# Real Numbers

## 2016

### Short Answer Type Questions I [2 Marks]

**Question 1.**

Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.

**Solution:**

Maximum capacity of a container, which can measure the petrol in exact number of times.

= HCF of (850 and 680)

$$\begin{array}{c|c} 2 & 850 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline & 17 \end{array} \qquad \begin{array}{c|c} 2 & 680 \\ \hline 2 & 340 \\ \hline 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

$850 = 2 \times 5 \times 5 \times 17$

$680 = 2 \times 2 \times 2 \times 5 \times 17$

HCF = (850 and 680) = $2 \times 5 \times 17 = 170$ litres.

**Question 2.**

Find the value of: $(-1) + (-1)^{2n} + (-l)^{2n+1} + (-l)^{4n+1}$ , where n is any positive odd integer.

**Solution:**

To find $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$

as '$n$' is any positive odd integer

$\Rightarrow 2n$ and $4n + 2$ are even positive integers.

Now

$$(-1)^n = -1$$
$$(-1)^{2n} = +1$$
$$(-1)^{2n+1} = (-1)^{2n}(-1)^1 = 1(-1) = -1$$
$$(-1)^{4n+2} = (-1)^{4n}(-1)^2 = 1(1) = 1$$

Put all values in the above expression ($i$)

We have $-1 + 1 - 1 + 1 = 0$

**Question 3.**

Find whether decimal expansion of 13/64 is a terminating or non-terminating decimal. If it terminates, find the number of decimal places its decimal expansion has.

**Solution:**

The given rational number is $\frac{13}{64}$

Now $\frac{13}{64} = \frac{13}{2^6} = \frac{13}{2^6 \times 5^0}$

The denominator of the given rational number is of the form $2^n \times 5^m$, i.e. $2^6 \times 5^0$

$\therefore$ The decimal expansion of $\frac{13}{64}$ is of the form of terminating.

The decimal expansion of $\frac{13}{64}$ terminates after 6 places of decimal.

$$\begin{array}{r|l} & 0.203125 \\ 64 & 13.000000 \\ & 128 \\ \hline & 200 \\ & 192 \\ \hline & 80 \\ & 64 \\ \hline & 160 \\ & 128 \\ \hline & 320 \\ & 320 \\ \hline & 0 \end{array}$$

### Short Answer Type Question II [3 Marks]

**Question 4.**

Explain whether the number 3 x 5 x 13 x 46 + 23 isa prime number or a composite number.

**Solution:**

We have $3 \times 5 \times 13 \times 46 + 23 = 23 \times (3 \times 5 \times 13 \times 2) + 23$

$= 23 \times (3 \times 5 \times 13 \times 2) + 23 = 23 \times 390 + 23$

It is clear that, the above number is a multiple of 23. Therefore, the number is $(3 \times 5 \times 13 \times 46 + 23)$ is not a prime number.

### Long Answer Type Question [4 Marks]

**Question 5.**

Prove that the product of any three consecutive positive integers is divisible by 6. Solution:

Let three consecutive numbers are n, n + 1, n + 2

**Solution:**

**1st Case:** If n is even

This means n + 2 is also even.

Hence n and n + 2 are divisible by 2

Also, product of n and (n + 2) is divisible by 2.

.’. n(n + 2) is divisible by 2.

This conclude n(n + 2) (n + 1) is divisible by 2 …(i)

As, n, n + 1, n + 2 are three consecutive numbers. n(n + 1) (n + 2) is a multiple of 3.

This shows n(n + 1) (n + 2) is divisible by 3. …(ii)

By equating (i) and (ii) we can say

n(n + 1) (n + 2) is divisible by 2 and 3 both.

Hence, n(n + 1) (n + 2) is divisible by 6.

**2nd Case:** When n is odd.

This show (n + 1) is even

Hence (n + 1) is divisible by 2. …(iii)

This conclude n(n + 1) (n + 2) is an even number and divisible by 2.

