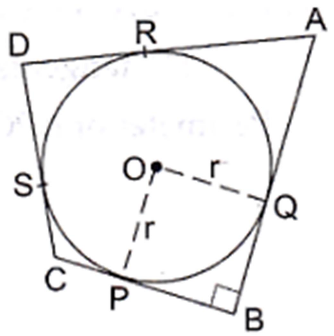
DPP – 04

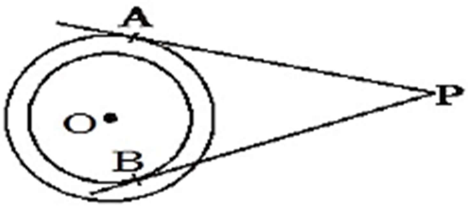
CLASS – 10^{th}

TOPIC – Theorem Based Questions

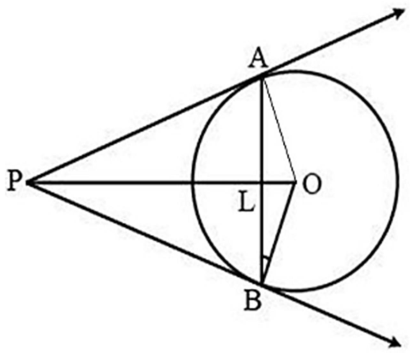
Q.1 In the fig. a circle is inscribed in a quadrilateral ABCD in which ∠B = 90° if AD = 23cm, AB = 29cm and DS = 5cm, find the radius of the circle.



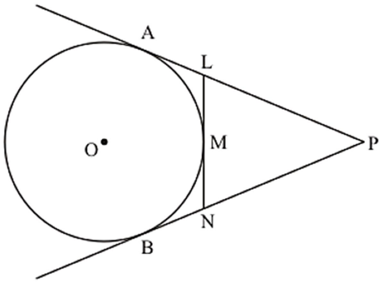
Q.2 In fig. there are two concentric circles with Centre O of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles if AP = 12cm, find the tangent length of BP.



Q.3 In the given figure, AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.



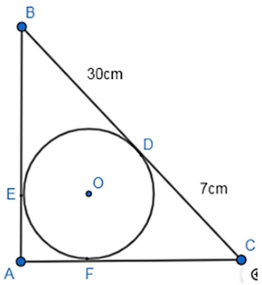
Q.4 In figure PA and PB are tangents from an external point P to the circle with centre O. LN touches the circle at M. Prove that PL + LM PN + MN



Q.5 In the given figure, BDC is a tangent to the given circle at point D such that BD = 30 cm and CD = 7 cm. The other tangents BE and CF are drawn respectively from B and C to the circle and meet when produced at A making BAC a right angle triangle. Calculate

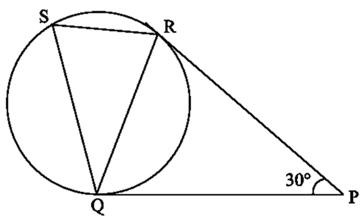
(i) AF

(ii) radius of thr circle



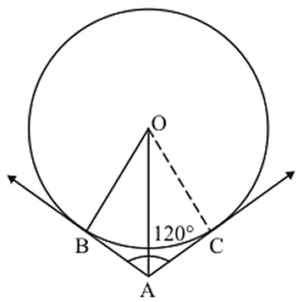
Q.6 If d1, $d_2(d_2 > d_1)$ be the diameters of two concentric circle s and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.

Q.7 In the given figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠ZRQS.



Q.8 From an external point P, tangents ∠PA = PB are drawn to a circle with centre or if ∠PAB=50°, then find ∠AOB

Q.9 In the given figure, two tangents AB and AC are drawn to a circle with centre O such that ∠BAC = 120°. Prove that OA = 2AB.



Q.10 The length of three concesutive sides of a quadrilateral circumscribing a circle are 4 cm, 5 cm, and 7 cm respectively. Determine the length of the fourth side.

Q.11 The common tangents AB and CD to two circles with centres O and O' intersect at E between their centres. Prove that the points O, E and O' are collinear.

