CHAPTER 05

Design of Doubly Reinforced Beam by Limit State Method

5.1 Introduction

After understanding the concept of singly reinforced beam sections wherein reinforcement is provided only on one side i.e. on tension side, we will now look at doubly reinforced beam sections wherein, apart from tension reinforcement, additional reinforcement is also provided in the compression zone of the beam. This increases the moment of resistance (or in other words, the moment carrying capacity) of the beam. But now the question arises:

"Why to provide compression reinforcement for enhancing the moment carrying capacity of a beam while we can always do this by increasing the effective depth (thereby overall depth) of the beam?"

The answer to above question lies in the fact that it is NOT always possible to increase the depth of the beam due to architectural considerations, head room restrictions etc. Moreover, increasing the depth of beam adds significantly to self-weight of the beam (as compared to weight of compression reinforcement).

Above all, the MOST important reason for providing doubly reinforced beams is that they provide safety against reversal of stresses in a structure due to wind, seismic forces, temperature stresses and many other reasons.

5.2 Doubly Reinforced Beam Section

Doubly reinforced beam sections are usually provided where there are restrictions in the beam depth due to architectural considerations or other restrictions like availability of head room etc. and where singly reinforced beam sections are not adequate/strong enough to carry the design moment coming over the beam.

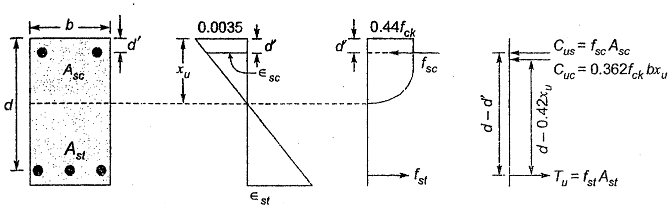
Doubly reinforced beam sections are also used where there is a possibility of occasional reversal of stresses due to lateral forces like wind or earthquake.

Provision of compression reinforcement is advantageous because it reduces the long term deflections due to shrinkage. All compression reinforcement is enclosed by ties to prevent their possible bucking under compression.

5.3 Hanger Bars v/s Compression Reinforcement

Hanger bars are of nominal diameter provided in the compression zone of a beam for holding the shear stirrups. Hanger bars are generally not considered as compression reinforcement. But in case, the area of hanger bars is significant (greater than 0.2%), then hanger bars are treated as compression reinforcement.

5.4 Analysis of Doubly Reinforced Rectangular Beam Sections



(a) Beam section (b) Strains (c) Stresses (d) Resultant force

Fig. 5.1 *Stress Strain distribution in doubly reinforced rectangular beam*

From the above figure, it can be seen that the stress strain distribution in doubly reinforced beams are similar to those of singly reinforced beams. The only difference in the present case is that there is additional stress (f_{sc}) in compression steel (A_{sc}) which also needs to be considered.

Now, the important thing which is to be noted that the stress in compression steel (f_{sc}) may or may not reach the design yield strength of steel ($0.87f_y$). The stress in compression steel (f_{sc}) depends on the corresponding compression strain ($\in_{sc}$).

From the above strain diagram

$$\frac{0.0035}{x_u} = \frac{\in_{sc}}{x_u - d'}$$

$$\Rightarrow \qquad \in_{sc} = 0.0035 \times \left(1 - \frac{d'}{x_u}\right)$$

where, d' = Distance of centroid of compression steel from the extreme compression fiber of concrete

In practical situations, d' is expressed in terms of the ratio (d'/d) which generally varies from 0.05 to 0.2.

Remember: In most cases, when Fe 250 is provided as compression reinforcement, then it will attain its design yield point ($0.87f_y$) but in case of Fe 415 and Fe 500 steel, it is not possible.

From static equilibrium,

$$C_{uc} + C_{us} = T_u$$

where, C_{uc} and C_{us} denote the compressive force in concrete and compression steel respectively.

Now, $\qquad C_{uc} = 0.362f_{ck}bx_u$

and $\qquad C_{us} = (f_{sc} - 0.45f_{ck})A_{sc}$

and $\qquad T_u = f_{st}A_{st}$

where, $\qquad f_{st} = 0.87f_y$ if $x_u \le x_{u\,lim}$

Thus, $\quad 0.362f_{ck}bx_u + (f_{sc} - 0.45f_{ck})A_{sc} = f_{st}A_{st}$

$$\Rightarrow \qquad x_u = \frac{f_{st}A_{st} - (f_{sc} - 0.45f_{ck})A_{sc}}{0.362f_{ck}b}$$

The above equation can be solved **explicitly** for x_u if $f_{st} = f_{sc} = 0.87f_y$.

For all other cases when f_{st} and f_{sc} are NOT equal to 0.87 f_y, the above equation has to be solved **implicitly**. Since in that case f_{st} and f_{sc} will in fact depend on x_u only (which is unknown and yet to be determined). This involves a trial and error procedure.

After having the value of x_u, the moment of resistance M_R can be determined as:

$$M_R = C_{uc}(d - 0.42\,x_u) + C_{us}(d' - d)$$

5.5 Limiting Moment of Resistance

The limiting moment of resistance can be obtained *when* x_u reaches $x_{u\,lim}$ and in that case, limiting moment of resistance of doubly reinforced beam section is given by:

$$M_{u\,lim} = 0.362f_{ck}bx_{u\,lim}(d - 0.42x_{u\,lim}) + (f_{sc} - 0.45f_{ck})A_{sc}(d - d')$$

Table 5.1: *Values of f_{sc}, when $x_u = x_{u\,lim}$ for different d'/d ratios*

Grade of steel	d'/d			
	0.05	0.1	0.15	0.2
Fe 250	217.5	217.5	217.5	217.5
Fe 415	355.1	351.9	342.4	329.2
Fe 500	423.9	411.3	395.1	370.3

Here the value of f_{sc} depends on the strain in compression steel ($\in_{sc}$). When $x_u = x_{u\,lim}$, f_{sc} can be calculated by interpolation from the above table for different $\frac{d'}{d}$ ratios.

