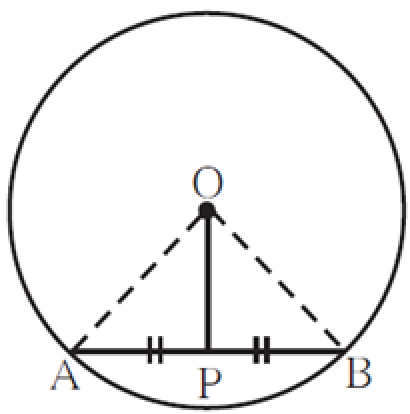
Circles

Practice set 6.1

Q. 1. Distance of chord AB from the center of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.

Answer :



Given that OP = 8 cm

And AB = 12 cm

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

∴ AP = PB = 6 cm

In the right angled ΔOAP using Pythagoras theorem,

⇒ $OA^2 = OP^2 + AP^2$

⇒ $OA^2 = 8^2 + 6^2$

⇒ $OA^2 = 64 + 36$

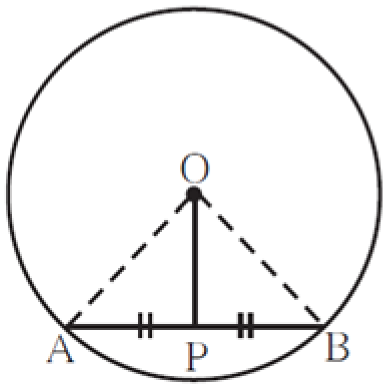
⇒ $OA^2 = 100$

⇒ OA = 10cm

So, the diameter of the circle is (2×10) = 20cm (Diameter = 2×Radius).

Q. 2. Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the center.

Answer :



Given that diameter = 26cm

Radius = Diameter / 2 = 26 /2 = 13cm

So, OA = 13cm

And AB = 24 cm

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

∴ AP = PB = 12 cm

In the right angled ΔOAP using Pythagoras theorem,

$\Rightarrow OA^2 = OP^2 + AP^2$

$\Rightarrow 13^2 = OP^2 + 12^2$

$\Rightarrow 169 = OP^2 + 144$

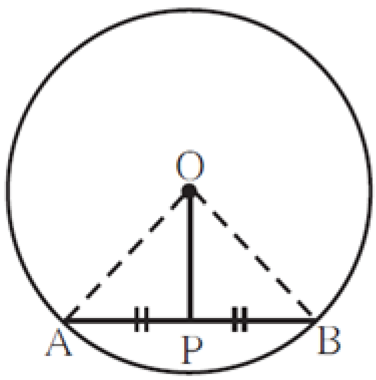
$\Rightarrow OP^2 = 25$

$\Rightarrow OP = 5cm$

So, the distance of chord from the center is 5cm.

Q. 3. Radius of a circle is 34 cm and the distance of the chord from the center is 30 cm, find the length of the chord.

Answer :



Given that

Radius = 34cm

So, OA = 34cm

And OP = 30 cm

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

∴ AP = PB,

AB = 2PB

In the right angled ΔOAP using Pythagoras theorem,

$\Rightarrow OA^2 = OP^2 + AP^2$

$\Rightarrow 34^2 = 30^2 + AP^2$

$\Rightarrow 1156 = 900 + AP^2$

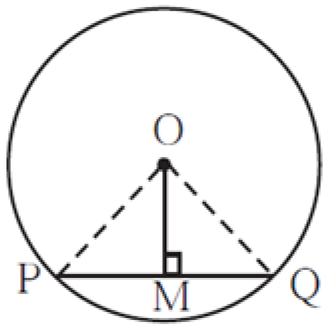
$\Rightarrow AP^2 = 256$

$\Rightarrow AP = 16cm$

(AB = 2AP)

Q. 4. Radius of a circle with center O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the center of the circle.

Answer :



Given that

Radius = 41 units

So, OP = 41 units

And PQ = 80 units

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

∴ PM = MQ = 40 cm

In the right angled ΔOAP using Pythagoras theorem,

$\Rightarrow OP^2 = OM^2 + PM^2$

$\Rightarrow 41^2 = OM^2 + 40^2$

$\Rightarrow 1681 = OM^2 + 1600$

$\Rightarrow OM^2 = 81$

$\Rightarrow$ OM = 9 units

So, the distance of chord from the center is 9 units.

Q. 5. In figure 6.9, center of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that AP = BQ

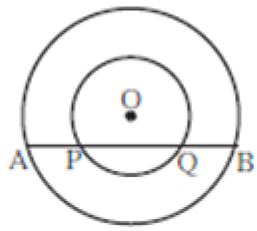
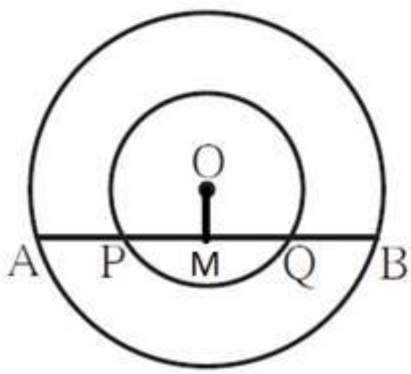


Fig. 6.9

Answer :



We draw a perpendicular on chord AB from O.

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

Therefore,

AM = MB …….(1)

OM is also perpendicular to chord PQ of smaller circle.

Therefore,

PM = MQ ………….(2)

Subtracting (2) from (1)

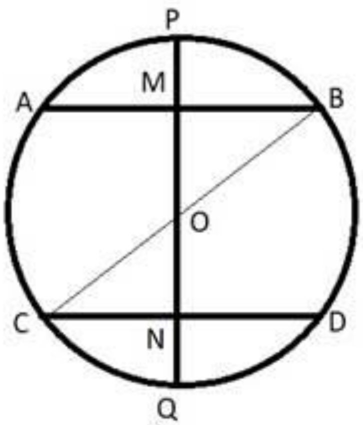
AM-PM = MB-MQ

⇒ AP = BQ

Hence Proved.

Q. 6. Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.

Answer :



We draw a circle with center O and AB, CD are the chords of this circle. Diameter PQ bisects AB and CD at M and N respectively.

We know that the line from the center bisecting the chord is perpendicular to the chord.