DPP – 04

CLASS – 10th

TOPIC – THEOREM BASED QUESTIONS

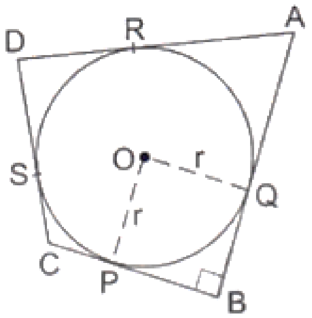
Sol.1

Given AD = 23 cm

AB = 29 cm

∠B = 90°

DS = 5cm



From fig in quadrilateral POQB

∠OPB = ∠OQB = 90° = ∠B = ∠POQ

and PO = OQ. ∴ POQB is a square PB = BQ = r

We know that

Tangents drawn from external point to circle are equal in length.

We know that

Tangents drawn from external point to circle are equal in length.

From A, AR = AQ .... (i)

From B, PB = QB .... (ii)

From C, PC = CS .... (iii)

From D, DR = DS .... (iv)

(i) + (ii) + (iv) ⇒ AR + DB + DR = AQ + QB + DS

⇒ (AR + DR) + r = (AQ + QB) + DS

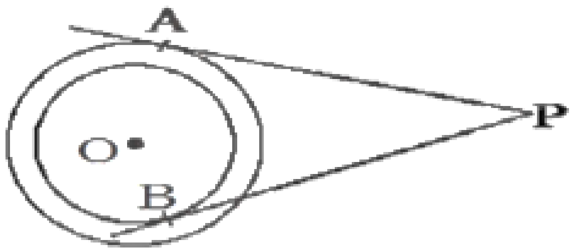
AD + r = AB + DS

⇒ 23 + r = 29 + 5

⇒ r = 34 – 23 = 11 cm

∴ radius = 11 cm

Sol.2



OA = 5 cm

OB = 3 cm

AP = 12 cm

BP = ?

We know that

At the point of contact, radius is perpendicular to tangent.

For circle 1, ΔOAP is right triangle

By Pythagoras theorem, $OP^2 = OA^2 + AP^2$

$\Rightarrow OP^2 = 5^2 + 12^2 = 25 + 144$

$= 169$

$\Rightarrow \text{OP} = \sqrt{169} = 13\ cm$

For circle 2, ΔOBP is right triangle by Pythagoras theorem,

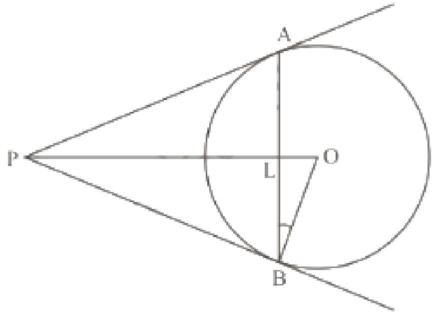
$OP^2 = OB^2 + BP^2$

$13^2 = 3^2 + BP^2$

$BP^2 = 169 - 9 = 160$

$BP = \sqrt{160} = 4\sqrt{10}\ cm$

Sol.3



From the property of tangents we know that the length of two tangents drawn form an external point will be equal. Therefore we have,

$PA = PB$

$OB = OA$ (They are the radii of the same circle)

PO is the common side

Therefore, from SSS postulate of congruency, we have,

ΔPOB and ΔPOA

Hence,

ΔOPA = ΔOPB ...... (1)

Now consider ΔPLA and ΔPLB. We have,

∠OPA = ∠OPB (From (1)

PA is the common side.

From the property of tangents we know that the length of two tangents drawn form an external point will be equal. Therefore we have,

$PA = PB$

From SAS postulate of congruent triangles, we have,

ΔPLA ≅ ΔPLB

Therefore,

$LA = LB$

It is given that AB = 16. That is,

$LA + LB = 16$

$LA + LA = 16$

$2LA = 16$

$LA = 8$

$LB = 8$

Also, ALB is a straight line. Therefore

∠ALB = 180°

That is,

$\angle PLA + \angle PLB = 180°$

Since $\Delta PLA = \Delta PLB$

$\angle PLA = \angle PLB$

Therefore,

$2\angle PLB = 180°$

$\angle PLB = 90°$

Now let us consider ΔOLB. We have,

$$OL^2 = OB^2 - OL^2$$

$$OL^2 = 10^2 - 8^2$$

$$OL^2 = 100 - 64$$

$$OL^2 = 36$$

OL=6

Consider ΔOPB Here,

$\angle OBP = 90°$ (Since the radius of the circle will always be perpendicular to the tangent at the point of contact)

Therefore,

$PB^2 = OP^2 - OB^2$ ...... (1)