A close look at the above values indicates that 217.5 N/mm^2 (=0.87 × 250 N/mm^2) is the design yield stress of Fe 250 which confines "Rememer" tip of art. 5.4.

5.6 Balanced Doubly Reinforced Sections

Due to IS 456: 2000 recommendation and also due to obvious reasons, over reinforced sections are undesirable in structural members. Thus the depth of neutral axis cannot exceed the limiting depth of neutral axis ($x_{u\,lim}$) for different grades of steel.

For singly reinforced sections which are obviously under reinforced, $x_u \le x_{u\,lim}$ and $p_t \le p_{t\,lim}$. In this case, if we further increase the compression resistance by introducing compression steel (doubly reinforced), then x_u will decrease further and section remains under reinforced.

But if the beam section is doubly reinforced with $p_t > p_{t\,lim}$ and $p_c > 0$, then the condition to avoid over reinforced sections i.e. $x_u \le x_{u\,lim}$ can be met by limiting $(p_t - p_{t\,lim})$ to a value which is in congruence to the value of p_c (percentage compression reinforcement).

Thus, $\qquad p_t = p_{t\,lim} + (p_t - p_{t\,lim})$

Here p_{tlim} corresponds to percentage tension steel for balanced singly reinforced sections and the corresponding tensile force in reinforcement steel is balanced completely by the ultimate compressive force in concrete i.e. $0.362f_{ck}bx_u$.

The remaining steel i.e. $(p_t - p_{t\,lim})$ and the associated tensile force is balanced by additional compressive force in compression steel C_{us}.

Let p_c^* = Percentage compression steel corresponding to balanced doubly reinforced section.

Thus, tensile force due to additional $(p_t - p_{t\,lim})$ steel = Compressive force due to additional compression steel

$$0.87f_y \frac{(p_t - p_{t\,lim})bd}{100} = (f_{sc} - 0.45f_{ck})\frac{p_c^*}{100}bd$$

$$p_c^* = \frac{0.87f_y}{(f_{sc} - 0.45f_{ck})}(p_t - p_{t\,lim})$$

The above expression gives the percentage compression reinforcement corresponding to balanced doubly reinforced section.

Remember: Thus p_c provided in beams MUST NOT be less than p_c^*. It implies that when $p_c > p_c^*$ then $x_u \le x_{u\,lim}$ and the section is under reinforced.

If however $p_c < p_c^*$ then section becomes over reinforced.

The moment of resistance of balanced doubly reinforced beam section is given by:

$$M_{ulim\text{-}DR} = 0.87f_y A_{st}(d - 0.42x_{ulim}) + (f_{sc} - 0.45f_{ck})A_{sc} \times (d - d')$$

$$\Rightarrow \quad M_{ulim\text{-}DR} = 0.87f_y\left[\frac{p_{t\,lim}}{100}bd(d - 0.42x_{u\,lim}) + \frac{(p_t - p_{t\,lim})bd}{100}(d - d')\right]$$

5.7 Design of Doubly Reinforced Rectangular Beam Section

Due to restrictions/limitations in the beam depth, the dimensions of the beam section are more or less already fixed/pre-determined and their design involves only the determination of tension reinforcement (A_{st}) and compression reinforcement (A_{sc}).

It is always to be kept in the mind that the depth of neutral axis (x_u) cannot exceed the limiting depth of neutral axis ($x_{u\,lim}$). This is achieved by resolving the design moment (M_u) into $M_{u\,lim}$ and ΔM_u.

Thus, $$M_u = M_{u\,lim-DR} = M_{u\,lim-SR} + \Delta M_u$$

Here, $M_{u\,lim-SR}$ is the limiting moment capacity of the singly reinforced beam section and ΔM_u is the additional moment which is required to be resisted by the compression steel (A_{sc}) and the corresponding additional tension steel ΔA_{st} to be provided in the tension zone.

The contribution of concrete in compression is entirely accounted for in $M_{u\,lim-SR}$ and thus it does not contribute to ΔM_u.

Alternatively, it can be expressed as,

$$p_t = p_{t\,lim} + \Delta p_t$$

$$\Rightarrow \quad A_{st} = A_{st\,lim} + \Delta A_{st}$$

Here $A_{st\,lim}$ is the tension steel corresponding to $M_{u\,lim-SR}$ and ΔA_{st} is corresponding to ΔM_u

Here ΔM_u is obtained from the couple formed by the compressive force in compression steel $(f_{sc} - 0.45f_{ck})A_{sc}$ and an equal (and opposite) tensile force of additional tensile steel i.e., $0.87f_y \Delta A_{st}$.

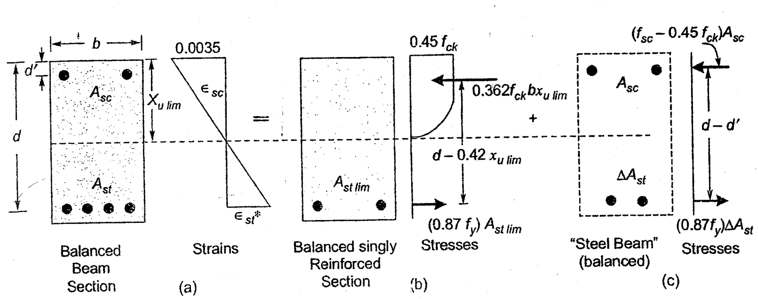


Fig. 5.2 *Balanced doubly reinforced beam section design*

The lever arm of this couple is $(d - d')$. The stress f_{sc} in the compression steel at ultimate limit state obviously depends on the strain in compression steel (ε_{sc}) at ultimate limit state.

