JEE 2023

Relations and Functions

DPP-02

1. Define a relation R on set $A = \{2, 3, 5, 6, 10\}$ as xRy if '$x < y$ and x divides y', then the domain of relation is:
(A) $\{2, 3, 5, 6, 10\}$ (B) $\{2, 3, 5\}$
(C) $\{2, 3, 5, 6\}$ (D) $\{2, 5, 10\}$

2. Consider the following with regard to a relation R on a set of real numbers defined by xRy if and only if $3x + 4y = 5$. Consider the following three statements:
(1) $0\,R1$ (2) $1R\frac{1}{2}$
(3) $\frac{2}{3}R\frac{3}{4}$
Which of the above are correct?
(A) 1 and 2 only (B) 1 and 3 only
(C) 2 and 3 only (D) 1, 2 and 3

3. A relation R is defined from $\{2, 3, 4, 5\}$ to $\{3, 6, 7, 10\}$ by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is:
(A) $\{2, 3, 5\}$ (B) $\{3, 5\}$
(C) $\{2, 3, 4\}$ (D) $\{2, 3, 4, 5\}$

4. The linear relation between the components of the ordered pairs of relation R given by:
$R = \{(0, 2), (-1, 5), (2, -4), \ldots\ldots\ldots\}$is
(A) $x + y = 2$ (B) $3x - y = 1$
(C) $x + 3y = 2$ (D) $3x + y = 2$

5. Let A be the set of first ten natural numbers and let R be a relation on A defined by
$(x, y) \in R \Leftrightarrow x + 2y = 10$, i.e.,
$R = \{(x, y) : x \in A, y \in A \text{ and } x + 2y = 10\}$. Then the domain and range of R is
(A) $\{2, 4, 6, 8\}$, $\{4, 3, 2, 1\}$ respectively
(B) $\{4, 3, 2, 1\}$, $\{2, 4, 6, 8\}$ respectively
(C) $\{1, 2, 3, 4\}$, $\{1, 2, 3, 4\}$ respectively
(D) None of these

6. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$. A relation $R : A \to B$ is defined by $R = \{(1, 3), (1, 5), (2, 1)\}$. Then R^{-1} is defined by:
(A) $\{(1,2), (3,1), (1,3), (1,5)\}$
(B) $\{(1, 2), (3, 1), (2, 1)\}$
(C) $\{(1, 2), (5, 1), (3, 1)\}$
(D) None of these

7. Let A and B are two sets such that $A \times B$ consists of 6 elements. If three elements of $A \times B = (1, 2), (2, 3), (4, 3)$ then the remaining order pairs of $A \times B$ are:
(A) $(1, 3), (2, 2), (4,2)$ (B) $(3, 1), (2, 2), (4, 2)$
(3) $(3, 1), (2, 4), (2, 2)$ (D) $(1, 3), (2, 2), (2, 4)$

8. Let $A = \{4, 5, 7\}$ and $B = \{2, 4, 6\}$ be two sets and let a relation R be a relation from A to B is defined by R: $\{(x, y) : x < y, x \in A, y \in B\}$ then the difference between the sum of elements of domain and range of R is:
(A) 2 (B) 3
(C) 4 (D) 5

9. For any two real numbers a and b, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is:
(A) Reflexive but not symmetric
(B) Symmetric but not transitive
(C) Transitive but not reflexive
(D) An equivalence relation.

10. Let S be the set of all real numbers. Then the relation $R = \{(a,b) : 1 + ab > 0\}$ on S is:
(A) Reflexive and symmetric but not transitive
(B) Reflexive, transitive and symmetric
(C) Reflexive and transitive but not symmetric.
(D) None of these.