Also product of three consecutive number is a multiple of 3.

n(n + 1)(n + 2) is divisible by 3. …(iv)

Equating (iii) and (iv) we can say

n(n + 1) (n + 2) is divisible by both 2 and 3 Hence, n(n + 1)(n + 2) is divisible by 6.

## 2015

### Short Answer Type Questions I [2 Marks]

### Question 6.

Apply Euclid's division algorithm to find HCF of numbers 4052 and 420.

**Solution:**

Given numbers are 4052 and 420.

On applying Euclid's division algorithm, we have

$$4052 = 420 \times 9 + 272$$
$$420 = 272 \times 1 + 148$$
$$272 = 148 \times 1 + 124$$
$$148 = 124 \times 1 + 24$$
$$124 = 24 \times 5 + 4$$
$$24 = 4 \times 6 + 0$$

So, HCF of 4052 and 420 = 4.

```
420 ) 4052 ( 9
      3780
      272 ) 420 ( 1
            272
            148 ) 272 ( 1
                  148
                  124 ) 148 ( 1
                        124
                         24 ) 124 ( 5
                              120
                                4 ) 24 ( 6
                                    24
                                     0
```

### Question 7.

Show that (√3+√5)² is an irrational number.

**Solution:**

We have to prove that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number, i.e. to prove $(8+2\sqrt{15})$ is an irrational number.

Let us suppose $8+2\sqrt{15}$ is a rational number.

∴ There exists coprime integers (say) $a$ and $b$, $b \neq 0$ such that

$$8 + 2\sqrt{15} = \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{15} = \frac{a}{b} - 8 \quad \Rightarrow \quad \sqrt{15} = \frac{a-8b}{2b}$$

$\frac{a-8b}{2b}$ being rational number so, $\sqrt{15}$ becomes rational, which is contradiction with the fact that $\sqrt{15}$ is irrational. We led to contradiction due to wrong supposition.

Hence, $(\sqrt{3}+\sqrt{5})^2$ is irrational.

## Short Answer Type Question [3 Marks]

### Question 8.

Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively. If they start tolling together, after what time will they next toll together?

**Solution:**

LCM of 12, 15, 18 = $2^2 \times 3^2 \times 5$

=4x9x5 = 180

So, next time the bells will ring together after 180 minutes.

| | |
|---|---|
| 2 | 12, 15, 18 |
| 2 | 6, 15, 9 |
| 3 | 3, 15, 9 |
| 3 | 1, 5, 3 |
| 5 | 1, 5, 1 |
| | 1, 1, 1 |

## 2014

## Short Answer Type Questions I [2 Marks]

**Question 9.**

If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m.

**Solution:**

On applying Euclid's division algorithm,

180 = 144 x 1 + 36

144 = 36 x 4 + 0

At the last stage, the divisor is 36.

∴ HCF of 144 and 180 is 36.

∵ 36 = 13 x 3 – 3

So, m = 3

On applying Euclid's division algorithm,
$180 = 144 \times 1 + 36$
$144 = 36 \times 4 + 0$
At the last stage, the divisor is 36.
∴ HCF of 144 and 180 is 36.
∵ $36 = 13 \times 3 - 3$
So, $m = 3$

$$\begin{array}{r} 144\,)\,180\,(\,1 \\ 144 \\ \hline 36\,)\,144\,(\,4 \\ 144 \\ \hline 0 \end{array}$$

**Question 10.**

Show that $9^n$ can not end with digit 0 for any natural number n.

**Solution:**

Since prime factorisation of $9^n$ is given by $9^n$= (3 x 3)$^n$ = 3271.

Prime factorisation of 9″ contains only prime number 3.

9 may end with the digit 0 for some natural number V if 5 must be in its prime factorisation, which is not present.

So, there is no natural number N for which $9^n$ ends with the digit zero.

**Question 11.**

Determine the values otp and q so that the prime factorisation of2520 is expressible as 23 X y X q x 7.

