Chapter 11

CONIC SECTIONS - 2 (PARABOLA)

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. The length of each of side of an equilateral triangle inscribed in the parabola $y^2 = 4x$ with one vertex at the vertex of the parabola is

(a) 8 (b) $8\sqrt{2}$

(c) $8\sqrt{3}$ (d) $\frac{8\sqrt{3}}{3}$

2. If the focus of parabola $(y-k)^2 = 4(x-h)$ always lies between the lines $x+y=1$ and $x+y=3$ then

(a) $0<h+k<2$ (b) $0<h+k<1$

(c) $1<h+k<2$ (d) $1<h+k<3$

3. The locus of a point such that the sum of it distances from the origin and the line $x = 2$ is 4 units is

(a) a circle of radius 4

(b) a parabola of latus rectum 8

(c) segments of two opposite parabola of laturecta 4 and 12

(d) an ellipse or a parabola

4. Parabolas $y^2 = 4a(x-c_1)$ and $x^2 = 4a(y-c_2)$ where c_1 and c_2 are variables, touch each other. Locus of their point of contact is

(a) $xy = a^2$ (b) $xy = 2a^2$

(c) $xy = 4a^2$ (d) None of these

5. A line bisecting the ordinate PN of a point $P(at^2, 2at)$, $t>0$, on the parabola $y^2 = 4ax$ is drawn parallel to the axis to meet the curve at Q. If NQ meets the tangent at the vertex at the point T, then the coordinates of T are

(a) $\left(0, \frac{4}{3}at\right)$ (b) $(0, 2at)$

(c) $\left(\frac{4}{3}at^2, at\right)$ (d) $(0, at)$

6. If the normal to the parabola $y^2 = 4ax$ at point t_1 cuts the parabola again at point t_2, then

(a) $2 \le t_2^2 \le 8$ (b) $t_2^2 \le 2$

(c) $t_2^2 \ge 8$ (d) None of these

7. The chord $x+y=1$ cuts the parabola $y^2 = 4ax$ in points A, B. The normals at A and B intersect at C. A third line from C cuts the parabola normally at D whose coordinates are

(a) $(a, -2a)$ (b) $(4a, 4a)$

(c) $(0, 0)$ (d) $(2a, -a)$

8. The triangle formed by the tangent to the parabola $y^2 = 4x$ at the point whose abscissa lies in the interval $[a^2, 4a^2]$, the ordinate and the x axis, has the greatest area equal to

(a) $12a^3$ (b) $8a^3$

(c) $16a^3$ (d) None of these

9. The straight line $y = mx + c$ $(m>0)$ touches the parabolas $y^2 = 8(x+2)$ then the minimum value taken by c is

(a) 4 (b) 8

(c) 12 (d) 6

10. PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than

(a) 40 (b) 45

(c) 32 (d) 48

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ

11. The equation of the parabola to which the line $m^2(y-10) - mx - 1 = 0$ is a tangent for any real value of m is

(a) $x^2 = -4y$ (b) $x^2 = -4(y-10)$

(c) $y^2 = 4(x-4)$ (d) $x^2 = y - 10$

12. If $P(2, -4)$ and Q are points on the parabola $y^2 = 8x$ and the chord PQ subtends a right angle at the vertex of the parabola, then the coordinates of the point of intersection of normal at P and Q is

(a) (30, 24) (b) (30, –24)

(c) (24, 30) (d) (24, –30)

13. If (h, k) is a point on the axis of parabola $2(x-1)^2 + 2(y-1)^2 = (x+y+2)^2$ from where three distinc normals can be drawn, then

(a) $h > 2$ (b) $h < 4$

(c) $h > 8$ (d) $h < 8$

14. If a chord PQ of a parabola $y^2 = 12x$ subtends a right angle at the vertex, then the locus of point of intersection of the normals at P and Q is

(a) $y^2 = 48(x+18)$ (b) $y^2 = 48(x+8)$

(c) $y^2 = 48x$ (d) $y^2 = 48(x-18)$

15. The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the coordinate axes lies in the first quadrant. If its area is 2, then the value of b is

(a) – 1 (b) 3

(c) – 3 (d) 1

16. If three normals can be drawn to a parabola from a point (h, k) which cut the parabola at P, Q and R, then the centroid of ΔPQR

(a) coincides with the vertex

(b) coincides with the focus

(c) lies at the axis

(d) lies at the directrix

17. If the normals from any point to the parabola $y^2 = 4x$ cut the line $x = 2$ in points whose ordinates are in A.P., then the slopes of tangents at the co-normal points are in

(a) H.P. (b) G.P.

(c) A.P. (d) None of these.

18. If the normal drawn from the point on the axis of the parabola $y^2 = 8ax$ whose distance from the focus is 8 a, and which is not parallel to either axis, makes an angle θ with the axis of x, then θ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of these.

19. Maximum number of common normal of $y^2 = 4ax$ and $x^2 = 4by$ may be equal to

(a) 2 (b) 4

(c) 5 (d) 0

20. The tangents at P and Q on the parabola $y^2 = 4x$ meet in T, S is the focus, SP, ST, SQ are equal to a, b, c, respectively. Then the roots of the equation $ax^2 + 2bx + c = 0$ are

(a) Real and different

(b) Real and equal

(c) non-real complex numbers

(d) Irrational

21. Minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is

(a) $\sqrt{5}$ (b) $\sqrt{21}$

(c) $\sqrt{28} - \sqrt{5}$ (d) $\sqrt{21} - \sqrt{5}$

22. If the tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x+b)$ at Q and R, then the mid-point of QR is

(a) $(x_1 + b, y_1 + b)$ (b) $(x_1 - b, y_1 - b)$

(c) (x_1, y_1) (d) $(x_1 + b, y_1 - b)$

23. If perpendiculars be drawn from any two fixed points on the axis of a parabola equidistant from the focus on any tangent to it, then the difference of their squares is [ℓ is the length of latus rectum and $2d$ is the distance between two points]

(a) ℓd (b) $2\ell d$

(c) $4\ell d$ (d) $\sqrt{\ell^2 + d^2}$

MARK YOUR RESPONSE	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ
	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ		

24. Let P be any point on the parabola $y^2 = 4ax$ whose focus is S. If normal at P meet x - axis at Q. Then ΔPSQ is always

(a) isosceles (b) equilateral

(c) right angled (d) None of these

25. The condition that the parabolas $y^2 = 4ax$ and $y^2 = 4c\,(x - b)$ have a common normal other than x -axis (a, b, c being distinct positive real numbers) is

(a) $\frac{b}{a-c} < 2$ (b) $\frac{b}{a-c} > 2$

(c) $\frac{b}{a-c} < 1$ (d) $\frac{b}{a-c} > 1$

26. The number of points with integral coordinates that lie in the interior of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 4x$ is

(a) 8 (b) 10

(c) 16 (d) none of these.

27. If three normals can be drawn from $(h, 2)$ to the parabola $y^2 = -4x$, then

(a) $h < -2$ (b) $h > 2$

(c) $-2 < h < 2$ (d) h is any real number

28. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1, 2) is

(a) $(x-1)^2 = 4\,(y+1)$ (b) $(x+1)^2 = 4\,(y+1)$

(c) $(x+1)^2 = 4\,(y-1)$ (d) $(x-1)^2 = 4\,(y-1)$

29. If two distinct chords drawn from the point (4, 4) on the parabola $y^2 = 4ax$ are bisected on the line $y = mx$, then the set of values of m is given by

(a) $\left(\frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2}\right)$ (b) $\boldsymbol{R}$

(c) $(0, \infty)$ (d) $(-2, 2)$

30. Set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 4$ touches parabola. $y^2 = 4ax$, is

(a) $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$

(b) $(-\infty, -1) \cup (1, \infty)$

(c) $(-1, 1)$

(d) $(-\infty, \infty)$

31. Minimum distance between the parabolas $y^2 - 4x - 8y + 40 = 0$ and $x^2 - 8x - 4y + 40 = 0$ is

(a) 0 (b) $\sqrt{3}$

(c) $2\sqrt{2}$ (d) $\sqrt{2}$

32. AB is a chord of the parabola $y^2 = 4ax$ with vertex A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(a) a (b) $2a$

(c) $4a$ (d) $8a$

33. If normals are drawn form a point $P(h, k)$ to the parabola $y^2 = 4ax$ then the sum of the intercepts which the normal cut off from the axis of the parabola is

(a) $(h + a)$ (b) $3(h + a)$

(c) $2(h + a)$ (d) $h + 2a$

34. Two parabolas have the same focus $(3, -2)$. Their directrices are the x-axis and the y-axis respectively. Then the slope of their common chord is

(a) -1 (b) $-\frac{1}{2}$

(c) $-\frac{\sqrt{3}}{2}$ (d) 1

35. The line $x - y = 1$ intersects the parabola $y^2 = 4ax$ at A and B Normals at A and B intersect at C. If D is the point at which line CD is normal to the parabola, then coordinates of D are

(a) $(4, -4)$ (b) $(4, 4)$

(c) $(-4, -4)$ (d) $(-4, 4)$

36. From a point 't' on the parabola $y^2 = 4ax$, a focal chord and a tangent is drawn. Two circles are drawn in which one circle is drawn taking focal chord as diameter and other is drawn by taking intercept of tangen between point 't' and directix as diameter. Then the locus of midpoint of common chord of the circles is

(a) $y = 3x + a$

(b) $9ax^2 - ay^2 - 2xy^2 + 6a^2x + a^3 = 0$

(c) $9ax^2 + ay^2 + 2xy^2 - 6a^2x - a^3 = 0$

(d) none of these

MARK YOUR RESPONSE	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ
	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ
	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ		

37. A focal chord of parabola $y^2 = 4x$ is inclined at an angle of $\frac{\pi}{4}$ with positive x-direction, then the slope of normal drawn at the ends of chord will satisfy the equation

(a) $m^2 - 2m - 1 = 0$ (b) $m^2 + 2m - 1 = 0$

(c) $m^2 - 1 = 0$ (d) $m^2 + 2m - 2 = 0$

38. Two mutually perpendicular chords OA and OB are drawn through the vertex 'O' of a parabolar $y^2 = 4ax$. Then the locus of the circumcentre of triangle OAB is

(a) $y^2 = 2ax - 4a$ (b) $y^2 = 2ax - 8a^2$

(c) $y^2 = 2ax - 2a$ (d) $y^2 = 4ax - 8a^2$

39. If the normals at three distinct points $(p^2, 2p)$, $(q^2, 2q)$ and $(r^2, 2r)$ of the parabola $y^2 = 4x$ are concurrent then

(a) $p + r = 2q$ (b) $p + r = q$

(c) $p + 2q + 3r = 0$ (d) $p + q + r = 0$

40. A chord *PP'* of a parabola cuts the axis of the parabola at *O*. The feet of the perpendiculars from *P* and *P'* on the axis are *M* and *M'* respectively. If *V* is the vertex then VM, VO, VM' are in

(a) A.P. (b) GP.