Now consider ΔPLB

$PB^2 = PL^2 + BL^2$ ...... (2)

Since the Left Hand Side of equation (1) is same as the Left Hand Side of equation (2), we can equate the Right Hand Side of the two equations. Hence we have,

$OP^2 - OB^2 = PL^2 + LB^2$ ...... (3)

From the figure we can see that,

$OP = OL + LP$

Therefore, let us replace OP with $OL + LP$ in equation (3). We have,

$$(OL + PL)^2 - OB^2 - PL^2 + LB^2$$

We have found that $OL = 6$ and $LB = 8$. Also it is given that $OB = 10$. Substituting all these values in the above equation, we get,

$$(6+PL)^2 - 10^2 = PL^2 + 8^2$$

$$36 + PL^2 + 2\times 6\times PL - 100 = PL^2 + 64$$

12PL=128

$$PL = \frac{32}{3}$$

Now, let us substitute the value of PL in equation (2). We get,

$$PB^2 = \left(\frac{32}{3}\right)^2 + 8^2$$

$$PB^2 = \frac{1024}{9} + 64$$

$$PB^2 = \frac{1600}{9}$$

$$PB = \sqrt{\frac{1600}{9}}$$

$$PB = \frac{40}{3}$$

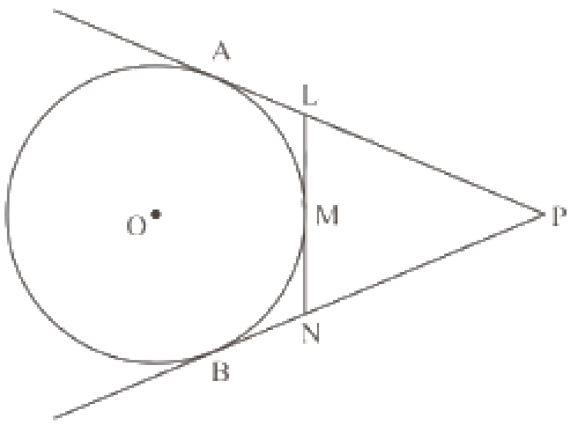
We know that tangents drawn from an external point will always be equal. Therefore,

$PB = PA$

Hence, we have,

$$PB = \frac{40}{3}$$

Sol.4



Given

O is Centre of circle

PA and PB are tangents

We know that

The tangents drawn from external point to the circle are equal in length.

From point P, PA = PB

⇒ PL + AL = PN + NB .... (i)

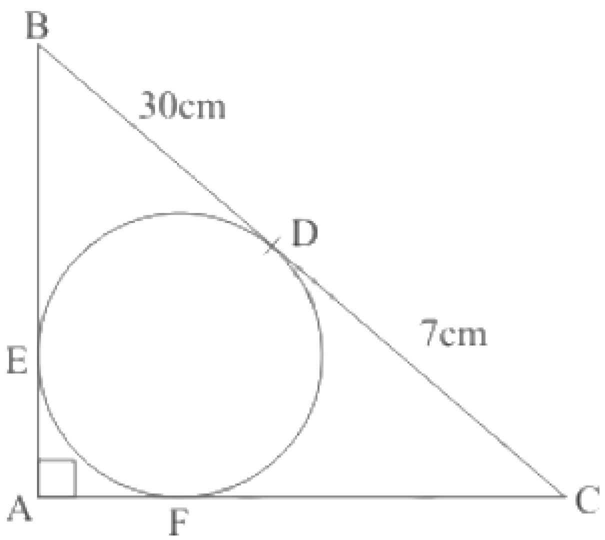
From point L & N, AL = LM and MN = NB } .... Substitute in (i)

PL + Lm = PN + MN

⇒ Hence proved.

Sol.5 (i)

The given figure is below



(i) The given triangle *ABC* is a right triangle where side BC is the hypotenuse. Let us now apply Pythagoras theorem. We have,

$AB^2 + AC^2 = BC^2$

Looking at the figure we can rewrite the above equation as follows.