Now, $$\Delta M_u = M_u - M_{u\,lim}$$

$$\Rightarrow \quad 0.87f_y \Delta A_{st}(d - d') = M_u - M_{ulim}$$

$$\Rightarrow \quad \Delta A_{st} = \frac{M_u - M_{ulim}}{0.87f_y(d - d')}$$

$$\Rightarrow \quad \frac{\Delta p_t}{100} = \frac{R_u - R_{u\,lim}}{0.87f_y\left(1 - \frac{d'}{d}\right)} \qquad \text{Where } R_u = \frac{M_u}{bd^2} \text{ and } R_{ulim} = \frac{M_{ulim}}{bd^2}$$

Thus, $$\Delta p_t = \frac{100(R_u - R_{u\,lim})}{0.87f_y\left(1 - \frac{d'}{d}\right)}$$

Also, $$A_{sc} = \frac{0.87f_y \Delta A_{st}}{f_{sc} - 0.447f_{ck}}$$

$$\Rightarrow \quad p_c^* = \frac{0.87f_y(p_t - p_{t\,lim})}{f_{sc} - 0.447f_{ck}} = \text{Percentage compression reinforcement.}$$

Table 5.2: Values of stress (in N/mm²) in compression steel (f_{sc}) at various strain levels

Fe415		Fe500		Stress Level
STRAIN	STRESS (N/mm²)	STRAIN	STRESS (N/mm²)	(N/mm²)
0.00144	288.7	0.00174	347.8	$0.8f_{yd}$
0.00163	306.7	0.00195	369.6	$0.85f_{yd}$
0.00192	324.8	0.00226	391.3	$0.9f_{yd}$
0.00241	342.8	0.00277	413	$0.95f_{yd}$
0.00276	351.8	0.00312	423.9	$0.975f_{yd}$
0.00380	360.9(= 0.87 × 415)	0.00417	434.8(= 0.87 × 500)	$1.0f_{yd}$

f_{yd} = Design yield stress of steel = $0.87f_y$.

If value of f_{sc} is not given then take f_{sc} = 350 N/mm² for Fe415 and f_{sc} = 420 N/mm² for Fe500.

5.8 Design Steps for a Given Factored Moment (M_u)

1. **Determine A_{st}:** For a given rectangular beam section (i.e. given b, d, f_{ck}, f_y), determine the limiting moment of resistance of the singly reinforced rectangular section (which is $0.148 f_{ck} bd^2$, $0.138 f_{ck} bd^2$ and $0.133 f_{ck} bd^2$ for Fe250, Fe415 and Fe500 respectively). If $M_{u\,lim}$ is greater than or equal to the factored moment M_u then section is designed as singly reinforced balanced section. If $M_{u\,lim} < M_u$ then section is designed as doubly reinforced section. Assume a suitable value of d′ and determine ΔA_{st} as derived in above steps. The total $A_{st\,reqd.}$ can be obtained as ($A_{st\,lim} + \Delta A_{st}$).
2. **Determine A_{sc}:** Using the value of ΔA_{st} actually provided, determine the value of $A_{sc\,reqd}$ from the expression of A_{sc} as derived above. The value of stress in compression steel (f_{sc}) is obtained from the value of strain in compression steel (ε_{sc}). The compression bars should be so selected that the area of compression steel provided is as close (but in any case must not be less than) to the compression steel required as possible.

5.9 Deflection Control in Doubly Reinforced Beams

For doubly reinforced beams, deflection check is normally not required because the limiting (l/d) ratio is usually satisfied due to the modification factor (k_c) for compression steel for deflection control. However, when beam is very shallow, check for deflection becomes necessary.

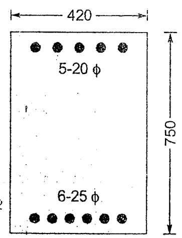
Example 5.1 **Calculate the moment of resistance of doubly reinforced beam section of size 420 × 750 mm. Reinforcement provided on the tension side is 6 nos. 25 mm dia. bars and on the compression side is 5 nos. 20 mm dia. bars. Use M 25 concrete and Fe 500 steel.**

Solution:

Width of the beam (b) = 420 mm

Overall depth of the beam (D) = 750 mm

Assuming an effective cover of 50 mm,



Effective depth of the beam (d) = 750 − 50 mm = 700 mm

$$\text{Area of tension reinforcement } (A_{st}) = 6\times\frac{\pi}{4}\times 25^2 = 2945 \text{ mm}^2$$

$$\text{Area of compression reinforcement } (A_{sc}) = 5\times\frac{\pi}{4}\times 20^2$$

$$= 1570.8 \text{ mm}^2 \approx 1571 \text{ mm}^2$$

Limiting depth of neutral axis

$$x_{u\,lim} = 0.46\,d$$
$$= 0.46 \times 700 \text{ mm} = 322 \text{ mm}$$

Actual depth of neutral axis

$$C = T$$
$$0.36 f_{ck} b x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$
$$\Rightarrow \quad 0.36 \times 25 \times 420 \times x_u + (f_{sc} - 0.45 \times 25) \times 1571 = 0.87 \times 500 \times 2945$$
$$\Rightarrow \quad 3780 x_u + 1571 (f_{sc}) = 1298749 \qquad \text{...(i)}$$

Let us solve by trial and error

Assume f_{sc} = 420 N/mm²

∴ x_u = 169 mm ... Ist trial

$$\epsilon_{sc} = \frac{x_u - d_c}{x_u}\times 0.035 = \frac{169-50}{169}\times 0.0035$$

$$\epsilon_{sc} = 0.00246$$

$$f_{sc} = 391.3 + \frac{(413-391.3)}{(0.00277-0.00226)}\times(0.00246-0.00226) = 399.8 \text{ N/mm}^2$$

In eq. (i), put the value of f_{sc}

∴ x_u = 177.34 mm ...IInd trial

$$\epsilon_{sc} = \frac{177.34-56}{177.34}\times 0.0035 = 0.00251$$

$$\therefore \quad f_{sc} = 391.3 + \frac{(413-391.3)}{(0.00277-0.00226)}\times(0.00251-0.00226)$$

$$f_{sc} = 402 \text{ N/mm}^2$$

Put this value of f_{sc} in eq. (i)

∴ x_u = 176.5 mm < $x_{u\,lim}$ (= 322 mm)

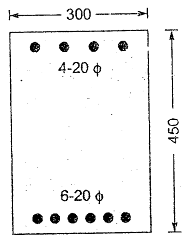
Take x_u = 177 mm and f_{sc} = 402 N/mm².