Answer Key

1. (B)
2. (C)
3. (D)
4. (D)
5. (A)
6. (C)
7. (A)
8. (B)
9. (D)
10. (A)

Hints and Solutions

1. **(B)**

Given $A = \{2, 3, 5, 6, 10\}$ and 'xRy if $x < y$ and x divides y'; $R \subseteq A \times A$

$\Rightarrow R = \{(2, 6), (2, 10), (3, 6), (5, 10)\}$

Domain of $R = \{2, 3, 5\}$

2. **(C)**

Given $3x + 4y = 5 \Rightarrow y = \dfrac{5-3x}{4}$

We have to find y for $x = 0, 1, \dfrac{2}{3}$ (by given options)

$\Rightarrow y = \dfrac{5}{4}$ for $x = 0$; $y = \dfrac{1}{2}$ for $x = 1$

and $y = \dfrac{3}{4}$ for $x = \dfrac{2}{3}$

$\Rightarrow 1R\dfrac{1}{2}$ and $\dfrac{2}{3}R\dfrac{3}{4}$ but $0 \not R 1$ So, 2 and 3 are true

3. **(D)**

$R = \{(x, y) : x$ is relatively prime to $y\}$

$= \{(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 7)\}$.

Domain of $R = \{2, 3, 4, 5\}$

4. **(D)**

Let $R = \{(x, y)\} = \{(0, 2), (-1, 5), (2, -4)\}$ and for linear relationship between x and y.

Let $y = ax + b$ …… (1)

Now, when $x = 0$ then $y = 2$

so by (1) $\Rightarrow \boxed{2 = b}$

When $x = -1$ then $y = 5$; so by (1)

$\Rightarrow 5 = -a + b \Rightarrow a = b - 5 = 2 - 5 = -3$

So, $y = -3x + 2 \Rightarrow \boxed{3x + y = 2}$

5. **(A)**

We have $(x, y) \in R \Leftrightarrow x + 2y$

$= 10 \Leftrightarrow y = \dfrac{10-x}{2}, x, y \in A$

where $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Now $x = 1, y = \dfrac{10-1}{2} = \dfrac{9}{2} \notin A$

This shows that 1 is not related to any element in A. similarly, we can observe that 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation. Further, we find that for

$x = 2, y = \dfrac{10-2}{2} = 4 \in A$ $\therefore (2, 4) \in R$

For $x = 4, y = \dfrac{10-4}{2} = 3 \in A$ $\therefore (4, 3) \in R$

For $x = 6, y = \dfrac{10-6}{2} = 2 \in A$ $\therefore (6, 2) \in R$

For $x = 8, y = \dfrac{10-8}{2} = 1 \in A$ $\therefore (8, 1) \in R$

Thus $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

Hence domain of $R = \{2, 4, 6, 8\}$ and Range of $R = \{4, 3, 2, 1\}$.

6. (C)

$(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}$ $\therefore R^{-1} = \{(3, 1), (5, 1), (1, 2)\}$

7. **(A)**

$A = \{1, 2, 4\}, B = \{2, 3\}$

$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$

8. **(B)**

$R = \{(4, 6), (5, 6)\}$, Dom $(R) = \{4, 5\}$,

Range$(R) = \{6\}$

9. **(D)**

$\sin^2 a + \cos^2 b = 1$

Reflexive: $\sin^2 a + \cos^2 a = 1 \Rightarrow aRb$

$\sin^2 a + \cos^2 b = 1 \Rightarrow 1 - \cos^2 a + 1 - \sin^2 b = 1$

$\Rightarrow \sin^2 b + \cos^2 a = 1 \Rightarrow aRb$

Hence, symmetric

Let aRb, bRc

So, $\sin^2 a + \cos^2 b = 1$ ….(i)

and $\sin^2 b + \cos^2 c = 1$ ….(ii)

Adding (i) and (ii) we get;

$\sin^2 a + \cos^2 c = 1$

Hence, transitive therefore equivalent relation

10. **(A)**

Since, $1+a.a=1+a^2>0, \forall a \in S$

$\therefore (a, a) \in R \quad \therefore R$ is reflexive.

Also, $(a,b) \in R \Rightarrow 1+ab>0 \Rightarrow 1+ba>0$

$\Rightarrow (b, a) \in R \quad \therefore R$ is symmetric

$(a, b) \in R$ and $(b, c) \in R$ need not imply $(a, c) \in R$

Hence, R is not transitive.