(c) H.P. (d) none of these

41. The parabola $y^2 + 4x$ and the circle $(x-6)^2 + y^2 = r^2$ will have no common tangent if 'r' is equal to

(a) $r > \sqrt{20}$ (b) $r < \sqrt{20}$

(c) $r > \sqrt{18}$ (d) $R \in (\sqrt{20}, \sqrt{28})$.

42. Radius of circle touching parabola $y^2 = x$ at (1, 1) and having directrix of $y^2 = x$ as its normal is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{5\sqrt{5}}{4}$

43. The ends of line segement are *P* (1, 3) and *Q* (1, 1). *R* is a point on the line segement *PQ* such that $PR : RQ = 1 : \lambda$. If *R* is an interior point of parabola $y^2 = 4x$, then

(a) $\lambda \in (0, 1)$ (b) $\lambda \in \left(\frac{-3}{5}, 1\right)$

(c) $\lambda \in (-1, 0)$ (d) $\lambda \in (1, \infty)$

44. The length of normal chord which subtend an anlge of $90°$ at the vertex of the parabola $y^2 = 4x$ is

(a) $6\sqrt{2}$ (b) $7\sqrt{2}$

(c) $8\sqrt{2}$ (d) $4\sqrt{2}$

45. If two circles $x^2 + y^2 - 6y - 6y + 13 = 0$ and $x^2 + y^2 - 8y + 9 = 0$ intersect at *A* and *B*. The focus of the parabola whose directrix is line *AB* and vertex at (0, 0) is

(a) $\left(\frac{3}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{3}{5}, \frac{1}{5}\right)$

(c) $\left(-\frac{3}{5}, -\frac{1}{5}\right)$ (d) $\left(\frac{3}{5}, -\frac{1}{5}\right)$

46. A moving varible parabola, having it's axis parallel to x-axis always touches the given equal parabola $y^2 = 4x$, the locus of the vertex of the moving parabola is;

(a) $y^2 = -4x$ (b) $y^2 = 8x$

(c) $y^2 = -8x$ (d) $x^2 = 4y$

47. From the focus of the parabola $y^2 = 8x$, tangents are drawn to the circle $(x-6)^2 + y^2 = 4$. Then the equation of circle through the focus and points of contact of the tangents is

(a) $x^2 + y^2 + 8x - 12 = 0$ (b) $x^2 + y^2 - 6x + 12 = 0$

(c) $x^2 + y^2 - 8x + 12 = 0$ (d) $x^2 + y^2 + 6x - 12 = 0$

48. Let *P* be any point on parabola $y^2 = 4ax$ between its vertex and +ve end of latus rectum. *M* is point of perpendicular from focus *S* to tangent at *P*, then maximum value of area of ΔPMS is

(a) a^2 (b) $\frac{a^2}{3}$

(c) $4\,a^2$ (d) $\frac{2a^2}{3}$

MARK YOUR RESPONSE					
	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ
	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ	46. ⓐⓑⓒⓓ
	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ			

B

COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

Normally, the various propositions you study, e.g. equation of tangent, normal, chord, focal chord, formula for focal distance etc., are derived for the parabola $y^2 = 4ax$. However, all the results with slight transformation are valid for any parabola.

Suppose we represent the equation of parabola $y^2 - 4ax = 0$ by $S(x, y, a) = 0$ and any equation dervied for this parabola by $P(x, y, a) = 0$.

Now, if the given parabola is $y^2 = -4ax$, i.e. $y^2 + 4ax = 0$ we can write it $S(x, y, -a) = 0$, so the corresponding equation of P will be $P(x, y, -a) = 0$.

Similarly for $x^2 = 4ay$ can be written as $S(y, x, a)$ and corresponding transformation is $P(y, x, a)$ (i.e. interchange x and y).

1. The focal distance of the point (x, y) on the parabola $x^2 - 8x + 16y = 0$ is

(a) $|y-4|$ (b) $|y-5|$
(c) $|y-2|$ (d) $|x-4|$

2. Normals are drawn from the point (7, 14) to the parabola $x^2 - 8x - 16y = 0$. The sum of the slopes of these normals is

(a) 0 (b) $\frac{3}{2}$
(c) $\frac{7}{3}$ (d) $-\frac{3}{2}$

3. The points on the axis of the parabola $x^2 + 2x + 4y + 13 = 0$ from where three distinct normals can be drawn are given by

(a) $(-1, k), k \in (-1, 2)$ (b) $(-1, k),\ k \in (2, \infty)$
(c) $(-1, k), k \in (-\infty, -5)$ (d) $(k, k), k > 0$

4. The line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $x^2 + 4a(y+a) = 0$ if

(a) $a = p\sec\alpha$ (b) $a\cos 2\alpha = p\sin\alpha$
(c) $a^2\cos\alpha + p^2\sin\alpha = 0$ (d) $a\tan\alpha = p\sec\alpha$

PASSAGE-2

Normal to a parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is given by $xt + y = 2at + at^3$. If it passes through a point

(h, k) then $at^3 + t(2a - h) - k = 0$ .....(1)

If t_1, t_2, t_3 be roots of (1) then three points P, Q, R are $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$, $(at_3^2, 2at_3)$ from which normals pass through the point (h, k). Points P, Q, R are called co-normal points. Putting $2at = y$, the ordinates of P, Q, R are the roots of

$$y^3 + 4a(2a - h)y - 8a^2k = 0 \quad ....(2)$$

Let the circle PQR be $x^2 + y^2 + 2gx + 2fy + c = 0$

Eliminating x from equations of the parabola and circle, we have

$$\frac{y^4}{16a^2} + y^2 + 2g\frac{y^2}{4a} + 2fy + c = 0,$$

i.e. $y^4 + y^2(16a^2 + 8ag) + 32a^2fy + 16a^2c = 0$ .....(3)

Equation (3) gives the ordinates of the points of intersection of the parabola and the circle.

5. The circle through co-normal points of a parabola passes through

(a) focus of parabola
(b) vertex of parabola
(c) origin
(d) point of intersection of axis and directrix of parabola

6. The equation of the circle through the feet of normals drawn to parabola $y^2 = 4ax$ from a point (h, k) is

(a) $x^2 + y^2 = a^2$
(b) $x^2 + y^2 - 2ax = 0$
(c) $x^2 + y^2 - 2ax + (k + a)y = 0$
(d) $x^2 + y^2 - (h + 2a)x - \frac{1}{2}ky = 0$

7. If the normals at the point Q, R on parabola $y^2 - 4ax = 0$ meet the parabola at the same point P, then the locus of the circumcentre of the trinagle PQR is

(a) $y = ax - a^2$ (b) $2(x^2 + y^2) = a^2$
(c) $x^2 - y^2 = ax$ (d) $2y^2 - ax + a^2 = 0$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ			

PASSAGE-3

A tangent is drawn at any point P(t) on the parabola $y^2 = 8x$ and on it is taken a point Q (α, β) from which pair of tangent QA and QB are drawn to the circle $x^2 + y^2 = 4$. Using the information answer the following questions :

8. The locus of the point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 4$ is

(a) $y^2 - 2x = 0$ (b) $y^2 - x^2 = 4$

(c) $y^2 + 2x = 0$ (d) $y^2 - 2x^2 = 4$

9. The points from which perpendicular tangents can be drawn both to the given circle and the parabola is/are

(a) $(4, \pm\sqrt{3})$ (b) $(-1, \sqrt{2})$

(c) $(-\sqrt{2}, -\sqrt{2})$ (d) $(-2, \pm\sqrt{2})$

10. The locus of circumcentre of $\Delta\, AQB$ if $t = 2$ is

(a) $x - 2y + 4 = 0$ (b) $x + 2y - 4 = 0$

(c) $x - 2y - 4 = 0$ (d) $x + 2y + 4 = 0$

PASSAGE-4

Let two parabolas $y^2 = 4ax$ and $x^2 = 4ay$ intersect at O and A (O being origin). A parabola P is drawn, whose directix is the common tangent to the two parabolas and whose focus is the point which divides OA internally in the ratio $(1+\sqrt{3}):(7-\sqrt{3})$

11. The equation of the common tangent to $y^2 = 4ax$ and $x^2 = 4ay$ is

(a) $x + y + a = 0$ (b) $x + y - a = 0$

(c) $x - y + a = 0$ (d) $x - y - a = 0$

12. The equation of the parabola P is

(a) $\left[x - \frac{(1+\sqrt{3})a}{2}\right]^2 + \left[y - \frac{(1+\sqrt{3})a}{2}\right]^2 = \frac{(x+y+a)^2}{2}$

(b) $\left[x - \frac{(1+\sqrt{3})a}{2}\right]^2 + \left[y + \frac{(1+\sqrt{3})a}{2}\right]^2 = \frac{(x+y+a)^2}{2}$

(c) $\left[x + \frac{(1+\sqrt{3})a}{2}\right]^2 + \left[y - \frac{(1+\sqrt{3})a}{2}\right]^2 = \frac{(x+y+a)^2}{2}$

(d) none of these

13. Parabola P always passes through the points

(a) $(a,0),(0,a)$ (b) $(a,0),(0-a)$

(c) $(-a,0),(0,-a)$ (d) $(-a,0),(0,a)$

MARK YOUR RESPONSE	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ
	13. ⓐⓑⓒⓓ				

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement-1** : Slope of tangents drawn from (4, 10) to parabola $y^2 = 9x$ are $\frac{1}{4}, \frac{9}{4}$

Statement-2 : The point (4, 10) lies on the latus rectum of the given parabola

2. Let t_1, t_2 be parameters of two points P and Q on the parabola $y^2 = 4ax$.

Statement 1 : If PQ is normal at P and passes through focus of parabola then $t_2 = -t_1$

Statement 2 : If PQ is a focal chord then $t_1 t_2 = -1$

3. **Statement 1** : The locus of the centre of circle which cuts orthogonally the parabola $y^2 = 4x$ at the point $P\,(1, 2)$ is a circle.