Therefore,

$\angle$ OMA = $\angle$ OMB = 90°

Also, $\angle$ ONC = $\angle$ OND = 90°

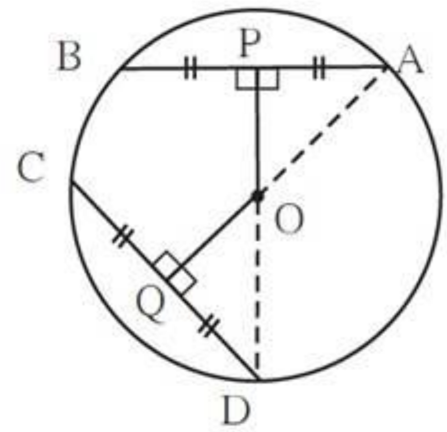
$\angle$ OMA + $\angle$ ONC = 90° + 90° = 180°

Hence the two chords, AB and CD are parallel to each other.

Practice set 6.2

Q. 1. Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the center of the circle?

Answer :



Given radius of circle is 10cm

OA = OD = 10cm

AB = CD = 16cm

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

CQ = QD = 8cm

In right angled ΔOQD using the Pythagoras theorem

$OD^2 = OQ^2 + QD^2$

$10^2 = OQ^2 + 8^2$

$100 = OQ^2 + 64$

$OQ^2 = 36$

OQ = 6cm

Therefore the chord CD is at 6cm from the center.

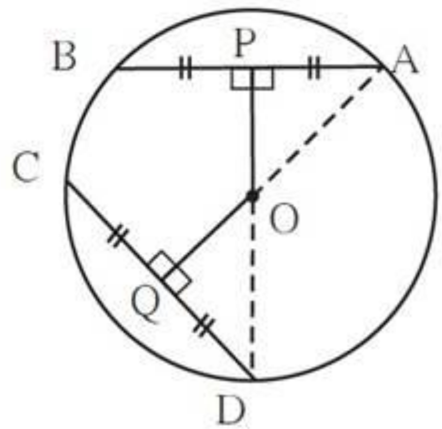
We know that Congruent chords of a circle are equidistant from the center of the circle.

As AB and CD are equal in length, they are equidistant.

∴ OP = OQ = 6cm

Q. 2 In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the center. Find the lengths of the chords.

Answer :



Given radius of circle is 13cm

OA = OD = 13cm

OQ = OP = 16cm

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

CQ = QD

CD = 2×QD

In right angled ΔOQD using the Pythagoras theorem

$OD^2 = OQ^2 + QD^2$

$13^2 = 5^2 + QD^2$

$169 = 25 + QD^2$

$QD^2 = 144$

QD = 12cm

Therefore the length of chord CD = 2×12 = 24cm

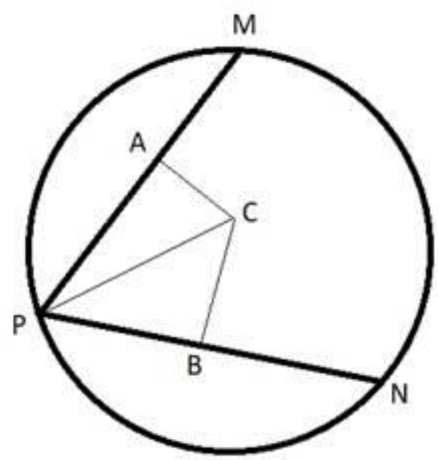
We know that The chords of a circle equidistant from the center of a circle are congruent

As AB and CD are equidistant, they are equal in length.

∴ AB = CD = 24cm

Q. 3. Seg PM and seg PN are congruent chords of a circle with center C. Show that the ray PC is the bisector of ∠NPM.

Answer :



Given that PM = PN

We know that Congruent chords of a circle are equidistant from the center of the circle.

Therefore, AC = CB ………………(1)

Also,

A perpendicular drawn from the centre of a circle on its chord bisects

the chord.

CB bisects PN as PB = BN,

Similarly, CA bisects PM as PA = AM.

In ΔAPC and ΔBPC,

∠CAP = ∠ CBP = 90°

PC = PC (common side)

AC = CB (From eq (1))

∴ ΔAPC≅ ΔBPC (RHS congruence)

∴ ∠ APC = ∠ BPC (by CPCT)

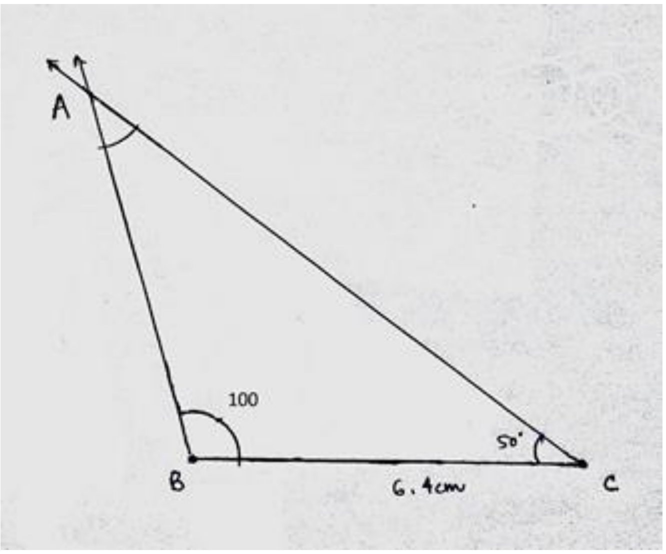
Hence proved that PC is the bisector of ∠ NPM.

Practice set 6.3

Q. 1. Construct ΔABC such that ∠B = 100^0, BC = 6.4, ∠C = 50^0 and construct its incircle.

Answer : Steps of Construction:

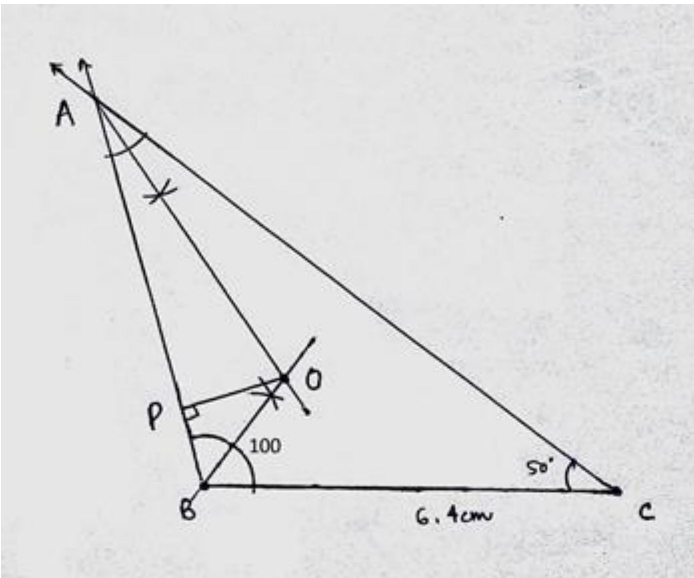
1.Construct ΔABC of given dimensions.