Moment of resistance

$$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$
$$= 0.36 \times 25 \times 420 \times 177(700 - 0.42 \times 177) + (402 - 0.45 \times 25)(700 - 50)$$
$$= 817.6 \text{ kNm}$$

Example 5.2 Calculate the moment of resistance of doubly reinforced beam section of size 300 × 450 mm. It is reinforced with 6-20φ bars in tension side and 4-20φ bars in compression side. Use M 20 concrete and Fe 250 steel.

Solution:



Width of the beam, b = 300 mm

Overall depth of the beam, D = 450 mm

f_{ck} = 20 N/mm²

f_y = 250 N/mm²

Let effective cover = 50 mm

∴ Effective depth of the beam, d = 450 − 50 mm = 400 mm

$$\text{Tension reinforcement, } A_{st} = 6\times\frac{\pi}{4}\times 20^2 = 1885 \text{ mm}^2$$

$$\text{Compression reinforcement, } A_{sc} = 4\times\frac{\pi}{4}\times 20^2 = 1256 \text{ mm}^2$$

$$C = T$$
$$\Rightarrow \quad 0.36 f_{ck} b x_u + (f_{sc} - f_{cc}) A_{sc} = f_{st} A_{st}$$

Let $f_{st} = f_{sc} = 0.87 f_y$ and neglecting 'f_{cc}', we have

$$x_u = \frac{0.87 f_y (A_{st} - A_{sc})}{0.36 f_{ck} b} = \frac{0.87(250)(1885-1256)}{0.36(20)(300)}$$

$$= 63.34 \text{ mm}$$

Limiting depth of neutral axis for Fe 250 $x_{u\,lim} = 0.53d = 0.53 \times 400 = 212 \text{ mm} > x_u$

$\because \quad x_u < x_{u\,lim}$

So secion is under reinforced and assumption of $f_{sc} = f_{st} = 0.87 f_y$ is correct.

Moment of resistance, $MOR = 0.36 f_{ck}\, b x_u (d - 0.42\, x_u) + f_{sc} A_{sc} (d - d')$

$$= 0.36 \times 20 \times 300 \times 63.34(400 - 0.42 \times 63.34)$$
$$+0.87 \times 250 \times 1256\,(400 - 50) \text{ Nmm}$$
$$= 146.7 \text{ kNm}$$

$\therefore$ MOR of doubly reinforced beam section is 146.7 kNm ≈ 146 kNm.

Example 5.3 Design a reinforced concrete beam of span 8 m which is being subjected to a live load of 30 kN/m. Overall depth of the beam is limited to 650 mm. Use M20 concrete and Fe 415 steel.

Solution:

Step-1: Loads

Let overall depth of beam, $D = 650$ mm

and width of beam, $B = 300$ mm

Self weight of beam, $= 25(0.3)(0.65) = 4.875$ kN/m

Live load $= 30$ kN/m (Given)

$\therefore$ Total load $= 34.875$ kN/m

$\therefore$ Factored load $w = 1.5 \times 34.875$

$= 52.3125$ kN/m

Step-2: Calculate design moment and design shear

Design moment $$M_u = \frac{wl^2}{8} = 52.3125 \times \frac{(8)^2}{8} = 418.5 \text{ kNm}$$

Design shear force $$V_u = \frac{wl}{2} = 52.3125 \times \frac{8}{2} = 209.25 \text{ kN}$$

Step-3: Calculate limiting MOR for singly reinforced beam section

For Fe 415, limiting moment of resistance

$$M_{u\,lim} = 0.138 f_{ck} b d^2$$

Let effective cover $= 50$ mm

Effective depth of beam $d = 650 - 50 \text{ mm} = 600$ mm

$$\therefore \quad M_{u\,lim} = 0.138(20)(300)(600)^2$$
$$= 298.08 \text{ kNm} < M_u (= 418.5 \text{ kNm})$$

Thus doubly reinforced beam section is required.

Step-4: Calculate amount of compression reinforcement required.

$$\Delta M_u = M_u - M_{u\,lim}$$
$$= 418.5 - 298.08 \text{ kNm} = 120.42 \text{ kNm}$$

$$\therefore \quad f_{sc} = \frac{0.0035(x_{u\,lim} - d')}{x_{u\,lim}} \times E_s$$

$$= \frac{0.0035(0.48 \times 600 - 50)}{0.48 \times 600} \times 2 \times 10^5 \text{ N/mm}^2$$

$$= 578.47 \text{ N/mm}^2$$

But $f_{sc} \not> 0.87 f_y = 0.87(415) = 361.05 \text{ N/mm}^2$

$$\therefore \quad A_{sc} = \frac{M_u - M_{u\,lim}}{f_{sc}(d-d')} = \frac{120.42 \times 10^6}{361.05(600-50)} = 606.413 \text{ mm}^2$$

Provide 2-20φ bars on compression side so that

$$A_{sc\,provided} = 628.3 \text{ mm}^2 > 606.413 \text{ mm}^2 \quad \text{(OK)}$$

Step-5: Calculate amount of tension reinforcement required

A_{st2} = Area of tension steel to balance A_{sc}

$$= \frac{f_{sc} A_{sc}}{0.87 f_y} = 606.413 \text{ mm}^2$$

$$A_{st1} = A_{st\,lim} = \frac{0.362 f_{ck} b x_{ulim}}{0.87 f_y}$$

$$= \frac{0.362(20)(300)(0.48 \times 600)}{0.87(415)} = 1732.5 \text{ mm}^2$$

Alternatively

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\frac{x_{ulim}}{d} = 41.61\left(\frac{20}{415}\right)(0.48) = 0.9625\%$$

$$\therefore \quad A_{st\,lim} = \frac{0.9625}{100} \times 300 \times 600 = 1732.5 \text{ mm}^2 \quad \text{(same as above)}$$

$$\therefore \quad A_{st} = A_{st1} + A_{st2}$$
$$= 1732.5 + 606.413 \text{ mm}^2$$
$$= 2338.913 \text{ mm}^2 \approx 2339 \text{ mm}^2$$

Provide 5-25φ bars so that $$A_{st\,provided} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.4 \text{ mm}^2 > 2339 \text{ mm}^2 \quad \text{(OK)}$$

$$p_{t\,provided} = 1.3636\%$$

Step-6: Design of shear reinforcement (more details are covered in chapter 6)