Statement 2 : The tangent at P to circle is a normal to the parabola at P.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ		

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. P is a point which moves in the $x-y$ plane such that the point P is nearer to the centre of a square than any of the sides. The four vertices of the square are $(\pm a, \pm a)$. The region in which P will move is bounded by parts of parabola of which one has the equation

(a) $y^2 = a^2 + 2ax$ (b) $x^2 = a^2 + 2ay$

(c) $y^2 + 2ax = a^2$ (d) $x^2 = a^2 - 2ay$

2. The ends of a line segment are $P(1, 3)$ and $Q(1, 1)$. R is a point on the line segment PQ such that $PR : QR = 1 : \lambda$. If R is an interior point of the parabola $y^2 = 4x$ then

(a) $\lambda \in (0, 1)$ (b) $\lambda \in \left(-\frac{3}{5}, 1\right)$

(c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$ (d) none of these

3. Consider the parabola $y^2 = 4ax$ and $x^2 = 4by$. The straight line $b^{1/3}y + a^{1/3}x + a^{2/3}\ b^{2/3} = 0$

(a) Touches $y^2 = 4ax$

(b) Touches $x^2 = 4by$

(c) Intersects both parabolas in real points.

(d) Touches first and intersects other

4. Three normals to the parabola $y^2 = x$ can be drawn through a point $(c, 0)$, if

(a) $c = \frac{3}{4}$ (b) $0 < c < \frac{1}{2}$

(c) $c > \frac{1}{2}$ (d) $c = \frac{1}{2}$

5. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is

(a) $\left(\frac{p}{2}, p\right)$ (b) $(2p, 2p)$

(c) $\left(\frac{p}{2}, -p\right)$ (d) $\left(-\frac{p}{8}, \frac{p}{2}\right)$

6. Tangents are drawn from $(-2, 0)$ to $y^2 = 8x$, radius of circle(s) that would touch these tangents and the corresponding chord of contact, can be equal to,

(a) $4(\sqrt{2}+1)$ (b) $4(\sqrt{2}-1)$

(c) $8\sqrt{2}$ (d) $4\sqrt{2}$

7. A quadrilateral is inscribed in a parabola, then

(a) quadrilateral may be cylic.

(b) diagonals of the quadrilateral may be equal.

(c) all possible pairs of adjacent sides may be perpendicular.

(d) none of these.

8. A normal drawn to parabola $y^2 = 4ax$ meet the curve again at Q such that the angle subtended by PQ at vertex is 90° then coordinates of P can be

(a) $(8a, 4\sqrt{2}a)$ (b) $(8a, 4a)$

(c) $(2a, -2\sqrt{2}a)$ (d) $(2a, 2\sqrt{2}a)$

9. From a point $(\sin\theta, \cos\theta)$ if three normals can be drawn to the parabola $y^2 = 4ax$ then the value of 'a' belong to

(a) $\left(\frac{1}{2}, 1\right)$ (b) $\left(-\frac{1}{2}, 0\right)$

(c) $\left(0, \frac{1}{2}\right)$ (d) $(1, \infty)$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

1. Observe the following columns :

	Column-I		Column-II
(A)	If two distinct chords of a parabola $y^2 = 4ax$ passing through the point $(a, 2a)$ are bisected by the line $x + y = 1$, then the length of the latus rectum can be	p.	-1
(B)	The parabola $y = x^2 - 5x + 4$ cuts the x-axis at P and Q. A circle is drawn through P and Q so that the orgin lies outside it. The length of a tangent to the circle from the origin is equal to	q.	0
(C)	If $y + b = m_1(x + a)$ and $y + b = m_2(x + a)$ are two tangents to $y^2 = 4ax$ then m_1m_2 is equal to	r.	1
(D)	If the point $(h, -1)$ is exterior to both the parabolas $y^2 = \|x\|$, then the integral part of h can be equal to	s.	2
		t.	3

2. Observe the following columns :

	Column-I		Column-II
(A)	The equations of the common normal(s) to the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is /are	p.	$x + a = 0$
(B)	A pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ intersects the coordinate axes in concylic points. The locus of P is	q.	$x = a$
(C)	The locus of point from which tangents drawn to parabolas $y^2 = 4a(x + a)$ and $y^2 = 8a(x + 2a)$ are mutually perpendicular is	r.	$x + y = 3a$
(D)	The chord of contact of a point P with respect to the parabola $y^2 + 4ax = 0$ subtends right angle at the vertex . The locus of P is	s.	$x + 3a = 0$
		t.	$x = 4a$

3. Observe the following columns :

	Column-I		Column-II
(A)	The normal chord at a point t on the parabola $y^2 = 4x$ subtends a right angle at the vertex, then t^2 is	p.	4
(B)	The area of the triangle inscribed in the curve $y^2 = 4x$, the parameter of coordinates of whose vertices are 1, 2 and 4 is	q.	2
(C)	The numberof distinct normals possible from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is	r.	3
(D)	The normal at $(a, 2a)$ on $y^2 = 4ax$ meets the curve again at $(at^2, 2at)$, then the value of $\|t - 1\|$ is	s.	6

MARK YOUR RESPONSE

1.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

2.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

3.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

4. Normal to parabola $y^2 = 4x$ at points P and Q of parabola meet at $R\,(x_2, 0)$ and tangents at P and Q meets at T $(x_1, 0)$. Let $x_2 = 3$ Match the entries of two columns.

	Column – I		**Column – II**
(A)	The area of quadrilateral $PTQR$ is	p.	3
(B)	If the quadrilateral PTQR can be inscribed in a circle then the value of $\dfrac{\text{circumferecnce}}{4\pi}$ is	q.	4
(C)	The number of nomals that can be drawn to the parabola from R is	r.	1
(D)	The square of the length PT is	s.	8

MARK YOUR RESPONSE

4.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

F NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010 etc.) or a single-digit integer, ranging from 0 to 9.

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to a question is 6092, then the correct darkening of bubbles will look like the given.

For single digit integer answer darken the extreme right bubble only.

0	**0**	0	0
1	1	1	1
2	2	2	**2**
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	**9**	9

1. A trapezium is inscribed in the parabola $y^2 = 4x$ such that its diagonal pass through the point (1, 0) and each has length $\frac{25}{4}$. If the area of trapezium be P then $4P$ is equal to

2. Three normals drawn from any point to the parabola $y^2 = 4ax$ cut the line $x = 2a$ in points whose ordinates are in arithmetic progression. If the slopes of the normals be m_1, m_2 and m_3 then $\left(\frac{m_1}{m_2}\right)\left(\frac{m_3}{m_2}\right)$ is equal to

3. Let the maximum and minimum values of the areas of the triangles formed by x-axis, tangent and normal at a point on the segment of parabola $y = x^2 + 1,\ 1 \le x \le 3$ be A_1 and A_2 respectively then $3A_1 + A_2$ is equal to

4. A chord is drawn from a point $P\,(1, t)$ to the parabola $y^2 = 4x$ which cuts the parabola at A and B. If $PA.PB = 3\,|\,t\,|$, then the maximum value of t is equal to

MARK YOUR RESPONSE

1., 2., 3., 4. (each grid):

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

5. The sides of trianlge ABC are tangents to the parabola $y^2 = 4ax$. Let D, E, F be the points of contact of side *BC, CA* and *AB* respectively. If lines *AD, BE* and *CF* are concurrent at focus of the parabola and the $\angle ABC$ of the $\Delta\, ABC$ is $\frac{\pi}{k}$ then k is equal to

6. If a point P on the parabola $y^2 = 4x$ is taken such that the point is at shortest distance from the circle $x^2 + y^2 + 2x - 2\sqrt{2}y + 2 = 0$, tangents are drawn to the circle and the parabola, then the area of the triangle PAB is $\sqrt{a}$ where *A* and *B* are the points of contact on two different lines on circle and parabola respectively, then a is equal to

7. Tangents at points *P* and *Q* of parabola $y^2 = 4x$ intersect each other at point *R* on parabola $y^2 = -x$. The normal at *P* and *Q* intersect at right angle at *S*. The diameter of circumcircle of qudrilateral $PQRS$ is equal to

MARK YOUR RESPONSE

5.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

6.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

7.

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(c)	**9**	(a)	**17**	(b)	**25**	(b)	**33**	(c)	**41**	(b)
2	(a)	**10**	(c)	**18**	(c)	**26**	(a)	**34**	(a)	**42**	(d)
3	(c)	**11**	(b)	**19**	(c)	**27**	(a)	**35**	(a)	**43**	(a)
4	(c)	**12**	(a)	**20**	(b)	**28**	(c)	**36**	(b)	**44**	(c)
5	(a)	**13**	(a)	**21**	(a)	**29**	(a)	**37**	(b)	**45**	(b)
6	(c)	**14**	(d)	**22**	(c)	**30**	(a)	**38**	(b)	**46**	(b)
7	(b)	**15**	(c)	**23**	(a)	**31**	(d)	**39**	(d)	**47**	(c)
8	(c)	**16**	(c)	**24**	(a)	**32**	(c)	**40**	(b)	**48**	(c)

B COMPREHENSION TYPE

1	(b)	**4**	(b)	**7**	(d)	**10**	(a)	**13**	(a)
2	(c)	**5**	(b)	**8**	(c)	**11**	(a)		
3	(c)	**6**	(d)	**9**	(d)	**12**	(a)		

C REASONING TYPE

1	(c)	**2**	(d)	**3**	(d)

D MULTIPLE CORRECT CHOICE TYPE

1	(a,b,c,d)	**3**	(a,b)	**5**	(a,c)	**7**	(a, b)	**9**	(b, c)
2	(a,c)	**4**	(a,c)	**6**	(a, b)	**8**	(c, d)		

E MATRIX-MATCH TYPE

1. **A - r, s, t; B - s; C - p; D - p, q**
2. **A - r, B - q, C - s, D - t**
3. **A - q; B - s; C - r; D - p**
4. **A - s; B - r; C - p; D - s**

F NUMERIC/INTEGER ANSWER TYPE

1	75	**3**	930	**5**	3	**7**	5
2	1	**4**	4	**6**	2		

Solutions

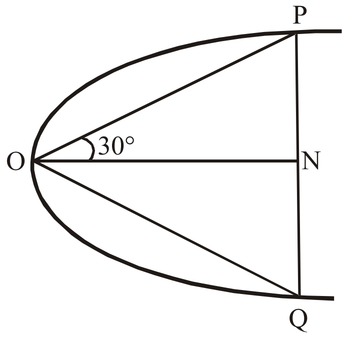
A SINGLE CORRECT CHOICE TYPE

1. **(c)** The vertex of the parabola $y^2 = 4x$ is the origin $O(0, 0)$. Let OPQ be the equilateral triangle inscribed in the given parabola. The triangle is symmetric about the x-axis, the axis of the parabola.