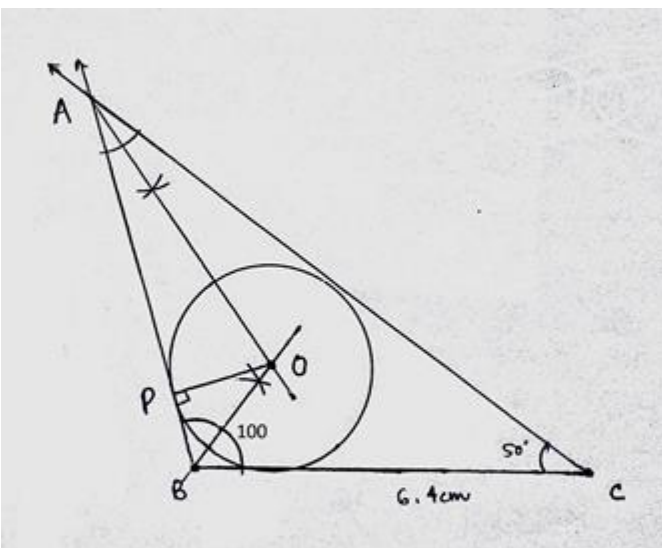
2.Draw bisectors of two angles, ∠A and ∠B.

3.Denote the point of intersection as O.

4.Draw perpendicular OP on AB.



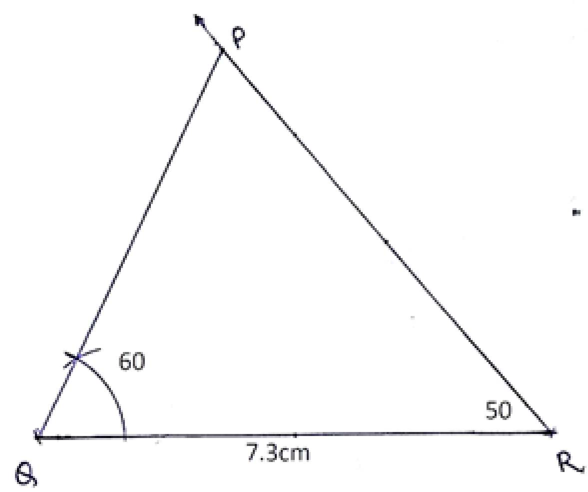
5. Draw a circle with O as center and OP as radius.



Q. 2. Construct ΔPQR such that ∠70^{0}, ∠R = 50^{0}, QR = 7.3cm, and construct its circumcircle.

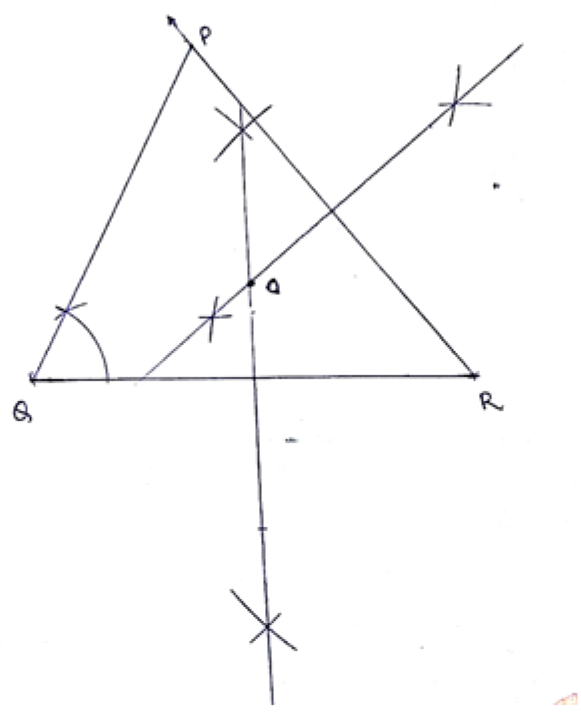
Answer : Steps of Construction:

1. Construct ΔPQR of given dimensions.

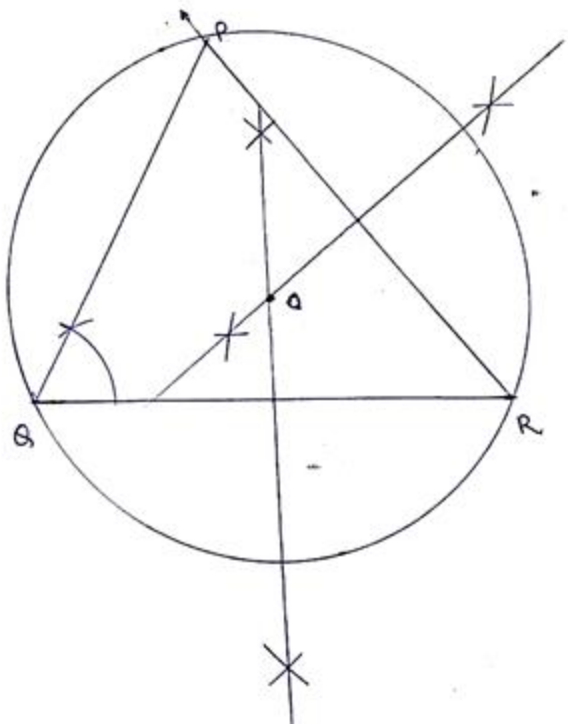


2.Draw perpendicular bisectors of two sides, QR and PR.

3.Denote the point of intersection as O.