**Solution:**

Prime factorisation of 2520 is given by

2520 = 23 x 32 x 5 x 7

Given that 2520 = 23 x 3p x q x 7

On comparing both factorisation we get p = 2 and q = 5.

**Question 12.**

Show that 2√2 is an irrational number.

**Solution:**

Let us assume that $2\sqrt{2}$ is rational.

∴ There exists coprime integers $a$ and $b$ $(b \neq 0)$ such that

$$2\sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{2b}$$

Since $a$ and $b$ are integers, we get $\frac{a}{2b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $2\sqrt{2}$ is rational.

Hence, we conclude that $2\sqrt{2}$ is irrational.

**Question 13.**

Show that any positive odd integer is of the form 4m + 1 or 4m + 3, where m is some integer.

**Solution:**

Let '$a$' be any positive integer and $b = 4$, then by Euclid's division algorithm, we have $a = 4q + r$, $0 \le r < 4$ where $q \ge 0$ and r = 0, 1, 2, 3.
Now '$a$' may be of the form of $4q$, $4q + 1$, $4q + 2$, $4q + 3$.

| | | | |
|---|---|---|---|
| When | When | When | When |
| $a = 4q$ | $a = 4q + 1$ | $a = 4q + 2$ | $a = 4q + 3$ |
| $= 2.2q$ | $= 2.(2q) + 1$ | $= 2.(2q) + 2$ | $= 2.2q + 2 + 1$ |
| $= 2m$ | $= 2m + 1$ | $= 2(2q + 1)$ | $= 2(2q + 1) + 1$ |
| where $m = 2q$ | = odd number | $= 2m$ | $= 2m + 1$ |
| = even number | | = even number | = odd number |

Clearly, it is seen that any positive odd integer is of the form $4m + 1$ or $4m + 3$ for some integer $m$.

## Short Answer Type Questions II [3 Marks]

**Question 14.**

By using, Euclid's algorithm, find the largest number which divides 650 and 1170.

**Solution:**

Given numbers are 650 and 1170.

On applying Euclid's division algorithm,

we get 1170 = 650 x 1 + 520

650 = 520 x 1 + 130

520 = 130 x 4 + 0

∵ At the last stage, the divisor is 130.

∴ The HCF of 650 and 1170 is 130.

$$\begin{array}{l} 650)\overline{1170}(1 \\ \quad 650 \\ \quad \overline{520)650}(1 \\ \qquad 520 \\ \qquad \overline{130)520}(4 \\ \qquad\quad 520 \\ \qquad\quad \overline{\ \ 0\ \ } \end{array}$$

**Question 15.**

Show that reciprocal of 3+2√2 is an irrational number

**Solution:**

We have to prove that $\frac{1}{3+2\sqrt{2}} = 3 - 2\sqrt{2}$ is an irrational number.

Let us assume that $3 - 2\sqrt{2}$ is rational.

∴ There exists coprime integers $a$ and $b$ ($b \neq 0$) such that

$$3 - 2\sqrt{2} = \frac{a}{b} \Rightarrow 2\sqrt{2} = 3 - \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{2} = \frac{3b-a}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{3b-a}{2b} = \frac{3}{2} - \frac{a}{2b}$$

Since $a$ and $b$ are integers, we get $\frac{3}{2} - \frac{a}{2b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.
This contradiction has arisen because of our incorrect assumption that $3 - 2\sqrt{2}$ is rational.
Hence, we conclude that $3 - 2\sqrt{2}$ is irrational.

## Long Answer Type Question [4 Marks]

**Question 16.**

Find HCF of 378,180 and 420 by prime factorisation method. Is HCF x LCM of three numbers equal to the product of the three numbers?

**Solution:**

378 = 2 x 33 x 7

180 = 22 x 32 x 5

420 = 22 x 3 x 5 x 7

∴ HCF (378, 180, 420) = 2 x 3 = 6.

No. HCF (p, q, r) x LCM (p, q, r) ≠ p x q x r. where p, q, r are positive integers.

