Pressure of the liquids at the bottom

$= 27.2$ cm of water column + 2 cm of Hg column

$= \dfrac{27.2}{13.6}$ of Hg column + 2 cm of Hg column

$= 4$ cm of Hg column

13. Let b be width of the glass wall. When the tank is half filled, then the average force on the glass wall is

$$F = \text{average pressure} \times \text{area}$$
$$= \left[\left(\frac{4}{2}\right)\rho_w g\right] \times \left(\frac{4}{2} \times b\right)$$

When tank is filled up to height 4 m, then

$$F' = (4\rho_w g)\,(4 \times b)$$
$$\frac{F'}{F} = \frac{4 \times 4}{2 \times 2} = 4$$

or $\quad F' = 4F$

14. Surface area, $A = 4\pi r^2$

or $\quad r = (A/4\pi)^{1/2}$

Volume, $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi\,(A/4\pi)^{3/2} = kA^{3/2}$

where, $\dfrac{4\pi}{3} \times \dfrac{1}{(4\pi)^{3/2}} = k =$ constant.

Using Boyle's law, we have

$$p_1 V_1 = p_2 V_2$$

or $\quad p_2 = \dfrac{p_1 V_1}{V_2} = \dfrac{(10 + h)\, kA_1^{3./2}}{kA_2^{3/2}}$

or $\quad p_2 = (10 + h)\left(\dfrac{A_1}{A_2}\right)^{3/2}$

As, $\quad p_2 = 10$ m of water, so

$$10 = \frac{10 + h}{8}$$

or $\quad 80 = 10 + h$

or $\quad h = 70$ m

15. $p_1 = \rho g d + p_0 = 3 \times 10^5$ Pa

$\therefore \quad \rho g d = 2 \times 10^5$ Pa

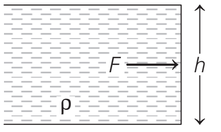
$$p_2 = 2\rho g d + p_0$$
$$= 4 \times 10^5 + 10^5 = 5 \times 10^5 \text{ Pa}$$

% increase $= \dfrac{p_2 - p_1}{p_1} \times 100$

$$= \frac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5} \times 100 = \frac{200}{3}\,\%$$

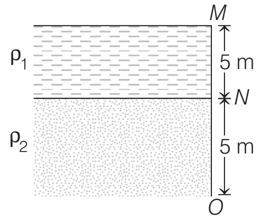
16. Since, density of iron is more than that of marble, the volume of iron is less than that of marble for the given mass. The upthrust of water on iron will be less than that on marble. Due to which, iron cube will weight more.

17. Force on a vertical surface of area A and dipped upto height h in a fluid of density ρ is



Force, $F = \dfrac{1}{2}(\rho g h \times A)$

So, in given case,



Force on portion $MN = \dfrac{\rho_1 g h}{2} \times A$

Force on portion NO

= Force due to pressure of liquid in MN portion + Force due to pressure of liquid in NO portion

$$= \rho_1 g h A + \frac{1}{2}(\rho_2 g h A)$$
$$= \rho_1 g h A + \frac{1}{2} \times 2\rho_1 \times g h A \qquad (\because\ \rho_2 = 2\rho_1)$$
$$= 2\rho_1 g h A$$

Required ratio, $\dfrac{F_{MN}}{F_{NO}} = \dfrac{\frac{1}{2}(\rho_1 g h A)}{2\rho_1 g h A} = \dfrac{1}{4}$

18. Fraction of volume immersed in the liquid, $V_{\text{in}} = \left(\dfrac{\rho}{\sigma}\right)V$

i.e., It depends upon the densities of the block and liquid, so there will be no change in it if system moves upward or downward with constant acceleration, *i.e.* uniform acceleration.

19. Given, $\quad 6\, g = \dfrac{V}{3} \times 10^3 \times g \quad$ ...(i)

and $\quad (6 + m)\, g = V \times 10^3 \times g \quad$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

or $\quad m = 18 - 6 = 12$ kg

20. The effective weight of the block in liquid will become less than 2 kg due to buoyancy of liquid. As a result of which A will read less than 2 kg.

As, the body immersed in liquid has some effective weight acting downwards so the reading of B will be more than 5 kg.

21. Volume of cylinder $= \dfrac{m}{\rho}$

Upthrust on cylinder $= \left(\dfrac{m}{\rho}\right)\sigma g$

From Newton's third law, the downward force exerted by cylinder on the liquid is $= \left(\dfrac{m}{\rho}\right)\sigma g$

$\therefore$ Increase in pressure $= \dfrac{m\sigma g}{\rho A}$

22. Given, size of the plate = 2m×5m and

Greatest and least depths of the plate are 6m and 4m.

We know that area of the plate $A = 2 \times 3 = 6\ \text{m}^2$

and depth of centre of the plate,

$$x = \frac{6+4}{2} = 5\,\text{m (mean depth)}$$

$\therefore$ Total thrust on the plate,

$$\begin{aligned}\rho &= \rho_w g\, A \cdot x\\ &= 10^3 \times 9.8 \times 6 \times 5\\ &= 294 \times 10^3\ \text{N}\end{aligned}$$

23. The speed of the body just before entering the liquid is $v = \sqrt{2gh}$. The buoyant force F_B of the lake (*i.e.*, upward thrust of liquid on the body) is greater than the weight of the body w, since $\sigma > \rho$. If V is the volume of the body and a is the acceleration of the body inside the liquid, then $F_B - w = ma$

or $\quad \sigma Vg - \rho Vg = \rho Va$

or $\quad (\sigma - \rho)\, g = \rho a$

or $\quad a = \dfrac{(\sigma - \rho)\, g}{\rho}$

Using the relation, $v^2 = u^2 + 2as$, we have

$$0 = (\sqrt{2gh})^2 - 2g\,\frac{(\sigma-\rho)}{\rho}\, s$$

or $\quad s = \dfrac{h\rho}{\sigma - \rho}$

24. Let D_1 be the inner diameter of the hemispherical bowl and D_2 be the outer diameter of the bowl. As, bowl is just floating so

$$\frac{4}{3}\pi\left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \frac{4}{3}\pi\left[\left(\frac{1}{2}\right)^3 - \left(\frac{D_1}{2}\right)^3\right] \times (2 \times 10^4)$$

or $\quad \dfrac{1.2 \times 10^3}{2 \times 10^4} = 1 - D_1^3$

$\Rightarrow \quad D_1 = \left(1 - \dfrac{1.2}{20}\right)^{1/3} = \left(\dfrac{18.8}{20}\right)^{1/3}$

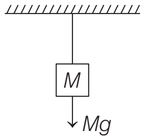
On solving, we get

$$D_1 = 0.98\ \text{m}$$

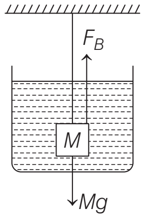
25. When load M is attached to wire, extension in length of wire is

$$\Delta l_1 = \frac{Mgl}{AY} \qquad \ldots\text{(i)}$$

where, Y is the Young's modulus of the wire.



When load is immersed in liquid of relative density 2, increase in length of wire as shown in the figure is



$$\Delta l_2 = \frac{(Mg - F_B)l}{AY}$$

where, F_B = buoyant force.

$\therefore \quad \Delta l_2 = \dfrac{\left(Mg - Mg\,.\,\dfrac{\rho_l}{\rho_b}\right)l}{AY} \qquad \left[\because F_B = V\rho_l g = \dfrac{Mg}{\rho_b}\rho_l g\right]$

Here given that, $\dfrac{\rho_l}{\rho_b} = \dfrac{2}{8} = \dfrac{1}{4}$

So, $\quad \Delta l_2 = \dfrac{\left(\dfrac{3}{4}Mg\right)l}{AY} \qquad \ldots\text{(ii)}$

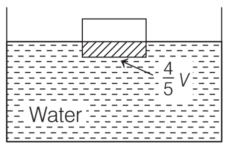
Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta l_2}{\Delta l_1} = \frac{3}{4}$$

$\Rightarrow \quad \Delta l_2 = \dfrac{3}{4} \times \Delta l_1 = \dfrac{3}{4} \times 4\ \text{mm} = 3\ \text{mm}$

26. For a floating body, upthrust = weight of the part of object, i.e. submerged in the fluid.

In first situation,



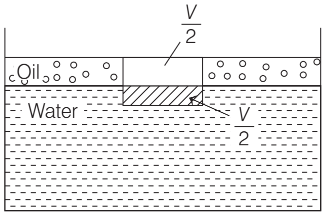
So, weight of block of volume V = weight of water of volume $\dfrac{4}{5}V \Rightarrow V\rho_b g = \dfrac{4}{5}V\rho_w g$

where, $\quad \rho_b$ = density of block

and $\quad \rho_w$ = density of water.