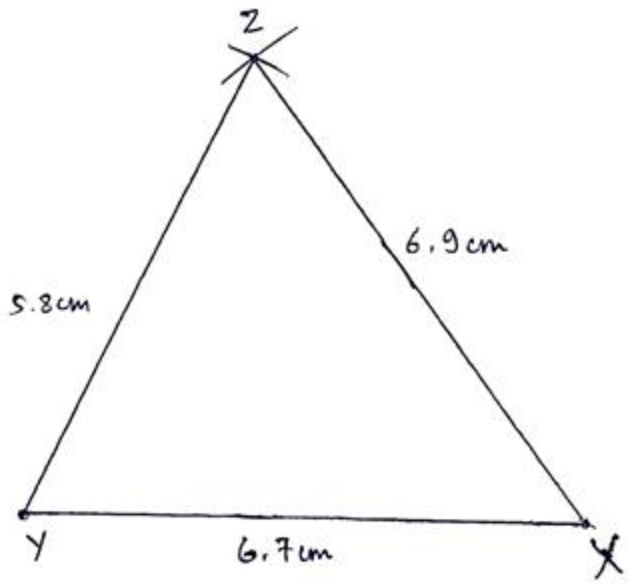
4.Draw a circle with O as center and OP as radius.



Q. 3. Construct ΔXYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle.

Answer : Steps of Construction:

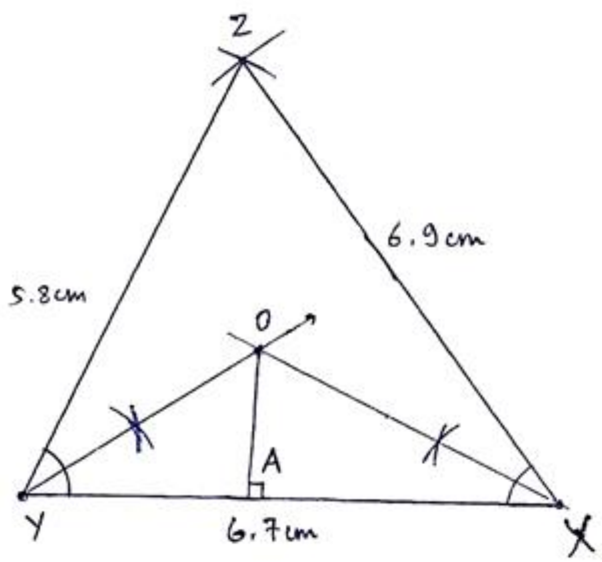
1.Construct ΔXYZ of given dimensions.



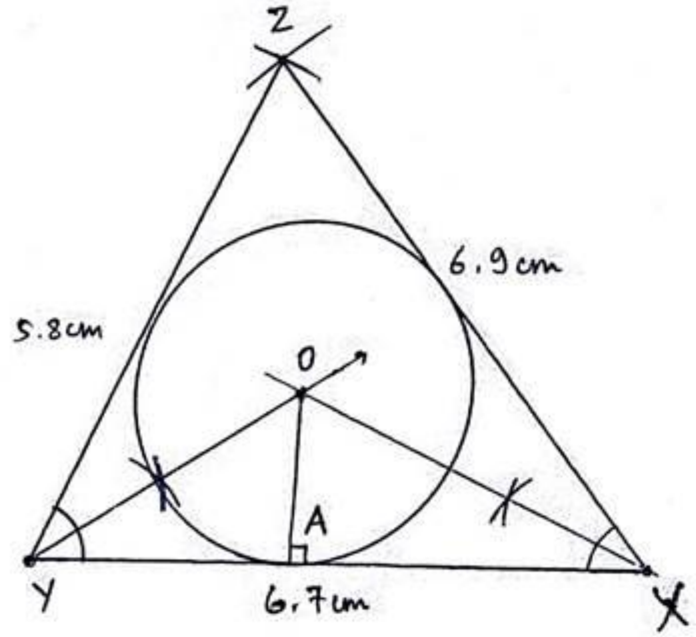
2.Draw bisectors of two angles, ∠X and ∠Y.

3.Denote the point of intersection as O.

4.Draw perpendicular OA on XY.



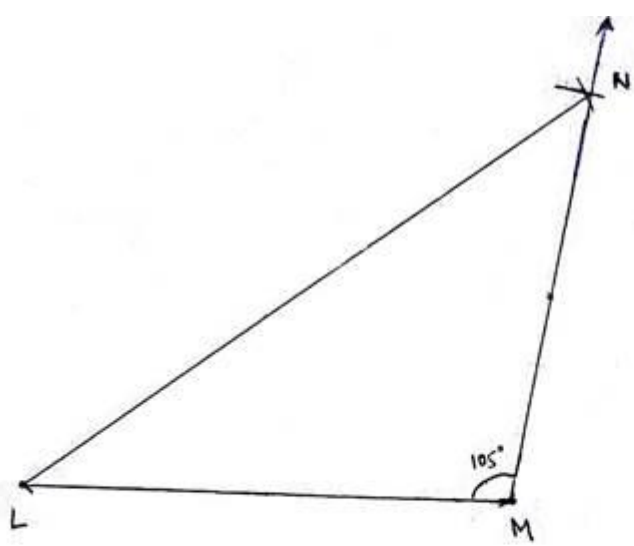
5.Draw a circle with O as center and OA as radius.



Q. 4. In ΔLMN, LM = 7.2cm, ∠M = 105^0, MN = 6.4cm, then draw ΔLMN and construct its circumcircle.

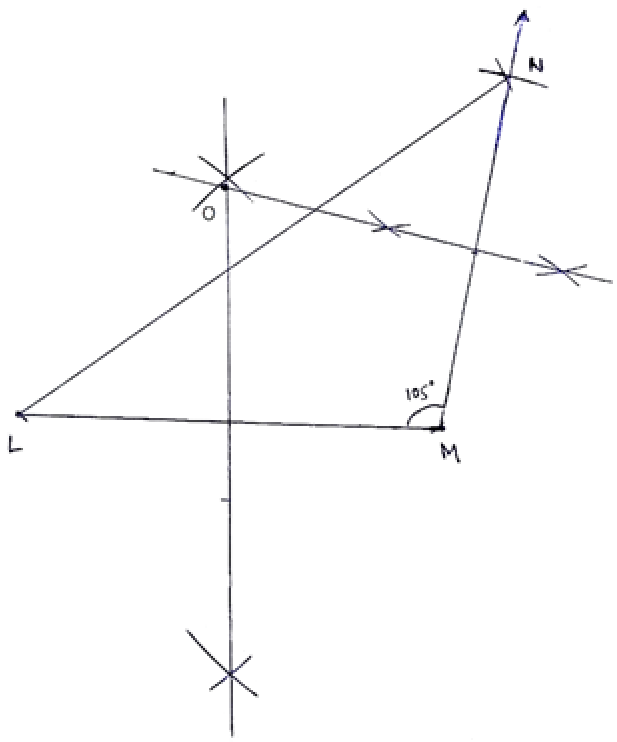
Answer : Steps of Construction:

1.Construct **ΔLMN** of given dimensions.