## 2013

## Short Answer Type Questions I [2 Marks]

**Question 17.**

Find the HCF of 255 and 867 by Euclid's division algorithm

**Solution:**

Given numbers are 255 and 867.

On applying Euclid's division algorithm, we have

867 = 255 x 3 + 102

255 = 102 x 2 + 51

102 = 51 x 2 + 0

∵ At the last stage, the divisor is 51

∴ The HCF of 255 and 867 is 51.

```
255)867(3
    765
    102)255(2
        204
         51)102(2
            102
              0
```

**Question 18.**

Find the HCF (865, 255) using Euclid's division lemma.

**Solution:**

Given numbers are 255 and 865.

On applying Euclid's division algorithm, we have

865 = 255 x 3 + 100

255 = 100 x 2 + 55

100 = 55 x 1 + 45

55 = 45 x 1 + 10

45 = 10 x 4 + 5

10 = 5 x 2 + 0

∵At the last stage, the divisor is 5

∴ The HCF of 255 and 865 is 5.

255)865(3
765
100)255(2
200
55)100(1
55
45)55(1
45
10)45(4
40
5)10(2
10
0

[Delhi]

## Short Answer Type Questions II [3 Marks]

**Question 19.**

Find HCF of 65 and 117 and find a pair of integral values of m and n such that HCF = 65m + 117n.

**Solution:**

Given numbers are 65 and 117.

On applying Euclid's division algorithm, we get

117 = 65 x 1 + 52

65 = 52 x 1 + 13

52 = 13 x 4 + 0

∵At the last stage, the divisor is 13.

∴ The HCF of 65 and 117 is 13.

The required pair of integral values of m and n is

(2,-1) which satisfies the given relation HCF = 65m + 117n.

65)117(1
65
52)65(1
52
13)52(4
52
0

**Question 20.**

By using Euclid's algorithm, find the largest number which divides 650 and 1170

**Solution:**

Since prime factorisation of $9^n$ is given by $9^n$= $(3 \times 3)^n$ = 3271.

Prime factorisation of 9″ contains only prime number 3.

9 may end with the digit 0 for some natural number V if 5 must be in its prime factorisation, which is not present.

So, there is no natural number N for which $9^n$ ends with the digit zero.

## 2012

## Short Answer Type Question I [2 Marks]

**Question 21.**

If $\frac{241}{4000}=\frac{241}{2^m5^n}$, find the values of m and n where m and n are non-negative integers.Hence write its decimal expansion without actual division.

**Solution:**

$$\frac{241}{4000} = \frac{241}{2^m \cdot 5^n} \Rightarrow 4000 = 2^m.5^n$$

$$2^5 \times 5^3 = 2^m.5^n \Rightarrow m = 5, n = 3$$

$$\frac{241}{4\times1000} = \frac{1}{4}\times0.241 = 0.06025 \text{ (Terminating).}$$

Short Answer Type Questions II [3 Marks]

**Question 22.**

Express the number 0.3178 in the form of rational number a/b.

**Solution:**

Let $x = 0.3\overline{178}$

$\Rightarrow \quad 10x = 3.\overline{178} \quad ...(i)$

$\Rightarrow \quad 10000x = 3178.\overline{178} \quad ...(ii)$

Subtracting (*i*) from (*ii*)

$\Rightarrow \quad 9990x = 3175$

$$x = \frac{635}{1998}$$

**Question 23.**

Using Euclid’s division algorithm, find whether the pair of numbers 847,2160 are coprimes or not.

**Solution:**

$$2160 = 847 \times 2 + 466$$
$$847 = 466 \times 1 + 381$$
$$466 = 381 \times 1 + 85$$
$$381 = 85 \times 4 + 41$$
$$85 = 41 \times 2 + 3$$
$$41 = 3 \times 13 + 2$$
$$3 = 2 \times 1 + 1$$
$$1 = 1 + 0$$

$\because \quad$ HCF $= 1$

$\therefore$ Numbers 847 and 2160 are coprimes.