$\Rightarrow \quad \dfrac{\rho_b}{\rho_w} = \dfrac{4}{5} \qquad \ldots\text{(i)}$

In second situation,



So, weight of block of volume V = weight of oil of volume $\frac{V}{2}$ + weight of water of volume $\frac{V}{2}$.

$$\Rightarrow \quad V\rho_b g = \frac{V}{2}\rho_o g + \frac{V}{2}\rho_w g$$

where, ρ_o = density of oil.

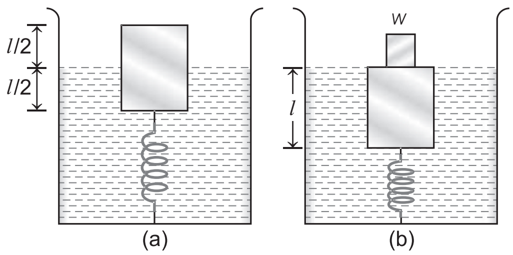
$$\Rightarrow \quad 2\rho_b = \rho_o + \rho_w$$

$$\Rightarrow \quad \frac{2\rho_b}{\rho_w} = \frac{\rho_o}{\rho_w} + 1$$

$$\Rightarrow \quad 2 \times \frac{4}{5} = \frac{\rho_o}{\rho_w} + 1 \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \rho_0/\rho_w = 8/5 - 1 = 3/5 = 0.6$$

27. Initially the position of wooden block is as shown in Fig. (a). Since, the density of block is half than that of water, hence half of its volume is immersed in water.



When weight w is put on the block, the remaining half of the volume of block is immersed in water, figure (b). Therefore,

$$w = \text{additional upthrust + spring force}$$
$$= l \times l \times \frac{l}{2} \times 2\rho \times g + k\left(\frac{l}{2}\right)$$
$$= l\left(l^2\rho g + \frac{k}{2}\right)$$

28. Mass of the cylinders = $AL(\rho_1 + \rho_2)$. As cylinders float with length $L/2$ outside the water, therefore length of cylinder inside the water = $3L/2$. When cylinders are floating, then, weight of cylinder = weight of water displaced by cylinder.

So, $\quad AL(\rho_1 + \rho_2)g = A(3L/2) \times 1 \times g$

or $\quad \rho_1 + \rho_2 = 3/2$

As, $\quad \rho_1 < \rho_2$

so, $\quad \rho_1 < 3/4$

29. Let v be the volume of ice-berg outside the sea water while floating. Therefore, volume of ice-berg inside the sea water = $(V - v)$. As ice-berg is floating, so weight of ice-berg = weight of sea water displaced by ice-berg.

i.e. $\quad V \times 0.9 \times g = (V - v) \times 1.1 \times g$

or $\quad 1.1\,v = 1.1\,V - 0/9\,V$

or $\quad v/V = 0.2\ /1.1 = 2/11$

30. Let l be the length of the cylinder in water when it is in the vertical position and A be the cross-sectional area of the cylinder. As cylinder is floating, so

weight of cylinder = upward thrust

or $\quad mg = Al\rho g$

or $\quad m = Al\rho$

When the cylinder is tilted through an angle θ, then length of cylinder in water = $\frac{l}{\cos\theta}$

Weight of water displaced $= \frac{l}{\cos\theta} A\rho g$

$$\therefore \quad \text{Restoring force} = \frac{lA\rho g}{\cos\theta} - lA\rho g$$
$$= lA\rho g\left[\frac{1}{\cos\theta} - 1\right]$$
$$= mg\left[\frac{1}{\cos\theta} - 1\right]$$

31. Fig. (i) is incorrect. From equation of continuity, the speed of liquid is larger at smaller area. According to Bernoulli's theorem, due to larger speed, the pressure will be lower at smaller area (large velocity) and therefore height of liquid column will also be at lesser height, while in Fig. (i) height of liquid column at narrow area in higher.

32. If the liquid is incompressible, then mass of liquid per second entering through left end should be equal to mass of liquid coming out from the right end.

$$M = M_1 + M_2$$
$$\Rightarrow \quad \rho A v_1 = \rho A v_2 + \rho(1.5A)v$$
$$\Rightarrow \quad Av_1 = Av_2 + 1.5\,Av$$
$$A \times 3 = A \times 1.5 + 1.5 \times v \times A$$
$$\Rightarrow \quad 1.5 = 1.5v$$
$$v = 1 \text{ m/s}$$

33. According to equation of continuity, av = constant. As v increases, a decreases.

34. Using theorem of continuity, we have

$$\pi D_p^2 v_p = \pi D_Q^2 v_Q$$
$$v_p = \left(\frac{D_Q}{D_P}\right)^2 v_Q$$
$$= \left(\frac{4 \times 10^{-2}}{2 \times 10^{-2}}\right) \times v_Q = 4\,v_Q$$

35. As, $Av = 2\,Av'$ or $v' = v/2$

For a horizontal pipe, according to Bernoulli's theorem,

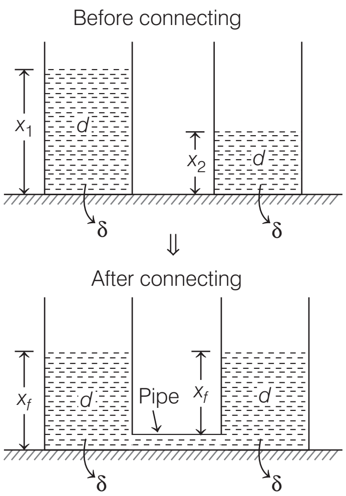
$$p + \frac{1}{2}\rho v^2 = p' + \frac{1}{2}\rho\left(\frac{v}{2}\right)^2$$

or $$p' = p + \frac{1}{2}\rho v^2\left(1 - \frac{1}{4}\right)$$

$\Rightarrow$ $$p' = p + \frac{3}{8}\rho v^2$$

36. When air stream is produced in between two suspended balls, the pressure there becomes less than the pressure on the opposite faces of the balls. Due to which the balls are pushed towards each other.

37.



By conservation of volume,

$$(V_{\text{system}})_{\text{initial}} = (V_{\text{system}})_{\text{final}}$$
$$Sx_1 + Sx_2 = Sx_f + Sx_f$$
$$x_1 + x_2 = x_f + x_f$$
$$x_1 + x_2 = 2x_f$$
$$x_f = \frac{x_1 + x_2}{2} \qquad \text{...(i)}$$

Now, initial energy of system,

$$(U_{\text{system}})_{\text{initial}} = M_1 g h_1 + M_2 g h_2$$
$$= dV_1 g h_1 + dV_2 g h_2$$
$$= dSx_1 g\left(\frac{x_1}{2}\right) + dSx_2 g\left(\frac{x_2}{2}\right)$$
$$= \frac{1}{2}dSgx_1^2 + \frac{1}{2}dSgx_2^2$$
$$= \frac{1}{2}dSg(x_1^2 + x_2^2) \qquad \text{...(ii)}$$

and final energy of system,

$$(U_{\text{system}})_{\text{final}} = M'gh' + M'gh'$$
$$= dV'gh' + dV'gh'$$
$$= dSx_f g\left(\frac{x_f}{2}\right) + dSx_f g\left(\frac{x_f}{2}\right)$$
$$= \frac{1}{2}dSgx_f^2 + \frac{1}{2}dSgx_f^2$$
$$= dSgx_f^2$$
$$= dSg\left(\frac{x_1 + x_2}{2}\right)^2 \qquad \text{...(iii) [Using eq. (i)]}$$

Therefore, the change in energy of system,

$$\Delta U_{\text{system}} = (U_{\text{system}})_{\text{final}} - (U_{\text{system}})_{\text{initial}}$$
$$= dSg\left(\frac{x_1 + x_2}{2}\right)^2 - \frac{1}{2}dSg\,(x_1^2 + x_2^2)$$
$$= dSg\left(\frac{x_1^2 + x_2^2 + 2x_1x_2}{4}\right) - dSg\left(\frac{x_1^2 + x_2^2}{2}\right)$$
$$= dSg\left[\left(\frac{x_1^2 + x_2^2 + 2x_1x_2}{4}\right) - \left(\frac{x_1^2 + x_2^2}{2}\right)\right]$$
$$= dSg\left[\frac{x_1^2 + x_2^2 + 2x_1x_2 - 2(x_1^2 + x_2^2)}{4}\right]$$
$$= \frac{dSg}{4}(x_1^2 + x_2^2 + 2x_1x_2 - 2x_1^2 - 2x_2^2)$$
$$= \frac{dSg}{4}(-x_1^2 - x_2^2 + 2x_1x_2)$$
$$= -\frac{dSg}{4}(x_1^2 + x_2^2 - 2x_1x_2) = -\frac{1}{4}\,gdS(x_2 - x_1)^2$$

So, energy of the system will be decreased by $\frac{1}{4}\,gdS(x_2 - x_1)^2$.

Hence, option (d) is correct.