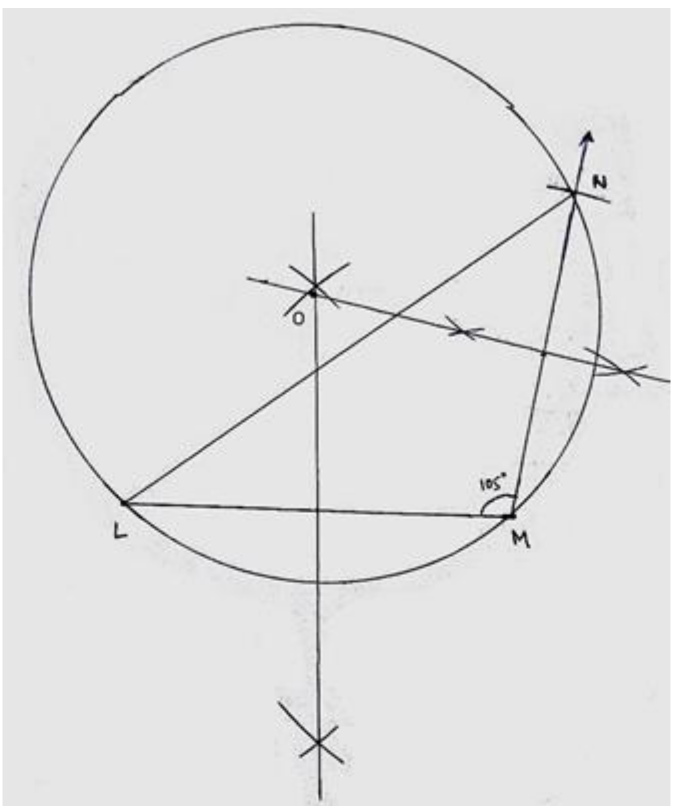


2.Draw perpendicular bisectors of two sides, LM and MN.

3.Denote the point of intersection as O.



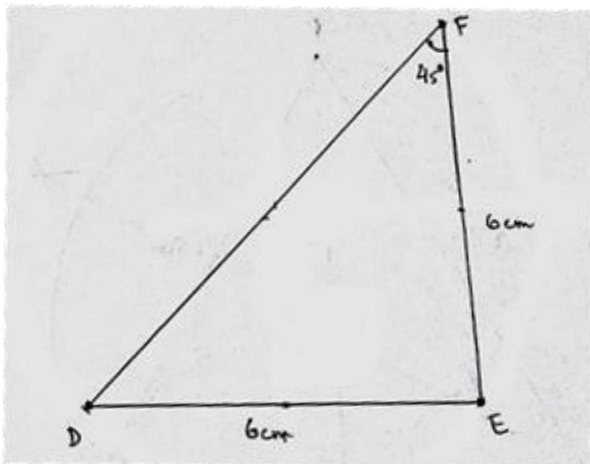
4.Draw a circle with O as center and OM as radius.



Q. 5. Construct ΔDEF such that DE = EF = 6 cm, ∠F = 45^0 and construct its circumcircle.

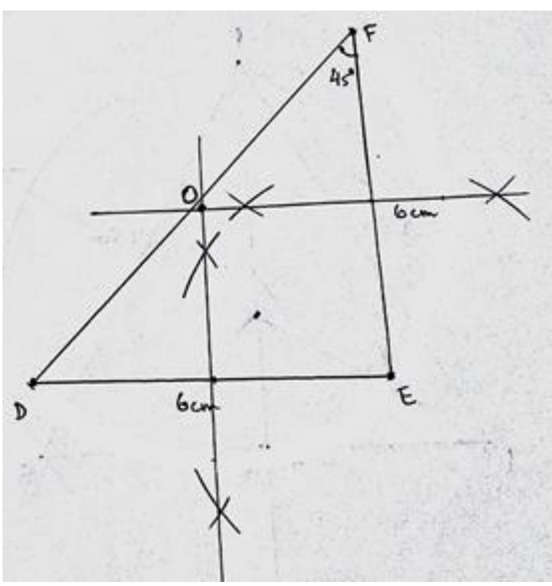
Answer : Steps of Construction:

1.Construct ΔDEF of given dimensions.

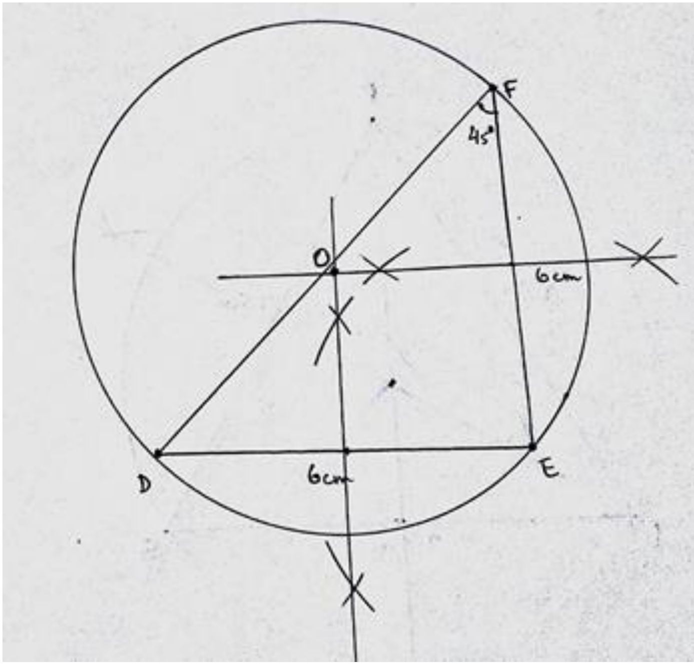


2.Draw perpendicular bisectors of two sides, DE and EF.