$(BE + EA)^2 + (AF + FC)^2 = (30 + 7)^2$ ...... (1)

From the property of tangents we know that the length of two tangents drawn from the same external point will be equal. Therefore we have the following, BE = BD

It is given that BD = 30 cm. Therefore,

BE = 30 cm

Similarly,

CD = FC

It is given that CD = 7 cm. Therefore,

FC = 7 cm

Also, on the same lines,

EA = AF

Let us substitute these in equation (1). We get,

$$(BE+EA)^2+(AF+FC)^2=(30+7)^2$$

$$(30+AF)^2+(AF+7)^2=37^2$$

$$\left(30^2+2\times AF+AF^2\right)+\left(AF^2+2\times 7\times AF+7^2\right)=1369$$

$$900+60AF+AF^2+AF^2+14AF+49=1369$$

$$2F^2+74AF-420=0$$

$$AF^2+37AF-210=0$$

$$AF^2(AF+42)-5(AF+42)=0$$

$$AF(AF+42)-5(AF+42)=0$$

$$(AF-5)(AF+42)=0$$

Therefore,

AF = 5

Or,

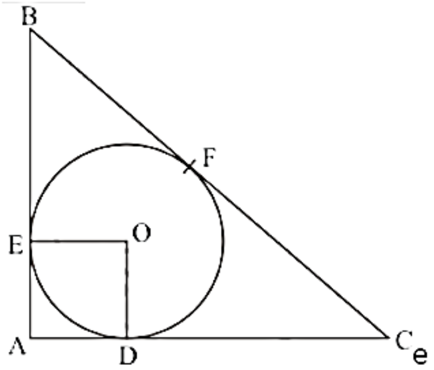
AF = – 42

Since length cannot have a negative value,

AF = 5

(ii)

Let us join the point of contact *E* with the centre of the circle say *O*. Also, let us join the point of contact *F* with the centre of the circle *O*. Now we have a quadrilateral *AEOF*.



In this quadrilateral we have,

$\angle EAD = 90^0$(Given in the problem)

$\angle oda = 90^0$(Since the radius will always be perpendicular to the tangent at the point of contact)

$\angle OEA = 90^0$(Since the radius will always be perpendicular to the tangent at the point of contact)

We know that the sum of all angles of a quadrilateral will be equal to 360^o. Therefore,

$\angle EAD + \angle ODA + \angle EOD + \angle OEA = 360^o$

$90^0 + 90^o + 90^o + \angle EOD = 360^o$

$\angle EOD = 90^o$

Since all the angles of the quadrilateral are equal to 90° and the adjacent sides are equal, this quadrilateral is a square. Therefore all the sides are equal. We have found that

AF = 5

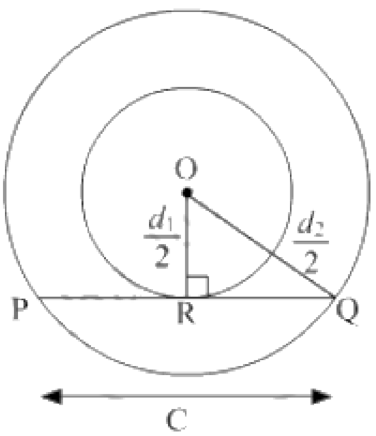
Therefore,

OD = 5

OD is nothing but the radius of the circle.

Thus we have found that *AF* = 5 cm and radius of the circle is 5 cm.

Sol.6



Let O be the centre of two concentric circles and PQ be the tangent to the inner circle that touches the circle at R.

Now, OQ= $\frac{1}{2}d_2$ and

OR= $\frac{1}{2}d_1$

Also, PQ = c

As, PQ is the tangent to the circle.

⇒ OR ⊥ PQ

⇒ QR = $\frac{1}{2}PQ = \frac{1}{2}c$

In Triangle OQR,

∴ By Pythagoras Theorem,

$$(OQ)^2 = (OR)^2 + (RQ)^2$$
$$\Rightarrow \left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$$
$$\Rightarrow (d_2)^2 = (d_1)^2 + c^2$$

Sol.7

It is given that, ∠RPQ = 30° and PQ and PR are tangents drawn to a circle from P to the same circle.

∴ PQ = PR ...(Tangents drawn from an external point to a circle are equal in length.)

In ΔPQR,

PQ = PR

∴ ∠PQR = ∠PRQ ...(Angles opposite to equal sides are equal.)

Now, In ΔPQR,

∠PQR + ∠PRQ + ∠RPQ = 180° ...(Angle sum property of a triangle)

∴ ∠PQR + ∠PQR + 30° = 180°

∴ 2∠PQR + 30° = 180°

∴ 2∠PQR = 180° − 30°

∴ 2∠PQR = 150°

∴ ∠PQR = 75°

So, ∠PQR = ∠QRS = 75° ...(Alternate angles)

∠PQR = ∠QSR = 75° ...(Alternate segment angles are equal)

and ∠QRS = ∠QSR = 75°

∴ ΔQRS is also an isosceles triangle.