Nominal shear stress $$\tau_v = \frac{V_u}{bd} = \frac{209.25 \times 1000}{300 \times 600} = 1.1625 \text{ N/mm}^2$$

For M 20 concrete and 1.3636% p_t, design shear strength of concrete as per table 19 of IS: 456-2000 $(\tau_c) = 0.7 \text{ N/mm}^2 < \tau_v (= 1.1625 \text{ N/mm}^2)$

$\therefore$ Shear reinforcement is required.

$$V_{us} = (\tau_v - \tau_c)\, bd$$

$$= (1.1625 - 0.7)\ 300 \times 600\ \text{N} = 83.25\ \text{kN}$$

Using 2-legged 8 mm dia. stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53\ \text{mm}^2$$

$$\therefore \quad S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87(415)100.53(600)}{83.25 \times 1000} = 261.6\ \text{mm c/c}$$

Maximum spacing of stirrups $\not> \begin{cases} 0.75d = 0.75(600)\text{mm} = 450\ \text{mm c/c} \\ 300\ \text{mm} \end{cases}$

(whichever is less)

$\therefore$ Provided 2-legged 8 mm diameter stirrups @ 250 c/c near the supports and spacing can be increased gradually towards the mid span of beam.

Step-7: Deflection control

$$\left(\frac{l}{d}\right)_{actual} = \frac{8000}{600} = 13.33$$

$$\left(\frac{l}{d}\right)_{maximum} = \left(\frac{l}{d}\right)_{basic} k_t k_c$$

$$A_{sc} = 628.3\ \text{mm}^2$$

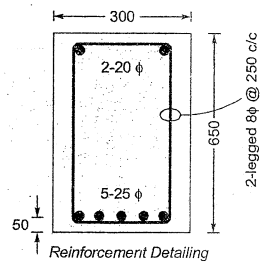
$$\therefore \quad p_c = \frac{628.3}{300 \times 600} \times 100 = 0.35\%$$

$$\therefore \quad k_t = 0.9 \quad \text{(fig. 4 of IS : 456-2000)}$$

$$k_c = 1.1 \quad \text{(fig. 5 of IS : 456-2000)}$$

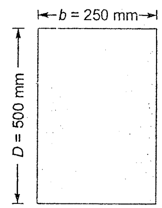
$$\therefore \quad \left(\frac{l}{d}\right)_{maximum} = 20 \times 0.9 \times 1.1$$

$$= 19.8 > 13.33 \quad \text{(OK)}$$



Reinforcement Detailing

Example 5.4 Design a RC beam of effective span 6 m to carry a superimposed load of 25 kN/m. The beam size is restricted to 250 × 500 mm. Use M20 and Fe415.



Solution:

Span (l) = 6 m

Effective cover = 40 mm

f_{ck} = 20 N/mm² (M20 concrete)

f_y = 415 N/mm² (Fe415 steel)

Self weight of beam = 0.25 × 0.5 × 25 = 3.125 kN/m

Effective depth (d) = 500 − 40 = 460 mm

Superimposed load = 25 kN/m

$\therefore$ Total load = 3.125 + 25 = 28.125 kN/m

$\therefore$ Factored load = 1.5 × 28.125 = 42.1875 kN/m

$$\therefore \quad \text{Factored moment} = \text{Design moment} = M_u = 42.1875 \times \frac{6^2}{8} = 189.84\ \text{kNm}$$

$$M_{u\,lim} = 0.138 f_{ck} bd^2 = 0.138\,(20)\,250\,(460)^2 = 146\ \text{kNm}$$

Thus, $M_u > M_{u\,lim}$

$\therefore$ Doubly reinforced beam is required since section can not be increased i.e. b and D are fixed.

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right) = 41.61\left(\frac{20}{415}0.479\right) = 0.961\%$$

$$\therefore \quad A_{st\,lim} = \frac{0.961}{100} \times 250 \times 460 = 1105.15\ \text{mm}^2$$

Using 20 mm diameter bars in compression, effective cover to compression

$$\text{reinforcement}\,(d') = 30\ \text{mm clear cover} + 8\ \text{mm dia shear stirrups} + \frac{20}{2}$$

$$= 30 + 8 + 10\ \text{mm} = 48\ \text{mm}$$

$$\therefore \quad \Delta A_{st\,reqd.} = \frac{M_u - M_{ulim}}{0.87 f_y (d - d')} = \frac{(189.84 - 146)10^6}{0.87(415)}(460 - 48) = 294.72\ \text{mm}^2$$

$$\therefore \quad \text{No. of 20 mm of bars required} = \frac{294.75}{\frac{\pi}{2} \times 20^2} = 0.94 \simeq 1(\text{say})$$

$$\therefore \quad \text{Total tension steel } (A_{st}) = A_{st\,lim} + \Delta A_{st\,reqd.} = 1105.15 + 294.72 = 1399.87\ \text{mm}^2$$

$$\therefore \quad \text{No. of 20 mm of bars required} = \frac{1399.87}{\frac{\pi}{4} \times 20^2} = 4.5 = 5\ \text{Nos. (say)}$$

Determining A_{sc}

Let $x = x_{u\,lim}$ (for balanced doubly reinforced section)

$$\frac{d'}{d} = \frac{48}{460} = 0.104 \simeq 0.1$$

$\therefore$ Stress in compression steel (f_{sc}) = 351.9 N/mm² ≠ 0.87 f_y = (0.87 × 415 = 361.05 N/mm²)

$$\therefore \quad A_{sc\,reqd.} = \frac{0.87 f_y \Delta A_{st}}{f_{sc} - 0.447 f_{ck}} = \frac{0.87(415)\frac{\pi}{4} \times 20^2 \times 1}{351.9 - 0.447 \times 20} = 330.73\ \text{mm}^2$$

$$\therefore \text{No. of 20 mm dia. bars required} = \frac{330.73}{\frac{\pi}{4} \times 20^2} = 1.05 \simeq 2\ \text{Nos.(say)}$$