**Question 24.**

The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

**Solution:**

Let HCF $= x$

$\therefore \quad$ LCM $= 14x$

A.T.Q. $\quad x + 14x = 600 \Rightarrow x = 40$

Now, 280 × other number = HCF × LCM = 40 × 560

Other number = 80

## 2011

### Short Answer Type Questions I [2 Marks]

### Question 25.

Prove that 15 + 17√3 is an irrational number.

**Solution:**

Let $15 + 17\sqrt{3}$ is a rational number.

$\therefore\ 15 + 17\sqrt{3} = \frac{a}{b}$, where $a$ and $b$ are coprime, $b \neq 0$

$\Rightarrow \quad 17\sqrt{3} = \frac{a}{b} - 15 \quad \Rightarrow \sqrt{3} = \frac{a - 15b}{17b}$

$\because$ $a$ and $b$ are integers

So, $\frac{a - 15b}{17b}$ is a rational number and so, $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational. This contradiction has arisen because of our incorrect assumption that $15 + 17\sqrt{3}$ is rational.

Hence, $15 + 17\sqrt{3}$ is irrational.

### Question 26.

Find the LCM and HCF of 120 and 144 by using Fundamental Theorem of Arithmetic.

**Solution:**

120 = 23 x 3 x 5

144 = 24 x 32

∴ HCF = 23 x 3 = 24

LCM = 24 x 5 x 32 = 720

## Short Answer Type Questions II [3 Marks]

### Question 27.

An army contingent of 1000 members is to march behind an army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

**Solution:**

1000 =2x2x2x5x5x5

56 = 2x2x2x7

HCF of 1000 and 56 = 8

Maximum number of columns = 8.

### Question 28.

Show that any positive odd integer is of the form 4q + 1 or 4q + 3 where q is a positive integer.

**Solution:**

Let $N$ be any positive integer and $b = 4$

Then by Euclid's division lemma, $N = 4q + r,\ 0 \leq r < 4;\ q > 0$

$\therefore\ N = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$

($i$) when $N = 4q = 2(2q) =$ even

($ii$) when $N = 4q + 1 = 2(2q) + 1 =$ even $+ 1 =$ odd

($iii$) when $N = 4q + 2 = 2(2q + 1) =$ even

($iv$) when $N = 4q + 3 = 4q + 2 + 1 = 2(2q + 1) + 1 =$ Even $+ 1 =$ odd

$\therefore$ When $N = 4q + 1$ or $4q + 3$, then it is odd

$\Rightarrow$ Any positive odd integer is of the form $4q + 1$ or $4q + 3$.

### Question 29.

Prove that 2√3/5 is irrational

**Solution:**

Let $\frac{2\sqrt{3}}{5}$ is rational.

$\therefore$ There exists coprime integers $a$ and $b (b \neq 0)$

Such that $\frac{2\sqrt{3}}{5} = \frac{a}{b} \Rightarrow \sqrt{3} = \frac{5a}{2b}$

$\because$ $a$ and $b$ are integers we get $\frac{5a}{2b}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{2\sqrt{3}}{5}$ is rational.

So, we conclude that $\frac{2\sqrt{3}}{5}$ is irrational.

## 2010

### Very Short Answer Type Questions [1 Mark]

**Question 30.**

Has the rational number $\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$, a terminating or a non-terminating decimal repressentation

**Solution:**

$\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$ is non-terminating decimal.

Since $q = 2^2 \times 5^7 \times 7^2$ is not of the form $2^m \times 5^n$.

**Question 31.**

Write whether $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

**Solution:**

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times5}+3\sqrt{4\times5}}{2\sqrt{5}} = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}} = \frac{12\sqrt{5}}{2\sqrt{5}}$$

$= 6$ which is rational number.

**Question 32.**

The HCF of 45 and 105 is 15. Write their LCM.