38. As, $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$ (from Bernoulli's equation)

or $$p_1 - p_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$
$$= \frac{1}{2} \times 1.3 \times (120^2 - 90^2)$$
$$= 4.095 \times 10^3\ \text{Nm}^{-2}$$

Gross lift on the wing $= (p_1 - p_2) \times \text{Area}$
$$= 4.095 \times 10^3 \times 10 \times 2$$
$$= 81.9 \times 10^3\ \text{N}$$

39. Vertical distance covered by water before striking ground $= (H - h)$. Time taken is, $t = \sqrt{2\,(H - g)\cdot g}$; Horizontal velocity of water coming out of hole at P, $u = \sqrt{2\,gh}$

$\therefore$ Horizontal range $= ut$
$$= \sqrt{2\,gh} \times \sqrt{2\,(H - g)/g}$$
$$= 2\sqrt{h\,(H - h)}$$

40. If h is the initial height of liquid in drum above the small opening, then velocity of efflux, $v = \sqrt{2\,gh}$. As the water drains out, h decreases, hence v decreases. This reduces the rate of drainage of water. Due to which, as the drainage continues, a longer time is required to drain out the same volume of water. So, clearly $t_1 < t_2 < t_3$.

41. Let the lower and upper surface of the wings of the aeroplane be at the same height h and speeds of air on the upper and lower surfaces of the wings be v_1 and v_2.

Speed of air on the upper surface of the wing,

$$v_1 = 70 \text{ m/s}$$

Speed of air on the lower surface of the wing,

$$v_2 = 63 \text{ m/s}$$

Density of the air, $\rho = 1.3 \text{ kg/m}^3$

Area, $A = 2.5 \text{ m}^2$

According to Bernoulli's theorem,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gh = p_2 + \frac{1}{2}\rho v^2 + \rho gh$$

or $$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$\therefore$ Lifting force acting on the wings,

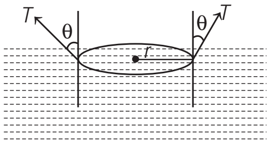
$$F = (p_2 - p_1) \times A = \frac{1}{2}\rho(v_1^2 - v_2^2) \times A \quad \left[\because \text{ pressure} = \frac{\text{force}}{\text{area}}\right]$$

$$= \frac{1}{2} \times 1.3 \times [(70)^2 - (63)^2] \times 2.5$$

$$= \frac{1}{2} \times 1.3 \, [4900 - 3969] \times 2.5$$

$$= \frac{1}{2} \times 1.3 \times 931 \times 2.5 = 1.51 \times 10^3 \text{ N}$$

42. As, weight of metal disc = total upward force



$= $ upthrust force + force due of surface tension

$= $ weight of displaced water $+ \, T\cos\theta \, (2\pi r)$

$= w + 2\pi r T \cos\theta$

43. Force on the ring due to surface tension of water

$$= (\pi D_1 + \pi D_2) S = mg$$

So, $$S = \frac{mg}{\pi (D_1 + D_2)} = \frac{3.47 \times 980}{(22/7) \times (8.5 + 8.7)}$$

$$= 62.96 \text{ dyne cm}^{-1}$$

44. Let R be the radius of the biggest aluminium coin which will be supported on the surface of water due to surface tension.

Then, $$mg = S \times 2\pi R$$

or $$\pi R^2 t \rho g = S \times 2\pi R$$

or $$R = 2S/\rho g t$$

45. As volume remains constant, *i.e.* $R^3 = 8000\, r^3$ or $R = 20\, r$

Now, $$\frac{\text{Surface energy of one big drop}}{\text{Surface energy of 8000 small drops}}$$

$$= \frac{4\pi R^2 T}{8000 \times 4\pi r^2 T} = \frac{R^2}{8000\, r^2}$$

$$= \frac{(20\, r)^2}{8000\, r^2} = \frac{1}{20}$$

46. If r is the radius of smaller droplet and R is the radius of bigger drop, then according to question,

$$\frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$$

or $$r = \frac{R}{100} = 0.01\, R$$

$$= 0.01 \times 10^{-2} \text{ m} = 10^{-4} \text{ m}$$

$\therefore$ Work done = Surface tension $\times$ Increase in area

$$= 35 \times 10^{-2} \times [(10^6 \times 4\pi \times (10^{-4})^2 - 4\pi \times (10^{-3})^2]$$

$$= 4.35 \times 10^{-2} \text{ J}$$

47. Increase in surface energy = surface tension $\times$ increase in surface area

$$= S\,(1000 \times 4\pi r^2 - 4\pi R^2)$$

$$\left(100 \times \frac{4}{3}\pi r^3 = \frac{4}{3}R^2 \text{ or } r = \frac{R}{10}\right)$$

$$= S \times 4\pi \left(1000 \times \frac{R^2}{100} - R^2\right) = 36\,\pi R^2 S$$

48. Let R and R' be the radius of bubble of volume V and $2V$ respectively, then

$$\frac{4}{3}\pi R^3 = V \text{ and } \frac{4}{3}\pi R'^3 = 2V$$

So, $$\frac{R'^3}{R^3} = 2 \quad \text{or} \quad R' = (2)^{1/3} R$$

As $$W = S \times (4\pi R^2)^2$$

and $$W' = S \times (4\pi R'^2)^2$$

$$\frac{W'}{W} = \frac{R'^2}{R^2} = 2^{2/3} = (4)^{1/3}$$

or $$W' = (4)^{1/3}\, W$$

49. As, $$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$\Rightarrow$ $$R = 10\, r$$

Surface energy of small drop, $E_1 = S \times 4\pi r^2$

Surface energy of large drop, $E_2 = S \times 4\pi\,(10\, r)^2$

$\therefore$ $$E_1/E_2 = 1/100$$

50. Since, the bubbles coalesce in vacuum and there is no change in temperature, hence its surface energy does not change. This means that the surface area remains unchanged. Hence,

$$4\pi a^2 + 4\pi b^2 = 4\pi R^2$$

or $$R = \sqrt{a^2 + b^2}$$

51. As, $T_c = T_0(1 - \alpha t)$, *i.e.* surface tension decreases with increase in temperature. Therefore, graph shown in option (b) is correct.

52. Change in surface area $= 2 \times 4\pi \, [(D/2)^2 - (d/2)^2]$

$$= 2\pi\,(D^2 - d^2)$$

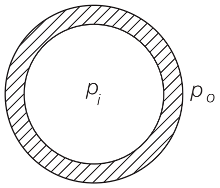
$\therefore$ Work done = surface tension $\times$ change in area

$$= 2\pi S\,(D^2 - d^2)$$

53. Excess of pressure inside a soap bubble is given by

$$\Delta p = p_i - p_o = \frac{4T}{R}$$

Given that, $(p_i)_{\text{I}} = 1.01$ atm
and $(p_i)_{\text{II}} = 1.02$ atm
As, $(p_o)_{\text{I}} = (p_o)_{\text{II}} = 1$ atm
We have, $(\Delta p)_{\text{I}} = 0.01$ atm and
$(\Delta p)_{\text{II}} = 0.02$ atm



Hence, $$\frac{(\Delta p)_{\text{I}}}{(\Delta p)_{\text{II}}} = \frac{4T/R_1}{4T/R_2} = \frac{R_2}{R_1}$$

$$\Rightarrow \quad \frac{0.01}{0.02} = \frac{R_2}{R_1}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{2}{1}$$

Now, ratio of volumes of soap bubbles is

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R_1^3}{\frac{4}{3}\pi R_2^3} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{2}{1}\right)^3 = 8:1$$

Hence, option (c) is correct.

54. Given, $\dfrac{T_{\text{Hg}}}{T_w} = 7.5, \; \dfrac{\rho_{\text{Hg}}}{\rho_w} = 13.6$

and $$\frac{\cos\theta_{\text{Hg}}}{\cos\theta_w} = \frac{\cos 135°}{\cos 0°} = \frac{1}{\sqrt{2}}$$

Height of the fluid inside capillary tube is given by

$$h = \frac{2T\cos\theta}{\rho g r}$$

According to given situation, $h_w = h_{\text{Hg}}$

$$\therefore \quad \frac{2\,T_w \cos\theta_w}{\rho_w\, g\, r_w} = \frac{2T_{\text{Hg}}\cos\theta_{\text{Hg}}}{\rho_{\text{Hg}}\, g\, r_{\text{Hg}}}$$

$$\therefore \quad \frac{r_{\text{Hg}}}{r_w} = \left(\frac{T_{\text{Hg}}}{T_w}\right)\left(\frac{\cos\theta_{\text{Hg}}}{\cos\theta_w}\right)\left(\frac{\rho_W}{\rho_{\text{Hg}}}\right)$$

Given, $r_{\text{Hg}} = r_1$ and $r_w = r_2$, then
Substituting the given values, we get

$$\frac{r_{\text{Hg}}}{r_w} = \frac{r_1}{r_2} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6}$$
$$= 0.4 = 2/5$$

55. By ascent formula, we have surface tension,

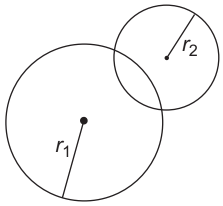
$$T = \frac{rhg}{2} \times 10^3 \;\frac{\text{N}}{\text{m}}$$
$$= \frac{dhg}{4} \times 10^3 \;\frac{\text{N}}{\text{m}} \qquad \left(\because r = \frac{d}{2}\right)$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \qquad \text{[given, } g \text{ is constant]}$$

So, percentage $$= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$$
$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \times 100$$
$$= 1.5\%$$

$$\therefore \quad \frac{\Delta T}{T} \times 100 = 1.5\%$$

56. The excess of pressure inside the first bubble of radius r_1 is $p_1 = 4S/r_1$ and in the second bubble of radius r_2 is $p_2 = 4S/r_2$.