Since $\angle POQ = 60°$, OP makes an angle of $30°$ with x-axis and hence the equation of OP is

$$y = x \tan 30° \text{ or } y = \frac{1}{\sqrt{3}}x.$$

So the coordinates of P are given by



$$\frac{1}{3}x^2 = 4x. \Rightarrow x = 12 \Rightarrow y = \frac{12}{\sqrt{3}}$$

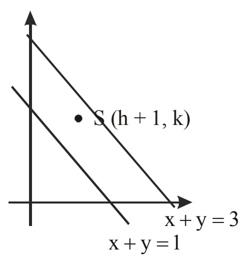
and then the coordinate of P are $\left(12, \frac{12}{\sqrt{3}}\right)$

Hence the length of the side of the triangle = PQ =

$$2\left(\frac{12}{\sqrt{3}}\right) = 8\sqrt{3}$$

2. **(a)** Coordinate of focus will be $(h + 1, k)$

Now focus should lie to the opposite side of origin with respect to line $x + y - 1 = 0$ and same side as origin with respect to line $x + y - 3 = 0$



Hence $h + k > 0$ and $h + k < 2$.

3. **(c)** Equation of the required locus is

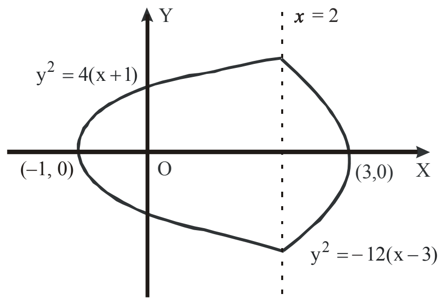
$$\sqrt{x^2 + y^2} + |x - 2| = 4$$

Case (1) Let $x \geq 2$, then locus is $\sqrt{x^2 + y^2} + x - 2 = 4$

$\Rightarrow \sqrt{x^2 + y^2} = 6 - x \Rightarrow y^2 = -12(x-3)$

Case (II) Let $x \leq 2$, then locus is

$$\sqrt{x^2 + y^2} - x + 2 = 4 \Rightarrow \sqrt{x^2 + y^2} = 2 + x$$



$\Rightarrow \quad y^2 = 4(x + 1).$

Both being parabola as shown in the adjacent figure.

4. **(c)** Let P (x, y) be the point of contact

$\therefore \quad 2y\frac{dy}{dx} = 4a$ and $2x = 4a\frac{dy}{dx}$

For the tangency of curves, $\frac{4a}{2y} = \frac{2x}{4a} \Rightarrow xy = 4a^2$,

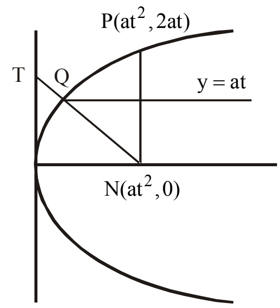
which is the required locus.

5. **(a)** Equation of the line parallel to the axis and bisecting the ordinate PN of the point $P(at^2, 2at)$ is $y = at$ which meets the parabola $y^2 = 4ax$ at the point Q $\left(\frac{1}{4}at^2, at\right)$.

Coordinates of N are $(at^2, 0)$.

Equation of NQ is $y = \dfrac{0 - at}{at^2 - \frac{1}{4}at^2}(x - at^2)$

Which meets the tangent at the vertex, $x = 0$ at the point $y = \frac{4}{3}at.$



6. **(c)** A normal at point t_1 cuts the parabola again at t_2, then $t_2 = -t_1 - \dfrac{2}{t_1} \Rightarrow t_1^2 + t_1 t_2 + 2 = 0$

Since, t_1 is real, discriminant $\geq 0 \Rightarrow t_2^2 - 8 \geq 0$

$\Rightarrow t_2^2 \geq 8$.

7. **(b)** Equation of a normal to the parabola $y^2 = 4ax$ can be written as

$y = mx - 2am - am^3$ at the point $(am^2, -2am)$

If the normal passes through C (h, k) then we have

$am^3 + (2a - h)m + k = 0$ ....(i)

Thus, if m_1, m_2, m_3 are the root of equation (i), then the coordinates of the feet of normals are

$A\,(am_1^2, -2am_1)$, $B\,(am_2^2, -2am_2)$ and $D(am_3^2, -2am_3)$.

According to given condition, A and B lie on the line $x + y = 1$. So,

$am_1^2 - 2am_1 = 1$ ....(ii) and

$am_2^2 - 2am_2 = 1$ ....(iii)

We get

$\Rightarrow m_1^2 - m_2^2 - 2(m_1 - m_2) = 0$

$\Rightarrow m_1 + m_2 - 2 = 0$ $[\because m_1 \neq m_2]$

$\Rightarrow -m_3 - 2 = 0 \Rightarrow m_3 = -2$

$[\because m_1 + m_2 + m_3 =]$

$\therefore$ The coordinates of D are $(4a, 4a)$.

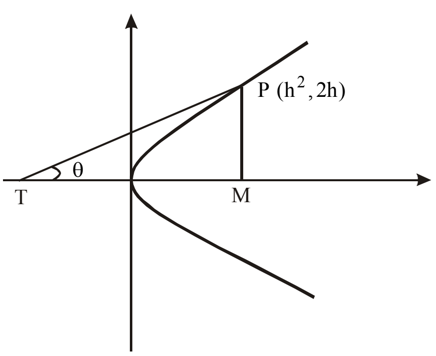
8. **(c)** Let $P \equiv (h^2, 2h)$.

Slope of tangent at $P = \tan\theta = \dfrac{1}{h}$

Thus the area of $\Delta PTM = \dfrac{1}{2} \times PM \times TM = \dfrac{1}{2} \times 2h \times 2h \cot\theta = 2h^3$

The area will be maximum when h is maximum

$\therefore a^2 \leq h^2 \leq 4a^2$



$\therefore$ for maximum h, $h = 2a$

$\therefore$ Maximum area $= 2\,(2a)^3 = 16a^3$.

9. **(a)** Tangent to $y^2 = 8\,(x + 2)$ is $y = m\,(x + 2) + \dfrac{2}{m}$

$c = 2m + \dfrac{2}{m} \Rightarrow \dfrac{c}{2} = \left(m + \dfrac{1}{m}\right)$

$\because m + \dfrac{1}{m} \geq 2 \Rightarrow \dfrac{c}{2} \geq 2 \Rightarrow c \geq 4 \Rightarrow$ The minimum value of $c = 4$.

10. **(c)** Length of focal chord having one extremity $(at^2, 2at)$ is $a\left(t + \dfrac{1}{t}\right)^2$

$\because \left|t + \dfrac{1}{t}\right| \geq 2 \Rightarrow a\left(t + \dfrac{1}{t}\right)^2 \geq 4a = 32 \Rightarrow$ length of focal chord $\not< 32$.

11. **(b)** Equation of the given line is $x = m\,(y - 10) - \dfrac{1}{m}$

compare with $x - h = m\,(y - k) + \dfrac{a}{m}$, we get

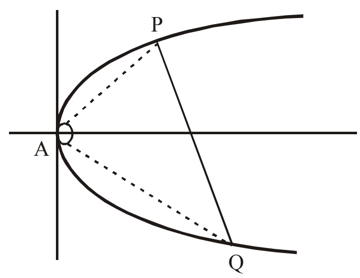
$h = 0, k = 10$ and $a = -1$

$\therefore$ The equation of parabola is $(x - h)^2 = 4a\,(y - k)$

$\Rightarrow x^2 = -4\,(y - 10)$.

12. **(a)** The normal at $P\,(at_1^2, 2a\,t_1)$ and $Q\,(at_2^2, 2a\,t_2)$ intersect at the point $(a\,(t_1^2 + t_2^2 + t_1t_2 + 2),\ -at_1t_2\,(t_1 + t_2))$.

In the given problem $a = 2$ Comparing P $(2, -4)$ with $(at_1^2, 2at_1)$, we get $t_1 = -1$



Again if A is the vertex,

then slope of $AP = \dfrac{2}{t_1}$ and slope of $AQ = \dfrac{2}{t_2}$

Since, $AP \perp AQ \Rightarrow t_1t_2 = -4 \Rightarrow t_2 = 4\ (\because t_1 = -1)$

$\therefore$ The point of intersection of normals at P and Q is $[2\,(1 + 16 - 4 + 2), -2 \times -1 \times 4\,(-1 + 4)]$

$\Rightarrow (30, 24)$.

The correct answer is (a).

13. **(a)** In the given equation of parabola the focus is (1,1) and equation of directrix is $x+y+2=0$

$\Rightarrow$ axis of parabola is $y=x$

Vertex of the parabola is (0, 0). Let a is the distance between vertex and focus = $\sqrt{2}$.

The distance of the point on the axis from, which three normal may be drawn will be at least $2a$ from the vertex which is equal to $2\sqrt{2}$. So coordinates of the point nearest to the vertex from which three normals may be drawn can be given as (2, 2) $\Rightarrow h>2$.

14. **(d)** Let the points P and Q are respectively $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.

Where $a = 3$. The coordinates (h, k) of the point of intersection of normals at P and Q are given by

$$h=a\ (t_1^2+t_2^2+t_1t_2+2) \qquad \text{..... (i)}$$

$$k=-\ at_1t_2\ (t_1+t_2) \qquad \text{..... (ii)}$$

Also $t_1t_2=-4$ [see solution of Q. N. 12] ..... (iii)

Putting $t_1\ t_2=-4$ in (i) and (ii), we get

$$h=3\ (t_1^2+t_2^2-2) \text{ and } k=12\ (t_1+t_2)$$

$$\therefore k^2=144\ (t_1^2+t_2^2+2t_1t_2)=144\left(\frac{h}{3}+2+2\times -4\right)$$

$$=144\left(\frac{h}{3}-6\right) \text{ or } k^2=48\ (h-18)$$

$\Rightarrow$ locus of (h, k) is $y^2 = 48\ (x-18)$

The correct answer is (d).

15. **(c)** $f(x)=x^2+bx-b,\ f'(x)=2x+b,\ f'(1)=2+b$

$\Rightarrow$ The slope of the tangent at (1, 1) is $2+b$

$\therefore$ the equation of the tangent to the curve at (1, 1) is $y-1=(2+b)(x-1)$

$$\Rightarrow \quad \frac{x}{\frac{1+b}{2+b}}-\frac{y}{1+b}=1$$

The intercepts formed are $\dfrac{1+b}{2+b}$ and $-(1+b)$

Area is $-\dfrac{1}{2}\left(\dfrac{1+b}{2+b}\right)(1+b)=2$ (given)

$$\Rightarrow \ (1+b)^2+4\,(2+b)=0 \Rightarrow b^2+6b+9=0$$

$$\Rightarrow (b+3)^2=0 \Rightarrow b=-3.$$

16. **(c)** Normal at $(at^2, 2at)$ is $tx+y=2at+at^3$

It passes through (h, k), so, $at^3+(2a-h)t-k=0$

If its roots are t_1, t_2, t_3 then $t_1+t_2+t_3=0$

Now, if the feet of the normal be (x_1, y_1), (x_2, y_2) and (x_3, y_3), then

$$y_1+y_2+y_3=2a(t_1+t_2+t_3)=0$$

$\therefore$ The centriod

$$\left(\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right)\equiv\left(\frac{x_1+x_2+x_3}{3}, 0\right)$$

lies on the x-axis i.e. the axis of parabola.