Check

Actual effective depth (d) = 460 mm (as computed above)

$$\text{Effective cover to compression reinforcement } (d') = 30 + 8 + \frac{20}{2} = 48\ \text{mm}$$

$$A_{st\ provided} = 5-20\phi \text{ bars} = 5\times\frac{\pi}{4}\times 20^2 = 1570.8 \text{ mm}^2$$

$$\therefore \quad p_{t\ provided} = \frac{1570.8}{250\times 460}\times 100 = 1.37\%$$

$$A_{sc\ provided} = 2-20\phi \text{ bars} = 2\times\frac{\pi}{4}\times 20^2 = 628.3 \text{ mm}^2$$

Minimum % of compression reinforcement required (p_c*)

$$= \frac{0.87f_y\left(p_t - p_{t\lim}\right)}{\left(f_{sc} - 0.447f_{cu}\right)} = \frac{0.87(415)(1.37-0.961)}{(351.9-0.447\times 20)} = 0.431\%$$

$$p_{c\ provided} = \frac{328.3}{250\times 460}\times 100 = 0.546\% > p_c^*(=0.431\%)$$ (OK)

Deflection check

$p_t = 1.37\%$

$$\therefore \quad f_{st} = 0.58f_y\frac{p_{t\ reqd.}}{p_{t\ provided}} = \frac{0.58(415)\dfrac{1399.87}{250\times 460}\times 100}{1.37}$$

$= 213.87$ N/m²

$k_t = 1.009$ (fig. 4 of IS 465: 2000)

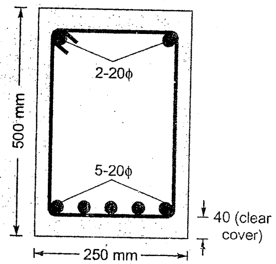
$p_c = 0.546\%$

$\therefore \quad k_c = 1.15104$ (fig. 5 of IS 456: 2000)

$$\therefore \quad \left(\frac{l}{d}\right)_{max} = \left(\frac{l}{d}\right)_{basic} k_t.k_c$$

$= 20 \times 1.009 \times 1.15104 = 23.23$

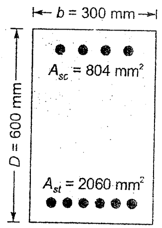
$$\left(\frac{l}{d}\right)_{provided} = \frac{6000}{460} = 13.04 < 23.23 \text{ (OK)}$$



Reinforcement details

Example 5.5 Find the MOR of a beam section 300 × 600 mm (overall depth) reinforcement with 804 sq. mm compression steel and 2060 sq. mm tension steel. Use M20 and Fe415. Take effective cover as 50 mm.

Solution:



Effective cover $= 50$ mm

$\therefore$ Effective depth (d) $= 600 - 50 = 550$ mm

$f_{ck} = 20$ MPa

$f_y = 415$ MPa

$d' = 50$ mm

$x_{u\lim} = 0.479d = 0.479(550) = 263.45$ mm

Let $f_{st} = f_{sc} = 0.87 f_y$

$\therefore \quad C_{us} + C_{uc} = T_u$

But $C_{uc} = 0.362 f_{ck} bx_u = 0.362(20)(300)x_u = 2172x_u$

$C_{us} = (f_{sc} - 0.447 f_{cu}) A_{sc} = (0.87\times 415 - 0.447\times 20)\,804$

$= 283096.44$ N

$\therefore \quad C_{us} + C_{uc} = T_u$

$\Rightarrow \quad 283096.44 + 2172x_u = 743763$

$\Rightarrow \quad x_u = 212.09 \text{ mm} < x_{u\lim} (= 263.45 \text{ mm})$

$\therefore$ Assumption of $f_{st} = 0.87 f_y$ is correct.

$$\text{Strain in compression steel } (\epsilon_{sc}) = 0.0035\left(1-\frac{d'}{x_u}\right) = 0.0035\left(1-\frac{50}{212.09}\right) = 0.0026749$$

$$\text{Yield strain } (\epsilon_y) = \frac{0.87f_y}{\epsilon_s} + 0.002 = \frac{0.87\times 415}{200000} + 0.002 = 0.00380525$$

Thus, $\epsilon_y > \epsilon_{sc}$

$\Rightarrow$ Compression steel has not yielded.

$\Rightarrow \quad f_{sc} \neq 0.87 f_y$

$\therefore \quad x_u \neq 212.09$ mm

Ist Iteration:

Let $\epsilon_{sc} = 0.0026749$

$\therefore \quad f_{sc} = 348.8$ N/mm²

$\therefore \quad C_{us} = (f_{sc} - 0.447 f_{ck}) A_{sc}$

$= (348.8 - 0.447\times 20)\,804$

$= 273247.44$ N

$$\therefore \quad x_u = \frac{743763 - 273247.44}{2172} = 216.63 \text{ mm}$$

$$\therefore \quad \epsilon_{sc} = 0.0035\left(1-\frac{50}{216.63}\right) = 0.002692$$

IInd Iteration:

$$\text{Let} \quad \epsilon_{sc} = \frac{0.0026749 + 0.002692}{2} = 0.00268345$$

$\therefore \quad f_{sc} = 349.032$ N/mm²

$\therefore \quad C_{us} = (349.032 - 0.447\times 20)\,804$

$= 273433.968$ N

$$\therefore \quad x_u = \frac{743763 - 273433.968}{2172} = 216.54 \text{ mm}$$

$$\therefore \quad \epsilon_{sc} = 0.0035\left(1-\frac{50}{216.54}\right) = 0.0026918 \simeq \epsilon_{sc} \text{ assumed}$$

$x_u = 216.54$ mm

$\therefore$ MOR $= C_{uc}(d - 0.42x_u) + C_{us}(d - d')$

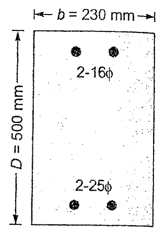
$= 2172 \times 216.54\,(550 - 0.42 \times 216.54) + 273433.968\,(550 - 50)$

$= 352.62$ kNm

Example 5.6 A RC beam of size 230 × 500 mm overall depth is having effective of 40 mm both in tension and compression side. 2 nos. 16 mm diameter bars are provided in compression zone and 2 nos. 25 mm diameter bars are provided in tension zone of the beam. Find the moment of resistance of this beam using M20 concrete and Fe415 steel.

