SEQUENCES AND SERIES

One mark questions

1. Find the 16th term of the sequence $a_n = 4n-3$ (K)

2. Find the 5th term of the sequence $a_n = \frac{n^2}{2^n}$ (K)

3. Find the 7th term of the sequence $a_n = (-1)^n n^3$ (K)

4. Find the 20th term of the sequence $a_n = \frac{n(n-2)}{n+3}$ (K)

5. If 3,n,8 are the three consecutive terms of the Fibonacci sequence, then find the value of n. (K)

6. Find the 15th term of the A.P. 1,4,7,…   (K)

7. If A,B,C are the angles of a triangle which are in A.P, then write the value of B.   (A)

8. Find the fifth term of the G.P. 2,6,18,…   (K)

9. Find the common difference of the A.P. $\frac{5}{4}, \frac{3}{4}, \frac{1}{4}$, …   (K)

10. Find the common ratio of the G.P. 2, $2\sqrt{3}$, 6,…   (K)

11. Find the arithmetic mean of the numbers 6 and 10.   (K)

12. Find the geometric mean of the numbers 4 and 9.   (K)

Three mark questions

13. Find the first three terms of the sequence $a_n = (-1)^{n-1} 5^{n+1}$ (K)

14. Find the first three terms of the sequence $a_n = \frac{n(n^2+5)}{4}$ (K)

15. Find the sum of the first three terms of the sequence defined by $a_1 = 3$, $a_n = 3a_{n-1}+2$(when n>1).   (K)

16. In an A.P. if m^{th} term is n and n^{th} term is m, where m≠n, show that p^{th} term is m+n-p.   (S)

17. In an A.P. if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, show that the sum of first pq terms is $\frac{1}{2}$(pq+1), where p≠q   (S)

18. If the sum of n terms of an A.P. is $nP + \frac{n(n-1)Q}{2}$, where P and Q are constants, find the common difference.   (A)

19. The sum of n terms of two arithmetic progressions are in the ratio (3n+8) : (7n+15). Find the ratio of their 12th terms.   (S)

20. The sum of n terms of two arithmetic progressions are in the ratio (5n+4) : (9n+6). Find the ratio of their 18th terms.   (S)

21. The income of a person is Rs 3,00,000 in the first year and he receives an increment of Rs 10,000 each year for the next 19 years. Find the total amount, he received in 20 years.   (A)

22. Find the 6 numbers to be inserted between 3 and 24 such that the resulting sequence is an A.P.(U)

23. Find the 5 numbers to be inserted between 8 and 26 such that the resulting sequence is an A.P.   (U)

24. Find the sum of odd integers from 1 to 2001.   (U)

25. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.   (A)

26. In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.   (S)

27. In the A.P. $-6, \frac{-11}{2}, -5, \ldots$ sum to n^{th} terms is -25. Find the value of n. (U)

28. In the A.P. 25,22,19,... sum to n^{th} terms is 116. Find the nth term. (U)

29. Find the sum to 10 terms of the A.P., whose n^{th} term is 5n+1. (A)

30. Find the sum to n terms of the A.P., whose k^{th} term is 5k+1. (A)

31. If the sum to n terms of an A.P is $3n+5n^2$, find the common difference. (A)

32. If the sum to n terms of an A.P is $pn+qn^2$, where p and q are constants. Show that the common difference is 2q. (A)

33. If the sum of first p terms of an A.P. is equal to the sum of first q terms and p, q are distinct, then show that the sum of first (p+q) terms is equal to 0. (S)

34. If the sum of n terms of an A.P is $3n^2 + 5n$ and its mth term is 164, find the value of m. (A)

35. A man starts repaying a loan as first instalment of Rs 1000. If he increases the instalment by Rs 50 every month, what amount he will pay in the 30th instalment? (A)

36. The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of sides of the polygon. (A)

37. In a G.P., the third term is 24 and the 6th term is 192. Find the 10th term. (U)

38. In the G.P. $3, \frac{3}{2}, \frac{3}{4}, \ldots$ if $S_n = \frac{3069}{512}$, find the value of n. (U)

39. In the G.P. $3, 3^2, 3^3, \ldots$ if $S_n = 120$, find the value of n. (U)

40. In the G.P. $2, 2\sqrt{2}, 4 \ldots$ if $a_n = 128$, find the value of n. (U)

41. In the G.P. $\sqrt{3}, 3, 3\sqrt{3} \ldots$ if $a_n = 729$, find the value of n. (U)

42. In the G.P. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ if $a_n = \frac{1}{729}$, find the value of n. (U)

43. Find the values of x, if the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P. Also find the common ratios (U)

44. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the common ratios. (A)

45. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratios. (A)

46. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own. (A)