**Solution:**

HCF $(45, 105) = 15$

$\therefore \quad \text{LCM} = \frac{45 \times 105}{15} = 315$

### Short Answer Type Questions II [3 Marks]

**Question 33.**

Prove that 2-3√5 is an irrational number.

**Solution:**

Let us assume, to contrary that $2 - 3\sqrt{5}$ is rational

Let $2 - 3\sqrt{5} = \frac{a}{b}$ where $a$ and $b$ are coprime numbers, $b \neq 0$

$$2 - \frac{a}{b} = 3\sqrt{5}$$

$$\frac{2b-a}{3b} = \sqrt{5}$$

Since $a$ and $b$ are integers, we get $\frac{2b-a}{3b}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $2 - 3\sqrt{5}$ is rational.

So, we conclude that $2 - 3\sqrt{5}$ is irrational.

## Question 34.

Prove that 2√3 – 1 is an irrational number.

**Solution:**

Let 2√3 – 1 is a irrational number.

$\therefore$ $2\sqrt{3} - 1 = \frac{a}{b}$, where $a$ and $b$ are coprimes and $b \neq 0$

$$\Rightarrow \quad 2\sqrt{3} = \frac{a}{b} + 1 \Rightarrow 2\sqrt{3} = \frac{a+b}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a+b}{2b} \quad \text{...}(i)$$

From ($i$), we notice

LHS is an irrational number and RHS is rational number, which is not possible. Hence, our supposition is wrong. Hence, $2\sqrt{3} - 1$ is an irrational number.

## Question 35.

Prove that √2 is irrational.

**Solution:**

Let us assume that $\sqrt{2}$ is rational.

$\therefore$ There exists coprime integers $a$ and $b$ ($b \neq 0$)

Such that

$$\sqrt{2} = \frac{a}{b} \quad \Rightarrow \sqrt{2}b = a$$

Squaring on both sides, we get

$$2b^2 = a^2 \quad \text{...}(i)$$

$\Rightarrow$ 2 divides $a^2$ $\Rightarrow$ 2 divides $a$

So, we can write

$$a = 2c \text{ for some integer } c \quad \text{...}(ii)$$

From ($i$) and ($ii$),

$$2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$$

$\Rightarrow$ 2 divides $b^2$ $\Rightarrow$ 2 divides $b$

$\therefore$ 2 is a common factor of $a$ and $b$.

But this contradicts the fact that $a$ and $b$ are coprimes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational.

## Question 36.

Prove that 7 – 2√3 is an irrational number.

**Solution:**

Let, $7-2\sqrt{3}$ be a rational number.

Let $7-2\sqrt{3} = \frac{a}{b}$, where $a$ and $b$ are coprimes and $b \neq 0$

$\Rightarrow \quad 7 - \frac{a}{b} = 2\sqrt{3} \Rightarrow \frac{7b-a}{b} = 2\sqrt{3} \Rightarrow \frac{7b-a}{2b} = \sqrt{3}$

We notice that LHS is a rational number, whereas RHS is an irrational number, which is a contradiction. Hence, our supposition is wrong.

Hence, $7-2\sqrt{3}$ is an irrational number.

**Question 37.**

Show that 5 + 3√2 is an irrational number.

**Solution:**

Let us assume 5 + 3√2 is an irrational number.

There exists coprime integers a and b (b≠0)

Such that

$$5 + 3\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{a}{b} - 5 \Rightarrow \sqrt{2} = \frac{a-5b}{3b}$$

$\because$ $a$ and $b$ are integers we get $\frac{a-5b}{3b}$ is rational number and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5+3\sqrt{2}$ is rational.

So, we conclude that $5+3\sqrt{2}$ is irrational.

## 2009

## Very Short Answer Type Questions [1 Mark]

**Question 38.**

The decimal expansion of the rational number $\frac{43}{2^4 . 5^3}$, will terminate after how many places of decimals.