$\therefore$ Excess pressure, $$p = \frac{4S}{r} = \frac{4S}{r_1} - \frac{4S}{r_2}$$

$$\Rightarrow \quad \frac{1}{r} = \frac{r_2 - r_1}{r_1 r_2}$$

$$\Rightarrow \quad r = \frac{r_1 r_2}{r_2 - r_1}$$

57. Let at a time t, dV be the decrease in volume of water in vessel in time dt. Therefore, rate of decrease of water in vessel = rate of water flowing out of narrow tube

So, $$\frac{dV}{dt} = \frac{\pi\,(p_1 - p_2)\,r^4}{8\eta l}$$

But, $$p_1 = p_2 = h\rho g$$

$$\therefore \quad -\frac{dV}{dt} = \frac{\pi\,(h\rho g)\,r^4}{8\eta l} = \frac{(\pi\rho g r^4)}{8\eta l \times A} \times (h \times A)$$

where $h \times A$ = volume of water in vessel at a time $t = V$

$$\therefore \quad dV = -\left(\frac{\pi\rho g r^4}{8\eta l A}\right) \times V\,dt = -\lambda V\,dt$$

or $$\frac{dV}{V} = -\lambda\,dt$$

where, $$\frac{\pi\rho g r^4}{8\,\eta l A} = \lambda = \text{constant}$$

Integrating it within the limits as time changes from 0 to t, volume changes from V_0 to V.

or $$\log_e \frac{V}{V_0} = -\lambda t \text{ or } V = V_0 e^{-\lambda t}$$

where, V_0 = initial volume of water in vessel = Ah_0
Therefore, $h \times A = h_0 A e^{-\lambda t}$ or $h = h_0 e^{-\lambda t}$
Thus, the variation of h and t will be represented by exponential curve as given by (a).

58. Volume of liquid flowing per second through each of the two tubes in series will be the same. So,

$$V = \frac{\pi r_1 R^4}{8\eta L} = \frac{\pi p_2 (R/2)^4}{8\eta\ (L/2)} \text{ or } \frac{p_1}{p_2} = \frac{1}{4}$$

59. Rate of flow of liquid, $V = \frac{p}{R}$

where liquid resistance, $R = \frac{8\eta l}{\pi r^2}$

For another tube, liquid resistance,

$$R' = \frac{8\eta l}{\pi \left(\frac{r}{2}\right)^4} = \frac{8\eta l}{\pi r^4} \cdot 16 = 16\,R$$

For series combination,

$$V_{\text{new}} = \frac{p}{R + R'} = \frac{p}{R + 16\,R} = \frac{p}{17\,R} = \frac{V}{17}$$

60. As, $V = \frac{\pi p r^4}{8\eta l}$ and $V' = \frac{\pi\,(3p + p)\,(r/2)^4}{8\eta l}$

$\therefore \quad \frac{V'}{V} = 4 \times (1/2)^4 = \frac{1}{4}$

or $\quad V' = \frac{V}{4}$

61. Excess pressure at common surface is given by

$$p_{\text{ex}} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T}{r}$$

$\therefore \quad \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$

$$r = \frac{ab}{b - a}$$

62. The meniscus of liquid in a capillary tube will be convex upwards if angle of contact is obtuse. It is so when one end of glass capillary tube is immersed in a trough of mercury.

63. As excess pressure $p \propto 1/r$, therefore pressure inside C is highest and pressure inside B is lowest. The pressure inside A is in between of B and C. Therefore, C starts collapsing with volume of A and B increasing.

64. When lift is accelerated downwards, the observed weight of body in a lift decreases. Hence, to counter balance the upward pull due to surface tension on the liquid meniscus, the height through which the liquid rises must increase.

65. There will be no over flowing of liquid in a tube of insufficient height but there will be adjustment of the radius of curvature of meniscus so that $hR = a$ finite constant.

Here, the maximum height upto which the fluid can rise is 16.3 cm. If the height of the tube is only 12 cm, Water will be able to rise only upto 12 cm because no tube means no adhesion pulling further upwards.

66. Height, $h \propto 1/R$

So, $h_1/h_2 = R_2/R_1 = 0.4/0.2 = 2$

67. Due to surface tension, water rises in the capillary tube upto a height, h with concave meniscus on both the sides. Therefore, the total height of water column in the capillary tube $= h + h = 2\,h$.

68. As, $\quad h = \frac{2S\cos\theta}{r\rho g} \qquad$ (height raised = h)

or $\quad S = \frac{hr\rho g}{2\cos\theta}$ or $S \propto \frac{h\rho}{\cos\theta}$

$$\therefore \quad \frac{S_w}{S_{\text{Hg}}} = \frac{h_1}{h_2} \times \frac{\cos\theta_2}{\cos\theta_1} \times \frac{\rho_1}{\rho_2}$$

$$= \frac{10}{(-3.42)} \times \frac{\cos 135°}{\cos 0°} \times \frac{1}{13.6}$$

$$= \frac{10}{3.42} \times \frac{0.707}{13.6} = \frac{1}{6.5}$$

69. The velocity gradient, $\frac{\Delta V}{\Delta r} = \frac{8}{0.1} = 80\ \text{s}^{-1}$

70. Given, $\quad v = \frac{2\,r^2 \rho g}{9\eta} \qquad$ ...(i)

Mass $= \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi\,(2\,r)^3 \rho_1$ or $\rho_1 = \rho/8$

Terminal velocity of second ball is

$$v_1 = \frac{2\,(2\,r)^2\,(\rho/8)\,g}{8\eta} = \frac{v}{2}$$

71. Terminal velocity, $v = \frac{2R^2\,(\rho - \rho_0)\,g}{9\eta}$

or $\quad \frac{v}{R^2} = \frac{2\,(\rho - \rho_0)\,g}{9\eta} = \text{constant}$

72. Terminal speed of a sphere falling in a viscous fluid is

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_f)g$$

where, η = coefficient of viscosity of fluid,

ρ_0 = density of falling sphere

and $\quad \rho_f$ = density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $\quad v_T \propto r^2 \qquad$ ...(i)

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then

Volume of sphere of radius $R = 27 \times$ Volume of sphere of radius r

$\Rightarrow \quad \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad R = 3r$

$\Rightarrow \quad r = \frac{R}{3}$

So, from Eq. (i), we have

$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

73. When a round pebble is dropped from the top of a tall cylinder, filled with viscous oil the pebble acquires terminal velocity (*i.e.,* constant velocity) after some time.

Hence, graph in option (c) represents velocity (v) of the pebble as a function of time (t).

74. When terminal velocity v is reaching, then

$$F = 2\times 10^{-5}\ v = \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow \quad 2\times 10^{-5}\ v = \frac{4}{3}\times\frac{22}{7}\times(1.5\times 10^{-3})^3\times 10^3\times 10$$

On solving, $v = 7.07\ \text{ms}^{-1} \approx 7\ \text{ms}^{-1}$

75. Gravitational force remains constant on the falling spherical ball. It is represented by straight line P. The viscous force ($F = 6\pi\eta r v$) increases as the velocity increases with time. Hence, it is represented by curve Q. Net force = gravitational force – viscous force. As viscous force increases, net force decreases and finally becomes zero. Then the body falls with a constant terminal velocity. It is thus represented by curve R.

76. Work done against air friction is the average gain in kinetic energy before attaining the terminal velocity

$$W_1 = \frac{0 + \frac{1}{2}mv_{\text{ter}}^2}{2} = \frac{1}{4}mv_{\text{ter}}^2$$

Work done against air friction after attaining terminal velocity is

$$W_2 = \frac{1}{2}mv_{\text{max}}^2$$

$$\therefore \quad W_2 > W_1$$

77. If v is the terminal velocity, then equation of force,

$$xg - yg = 6\pi\eta r v$$

or $$v = \frac{(x-y)}{r}\frac{g}{6\pi\eta} \text{ or } v \propto \frac{(x-y)}{r}$$

Round II

1. Angle of contact increases due to less adhesive force between wax molecule and liquid molecule, inside the capillary. Hence, height is reduced.

2. The nature of flow is determined by Reynolds number.

$$R_e = \frac{\rho v D}{\eta}$$

where, ρ = density of fluid,
η = coefficient of viscosity,
v = velocity of flow
and D = diameter of pipe.