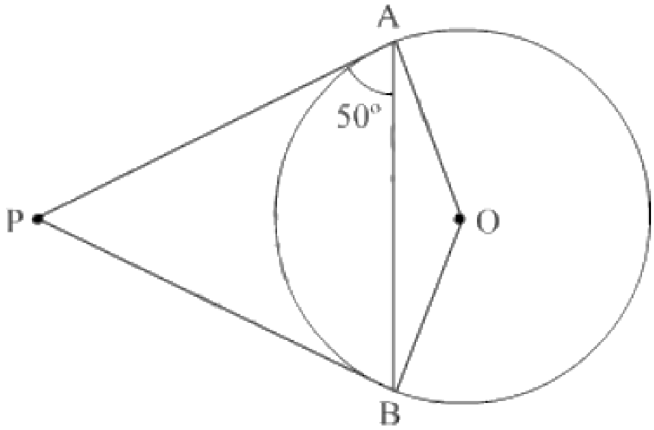
∴ ∠QRS + ∠QSR + ∠RQS = 180°

∴ 75° + 75° + ∠RQS = 180°

∴ ∠RQS = 180° − 150°

∴ ∠RQS = 30°

Sol.8



It is given that PA and PB are tangents to the given circle.

$\therefore \angle PAO = 90^o$ (Radius is perpendicular to the tangent at the point of contact.)

Now,

$\angle PAB = 50^o$ (Given)

$\therefore \angle OAB = \angle PAO - \angle PAB = 90^o - 50^o = 40^o$

In ΔOAB,

OB = OA (Radii of the circle)

$\therefore \angle OAB = \angle OBA = 40^o$ (Angles opposite to equal sides are equal.)

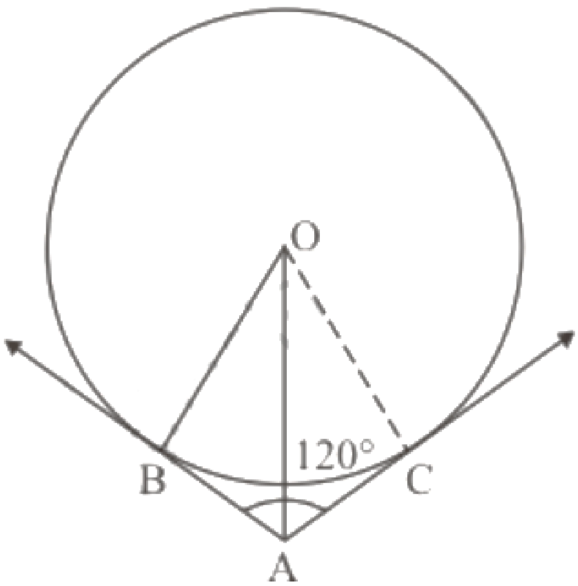
Now,

$\angle AOB + \angle OAB + \angle OBA = 180^o$ (Angle sum property)

$\Rightarrow \angle AOB = 180^o - 40^o - 40^o = 100^o$

Sol.9

Consider Δ OAB and Δ OAC.



We have,

$OB = OC$ (Since they are radii of the same circle)

$AB = AC$ (Since length of two tangents drawn from an external point will be equal)

OA is the common side.

Therefore by SSS congruency, we can say that Δ OAB and Δ OAC are congruent triangles.