**Solution:**

The decimal expansion of the rational number $\frac{43}{2^4 . 5^3}$, will terminate after 4 places of decimal.

$$\frac{43}{2^4 . 5^3} = \frac{43}{16 \times 125} = \frac{43}{2000} = 0.0215$$

**Question 39.**

Find the [HCF X LCM] for the numbers 100 and 190.

**Solution:**

HCF x LCM = one number x another number

= 100 x 190 = 19000

**Question 40.**

Find the [HCF and LCM] for the numbers 105 and 120. [All India]

**Solution:**

105 = 5 x 7 x 3

120 = 2x2x2x3x5

HCF = 3 X 5 = 15

LCM = 5x7x3x2x2x2 = 840

**Question 41.**

Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non terminating repeating decimal expansion.

**Solution:**

$\frac{51}{1500} = \frac{17}{500}$

Prime factorisation of 500 = $2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$

Its denominator has prime factors of the form $2^m \times 5^n$

So, it has terminating decimal expansion.

**Question 42.**

The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number.

**Solution:**

Let another number = $x$

$$\begin{aligned} \text{HCF } (45, x) &= 9 \\ \text{LCM } (45, x) &= 360 \\ \text{HCF} \times \text{LCM} &= 45 \times x \\ 9 \times 360 &= 45 \times x \\ x &= \frac{360}{5} = 72 \end{aligned}$$

## Short Answer Type Questions II [3 Marks]

**Question 43.**

Show that 5 – 2√3 is an irrational number.

**Solution:**

Let us assume that $5 - 2\sqrt{3}$ is rational number.

$\therefore$ There exists coprime integers $a$ and $b$ $(b \neq 0)$ such that

$5 - 2\sqrt{3} = \frac{a}{b}$

$\Rightarrow 2\sqrt{3} = 5 - \frac{a}{b} \Rightarrow \sqrt{3} = \frac{5b - a}{2b}$

$\because$ $a$ and $b$ are integers, we get $\frac{5b-a}{2b}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - 2\sqrt{3}$ is rational.

So, we conclude that $5 - 2\sqrt{3}$ is irrational.

**Question 44.**

Show that 3 + 5√2 is an irrational number.

**Solution:**

Let us assume that $3+5\sqrt{2}$ is rational number.

$\therefore$ There exists coprime integers $a$ and $b$ $(b \neq 0)$

Such that $3+5\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad 5\sqrt{2} = \frac{a}{b} - 3 \quad \Rightarrow \sqrt{2} = \frac{a-3b}{5b}$

$\because$ $a$ and $b$ are integers we get $\frac{a-3b}{5b}$ is rational number and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3+5\sqrt{2}$ is rational.

So, we conclude that $3+5\sqrt{2}$ is irrational.

**Question 45.**

Show that the square of any positive odd integer is of the form 8m + 1, for some integer m.

**Solution:**

Let $a = 2q + 1$ be any positive odd integer.

Now,

$$\begin{aligned} a^2 &= (2q+1)^2 \\ &= 4q^2 + 4q + 1 \\ &= 4q(q+1) + 1 \\ &= 4(2m) + 1 \\ &= 8m + 1 \end{aligned}$$

[$\because$ $q$ and $(q+1)$ are consecutive numbers, so one of them must be even and of the form $2m$.]

$\therefore$ Square of any positive odd integer is of the form $8m + 1$, for some integer $m$.

**Question 46.**

Prove that 7 + 3√2 is not a rational number.

**Solution:**

Let $7 + 3\sqrt{2}$ be rational

$\Rightarrow \quad 7 + 3\sqrt{2} = \frac{p}{q}$, where $q \neq 0$ and $p$ and $q$ are coprimes

$\Rightarrow \quad 3\sqrt{2} = \frac{p}{q} - 7$

$\Rightarrow \quad \sqrt{2} = \frac{p-7q}{3q}$

Here, on RHS $\frac{p-7q}{3q}$ is rational, whereas $\sqrt{2}$ is irrational.

Therefore, our assumption is wrong.

Hence, $7 + 3\sqrt{2}$ is irrational.