3.Denote the point of intersection as O.



4.Draw a circle with O as center and OD as radius.



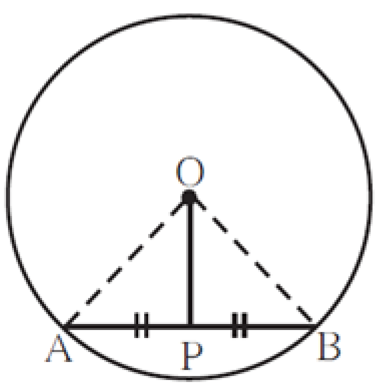
Problem set 6

Q. 1 A. Choose correct alternative answer and fill in the blanks.

Radius of a circle is 10 cm and distance of a chord from the center is 6 cm. Hence the length of the chord is ....

A. 16 cm
B. 8 cm
C. 12 cm
D. 32 cm

Answer :



Given that

Radius = 10cm

So, OA = 10cm

And OP = 6 cm

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

∴ AP = PB,

AB = 2PB

In the right angled ΔOAP using Pythagoras theorem,

$\Rightarrow OA^2 = OP^2 + AP^2$

$\Rightarrow 10^2 = 6^2 + AP^2$

$\Rightarrow 100 = 36 + AP^2$

$\Rightarrow AP^2 = 64$

⇒ AP = 8cm

So, the length of chord is 8× 2 = 16cm (AB = 2AP)

The correct answer is A.

Q. 1 B. Choose correct alternative answer and fill in the blanks.

The point of concurrence of all angle bisectors of a triangle is called the ......

A. centroid
B. circumcenter
C. incentre
D. orthocenter

Answer : Incenter is defined as the point of occurrence of all angle bisectors of a triangle. Incircle is the corresponding circle formed with incenter as the center.

Q. 1 C. Choose correct alternative answer and fill in the blanks.

The circle which passes through all the vertices of a triangle is called .....
A. circumcircle

B. incircle
C. congruent circle
D. concentric circle

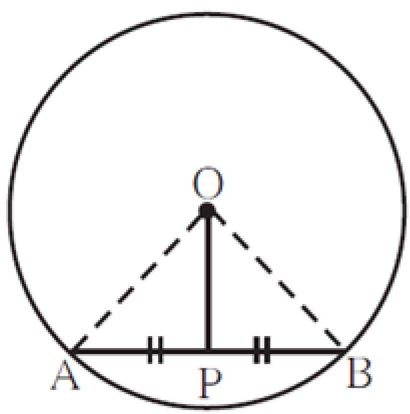
Answer : Circle passing through all the vertices of a triangle is called circumcircle of the triangle and the center of the circle is called the circumcenter of the triangle.

Q. 1 D. Choose correct alternative answer and fill in the blanks.

Length of a chord of a circle is 24 cm. If distance of the chord from the center is 5 cm, then the radius of that circle is ……..

A. 12 cm
B. 13 cm
C. 14 cm
D. 15 cm

Answer :



Given that OP = 5 cm

And AB = 24cm

We know that a perpendicular drawn from the center of a circle on its chord bisects

the chord.

∴ AP = PB = 12 cm

In the right angled ΔOAP using Pythagoras theorem,

$\Rightarrow OA^2 = OP^2 + AP^2$

⇒ $OA^2 = 5^2 + 12^2$

⇒ $OA^2 = 25 + 144$

⇒ $OA^2 = 169$

⇒ OA = 13cm

Hence, The correct option is B.

Q. 1 E. Choose correct alternative answer and fill in the blanks.

The length of the longest chord of the circle with radius 2.9 cm is .....

A. 3.5 cm
B. 7 cm
C. 10 cm
D. 5.8 cm

Answer : The longest chord of a circle is its diameter.

The length of diameter is 2× radius = 2×2.9 = 5.8cm

Hence, The correct answer is D.

Q. 1 F. Choose correct alternative answer and fill in the blanks.

Radius of a circle with center O is 4 cm. If l(OP) = 4.2 cm, say where point P will lie.

A. on the center
B. Inside the circle
C. outside the circle
D. on the circle

Answer : The longest distance of a point, from the center, within the circle is its radius = 4cm.

For distance = 4cm , it is on the circle.

For distance>4.2cm, it is outside the circle.

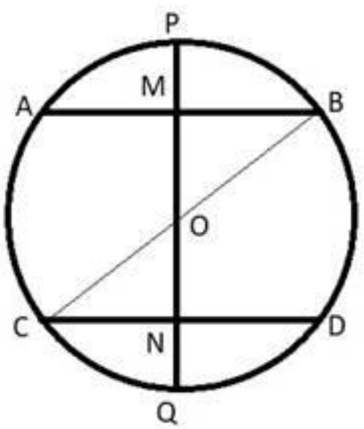
Hence, The correct answer is C.

Q. 1 G. Choose correct alternative answer and fill in the blanks.

The lengths of parallel chords which are on opposite sides of the center of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is .....

A. 2 cm
B. 1 cm
C. 8 cm
D. 7 cm

Answer :



Let, length of AB = 6cm and length of CD = 8cm

Radius of circle = 5cm

OB = OC = 5cm

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

AM = MB = 3cm

Also, CN = ND = 4cm

In ΔOMB,

$\Rightarrow OB^2 = OM^2 + MB^2$

$\Rightarrow 5^2 = OM^2 + 3^2$

$\Rightarrow OM^2 = 25-9$

$\Rightarrow OM^2 = 16$

$\Rightarrow OM = 4cm$

In ΔONC,

$\Rightarrow OC^2 = ON^2 + CN^2$

$\Rightarrow 5^2 = ON^2 + 4^2$

$\Rightarrow ON^2 = 25-16$

$\Rightarrow ON^2 = 9$

$\Rightarrow ON = 3cm$

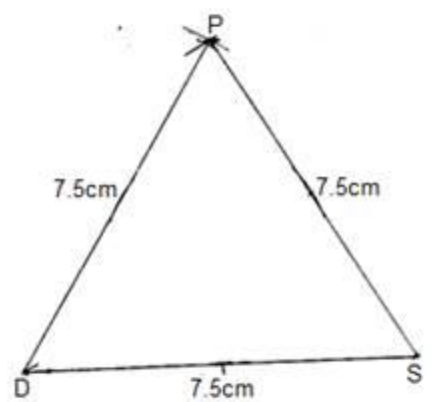
Distance between AB and CD = OM + ON = 4 + 3 = 7cm

Hence the correct option is D.