Table: Stress in compression

Fe415	d'/d			
	0.05	0.1	0.15	0.2
f_{sc} (MPa)	355	353	342	329



Solution:

Effective cover = 40 mm (for both tension and compression)

M20 concrete and Fe 415 steel

Effective depth (d) = 500 − 40 = 460 mm

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$A_{st} = 2 \times \frac{\pi}{4} \times 25^2 = 981.75 \text{ mm}^2$$

$x_{u\,lim} = 0.479\,d = 0.479\,(460) = 220.34$ mm

Let $f_{st} = f_{sc} = 0.87\,f_y$

$\therefore$ $C_{uc} = 0.362\,f_{ck}\,b\,x_u = 0.362\,(20)\,230\,(x_u) = 1665.2\,x_u$

$C_{us} = (0.87f_y - 0.447\,f_{ck})\,A_{sc}$

$= (0.67 \times 415 - 0.447 \times 20)\,402.12 = 141590.47$ N

$T_u = 0.87\,f_y A_{st} = 0.87\,(415)\,981.75 = 354460.84$ N

$C_{uc} + C_{us} = T_u$

$\Rightarrow$ $1665.2\,x_u + 141590.47 = 354460.84$

$\therefore$ $x_u = 127.83 \text{ mm} < x_{u\,lim}\,(= 220.34 \text{ mm})$

$\therefore$ Assumption of $f_{st} = 0.87\,f_y$ is correct.

Strain in compression steel (ϵ_{sc}) $= 0.0035\left(1 - \dfrac{40}{127.83}\right) = 0.002404795$

Yield strain (ϵ_y) $= \dfrac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.00380525 > \epsilon_s$

$\therefore$ Compression steel has not yielded.

$\Rightarrow$ $f_{sc} \neq 0.87\,f_y$

Thus, $x_u \neq 127.83$ mm

$$\frac{d'}{d} = \frac{40}{460} = 0.0869$$

$\therefore$ For $x_u = x_{u\,lim}$, $f_{sc} = 353.524$ N/mm²

$\therefore$ $M_{u\,lim} = 0.362\,f_{ck}\,b\,x_{u\,lim}(d - 0.42\,x_{u\,lim}) + (f_{sc} - 0.447\,f_{ck})\,A_{sc}\,(d - d')$

$= 0.362\,(20)\,230\,(220.34)\,(460 - 0.42 \times 220.34)$

$+ (353.524 - 0.447 \times 20)\,402.12\,(460 - 40)$

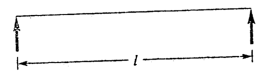
$= 193$ kNm

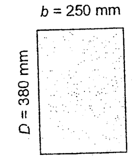
Example 5.7 A simply supported beam of 4.5 m effective span is carrying an imposed load of 25 kN/m. The size of the beam is restricted to 250 × 380 mm. Design the beam using M20 and Fe415.

Solution:

l = 4.5 m

LL = 25 kNm





M20 concrete and Fe415 steel

Self weight of beam = 0.38 × 0.25 × 25 = 2.375 kN/m

$\therefore$ Total load = 2.375 + 25 = 27.375 kN/m

$\therefore$ Factored load (w_u) = 1.5 × 27.375 = 41.0625 kN/m

Assume an effective cover of 50 mm both at the top and bottom

$$\frac{d'}{d} = \frac{50}{330} = 0.15$$

$$\therefore \quad \epsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_{ulim}}\right) = 0.0035\left(1 - \frac{50}{0.479 \times 330}\right) = 0.002393$$

$$\epsilon_y = \frac{0.87 \times 415}{200000} + 0.002 = 0.00380525 > \epsilon_{sc}$$

$\therefore$ Compression steel has not yielded.

Let $x_u = x_{u\,lim}$ and for $\dfrac{d'}{d} = 0.15$, $f_{sc} = 342$ N/mm²

$$\therefore \quad A_{sc\,reqd.} = \frac{0.87 f_y \Delta A_{st}}{(f_{sc} - 0.447 f_{ck})} = \frac{0.87(415)\frac{\pi}{4} \times 20^2}{(342 - 0.447 \times 20)} = 340.56 \text{ mm}^2$$

$\therefore$ No. of 20 mm ϕ bars required $= \dfrac{340.56}{\frac{\pi}{4} \times 20^2} = 1.08 = 2$ (say)

$\therefore$ Factored moment (M_u) $= w_u \dfrac{l^2}{8} = 41.0625 \times \dfrac{4.5^2}{8} = 103.94$ kNm

Now, $M_{u\,lim} = 0.138\,f_{ck}\,bd^2$

Let Effective cover = 50 mm

$\therefore$ $d = 380 - 50 = 330$ mm

$\therefore$ $M_{u\,lim} = 0.138\,(20)\,250\,(330)^2 = 75.141 \text{ kNm} < M_u$

$\therefore$ Doubly reinforced beam is required.

$$p_{t\,lim} = 41.61\left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{ulim}}{d}\right) = 0.961\%$$

$$\therefore \quad A_{st\,lim} = \frac{0.961}{100} \times 250 \times 330 = 792.825 \text{ mm}^2$$

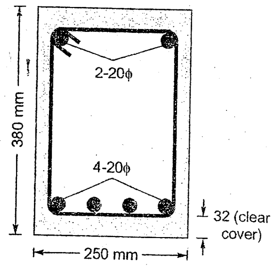
Let effective cover to compression reinforcement (d') = 50 mm

$$\therefore \quad \Delta A_{st\,reqd.} = \frac{M_u - M_{u\,lim}}{0.87 f_y (d - d')} = \frac{(103.94 - 75.141)10^6}{0.87(415)(330 - 50)} = 284.87 \text{ mm}^2$$

$$\therefore \quad \text{No. of } 20\phi \text{ bars required} = \frac{284.87}{\frac{\pi}{4} \times 20^2} = 0.9 = 1 \text{ (say)}$$

$$\therefore \quad A_{st} = A_{st\,lim} + \Delta A_{st\,reqd.}$$
$$= 792.825 + 284.87$$
$$= 1077.695 \text{ mm}^2$$

$$\therefore \quad \text{No. of } 20\,\phi \text{ bars required} = \frac{1077.695}{\frac{\pi}{4} \times 20^2} = 3.43 = 4 \text{ (say)}$$



Reinforcement details

Objective Brain Teasers

Q.1 Providing doubly reinforced beam is a better recourse than to increase the depth of beam because:

(i) Self-weight of the beam does not increase much by providing doubly reinforced beam.