17. **(b)** Equation of the normal to the parabola $y^2=4x$ is given by $y=-tx+2t+t^3$ .... (i)

Since it intersects $x=2$ we get $y=t^3$

let the three ordinates be t_1^3, t_2^3, t_3^3 in A.P.

$$\Rightarrow \ 2t_2^3=t_1^3+t_3^3=(t_1+t_3)^3-3t_1t_3(t_1+t_3) \qquad \text{... (ii)}$$

From equation (i) we have $t_1+t_2+t_3=0$

$\Rightarrow t_1+t_3=-t_2$

Hence equation (ii) reduces to

$$2t_2^3=(-t_2)^3-3t_1t_3(-t_2)=-t_2^3+3t_1t_2t_3$$

$$\Rightarrow \ 3t_2^3=3t_1t_2t_3 \Rightarrow t_2^2=t_1t_3$$

$\Rightarrow t_1, t_2, t_3$ are in G.P.

Hence slopes of tangents $\dfrac{1}{t_1}, \dfrac{1}{t_2}, \dfrac{1}{t_3}$ are in G.P.

18. **(c)** The focus of the Parabola $y^2=8ax$ is $(2a, 0)$

So the coordinates of the point on the axis of the parabola at a distance $8a$ from the focus is $(10a, 0)$.

Equation of a normal to the parabola $y^2=8ax$ is $y=mx-4am-2am^3$.

Since it passes through $(10a, 0)$

$$0=10am-4am-2am^3 \Rightarrow 2am\,(3-m^2)=0$$

$$\Rightarrow \ m^2=3 \qquad [\because m\neq 0]$$

$$\Rightarrow \ m=\pm\sqrt{3}=\tan\left(\pm\frac{\pi}{3}\right).$$

19. **(c)** Equation of normal to $y^2=4ax$ and $x^2=4by$ in terms of m are given by

$$y=mx-2am-am^3 \text{ and } y=mx+2b+\frac{b}{m^2}$$

For common normal $2b+\dfrac{b}{m^2}+2am+am^3=0$

$$\Rightarrow am^5+2am^3+2bm^2+b=0.$$

So, a maximum of 5 normals are possible.

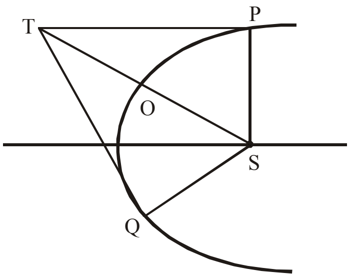
20. **(b)** If $P(t_1), Q(t_2)$, then $T=(t_1 t_2, t_1+t_2)$, $S=(1,0)$ for $y^2=4x$

$a = SP = (1+t_1^2)$ and $c = SQ = (1+t_2^2)$

Now $b^2 = ST^2 = (t_1 t_2 - 1)^2 + (t_1 + t_2)^2$

$= t_1^2 + t_2^2 + 1 + t_1^2 t_2^2 = (1+t_1^2)(1+t_2^2)$

$\Rightarrow b^2 = ac$



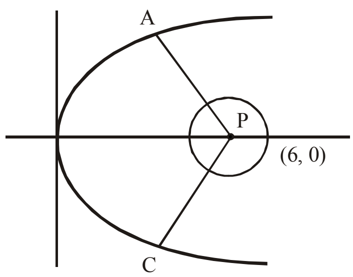
$\therefore$ discriminant of the equation $= 4b^2 - 4ac = 0$

$\therefore$ roots of $ax^2 + 2bx + c = 0$ are real and equal.

21. **(a)** Centre and radius of the given circle is $P(6, 0)$ and $\sqrt{5}$ respectively.
Equation of normal for $y^2 = 4x$ at $(t^2, 2t)$ is

$y = -tx + 2t + t^3$, it must pass though (6, 0) in order that it gives minimum distance between the two curves.

$\therefore$ $0 = t^3 - 4t \Rightarrow t = 0$ or $t = \pm 2$



$\therefore$ $A(4, 4)$ and $C(4, -4)$

$PA = PC = \sqrt{20} = 2\sqrt{5}$

$\therefore$ required minimum distance $= 2\sqrt{5} - \sqrt{5} = \sqrt{5}$.

22. **(c)** Equation of the tangent at $P(x_1, y_1)$ to $y^2 = 4ax$ is $yy_1 - 2ax - 2ax_1 = 0$ .... (i)
Equation of the chord of $y^2 = 4a(x+b)$ whose mid-point is (x', y') is
$yy' - 2ax - 2ax' - 4ab = y'^2 - 4ax' - 4ab$ or $yy' - 2ax - (y'^2 - 2ax') = 0$ .... (ii)
Equation (i) and (ii) represent the same line.

$\therefore$ $\dfrac{y_1}{y'} = \dfrac{2a}{2a} = -\dfrac{2ax}{y'^2 - 2ax'}$

This gives $y' = y_1$ and then $2ax_1 = y'^2 - 2ax' = y_1^2 - 2ax' = 4ax_1 - 2ax'$ $\therefore$ $x' = x_1$
$\therefore$ mid-point $(x', y') = (x_1, y_1)$.

23. **(a)** Suppose any two points on the axis equidistant from the focus $(a, 0)$ be $(a+d, 0)$ $(a-d, 0)$ where d is constant. Equation of any tangent to the parabola is $y = mx + \dfrac{a}{m}$ or $m^2x - my + a = 0$ if P_1 and P_2 be the perpendiculars on it, then

$$P_1^2 - P_2^2 = \left[\frac{m^2(a+d)+a}{\sqrt{m^4+m^2}}\right]^2 - \left[\frac{m^2(a-d)+a}{\sqrt{m^4+m^2}}\right]^2$$

$$= \frac{1}{m^2(1+m^2)}\left[\left\{a(m^2+1)+dm^2\right\}^2 - \left\{a(m^2+1)-dm^2\right\}^2\right]$$

$$= \frac{1}{m^2(1+m^2)}\left[4a(m^2+1)dm^2\right] = 4ad = \ell d,$$

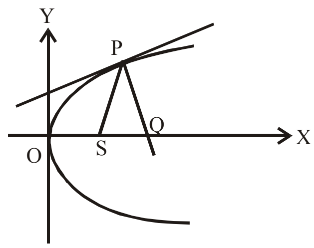
where $\ell = 4a =$ latus rectum

24. **(a)** Let co-ordinates of P be $(at^2, 2at)$,

then equation of normal is $at^3 + (2a - x)t - y = 0$

$\because$ PQ meet x - axis at Q

$\therefore$ co-ordinates of Q are $(at^2 + 2a, 0)$



co-ordinates of S are $(a, 0)$

$\therefore$ $SQ = at^2 + a = a(1+t^2)$

and $PS = a\sqrt{(t^2-1)^2 + 4t^2} = a(1+t^2) = QS$

$\therefore$ ΔPSQ is isosceles.

25. **(b)** For $y^2 + 4ax$, Normal : $y = mx - 2am - am^3$ ... (i)
For $y^2 = 4c(x-b)$, normal : $y = m(x-b) - 2cm - cm^3$ ... (ii)
If two parabolas have common normal :
Then (i) & (ii) must be identical
After comparing the coeffecients we get

$$m = \pm\sqrt{\frac{2(a-c)-b}{(c-a)}}$$

which is real of $-2 - \dfrac{b}{c-a} > 0 \Rightarrow \dfrac{b}{a-c} > 2$.

26. **(a)** If (λ, μ) is interior to both the curves then if

$\lambda^2 + \mu^2 - 16 < 0$ and $\mu^2 - 4\lambda < 0$.

Now, $\mu^2 - 4\lambda < 0 \Rightarrow \lambda > \left(\frac{\mu}{2}\right)^2$.

Hence, if $\mu = 0$, $\lambda = 1, 2, 3, \ldots$; if $\mu = 1$, $\lambda = 1, 2, 3, \ldots$;
if $\mu = 2$, $\lambda = 2, 3, \ldots$; if $\mu = 3$, $\lambda = 3, 4, \ldots$;

Also $\lambda^2 + \mu^2 - 16 < 0 \Rightarrow \lambda^2 < 16 - \mu^2$.

Hence, if $\mu = 0$, $\lambda = 1, 2, 3$; if $\mu = 1$, $\lambda = 1, 2, 3$; if $\mu = 2$, $\lambda = 2, 3$; if $\mu = 3$, λ has no integral value.

$\therefore$ (1, 0), (2, 0), (3, 0), (1, 1), (2, 1), (3, 1), (2, 2), (3, 2) are the possible points.

27. **(a)** Any normal to the parabola, $y^2 = -4ax$ of slope m is $y = mx + 2m + m^3$. If it passes through $(h, 2)$, $m^3 + (h + 2)m - 2 = 0$.

Let $f(m) = m^3 + (h+2)m - 2$. Then $f'(m)$

$= 3m^2 + (h+2)$.

If $f(m) = 0$ has distinct real roots, then necessary conditions is that $f'(m) = 0$ should have 2 distinct real roots i.e. $h < -2$.

28. **(c)** Any point on the given parabola is $(t^2, 2t)$. The equation of the tangent at (1, 2) is $x - y + 1 = 0$

The image (h, k) of the point $(t^2, 2t)$ in $x - y + 1 = 0$ is

given by $\frac{h - t^2}{1} = \frac{k - 2t}{-1} = -\frac{2(t^2 - 2t + 1)}{1 + 1}$

$\therefore \quad h = t^2 - t^2 + 2t - 1 = 2t - 1$ and

$k = 2t + t^2 - 2t + 1 = t^2 + 1$

Eliminating t from $h = 2t - 1$ and $k = t^2 + 1$,

We get $(h+1)^2 = 4(k-1)$

The required equation of reflection is

$(x+1)^2 = 4(y-1)$.