If $R_e < 1000$ = flow is steady
$1000 < R_e < 2000$ = flow becomes unsteady
$R_e > 2000$ = flow is turbulent

$$(R_e)_{\text{initial}} = 10^3 \times \frac{0.18\times 10^{-3}}{\pi\times(0.5\times 10^{-2})^2\times 60}\times\frac{1\times 10^{-2}}{10^{-3}}$$
$$= 382.16$$

$$(R_e)_{\text{final}} = 10^3 \times \frac{0.48\times 10^{-3}}{\pi\times(0.5\times 10^{-2})^2\times 60}\times\frac{1\times 10^{-2}}{10^{-3}}$$
$$= 1019.09$$

So, option (d) is correct.

3. As, work done = surface tension × increase in area

$$\Rightarrow \quad W = \text{surface tension} \times [0.10\times 0.006 - 0.10\times 0.005]\times 2$$
$$= 7.2\times 10^{-2}\times 0.10\times 0.001\times 2$$
$$= 1.44\times 10^{-5}\ \text{J}$$

4. As we know according to equation of continuity, when cross-section of duct decreases, the velocity of flow of liquid increases and in accordance with Bernoulli's theorem, in a horizontal pipe, the place where speed of liquid is maximum, the value of pressure is minimum. Hence, the 2nd graph correctly represents the variation of pressure.

5. When a coin placed on the top of a wooden box floating in water falls in water, upthrust on the block decreases. Due to it, l decreases as well as h decreases.

6. Excess pressure $\frac{4S}{r_1} - \frac{4S}{r_2} = \frac{4S}{r}$

or $$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

or $r = 20$ cm

7. The effective weight of ball in liquid w_3 becomes less then w_1 due to buoyancy of liquid. As, the ball immersed in liquid has some effective weight acting vertically downwards, so, $w_4 > w_2$.

8. As, net force = Averge pressure × Area – $T\times 2R$

$$= \left(p_0 + \rho g\frac{h}{2}\right)(2Rh) - T2R$$
$$= |2p_0Rh + R\rho gh^2 - 2RT|$$

9. Let H be the height above O at which the total force F would have to act to produce the given torque. Then

$$H\times F = \tau$$

or $$H = \frac{\tau}{F}$$

$$H = \frac{\rho g l h^3/6}{(\rho g l h^2/2)} = \frac{h}{3}$$

10. As, $v_1 = \frac{V}{A_1} = \frac{12\times 10^{-6}}{6\times 10^{-6}} = 2\ \text{ms}^{-1} = 200\ \text{cms}^{-1}$

and $v_2 = \frac{V}{A_2} = \frac{12\times 10^{-6}}{3\times 10^{-6}} = 4\ \text{ms}^{-1} = 400\ \text{cms}^{-1}$

Now, $p_A - p_B = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2)$

$$= 1\times 1000(100) + \frac{1}{2}(16\times 10^4 - 4\times 10^4)$$
$$= 10^5 + 6\times 10^4$$
$$= 1.6\times 10^5\ \text{dyne cm}^{-2}$$

11. As the water tries to enter the hole, it forms a liquid surface through the hole with its concave surface downward. Due to which it can withstand the pressure of the liquid upto which the canister is lowered.

$\therefore$ In equilibrium, $\frac{2T}{r} = h\rho g$

$$\Rightarrow \qquad h = \frac{2\sigma}{r\rho g}$$

Putting the given values, we get

$$r = \frac{2 \times 73.5}{40 \times 1 \times 980}$$
$$= 0.00375 \text{ cm}$$
$$= 0.0375 \text{ mm}$$

12. Let V_c be the volume of cavity and V is the actual volume of gold piece [excluding volume of cavity]

$$\therefore \qquad V = \frac{50}{19.3} = 2.6 \text{ cm}^3$$

Now, loss in weight of gold in water = Thrust due to water

$$\Rightarrow \qquad 50\,g - 45\,g = [V + V_c]\,\rho_w g$$
$$\Rightarrow \qquad 5 = (2.6 + V_c) \times 1$$
$$\Rightarrow \qquad V_c = 2.4 \text{ cm}^3$$

13. The velocity of efflux $= \sqrt{2gh}$

The rate of flow of liquid out of hole $= Av = \pi r^2 \sqrt{2gh}$

By using equation of continuity

$$(Av)_{\text{container}} = (Av)_{\text{hole}}$$
$$\pi \frac{D^2}{4} v = \pi r^2 \sqrt{2gh}$$
$$v = \frac{4r^2}{D^2} \sqrt{2gh}$$

$\therefore$ Speed with which water level falls $= \frac{4r^2}{D^2} \sqrt{2gh}$

14. Pressure on left end of horizontal tube, $p_1 = p_0 + h_1 \rho g$

Pressure on right end of horizontal tube, $p_2 = p_0 + h_2 \rho g$

As $p_1 > p_2$, so acceleration should be towards right hand side. If A is the area of cross-section of the tube in the horizontal portion of U-tube, then

$$p_1 A - p_2 A = (lA\rho)a$$

or $\qquad (h_1 - h_2)\rho\, g\, A = lA\rho a$

or $\qquad a = \frac{g(h_1 - h_2)}{l}$ towards right

15. Let A be the area of cross-section of through and ρ be the density of mercury.

Initial mass of mercury in trough $= A \times 3.6 \times \rho$

Final mass of mercury in trough $= Ah'\rho$

$$= (A \times 3.6 \times \rho) \times 2$$

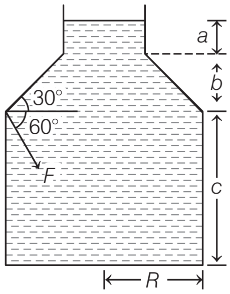
or $\qquad h' = 7.2 \text{ cm}$

16. As, $\qquad h\rho\, g = \frac{2S}{r}$

or $\qquad h = \frac{2S}{r\rho\, g}$

$$= \frac{2 \times 75 \times 10^{-3}}{\left(\frac{1}{2} \times 10^{-3}\right) \times 10^3 \times 10}$$
$$= 0.03 \text{ m} = 3 \text{ cm}$$

17. When jar is placed in vacuum, the liquid level rises up to the top of jar. The force exerted by liquid on the base of jar = force due to vertical column of liquid of height $(a + b + c)$ + vertical downward.



Component of thrust F acting on the portion BC of jar

$$= (a + b + c)\rho g \times \pi R^2 + F \sin 60°$$
$$= \text{greater than } (a + b + c)\rho\, g \times \pi R^2$$

18. As, level of water in tank remains constant with time, so (water inflow rate) = (outflow rate of water)

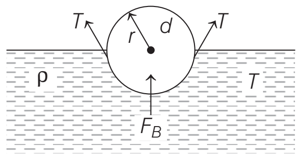
$\Rightarrow 10^{-4} \text{ m}^3\text{s}^{-1}$ = Area of orifice $\times$ Velocity of outflow

$\Rightarrow 10^{-4} \text{ m}^3\text{s}^{-1} = 10^{-4} \times \sqrt{2gh}$

where, h = Height of water above the orifice or hole.

$$\Rightarrow \qquad \sqrt{2gh} = 1 \text{ or } 2 \times 9.8 \times h = 1$$
$$\Rightarrow \qquad h = \frac{1}{19.6} \text{ m} = \frac{100}{19.6} \text{ cm or } h = 5.1 \text{ cm}$$

19. Weight of the drop is balanced by upthrust or buoyant force (F_B) and surface tension (T).



So, weight, $w = F_B + T \times 2\pi r$

$$\Rightarrow \qquad d \cdot V \cdot g = \rho \cdot \frac{V}{2} g + T \times 2\pi r$$

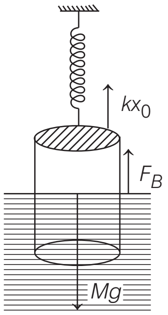
$$\left[\because w = dVg \text{ and } F_B = \rho \frac{V}{2} g\right]$$

$$\Rightarrow d \cdot \frac{4}{3}\pi r^3 \cdot g = \rho \cdot \frac{2}{3}\pi r^3 \cdot g + T \times 2\pi r \qquad \left[\because V = \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \qquad r = \sqrt{\frac{3T}{(2d - \rho)g}}$$

20. In equilibrium, upward force = downward force

$$kx_0 + F_B = mg$$



Here, kx_0 is restoring force of spring and F_B is buoyancy force.