Q. 2. Construct incircle and circumcircle of an equilateral ΔDSP with side 7.5 cm. Measure the radii of both the circles and find the ratio of radius of circumcircle to the radius of incircle.

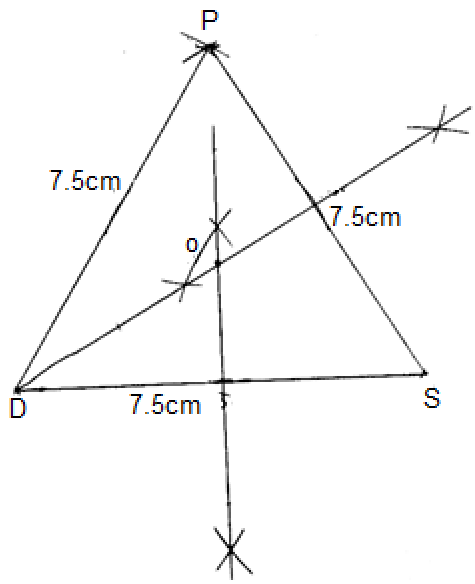
Answer : Steps of Construction:

1.Construct ΔDSP of given dimensions.

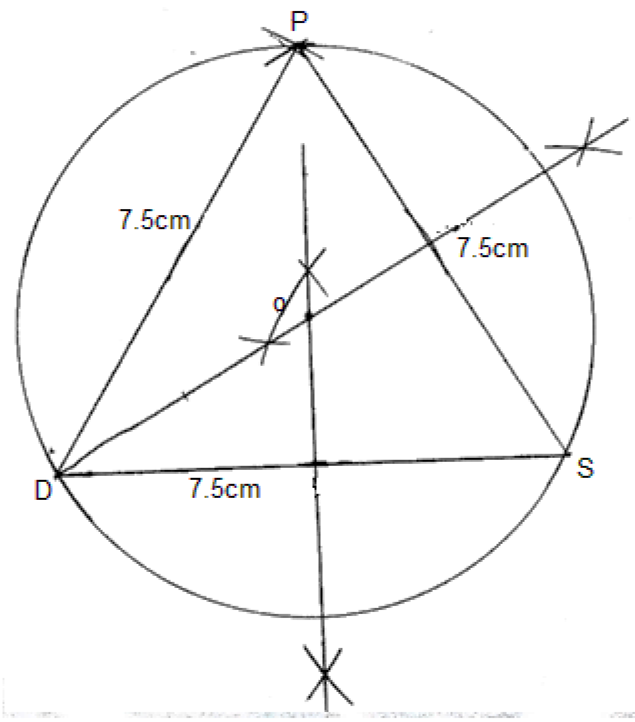


2.Draw perpendicular bisectors of two sides, DS and SP.

3.Denote the point of intersection as O.

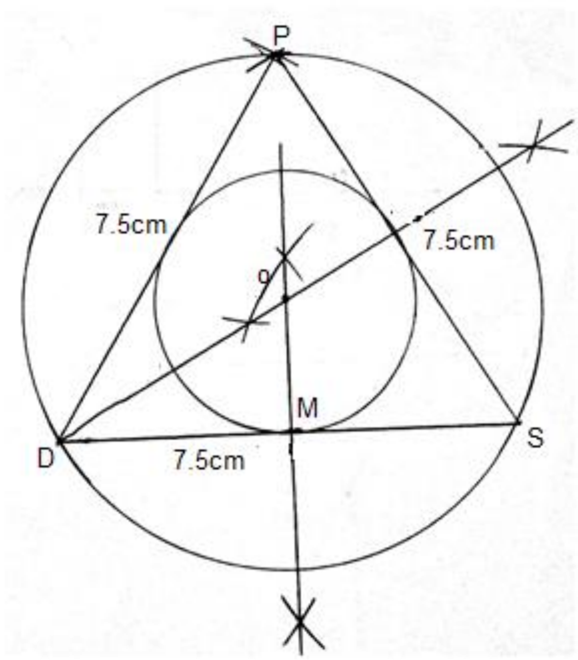


4.Draw a circle with O as center and OD as radius.



As we know that for an equilateral triangle, the incenter is same as the circumcenter.

5.Taking O as center, and the OM as the radius draw a circle.



Circumradius = OP = 4.3cm

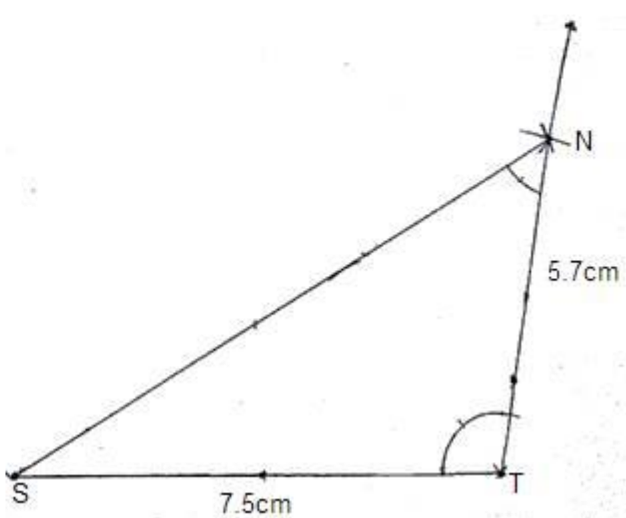
Inradius = OM = 2.1cm

For an equilateral triangle, Circumradius = 2×Inradius.