29. **(a)** Any point on the line $y = mx$ can be of taken as (t, mt). Equation of chord of parabola with this as mid point will be $ymt - 2(x + t) = m^2t^2 - 4t$.
It passes through (4, 4), so $4mt - 2(4 + t) = m^2t^2 - 4t$
$\Rightarrow m^2t^2 - 2[2m + 1]t + 8 = 0$
since we want 2 such chords, so $D > 0$

$(2m+1)^2 - 8m^2 > 0 \Rightarrow 4m^2 - 4m - 1 < 0$

$\Rightarrow \frac{1 - \sqrt{2}}{2} < m < \frac{1 + \sqrt{2}}{2}$

30. **(a)** Equation of tangent to the parabola with slope m is

$y = mx + \frac{1}{m}$

For this line to be chord of the circle $x^2 + y^2 = 4$,

$$\left| \frac{\frac{1}{m}}{\sqrt{1 + m^2}} \right| < 2$$

$\therefore 1 < 4m^2(1 + m^2)$

$4m^4 + 4m^2 - 1 > 0 \Rightarrow m^2 > \frac{-1 + \sqrt{2}}{2}$

$$\Rightarrow m \in \left(-\infty, -\sqrt{\frac{-1+\sqrt{2}}{2}}\right) \cup \left(\sqrt{\frac{-1+\sqrt{2}}{2}}, \infty\right)$$

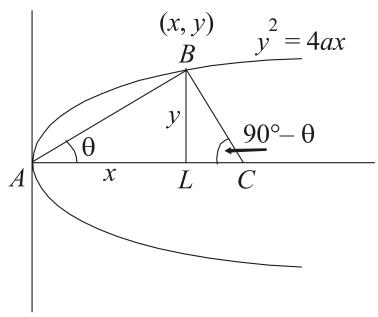
31. **(d)** Since two parabolas are symmetrical about $y = x$.
So, minimum distance is zero if they intersect on $y = x$
on solving $y = x$ & $y^2 - 4x - 8y + 4 = 0$
we get $x^2 - 12x + 40 = 0$ which has no real solution so they do not intersect.
Minimum distance is between tangents to the parabola parallel to $y = x$.
Differentiating $x^2 - 8x - 4y + 40 = 0$ with respect to x,

we get $2x - 8 - 4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x - 4}{2} = 1$ (slope of tangent).

So, $x = 6$ and $y = 7$
Point on parabola $(x - 4)^2 = 4(y - 6)$ is (6, 7) and corresponding point on $(y - 4)^2 = 4(x - 6)$ is (7, 6)
So, minimum distance $= \sqrt{2}$.

32. **(c)** $\tan\theta = \frac{y}{x}$ (See figure)



Projection of BC on the x-axis

$= LC = \frac{y}{\tan(90° - \theta)} = y\tan\theta$

$= \frac{y^2}{x} = 4a$.

33. **(c)** Equation of normal : $y = mx - 2am - am^3$

Put $y = 0$

We get $$x_1 = 2a + am_1^2$$ $$x_2 = 2a + am_2^2$$ $$x_3 = 2a + am_3^2$$

where x_1, x_2, x_3 are the intercepts on the axis of the parabola, the normal passes through (h, k)

$\Rightarrow \quad am^3 + (2ah - h)m + k = 0$

$m_1 + m_2 + m_3 = 0$

$m_1m_2 + m_2m_3 + m_3m_1 = \dfrac{2a-h}{a}$

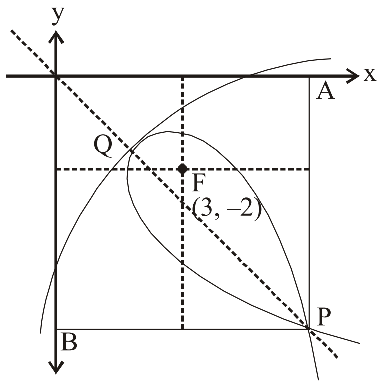
$\Rightarrow \quad m_1^2 + m_3^2 + m_3^2 = (m_1 + m_2 + m_3)^2$

$-2(m_1m_2 + m_2m_3 + m_3m_1) = -2\dfrac{(2a-h)}{a}$

$\Rightarrow \quad x_1 + x_2 + x_3 = 6a + a\left(m_1^2 + m_2^2 + m_3^2\right)$

$= 6a - 2(2a - h) = 2(h + a)$

34. **(a)**



Let the focus be F. The parabolas are open down and open right respectively. Let the parabolas intersect at points P and Q. From P draw perpendiculars on the x-axis and y-axis at A and B respectively, then

$PA = PF = PB \quad \Rightarrow P$ lies on the line $y = -x$

Similarly, Q lies on the line $y = -x$

$\Rightarrow$ Slope of $PQ = -1$.

35. **(a)** Since $y = x - 1$

solving with the equation of parabola

$(x-1)^2 = 4x \;\Rightarrow\; x^2 - 6x + 1 = 0 \;\Rightarrow\; x = 3 \pm \sqrt{8}$

$\therefore \quad y = 2 \pm \sqrt{8}$

Suppose point D is (x_1, y_3) then

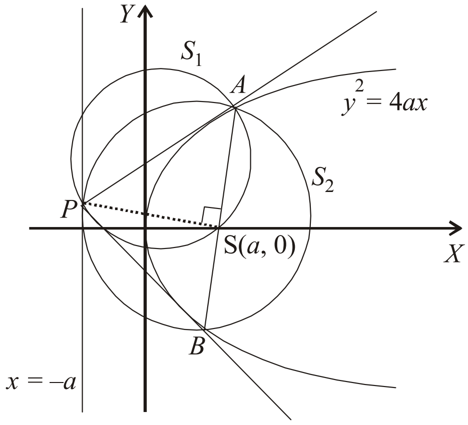
$y_1 + y_2 + y_3 = 0$

$2 + \sqrt{8} + 2 - \sqrt{8} + y_3 = 0$

$\Rightarrow \quad y_3 = -4$ then $x_3 = 4$

$\therefore$ The point is $(4, -4)$,

36. **(b)** Circle S_2, taking focal chord AB as diameter will touch directrix at point P and circle S_1, taking tangent AP as diameter will pass through focus S (since AP subtends angle 90° at focus of parabola).



Hence common chord of given circles is line AP (which is intercept of tangents at point 'A' between points A and directrix.)

Equation of tangents at A ('t') is

$yt = x + at^2$

$\Rightarrow \quad$ point $P = \left(-a, \dfrac{a(t^2-1)}{t}\right)$

Let midpoint of AP is $R\ (h, k)$

$$h = \frac{at^2 - a}{2}, k = \frac{\frac{a(t^2-1)}{t} + 2at}{2}$$

$\Rightarrow \quad$ locus of R is (by eliminating t)

$9ax^2 - ay^2 - 2xy^2 + 6a^2x + a^3 = 0$

37. **(b)** For focal chord $t_1t_2 = -1$,

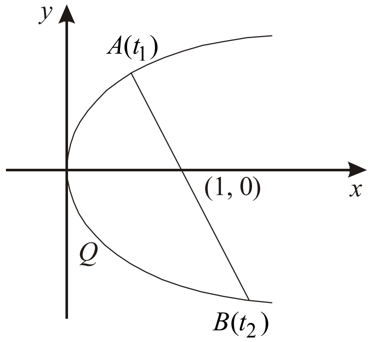
also slope of $AB = \dfrac{2}{t_1 + t_2} = 1$ (given)

$\Rightarrow \quad t_1 + t_2 = 2, t_1t_2 = -1$

Hence the required equation

$m^2 + 2m - 1 = 0.$

Since slope of normals drawn at A and B are $-t_1, -t_2$ respectively.



38. **(b)** Let $(-g,-f)$ be the circumcentre of ΔOAB.

Since OA is perpendicular to OB

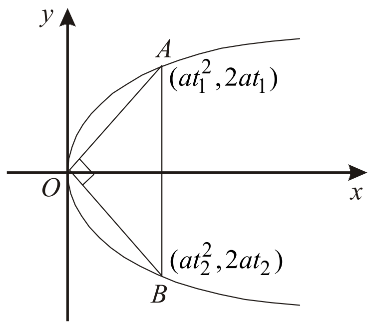
$\Rightarrow \quad t_1t_2 = -4$ ...(1)

Clearly $(-g-f)$ is the mid-point of AB.

$\Rightarrow \quad -g = \dfrac{a(t_1^2+t_2^2)}{2}$ and $-f = (t_1+t_2)$ ...(2)

From (1) and (2), we get

$$(t_1+t_2)^2 = t_1^2+t_2^2+2t_1t_2$$



$\Rightarrow \dfrac{f^2}{a^2} = -\dfrac{2g}{a} - 8 \Rightarrow$ required locus $y^2 = 2ax - 8a^2$.

39. **(d)** Equations of thee normals are

$$px + y - 2(p^3+2p) = 0$$

$$qx + y - 2(q^3+2q) = 0$$

$$rx + y - 2(r^3+2r) = 0$$

Since these lines are concurrent

$$\begin{vmatrix} p & 1 & p^3+2p \\ q & 1 & q^3+2q \\ r & 1 & r^3+2r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p & 1 & p^3 \\ q & 1 & q^3 \\ r & 1 & r^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & p & p^3 \\ 1 & q & q^3 \\ 1 & r & r^3 \end{vmatrix} = 0$$

$\Rightarrow \quad (p+q+r)(p-q)(q-r)(r-p) = 0$

$\Rightarrow \quad p+q+r = 0$

40. **(b)** Let the parabola be $y^2 = 4ax$ and $P = (a_1^2, at_1)$ and $P' = (at_2^2, at_2)$ are respectively

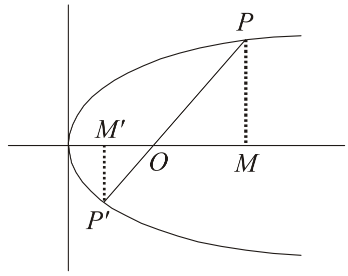
$\therefore \quad VM = at_1^2, VM' = at_2^2$

Let $VO = k$ then

$$\begin{vmatrix} at_1^2 & at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ k & 0 & 1 \end{vmatrix} = 0 \qquad (\because \; P', P, O \text{ are collinear})$$

$\Rightarrow \quad k + at_1t_2 = 0$

$\therefore \quad VM.VM' = (at_1t_2)^2 = k^2 = OV^2$

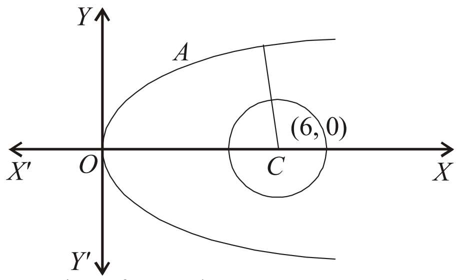


41. **(b)** Any normal of parabola is $y = -tx + 2t + t^3$. If it passes through (6, 0), then $-6t + 2t + t^3 = 0$

$\Rightarrow \quad t = 0, t^2 = 4, A \equiv (4,4)$

thus for non common tangents $AC > r$

$\Rightarrow \quad \sqrt{4+16} > r \;\Rightarrow\; r < \sqrt{20}$



42. **(d)** Equation of normal at P

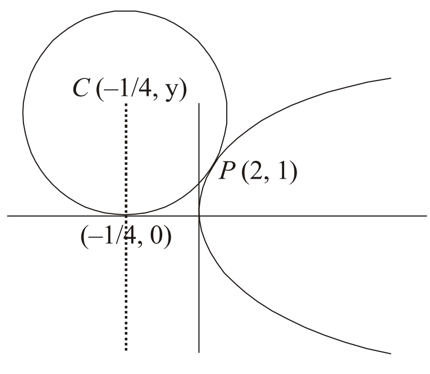
$y - 1 = -2(x-1)$

$\Rightarrow \quad y + 2x = 3$

at $x = -\dfrac{1}{4}, \qquad y = 3 + \dfrac{1}{2} = \dfrac{7}{2}$.