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$x_0 = \frac{Mg - \dfrac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

21. $v = a_1 a_2 \sqrt{\dfrac{2(p_1 - p_2)}{\rho(a_1^2 - a_2^2)}}$

$$= \pi r_1^2 \times \pi r_2^2 \sqrt{\frac{2(p_1 - p_2)}{\rho[(\pi r_1^2)^2 - (\pi r_2^2)^2]}}$$

$$= \pi r_1^2 r_2^2 \sqrt{\frac{2(p_1 - p_2)}{\rho(r_1^4 - r_2^4)}}$$

$$= \frac{22}{7} \times (0.1)^2 \times (0.04)^2 \sqrt{\frac{2 \times 10}{(1.25 \times 10^3)[(0.1)^4 - (0.04)^4]}}$$

$$= 6.4 \times 10^{-4} \text{m}^3\text{s}^{-1}$$

22. Let V be the volume of wooden ball. The mass of ball is $m = V\rho$.

Upward acceleration,

$$a = \frac{\text{upward thrust} - \text{weight of ball}}{\text{mass of ball}}$$

$$\Rightarrow \qquad a = \frac{V\rho_0 g - V\rho g}{V\rho} = \frac{(\rho_0 - \rho)g}{\rho}$$

If v' is the velocity of ball on reaching the surface after being released at depth h is

$$v = \sqrt{2as} = \left[2\left(\frac{p_0 - \rho}{\rho}\right)gh\right]^{1/2}$$

If h' is the vertical distance reached by ball above the surface of water, then

$$h' = \frac{v^2}{2\,g} = \frac{2(\rho_0 - \rho)}{\rho} gh \times \frac{1}{2\,g}$$

$$= \left(\frac{\rho_0 - \rho}{\rho}\right)h = \left(\frac{\rho_0}{\rho} - 1\right)h$$

23. In time Δt, momentum of water entering the hydrant

$$\mathbf{p}_1 = (\rho L \Delta t) v \hat{\mathbf{j}}$$

Momentum of water while leaving the hydrant in time Δt is

$$\mathbf{p}_2 = (\rho L \Delta t) v(-\hat{\mathbf{i}})$$

Change in momentum in time Δt is

$$\Delta\,\mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 = \rho\,L\,t\,v(-\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

$$|\Delta\,\mathbf{p}| = \rho\,L\,\Delta\,t\,v\sqrt{(-1)^2 + (-1)^2} = \sqrt{2}\,\rho\,L\Delta\,t\,v$$

Force exerted by water, $F = \dfrac{|\Delta\mathbf{p}|}{\Delta\,t} = \sqrt{2}\rho L v$

24. If v_1 and v_2 are the speeds of air on the lower and upper surface of the wings of aeroplane and p_1, p_2 are the pressures there, then

Assume difference,

$$p_1 - p_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

or $\qquad \Delta p = \rho\left(\dfrac{v_2 + v_1}{2}\right)(v_2 - v_1) = \rho v_{\text{av}}(v_2 - v_1)$

Here, $\quad v_{\text{av}} = 360 \text{ kmh}^{-1} = 100 \text{ ms}^{-1}$

$$\Rightarrow \quad \frac{v_2 - v_1}{v_{\text{av}}} = \frac{\Delta p}{\rho v_{\text{av}}^2} = \frac{mg / A}{\rho v_{\text{av}}^2} = \frac{3250 \times 10 / 50}{1 \times (100)^2}$$

$$= 0.065 = 6.5\%$$

25. The rate of flow of liquid (V) through capillary tube is

$$V = \frac{\pi\,pr^2}{8\eta hl} = p\left(\frac{\pi r^4}{8\eta l}\right) = \frac{p}{R} = \frac{\text{pressure difference}}{\text{resistance}}$$

where, $\quad T = \dfrac{8\eta hl}{\pi r^4}$

When two tubes are in series,

total resistance, $R = R_1 + R_2$

Rate of flow of liquid, $V' = \dfrac{p}{R_1 + R_2}$

$$= \frac{p}{\dfrac{8\eta}{\pi}\left[\dfrac{l_1}{r^4} + \dfrac{l_2}{r_2^4}\right]}$$

$$= \frac{\pi\,p}{8\eta}\left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right]^{-1}$$

26. Let V_0, V_t = volume of the metal ball at 0° C and t° C respectively, $\rho_0\,\rho_2$ = density of alcohol at 0° C and t° C respectively. Then

$$W_1 = W_0 - V_0\,\rho_0 g$$

$$W_2 = W_t - V_t\,\rho_t g$$

where, $\quad V_t = V_0(1 + \gamma_m t)$ and $\rho_t = \dfrac{\rho_0}{(1 + \gamma_a l)}$

Upthrust at t° $C = V_t \rho_t g$

$$= V_0(1 + \gamma_m t) \times \frac{\rho_0}{(1 + \gamma_a t)} g$$

$$= V_0 \rho_0 \frac{(1 + \gamma_m l)}{(1 + \gamma_a l)} g$$

As $\gamma_m < \gamma_a$, hence upthrust at t° C is less than at 0° C. It means upthrust has been decreased with increase in temperature. Due to which $W_2 > W_1$.

27. In the given figure, total force on the ring due to surface tension of soap film $= (2\pi b) \times 2S \sin\theta$

Mass of air entering per second the bubble

$$= \text{volume} \times \text{density}$$
$$= (Av)\rho = \pi b^2 \times v\rho$$

Momentum of air entering per sec,

$$\pi b^2 v\rho \times v = \pi^2 b^2 v^2 \rho$$

The soap bubble will separate from the ring, when force of surface tension of ring equal to the force

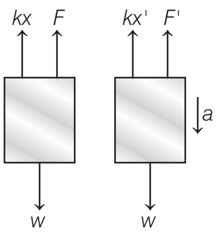
or $$2\pi b \times 2S \times \frac{b}{R} = \pi b^2 v^2 \rho$$

or $$R = \frac{4S}{\rho v^2}$$

28. Let k be the spring constant of spring and it gets compressed by length x in equilibrium position. Let m be the mass of the block and F be the upward thrust of water on block. When the block is at rest,

$$w = kx + F$$

or $$w - F = kx \quad \ldots\text{(i)}$$



When the vessel moves downward with accleration a ($< g$), the effective downward acceleration $= g - a$. Now, upthrust is reduced and becomes F'

where $$F' = \frac{F}{g}(g - a)$$

In figure, then

$$w - kx' - F' = ma$$

or $$w - kx' - \left(\frac{g-a}{g}\right)F = \frac{wa}{g}$$

or $$(w - F) - kx' + \frac{a}{g}F = \frac{wa}{g}$$

or $$kx - kx' + \frac{a}{g}F = \frac{wa}{g}$$

or $$x' = x + (F - w)\frac{a}{gk}$$

Hence, the spring length will increase.

29. Let $p_0 =$ atmospheric pressure, then

$$p_1 V_1 = p_2 V_2$$

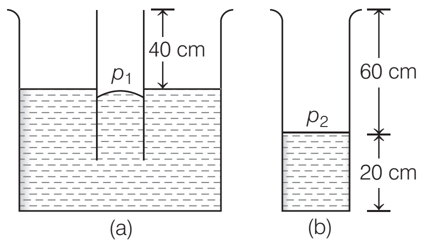
or $$p_2 = p_1 \frac{V_1}{V_2}$$

or $$p_2 = p_0\left(\frac{40}{60}\right) = \frac{2}{3}p_0$$

Now, $p_2 +$ (20cm of Hg) $= p_0$

or $\frac{2}{3}p_0 +$ (20cm of Hg) $= p_0$

or $$\frac{p_0}{3} = 20 \text{ cm of Hg}$$
$$p_0 = 60 \text{ cm of Hg}$$



30. Volume of water in the vessel of base area A' and height h is $V = A'h$. Average velocity of out flowing water when height of water changes from h to 0 is

$$v = \frac{\sqrt{2gh} + 0}{2} = \frac{\sqrt{2gh}}{2}$$

$\therefore$ $$V = Avt$$

When vessel is filled to height $4h$, the volume in vessel

$$= 4V = 4Avt = 4A\frac{\sqrt{2gh}}{2} \times t$$

If t is the time taken for the out flowing liquid and v_1 is the averege velocity of out flowing liquid, then

$$4V = Av_1 t_1$$

or $$t_1 = \frac{4V}{Av_1} = \frac{4A\sqrt{2gh} \times t \times 2}{2 \times A \times \sqrt{2g \times 4h}} = 2t$$

31. Excess pressure inside the soap bubble $= \frac{4S}{r}$. So, the pressure inside the soap bubble $= p_{\text{atm}} + \frac{4S}{r}$

From ideal gas equation, $pV = nRT$

$$\frac{p_A V_A}{p_B V_B} = \frac{n_A}{n_B}$$

$$\Rightarrow \quad \frac{\left(8 + \frac{4S}{r_A}\right)\frac{4}{3}\pi r_A^3}{\left(8 + \frac{4S}{r_B}\right)\frac{4}{3}\pi r_B^2} = \frac{n_A}{n_B} \quad \ldots\text{(i)}$$