(ii) It takes care of occasional reversal of stresses in framed structures.

Which of the above statement(s) is/are false?

(a) (i) only (b) (ii) only
(c) Both (i) and (ii) (d) None of (i) or (ii)

Q.2 Hanger bars are considered as compression reinforcement in beams when the area of hanger bars is:

(a) Less than 0.1% (b) Greater than 0.2%
(c) Greater than 0.5% (d) Less than 0.05%

Q.3 Which of the following grade of steel generally attains its yield value when provided as compression reinforcement?

(a) Fe 250 (b) Fe 415
(c) Fe 500 (d) All of the above

Q.4 Consider the following for a rectangular beam:

p_t = Percentage tension reinforcement

$p_{t\,lim}$ = Limiting percentage of tension reinforcement corresponding to balanced singly reinforced beam section

p_c = Percentage of compression reinforcement

p_c^* = Percentage of compression reinforcement corresponding to balanced doubly reinforced beam section

It is given that $p_t > p_{t\,lim}$ and $p_c > 0$ then in order to avoid over reinforced section, we must have:

(a) $p_c < p_c^*$ (b) $p_c > p_c^*$
(c) $p_c = 0$ (d) $p_c < (p_t - p_{t\,lim})$

Q.5 If it is known that in a doubly reinforced beam, the stress in compression steel (f_{sc}) and stress in tension steel (f_{st}) has attained the yield value then depth of neutral axis for this beam can be computed:

(a) Explicitly
(b) Implicitly
(c) Both (a) and (b) can be used
(d) Data insufficient

Q.6 Deflection check for doubly reinforced beams is generally not required due to the following factor:

(a) Modification factor k_t
(b) Modification factor k_c
(c) Stress in compression steel (f_{sc})
(d) All of the above

Q.7 It is essential to provide percentage compression reinforcement as close as possible (but not less than) the percentage compression reinforcement for balanced doubly reinforced section because otherwise

(a) the beam will fail in deflection
(b) the beam will fail in shear
(c) the beam will become over-reinforced
(d) All of the above

Q.8 In a framed RCC structure, beam sections are generally

(a) Singly reinforced without hanger bars
(b) Singly reinforced with hanger bars
(c) Doubly reinforced
(d) It depends on the type of loading

Q.9 A doubly reinforced concrete beam of size 300 mm × 500 mm effective depth is reinforced with 2200 mm² steel in tension zone and 628 mm² steel in compression zone. The effective cover to compression steel is 50 mm. The steel on both tension and compression side yield and concrete used is of M20 grade and steel of Fe250 grade. The depth of the neutral axis is

(a) 161 mm (b) 205 mm
(c) 185 mm (d) 215 mm

Q.10 The moment of resistance for the above beam is

(a) 298 kNm (b) 270 kNm
(c) 209 kNm (d) 238 kNm

Q.11 In a doubly reinforced RC beam, the maximum strain in concrete at the level of compression steel is

(a) $0.0035\left(1 - \frac{d'}{x_u}\right)$ (b) $0.0035\left(1 - \frac{x_u}{d'}\right)$

(c) 0.0035 (d) None of these

Q.12 Doubly reinforced beam is uneconomical as compared to singly reinforced beam because

(a) compressive steel is not stressed upto $0.87 f_y$
(b) more steel is required in doubly reinforced beams
(c) compression concrete is not stressed fully
(d) All of the above

Answers

1. (d)	2. (b)	3. (a)	4. (b)	5. (a)
6. (b)	7. (c)	8. (c)	9. (a)	10. (c)
11. (a)	12. (a)			

Hints:

10. (c)

$$C_{uc} = 0.36 f_{ck}\, b x_u$$
$$C_{us} = (f_{sc} - 0.45 f_{ck}) A_{sc}$$
$$T_u = 0.87 f_y A_{st} \quad (\because \text{ steel yields})$$
$$\therefore \quad C_{uc} + C_{us} = T_u$$
$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - 0.45 f_{cu}) A_{sc}}{0.36 f_{ck} b}$$

Let $f_{sc} = 0.87 f_y$

$$\therefore \quad x_u = 160.9 \text{ mm}$$
$$x_{u\,lim} = 0.53\, d = 265 \text{ mm}$$

$\therefore$ $x_u < x_{u\,lim}$ and thus $f_{st} = 0.87 f_y$ is true.

Moment of resistance

$$(M_u) = C_{uc}(d - 0.42\, x_u) + C_{us}(d - d')$$
$$= 209 \text{ kNm}$$

Conventional Practice Questions

Q.1 Design a doubly reinforced beam to resist a factored moment of 375 kNm with b = 250 mm, d = 500 m and d' = 50 mm. Use M30 and Fe415. Use LSM.

Q.2 Find the MOR of a doubly reinforced beam of width 300 mm, effective depth 600 mm, d' = 75 mm. Compression reinforcement consists of 5-25 φ bars and tension reinforcement consists of 4-32 φ + 4-16 φ bars. Use M30 and Fe500.

Q.3 Design simply supported beam of effective span 8 m subjected to a live load of 35 kN/m. The beam dimensions are restricted to 300 × 650 mm. Use M 25 and Fe 415.

Q.4 Find the ultimate moment capacity of a doubly reinforced beam of width 350 mm, effective depth 600 mm and d' = 50 mm. 6-25 φ bars are provided on tension side and 4-20φ bars on compression side. Use M20 and Fe415.

■■■■