Therefore,

∠OAC = ∠OAC

It is given that,

$\angle OAB + \angle OAC = 120^o$

$2\angle OAB = 120^o$

$\angle OAB = 60^o$

We know that,

$\cos \angle OAB = \frac{AB}{OA}$

$\cos 60^o = \frac{AB}{OA}$

We know that,

$\cos 60^o = \frac{1}{2}$

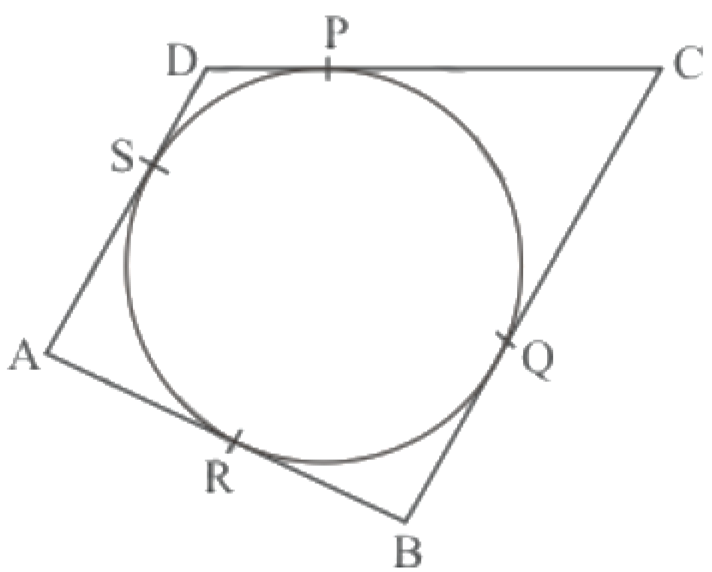
Therefore,

$\frac{1}{2} = \frac{AB}{OA}$

$OA = 2AB$

Sol.10

Let us first put the given data in the form of a diagram.



From the property of tangents we know that the length of two tangents drawn from the same external point will be equal. Therefore we have,

$AR = SA$

Let us represent AR and SA by 'a'.

Similarly,

$QB = RB$

Let us represent SD and DP by 'b'

$PC = CQ$

Let us represent PC and PQ by 'c'

$SD = DP$

Let us represent QB and RB by 'd'

It is given that,

$AB = 4$

$AR + RB = 4$

$a + b = 4$

$b = 4 - a$ ...... (1)

Similarly,

$BC = 5$

That is,

$b + c = 5$

Let us substitute for b from equation (1). We get,

$4 - a + c = 5$

$c - a = 1$

$c = a + 1$ ...... (2)

$CD = 7$

$c + d = 7$

Let us substitute for c from equation (2). We get,

$a + 1 + d = 7$

$a + d = 6$

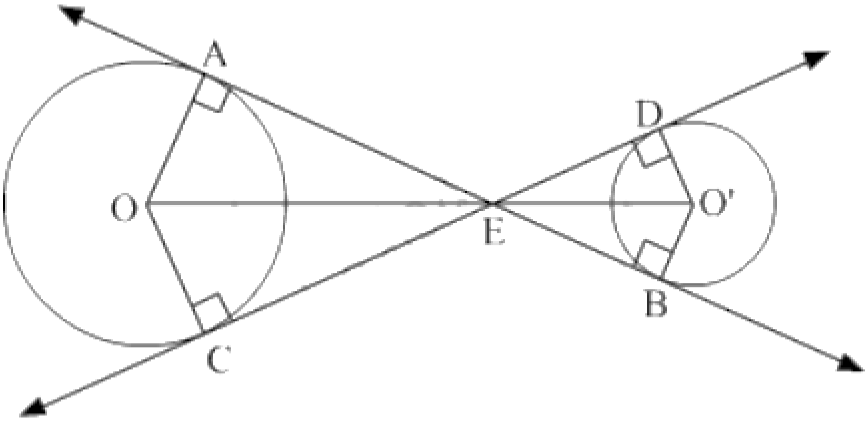
In the previous section we had represented AS and SR with 'a' and SD and DP with 'b'. We shall now put AS in place of 'a' and SD in place of 'd'. We get,

$AS + SD = 6$

$AD = 6$ cm

Therefore, the length of the fourth side of the quadrilateral is 6 cm.

Sol.11



Here Angle AEC and DEB are equal (vertically opposite angles)

Join OA and OC,

So in triangle OAE and OCE, we have

OA = OC (radii of same circle)

OE = OE (common)

$\angle OAE = \angle OCE$ [90 each, as tangent is always perpendicular to its radius at point of contact]

$\triangle OEA \cong \triangle OCE\,(RHS)$

$So, \angle AEO = \angle CEO\,(CPCT)$

Similarly, for the other circle we have

$\angle DEO' = \angle BEO'$

$Now \angle AEC = \angle DEB$

$\Rightarrow \frac{1}{2}\angle AEC = \frac{1}{2}\angle DEB$

$\Rightarrow \angle AEO = \angle CEO = \angle DEO' = \angle BEO'$

So, all four angles are equal and bisected by OE and OE'.

Hence, O, E' and O' are collinear.