47. Find the 3 numbers to be inserted between 1 and 256 such that the resulting sequence is an G.P. (U)

48. Find the 3 numbers to be inserted between 1 and 243 such that the resulting sequence is an G.P. (U)

49. If A and G respectively represents the Arithmetic mean and Geometric mean of two positive real numbers, then show that A≥G. (S)

50. If A.M. and G.M. of two positive numbers 'a' and 'b' are 10 and 8, respectively, find the numbers. (A)

51. Find the 12th term of the G.P. whose 8th term is 192 and common ratio is 2. (U)

52. The 5th, 8th and 11th terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$. (A)

53. The 4th term of a G.P. is square of its second term, and the first term is -3. Determine its 7th term. (U)

54. The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term and the common ratio. (A)

55. Given a G.P with a=729 and 7th term is 64, Determine S_7. (U)

56. In a G.P. if sum of the first two terms is -4 and the fifth term is 4 times the third term. Find the first terms of the G.P. (A)

57. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P. (A)

Show that the products of the corresponding terms of the sequence, $ar, ar^2, ... ar^{n-1}$ and $A, AR, AR^2, ... AR^{n-1}$ form a G.P. and find the common ratio.

58. Find the sum of the products of the corresponding terms of the sequence
   $2, 4, 8, 16, 32$ and $128, 32, 8, 2, \frac{1}{2}$. (U)

59. Find four numbers forming a G.P. in which the third term is greater than the first term by 9, and the second term is greater than the fourth by 18. (S)

60. If m^{th}, p^{th} and q^{th} terms of a G.P are x, y and z respectively. Prove that $x^{p-q}y^{q-m}z^{m-p} = 1$ (S)

61. If the first and n^{th} term of a G.P. are a and b, respectively, and if P is the product of first n terms, Prove that $P^2 = (ab)^n$. (S)

62. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$. (S)

63. If a, b, c and d are in G.P. show that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$. (S)

64. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 8th hour? (A)

65. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually? (A)

66. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation. (S)

67. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, prove that x, y, z are in A.P. (S)

68. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A.P. is equal to twice the mth term. (A)

69. If the sum of three numbers in A.P., is 24 and their product is 440, find the numbers. (S)

70. Let the sum of n, 2n, 3n terms of an A.P be S_1, S_2 and S_3 respectively, show that $S_3 = 3(S_2 - S_1)$. (S)

71. Find the sum of all numbers between 200 and 400 which are divisible by 7. (A)

72. Find the sum of all two digit numbers which when divided by 4, yields 1 as reminder. (A)

73. If f is a function satisfying f(x+y)=f(x).f(y) for all x,y ∈ N such that f(1)=3 and $\sum_{x=1}^{n} f(x) = 120$, find the value of n. (S)

74. The sum of some terms of G.P is 315 whose first term and the common ratio are 5 and 2 respectively. Find the last term and the number of terms. (U)

75. The first term of a G.P is 1. The sum of third and fifth term is 90. Find the common ratio of G.P. (U)

76. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find the common ratio. (S)

77. If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ (x≠ 0), then show that a, b, c, d are in G.P. (S)

78. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. Prove that a, c, e are in G.P. (S)

79. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1+8S_1)$. (A)

80. A person writes a letter to four office friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on postage when 8th set of letter is mailed. (A)

81. A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th yearsince he deposited the amount and also calculate the total amount after 20 years. (A)
82. A manufacturer reckons that the value of a machine, which costs him Rs 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years. (A)

Four mark questions

83. Find the sum to n terms of the series :   5+11+19+29+41+.... (S)
84. Find the sum of the first n terms of the series: 3+7+13+21+31+... (S)
85. Find the sum to n terms of the series whose nth term is n(n+3) and hence find S_{10}. (K)
86. Find the sum to n terms of the series: 1x2 + 2x3 + 3x4 + 4x5 + ...and hence find S_{12}. (U)
87. Find the sum to n terms of the series: 1x2x3 + 2x3x4 + 3x4x5 + ... (U)
88. Find the sum to n terms of the series: $3x1^2 + 5x2^2 + 7x3^2 +$ .... (U)
89. Find the sum to n terms of the series 3x8 + 6x11 + 9x14 + ... and hence find S_9. (U)
90. Find the sum to n terms of the series: $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) +$ .... (U)
91. Find the sum to n terms of the series: $\frac{1}{1x2} + \frac{1}{2x3} + \frac{1}{3x4} +$ ... (S)
92. Find the sum to n terms of the series whose nth term is given by n(n+1)(n+4). (U)
93. Find the sum to n terms of the series whose nth term is given by n^2+2^n.Hence find S_4. (U)
94. Find the sum to n terms of the series whose nth term is given by $(2n-1)^2$.Hence find S_5. (U)
95. Find the sum of the following series up to n terms: $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots$ (A)

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