$\Rightarrow$ radius $= \sqrt{\left(1+\dfrac{1}{4}\right)^2 + \left(1-\dfrac{7}{2}\right)^2}$

$= \sqrt{\dfrac{25}{16} + \dfrac{25}{4}} = \dfrac{5\sqrt{5}}{4}$.



43. **(a)** Clearly $R\left(1,\frac{1+3\lambda}{1+\lambda}\right)$. It is an interior point.

$$\left(\frac{1+3\lambda}{1+\lambda}\right)^2-4<0$$

$$\Rightarrow \frac{-3}{5}<\lambda<1, \text{ but } \lambda>0$$

$$\Rightarrow 0<\lambda<1.$$

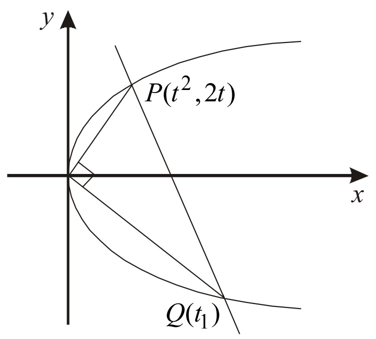
44. **(c)** $t_1=-t-\frac{2}{t}$ also, $tt_1=-4$

$$\Rightarrow t_1=-\frac{4}{t} \Rightarrow -\frac{4}{t}=-2-\frac{2}{t}$$

$$\Rightarrow \frac{2}{t}=2 \Rightarrow t=1$$

$$\Rightarrow t_1=-3 \Rightarrow Q\equiv(9,-6), P\equiv(1,2)$$

$$\Rightarrow PQ=\sqrt{8^2+8^2}=8\sqrt{2}.$$



45. **(b)** Common chord is $S_1-S_2=0 \Rightarrow 3x-y=2$ which is directrix of parabola whose vertex is (0, 0).
The axis of the parabola is $x+3y=0$.

Point of intersection of axis and directix $A\equiv\left(\frac{3}{5},-\frac{1}{5}\right)$.

Let the focus be (x_1,y_1) then

$$\frac{x_1+\frac{3}{5}}{2}=0, \frac{y_1-\frac{1}{5}}{2}=0 \qquad x_1=-\frac{3}{5}, y=\frac{1}{5}.$$

46. **(b)** Let the moving parabola be $(y-k)^2=-4(x-h)$ (Here we are taking –ve sign as the parabolas has to open in opposite directions). Moving parabola touches the given parabola $y^2=4x$, hence $(y-k)^2=-y^2+4h$ should have equal roots

$\Rightarrow 2y^2-2yk+(k^2-4h)=0$ should have equal roots.

$$\Rightarrow 4k^2-8(k^2-4h)=0 \Rightarrow k^2=8h$$

thus the required locus is $y^2=8x$

47. **(c)** Focus of the parabola is (2, 0)
Centre of the given circle is C (6, 0)
Now, circle through S, P and Q will also pass through C.

$\Rightarrow$ Required circle is $(x-2)(x-6)+y^2=0$

$$\Rightarrow x^2+y^2-8x+12=0$$

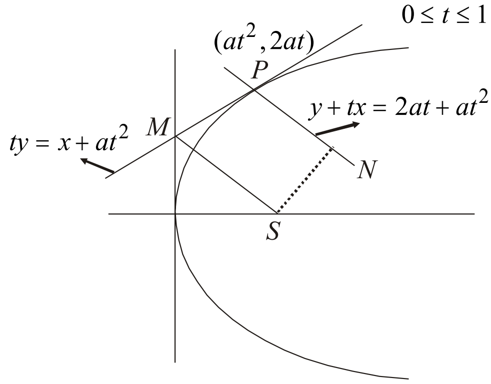
48. **(c)** Let PN be normal at P and SN be $\perp$ from S to PN then
$SM\times PM=SM\times SN$

$$A(t)=\frac{a(1+t^2)}{\sqrt{1+t^2}}\times\frac{a(t+t^3)}{\sqrt{1+t^2}}=a^2(t+t^3)$$

$$\frac{dA(t)}{dt}=a^2(1+3t^2)>0\forall t$$

So, A (t) will be maximum at t=1

$0\le t\le 1$



B Comprehension Type

1. **(b)** The equation can be written as
$(x-4)^2=-16(y-1)$ .....(1)
We know that focal distance of any point (x, y) on $y^2=4ax$ is $|x+a|$.
So, the focal distance of point (x, y) on the parabola (1) is $|y-1-4|=|y-5|$

2. **(c)** The parabola is $(x-4)^2=16(y+1)$
The equation of normal to $y^2=4ax$ is $y=mx-2am-am^3$
So, the equation of normal to $(x-4)^2=16(y+1)$ is $x-4=m(y+1)-8m-4m^3$
It passes through (7, 14), so, $3=15m-8m-4m^3+6$

$$\Rightarrow 4m^3-7m+3=0 \Rightarrow (m-1)(2m-1)(2m+3)=0$$

$$\therefore m=1, \frac{1}{2}, -\frac{3}{2}$$

So, the slope of normals is $\frac{1}{m}=1, 2, -\frac{2}{3}$

(**NOTE :** here slope $\neq m$)

3. (c) The equation of the parabola is $(x+1)^2 = -4(y+3)$ whose axis is $x = -1$.
The equation of a normal to the parabola is
$x+1 = m(y+3)+2m+m^3$
It passes through a point $(-1, k)$ on the axis if ,
$0 = m(k+3)+2m+m^3 \Rightarrow m(m^2+k+5) = 0$
giving $m = 0, \pm\sqrt{-(k+5)}$. Hence for three real, distinct normals $k+5<0 \Rightarrow k<-5$.

4. (b) The equation of a tangent to parabola $x^2 = -4a(y+a)$
will be of the form $x = m(y+a) - \frac{a}{m}$ ....(1)
The given equation is $x = -y\tan\alpha + p\sec\alpha$....(2)
Comparing (1) and (2), we get $m = -\tan\alpha$ and
$am - \frac{a}{m} = p\sec\alpha$
Eliminating m, we get
$-a\tan\alpha + a\cot\alpha = p\sec\alpha \Rightarrow a\cos 2\alpha = p\sin\alpha$

5. (b) Let y_1, y_2, y_3, y_4 be the roots of the equation (3) then
$$y_1+y_2+y_3+y_4 = 0$$
Also, if y_1, y_2, y_3 be the ordinates of the conormal points then
$$y_1+y_2+y_3 = 0 \Rightarrow y_4 = 0.$$
So, one root of the equation (3) is zero $\Rightarrow c = 0$
Hence the circle passes through origin, which is the vertex of the parabola.
NOTE : Origin is not the general answer.

6. (d) Putting $c = 0$, the equation (3) becomes
$y^3 + (16a^2+8ag)y + 32a^2 f = 0,$
which must be same as the equation (2), so,
$4a(2a-h) = 16a^2+8ag$ and $-8a^2k = 32a^2 f$
Solving, we get
$2g = -(h+2a)$ and $2f = -\frac{k}{2}$
So, the equation of the circle is
$x^2+y^2-(h+2a)x - \frac{1}{2}ky = 0$

7. (d) If (α, β) be the coordinates of the circumcentre then
$$\alpha = a + \frac{h}{2} \text{ and } \beta = \frac{k}{4}$$
Now (h, k) lies on parabola $y^2 = 4ax$, so
$$k^2 = 4ah \Rightarrow (4\beta)^2 = 4a(2\alpha - 2a)$$
$\therefore$ Locus of (α, β) is $2y^2 - ax + a^2 = 0$

8. (c) Eqaution of the tangent at point P of the parabola $y^2 = 8x$ is
$$yt = x + 2t^2 \quad ...(1)$$
Equation of the chord of contact of the circle $x^2+y^2 = 4$ is
$$xa + y\beta = 4 \quad ...(2)$$
$\because$ (α, β) lies on (1),
hence $\beta t = \alpha + 2t^2$ ...(3)
$$x\alpha + y\left(\frac{\alpha}{t} + 2t\right) - 4 = 0 \quad \text{[from (2) and (3)]}$$
$$\Rightarrow 2(ty-2) + \alpha\left(x + \frac{y}{t}\right) = 0$$
For point of concurrency
$$x = -\frac{y}{t} \text{ and } y = \frac{2}{t}$$
$\therefore$ locus is $y^2 + 2x = 0$.

9. (d) Required point will lie on the director circle of the given circle as well as on the directrix of parabola
$\Rightarrow x_1^2 + y_1^2 = 8$ and $x_1 + 2 = 0$.
$\Rightarrow 4 + y_1^2 = 8$
$\Rightarrow y_1 = \pm\sqrt{2}$
$\therefore$ Points are $(-2, \pm\sqrt{2})$.

10. (a) Equation of circumcentre of ΔAQB is
$$x^2+y^2-4+\lambda(x\alpha + y\beta - 4) = 0$$
$\because$ it passes through (0, 0) i.e. centre of circle
$\Rightarrow \lambda = -1$
Let circumcentre be (h, k)
$\therefore h = \frac{\alpha}{2}, k = \frac{\beta}{2} \Rightarrow \alpha = 2h, \beta = 2k$
Also, $\beta t = \alpha + 2t^2$
or $\alpha - \beta t + 8 = 0 \quad \because t = 2$
Substituting $\alpha = 2h$ and $\beta = 2k$ we get
$$h - 2k + 4 = 0$$
$\therefore$ locus is $x - 2y + 4 = 0$.