Q. 3. Construct ΔNTS where NT = 5.7 cm, TS = 7.5 cm and ∠NTS = 110^0 and draw incircle and circumcircle of it.

Answer : Steps of Construction:

1. Construct ΔNTS of given dimensions.

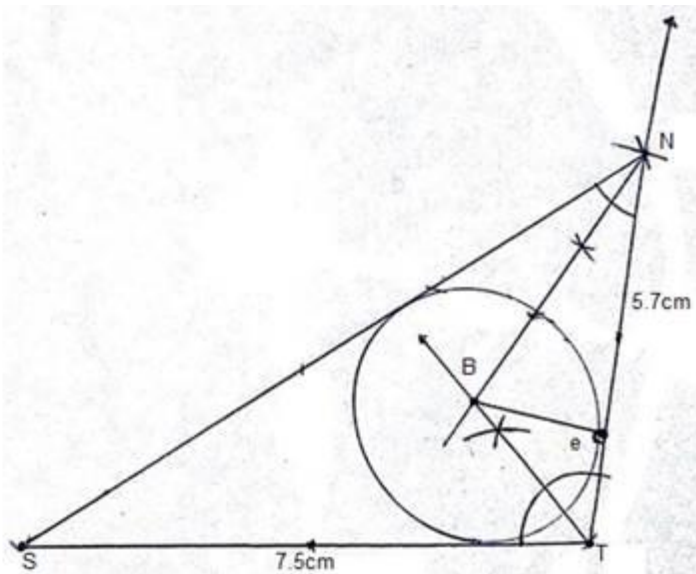


2.Draw bisectors on ∠T and ∠S.

3.The point of intersection as B.

4.Draw perpendicular on NT from B.

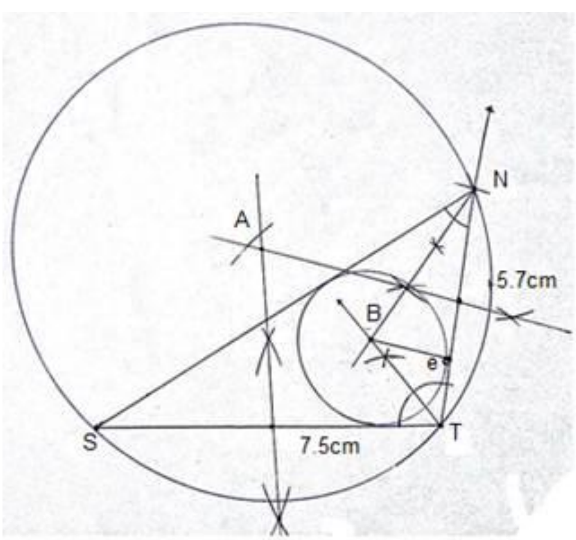
5. With B as center and length of perpendicular as radius, draw a circle.



6.Draw perpendicular bisectors of ST and TN.

7.The point of intersection is A.

8. With A as center, and AS as radius draw a circle.



Q. 4. In the figure 6.19, C is the center of the circle. seg QT is a diameter CT = 13, CP = 5, find the length of chord RS.

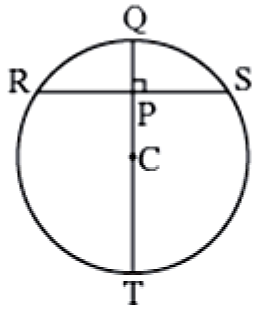


Fig. 6.19

Answer : We join C to S to form ΔCPS.

CP = 5 (given)

Radius of circle = CT = 13 (given)

Therefore CS = 13 (radius of the same circle)

In ΔCPS,

$\Rightarrow CS^2 = CP^2 + PS^2$

$\Rightarrow 13^2 = 5^2 + PS^2$

$\Rightarrow PS^2 = 169-25$

$\Rightarrow PS^2 = 144$

$\Rightarrow$ PS = 12 units

We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

PS = RP = 12 units

RS = 2×PS = 2×12 = 24 units

Q. 5. In the figure 6.20, P is the center of the circle. chord AB and chord CD intersect on the diameter at the point E If ∠AEP ≅ ∠DEP then prove that AB = CD.

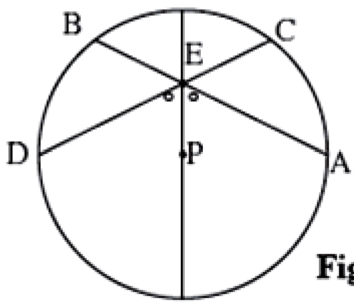
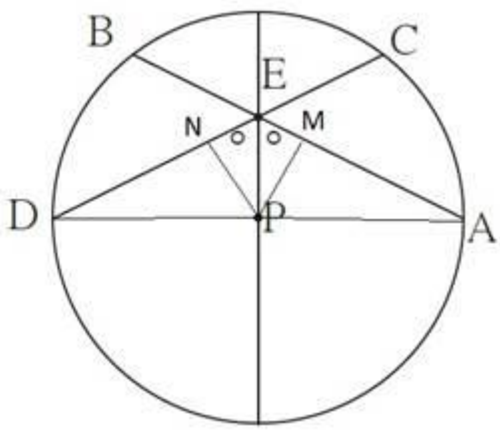


Fig. 6.20

Answer :



In the figure, we join PA and PD. Draw perpendiculars on AB and CD from P as PM and PN respectively.

∠AEP = ∠ DEP (given)

So, ∠PEN = ∠PEM (M and N are points on line AE and ED respectively)

In ΔPEN and ΔPEM,

∠PNE = ∠PME = 90°

∠PEN = ∠PEM

PE = PE (common)

Therefore, Δ PEN ≅ ΔPEM (by AAS congruence)

∴ PN = PM (by CPCT)

We know that The chords of a circle equidistant from the center of a circle are congruent.

So, AB = CD.

Hence proved.

Q. 6. In the figure 6.21, CD is a diameter of the circle with center O. Diameter CD is perpendicular to chord AB at point E. Show that ΔABC is an isosceles triangle.

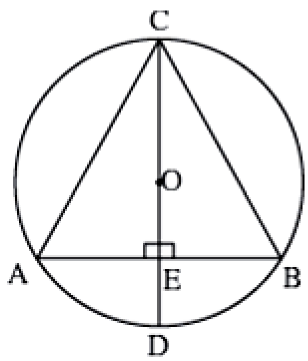


Fig. 6.21

Answer : We know that a perpendicular drawn from the center of a circle on its chord bisects the chord.

So, AE = EB

In ΔACE and ΔBCE,

AE = EB

$\angle AEC = \angle BEC = 90°$

CE = CE (common)

Δ ACE ≅ Δ BCE (By SAS congruence)

Therefore, AC = BC (by CPCT)

Hence proved that ABC is an isosceles triangle.