Substituting, $S = 0.04$ N/m, $r_A = 2$ cm, $r_B = 4$ cm in Eq. (i), we get

$$\frac{n_A}{n_B} = \frac{1}{6}$$

$\therefore$ $$\frac{n_B}{n_A} = 6$$

32. Given, surface tension of soap solution $(S) = 2.5 \times 10^{-2}$ N/m

Density of soap solution $(\rho) = 1.2 \times 10^3$ kg/m^3

Radius of soap bubble $(r) = 5.00$ mm

$$= 5.0 \times 10^{-3} \text{ m}$$

Atmospheric pressure $(p_0) = 1.01 \times 10^5$ Pa

Excess pressure inside the soap bubble $= \frac{4S}{r}$

$$= \frac{4 \times 2.5 \times 10^{-2}}{5.0 \times 10^{-3}} = 20 \text{ Pa}$$

Excess pressure inside the air bubble $= \frac{2S}{R}$

$$= \frac{2 \times 2.5 \times 10^{-2}}{5.0 \times 10^{-3}} = 10 \text{ Pa}$$

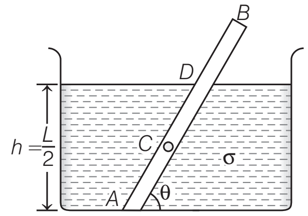
$\therefore$ Pressure inside the air bubble = Atmospheric pressure + Pressure due to 40 cm of soap solution column + Excess pressure inside the bubble

$$= (1.01 \times 10^5) + (0.40 \times 1.2 \times 10^3 \times 9.8) + 10$$
$$= (1.01 \times 10^5) + 4.704 \times 10^3 + 10$$
$$= 1.01 \times 10^5 + 0.04704 \times 10^5 + 0.00010 \times 10^5$$
$$= 1.05714 \times 10^5$$
$$= 1.06 \times 10^5 \text{ Pa}$$

33. As, $AB = L$, $AC = \frac{L}{2}$; $AD = l$ (say)

Let A = area of cross-section of the rod.

Weight of the rod = $AL\rho g$ acting vertically downwards at C.



Upthrust of liquid on rod = $Al\sigma g$ acting upwards through the mid-point of AD.

For rotational equilibrium of rod, net torque about point A should be zero. So,

$$(LA\rho g)\frac{L}{2}\cos\theta = (lA\sigma g)\frac{l}{2}\cos\theta$$

or $$\frac{l^2}{L^2} = \frac{\rho}{\sigma} \quad \text{or} \quad \sin\theta = \frac{1}{2}\sqrt{\frac{\sigma}{\rho}}$$

34. Height of liquid rise in capillary tube,

$$h = \frac{2T\cos\theta_c}{\rho r g}$$

$$\Rightarrow \quad h \propto \frac{1}{r}$$

So, when radius is doubled, height becomes half.

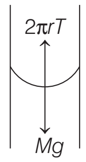
$$\therefore \quad h' = h/2$$

Now, density $(\rho) = \frac{\text{mass}(M)}{\text{volume}(V)}$

$$\Rightarrow \quad M = \rho \times \pi r^2 h$$
$$\therefore \quad M' = \rho\pi r'^2 h'$$
$$\therefore \quad \frac{M'}{M} = \frac{r'^2 h'}{r^2 h} = \frac{(2r)^2(h/2)}{r^2 h} = 2$$
$$\Rightarrow \quad M' = 2M$$

Alternate Solution

According to the given figure, force inside the capillary tube is



$$2\pi r T = Mg \Rightarrow M \propto r$$

When $\quad r' = 2r$, then

$$M' = 2M$$

35. Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho A v$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

$\therefore$ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right)v = \frac{1}{4}\rho A v^2 \quad \text{...(i)}$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) \times 2v = \frac{1}{2}\rho A v^2 \quad \text{...(ii)}$$

[$\because$ Net change in velocity is $= 2v$]

$\therefore$ Net pressure on the mesh is

$$p = \frac{F_{\text{net}}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \quad \left[\because F = \frac{dp}{dt}\right]$$

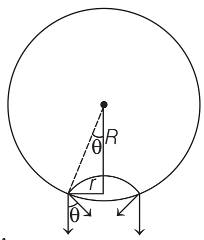
$\therefore$ From Eqs. (i) and (ii), we get

$$p = \frac{3}{4}\rho v^2 A/A = \frac{3}{4}\rho v^2$$

36. The bubble will detach if,

Buoyant force $\geq$ Surface tension force

$$\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$$



$\int \sin\theta\, T \times dl = T(2\pi r)\sin\theta$

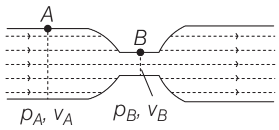
$$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$$

$$\sin\theta = \frac{r}{R}$$

Solving, we get $\quad r = \sqrt{\frac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\frac{2\rho_w g}{3T}}$

37. By equation of continuity for sections A and B, we have

$$A_A v_A = A_B v_B$$



$\Rightarrow \quad 40\, v_A = 20\, v_B$

$\Rightarrow \quad 2\, v_A = v_B$ `...(i)

Now, using Bernoulli's equation (for horizontal tube), we have

$$p_A + \frac{1}{2}\rho v_A^2 = p_B + \frac{1}{2}\rho v_B^2$$

$\Rightarrow \quad p_A - p_B = \frac{1}{2}\rho(v_B^2 - v_A^2)$

Here, $p_A - p_B = 700\ \text{Nm}^{-2}$ and $\rho = 1000\ \text{kg m}^{-3}$

$\Rightarrow \quad 700 = \frac{1}{2} \times 1000\ (v_B^2 - v_A^2)$

$\Rightarrow \quad v_B^2 - v_A^2 = 1.4$ ...(ii)

From Eqs. (i) and (ii), we have

$$3\, v_A^2 = 1.4$$

$\Rightarrow \quad v_A = \sqrt{0.467} = 0.68\ \text{ms}^{-1} = 68\ \text{cm s}^{-1}$

So, volume flow rate of water $= A_A \cdot v_A$

$$= 40 \times 68 = 2720\ \text{cm}^3/\text{s}$$

38. Let $m_1 g$ and $m_2 g$ be the mass of Cu and Zn respectively in alloy,

$\therefore \quad$ Volume of Cu $= \frac{m_1}{8.9}$ cc

and $\quad$ Volume of Zn $= \frac{m_2}{7.1}$ cc

$\therefore$ Total volume of alloy $= \left[\frac{m_1}{8.9} + \frac{m_2}{7.1}\right]$ cc

Now, loss of wt. in water = thrust due to water

$\Rightarrow \quad (16.8 - 14.7)g = \left[\frac{m_1}{8.9} + \frac{m_2}{7.1}\right] \times 1g$

$\Rightarrow \quad 2.1 = \frac{m_1}{8.9} + \frac{m_2}{7.1}$ ...(i)

Also, the total mass of alloy

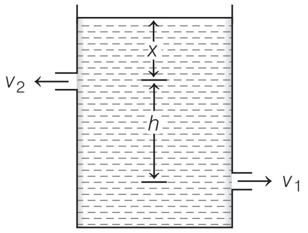
$m_1 + m_2 = 16.8$ g ...(ii)

Solving Eqs. (i) and (ii), we get

$m_1 = 9.345$ g

and $\quad m_2 = 7.455$ g

39. Here, $v_1 = \sqrt{2g(h+x)}; v_2 = \sqrt{2gx}$



Let, a = area of cross-section of each hole

ρ = density of the liquid

The momentum of the liquid flowing out per second through lower hole = mass × velocity

$$= a v_1 \rho \times v_1 = a\,\rho\, v_1^2$$

The force exerted on the lower hole towards left $= a\,\rho\, v_1^2$

Similarly, the force exerted on the upper hole towards right

$$= a\,\rho\, v_2^2$$

Net force on the tank, $F = a\,\rho(v_1^2 - v_2^2)$

$= a\,\rho[2g(h+x) - 2gx]$

$= 2a\rho g h$

$\Rightarrow \quad F \propto h$

40. If V is the volume of the body, its weight $= V\rho_1 g$. Velocity gained by body when it falls from a height $h_1 = \sqrt{2gh_1}$. The weight of liquid displaced by the body as body starts immersing into the liquid $= V\rho_2 g$. The net retarding force on the body when it starts going in the liquid, $F = V(\rho_2 - \rho_1)g$

$\therefore$ Retardation, $\quad a = \frac{F}{V\rho_1} = \left[\frac{V(\rho_2 - \rho_1)g}{V\rho_1}\right]$

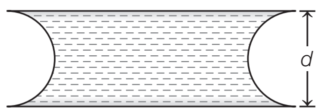
The time of immersion of the body is that time in which the velocity of the body becomes zero. Using the relation $v = u + at$, we have $v = 0, u = \sqrt{2gh_1}$,

$$a = \frac{v(\rho_2 - \rho_1)g}{V\rho_1} = -\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)g$$

we have $\quad 0 = \sqrt{2gh_1} = \left(\frac{\rho_2 - \rho_1}{\rho_1}\right)g$

or $\quad t = \sqrt{\frac{2h_1}{g}} \times \left(\frac{\rho_1}{\rho_2 - \rho_1}\right)$

41. Let A be the circular area over which the liquid wets the plate and d be the distance between two plates. Mass of liquid drop, $m = Ad\rho$. If S is the force of surface tension of water, then excess of pressure inside the liquid film in excess of atmospheric pressure is given by



$$p = \frac{S}{r} = \frac{S}{d/2} = \frac{2S}{d}$$

Force of attraction between the plates,

$F = \frac{2S}{d} A \qquad \left[\therefore p = \frac{F}{A}\right]$

$F = \frac{2S}{\rho d^2} \times A\rho d = \frac{2Sm}{\rho d^2}$

$\Rightarrow \quad \frac{2 \times 0.07 \times (80 \times 10^{-6})}{10^3 \times (4 \times 10^{-8})} = 0.28$ N

42. When, there is an equal level of liquid in two arms of U-tube, the height of liquid in each arm of U-tube $= \dfrac{h_1 + h_2}{2}$. We may consider that a height, $h_1 - \dfrac{(h_1 + h_2)}{2} = \dfrac{h_1 - h_2}{2}$ of the liquid has been transferred from left arm to right arm of U-tube $= \left(\dfrac{h_1 - h_2}{2}\right) A\rho$

where, A = area of cross-section of tube and ρ = density of liquid.

The decrease in height of this liquid $= \left(\dfrac{n_1 - n_2}{2}\right)$

Loss in potential energy of this liquid $= \left(\dfrac{h_1 - h_2}{2}\right)^2 A\rho g$

The mass of the entire liquid in U-tube

$$= (h_1 + h_2 + h)\rho A$$

If this liquid moves with velocity v, then its

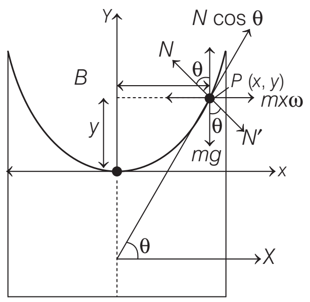
$$\text{KE} = \frac{1}{2}(h_1 + h_2 + h)\rho\, A\, v^2$$

Using law of conservation of energy, we have

$$\frac{1}{2}(h_1 + h_2 + h)\rho\, A\, v^2 = \left(\frac{h_1 - h_2}{2}\right)^2 A\,\rho\, g$$

or $$v = (h_1 - h_2)\sqrt{\frac{g}{2(h_1 + h_2 + h)}}$$

43. Let tangent at any point $P(x, y)$ makes angle θ with X-axis as shown below



In rotating frame of vessel, let N be normal reaction. Then,

$$N\cos\theta = mg \quad \text{and} \quad N\sin\theta = mx\omega^2$$

$$\Rightarrow \quad \tan\theta = \frac{\omega^2 x}{g}$$

But $\tan\theta = \dfrac{dy}{dx}$ = slope of tangent

$$\therefore \quad \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\Rightarrow \quad y = \int dy = \int \frac{\omega^2 x}{g}\cdot dx$$

$$\Rightarrow \quad y = \frac{\omega^2}{g}\cdot\frac{x^2}{2}$$

At topmost point of profile of rotating fluid body, $y = h$ and $x = R$

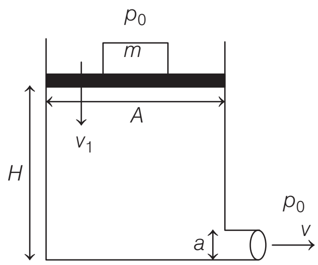
$$\Rightarrow \quad h = \frac{\omega^2 R^2}{2g}$$

Given, $R = 5$ cm

$$\Rightarrow \quad h = \frac{\omega^2 (5)^2}{2g} = \frac{25\,\omega^2}{2g}\text{ cm}$$

Hence, correct option is (a).

44.



Given,
$$m = 24\text{ kg}$$
$$A = 0.4\text{ m}^2$$
$$a = 1\text{ cm}^2$$
$$H = 40\text{ cm}$$

Using Bernoulli's equation,

$$\Rightarrow \quad \left(p_0 + \frac{mg}{A}\right) + \rho g H + \frac{1}{2}\rho v_1^2$$

$$\Rightarrow \quad p_0 + 0 + \frac{1}{2}\rho v^2 \quad \text{...(i)}$$

Neglecting v_1, we get

$$\Rightarrow \quad v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow \quad v = \sqrt{8 + 1.2}$$

$$\Rightarrow \quad v = 3.033\text{ m/s}$$

$$\Rightarrow \quad v = 3\text{ m/s}$$

45. Let V and ρ be volume and density of solid respectively and ρ' be the density of water, *i.e.* $p' = 10^3$ kg m^{-3}

Weight of body $= V\rho g$

Volume of solid body outside water $= V/4$

$\therefore$ Volume of solid body inside water

$$= V - V/4 = 3V/4$$

Weight of water displaced by solid

$$= \frac{3V}{4}\times 10^3 \times g$$

As solid body is floating, then

Weight of body = Weight of water displaced by it

$$V\rho g = \frac{3V}{4}\times 10^3\, g$$

$$\rho = \frac{3}{4}\times 1000 = 750\text{ kg m}^{-3}$$

46. Here, $r = 2 \times 10^{-3}$m, $D = 2r = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$ m;

$\eta = 2.084 \times 10^{-3}$ Pa-s;

$\rho = 1.06 \times 10^{3}$ kg m^{-3}

For flow to be laminar, $N_R = 2000$

Now, $$v = \frac{N_R \eta}{\rho D} = \frac{2000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^{3}) \times (4 \times 10^{-3})}$$
$$= 0.98 \text{ ms}^{-1}$$

47. Here, r = 30.0 mm = 3 cm, F = 3.03 gf = 3.03×980 dyne. Since, the liquid is touching the ring, both inside as well as outside therefore, force acting on the ring due to surface tension is given by

$$F'' = 2\,(S \times \text{circumference of ring})$$
$$= 2(S \times 2\pi r) = 4S\pi r$$
$$= 4 \times S \times \frac{22}{7} \times 3 \text{ dyne}$$

As, $$F' = F = 4 \times S \times \frac{22}{7} \times 3 = 3.03 \times 980$$
$$S = \frac{3.03 \times 980 \times 7}{4 \times 22 \times 3} = 78.76 \text{ dyne cm}^{-1}$$

48. Here, side of the square plate, $l = 10$ cm

Area of the plate = 100 cm^2, dv = 10 cms^{-1},

$F = 200$ dyne,

$\eta = 0.01$ poise, $dx = ?$

As, $F = \eta A \dfrac{dv}{dx}$

$$dx = \frac{\eta A dv}{F} = \frac{0.01 \times 100 \times 10}{200} = 0.05 \text{ cm}$$
$$= 5 \times 10^{-2}\text{m}$$
$$= x \times 10^{-2}\text{m (given)}$$

$\therefore \quad x = 5$

49. Here, density of ice, $\rho = 917$ kgm^{-3}

Density of fresh water, $\rho = 1000$ kgm^{-3}

Let V be the total volume of the ice and v be the volume of ice above the water. Then volume of the water displaced by the immersed part of ice = $(V - v)$

According to law of floatation,

weight of ice = weight of the water displaced

$$V \times 917 \times g = (V - v) \times 1000 \times g$$

or $$1000\,v = 1000\,V - 917\,V = 83\,V$$

or $$\frac{v}{V} = \frac{83}{1000} = 0.083$$

50. Height of liquid in capillary tube,

$$h = \frac{2S\cos\theta}{\rho g r}$$

$\Rightarrow$ $$S = \frac{\rho g r h}{2\cos\theta}$$

Here, $\rho = 900$ kgm^{-3},

$g = 10$ ms^{-2},

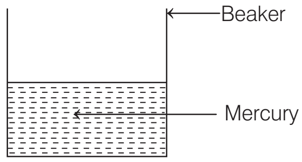
$r = 0.015$ cm $= 15 \times 10^{-5}$ m,

$h = 15$ cm $= 15 \times 10^{-2}$ m and $\theta \approx 0°$

$\Rightarrow \quad \cos\theta = 1$

Hence, $$S = \frac{900 \times 10 \times 15 \times 10^{-5} \times 15 \times 10^{-2}}{2 \times 1}$$
$$= 101.25 \times 10^{-3} \text{ N/m} \approx 101 \text{ mN/m}$$

51.



Volume of empty part = Volume of beaker − Volume of mercury

Also it is given that, there is no change in unfilled volume of the beaker with the varying temperature.

So, change in volume of beaker = change in volume of mercury

$\Rightarrow$ $$V_b \gamma_b \Delta T = V_m \gamma_m \Delta T$$

$\Rightarrow$ $$V_b \gamma_b = V_m \gamma_m$$

$\Rightarrow$ $$V_m = \frac{V_b \gamma_b}{\gamma_m} = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}}$$
$$= 20 \text{ cm}^3 = 20 \text{ cc}$$