11. **(a)** Let the common tangent to the parabolas is

$$y = mx + \frac{a}{m}$$

Solving it with $x^2 = 4ay$, we get,

$$x^2 = 4a(mx + a/m)$$

$$\Rightarrow \quad x^2 - (4\,\text{am})x - \frac{4a^2}{m} = 0$$

Discriminant = 0 gives, $m = -1$

So, the common tangent would be $x + y + a = 0$

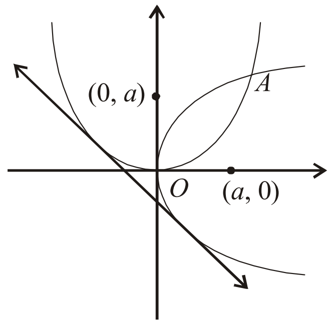
12. **(a)** Solving $y^2 = 4ax$ and $x^2 = 4ay$ simultaneously

we get, $A \equiv (4a, 4a)$

So, the point dividing OA internally in $(1+\sqrt{3}):(7-\sqrt{3})$ is

$$S \equiv \left[\frac{(1+\sqrt{3}).4a}{(1+\sqrt{3})+(7-\sqrt{3})}, \frac{(1+\sqrt{3})4a}{(1+\sqrt{3})+(7-\sqrt{3})}\right]$$

i.e. $S \equiv \left[\frac{(1+\sqrt{3})}{2}a, \frac{1+\sqrt{3}}{2}a\right]$



So, the equation of the parabolla with focus at S and directrix as $x + y + a = 0$ is

$$\left[x - \frac{(1+\sqrt{3})a}{2}\right]^2 + \left[y - \frac{(1+\sqrt{3})a}{2}\right]^2 = \frac{(x+y+a)^2}{2}$$

13. **(a)** Putting $(a, 0)$ and $(0, a)$, the foci of the given parabolas, in P, we see the points satisfy the equations. Hence parabola P pass through the points $(0, a)$ and $(a, 0)$.

C REASONING TYPE

1. **(c)** $y = mx + \frac{a}{m}$

$$10 = 4m - \frac{9/4}{m} \Rightarrow 16m^2 - 40m + 9 = 0$$

$$m_{12} + m_2 = \frac{40}{16} = \frac{5}{2};$$

$$m_1 m_2 = \frac{9}{16} \Rightarrow m_1 = \frac{1}{4}, m_2 = \frac{9}{4}$$

2. **(d)** $t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_1 t_2 + t_1^2 + 2 = 0$

$\because \quad t_1 t_2 = -1 \Rightarrow t_1^2 + 1 = 0$

$\Rightarrow$ No such chord is possible.

3. **(d)** The tangents at P to parabola is

$$y.2 = 2(x+1) \Rightarrow y = x + 1.$$

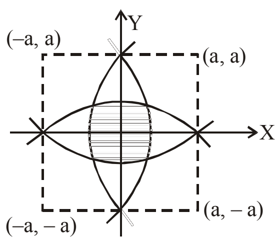
It must pass through the centre of the circle so the line $y = x + 1$ is the desired locus.

D MULTIPLE CORRECT CHOICE TYPE

1. **(a,b,c,d)** If $P = (x, y)$, then

$$\sqrt{x^2+y^2} < |x-a|, \sqrt{x^2+y^2} < |a+x|$$

$$\sqrt{x^2+y^2} < |a-y|, \sqrt{x^2+y^2} < |a+y|$$



$\therefore$ The region is bounded by the curves

$x^2 + y^2 = (a-x)^2, x^2 + y^2 = (a+x)^2$

$x^2 + y^2 = (a-y)^2,\ x^2 + y^2 = (a+y)^2$

or, $y^2 = a^2 - 2ax,\ y^2 = a^2 + 2ax, x^2 = a^2 - 2ay,$

$x^2 = a^2 + 2ay$

2. **(a,c)** $R = \left(1, \frac{1+3\lambda}{1+\lambda}\right)$. It is an interior point of

$$y^2 - 4x = 0 \text{ if } \left(\frac{1+3\lambda}{1+\lambda}\right)^2 - 4 < 0$$

Therefore, $-\frac{3}{5} < \lambda < 1$. But $\lambda > 0 \Rightarrow 0 < \lambda < 1$

3. **(a,b)** The equation of the line can be written in the slope form as

$$y=-\frac{a^{1/3}}{b^{1/3}}x+\frac{a}{\left(-a^{1/3}/b^{1/3}\right)} \text{ i.e. } y=mx+\frac{a}{m}$$

where $m=-\dfrac{a^{1/3}}{b^{1/3}}$.

So it touches the parabola $y^2=4ax$.

The equation of the line can also be written in the form

$$x=-\frac{b^{1/3}}{a^{1/3}}y+\frac{b}{\left(-b^{1/3}/a^{1/3}\right)}$$

i.e. $x=my+\dfrac{b}{m}$ where $m=\dfrac{-b^{1/3}}{a^{1/3}}$.

So it touches the parabola $x^2=4by$ also.

4. **(a,c)** Any normal to the parabola $y^2=x$ is

$$y=mx-\frac{1}{2}m-\frac{1}{4}m^3.$$

It passes through $(c,0)$, hence, $0=mc-\dfrac{1}{2}m-\dfrac{1}{4}m^3$

$$\Rightarrow m=0 \text{ or } \pm\sqrt{c-\frac{1}{2}}$$

For three normals all three values of m should be real and distinct i.e. $c-\dfrac{1}{2}>0\Rightarrow c>\dfrac{1}{2}$.

5. **(a,c)** The focus of the parabola is at $\left(\dfrac{p}{2},0\right)$ and directrix is $x=-\dfrac{p}{2}$.

Centre of the circle is $\left(\dfrac{p}{2},0\right)$ and radius

$$=\frac{p}{2}-\left(-\frac{p}{2}\right)=p$$

Equation of circle is $\left(x-\dfrac{p}{2}\right)^2+(y-0)^2=p^2$

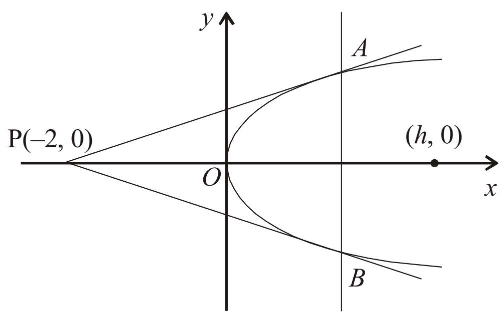
$$\Rightarrow x^2+y^2-px-\frac{3p^2}{4}=0$$

Solving this equation with $y^2=2px$, we obtain

$x=\dfrac{p}{2}$ and $y=\pm p$

$\therefore$ The points of intersection of the circle and the parabola are $\left(\dfrac{p}{2},p\right)$ and $\left(\dfrac{p}{2},-p\right)$.

6. **(a, b)**



Point 'P' clearly lies on the directrix of $y^2=8x$.

Thus slope of PA and PB are 1 and – 1 respectively.

Equation of $PA: y=x+2$,

Equation of $PB: y=-x-2$,

Equation of $AB: x=2$.

Let the centre of the circle be $(h,0)$ and radius be 'r'

$$\Rightarrow \frac{|h+2|}{\sqrt{2}}=\frac{|h-2|}{1}=r$$

$$\Rightarrow h^2+4+4h=2(h^2+4-4h)$$

$$\Rightarrow h^2-12h+4=0$$

$$h=\frac{12\pm8\sqrt{2}}{2}=6\pm4\sqrt{2}$$

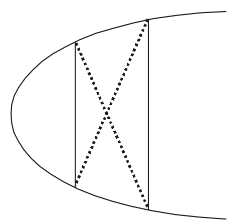
$$\Rightarrow |h-2|=4(\sqrt{2}-1),4(\sqrt{2}+1).$$

7. **(a, b)**

As a circle can itersect a parabola in four points, so quadrilateral may be cyclic. The diagonals of the quadrilateral may be equal as the quadrilateral may be an isoceles trapezium.

A rectangle cannot be inscribed in a parabola.

So (C) is wrong.



8. **(c, d)** $t_2=-t_1-\dfrac{2}{t_1}$ also $\dfrac{2at_1}{at_1^2}\times\dfrac{2at_2}{at_2^2}=-1$

$$\Rightarrow t_1t_2=-4$$

$$\therefore \frac{4}{t_1}=-t_1-\frac{2}{t_1}\Rightarrow t_1^2+2=4 \text{ and } t_1=\pm\sqrt{2}$$

so point can be $(2a, \pm 2\sqrt{2})$.

9. (b, c)

Let $y = mx - 2am - am^3$ be normal

$\Rightarrow \quad \cos\theta = m\sin\theta - 2am - am^3$

$\Rightarrow \quad am^3 + 2am - m\sin\theta + \cos\theta = 0$

$\Rightarrow \quad am^3 + m(2a - \sin\theta) + \cos\theta = 0$

Above equation will have three roots if its derivative will have two roors so

$$3am^2 + (2a - \sin\theta) = 0$$

$$m^2 = \frac{-2a + \sin\theta}{3a}$$

Here, $\frac{\sin\theta - 2a}{3a} > 0$ and $a \neq 0$.

If $a > 0, \sin\theta - 2a > 0 \Rightarrow 2a < \sin\theta$

$\Rightarrow \quad a < \frac{\sin\theta}{2} \Rightarrow a < \frac{1}{2} \Rightarrow a \in \left(0, \frac{1}{2}\right)$

If $a < 0, \sin\theta - 2a < 0 \Rightarrow 2a > \sin\theta \Rightarrow a > \frac{\sin\theta}{2}$

$\Rightarrow \quad a > -\frac{1}{2} \Rightarrow a \in \left(-\frac{1}{2}, 0\right)$

$\Rightarrow \quad a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$.

E MATRIX-MATCH TYPE

1. A - r, s, t; B - s; C - p; D - p, q

(A) Any point on the line is $(t, 1-t)$. The chord with this as mid point is $y(1-t) - 2a(x+t) = (1-t)^2 - 4at$ it passes through $(a, 2a) \Rightarrow (1-t)^2 = 2a(1-a) > 0 \Rightarrow 0 < a < 1$

$\therefore$ L.R. $\in (0,4)$

(B) The points P and Q are (1,0) and (4,0) and the circle is $(x-1)(x-4) + y^2 + \lambda y = 0$

The length of the tangent from (0,0) is $\sqrt{4} = 2$

(C) Solving the equation of the given lines, we get $x + a = 0$ so, the tangents intersect at the directrix

$\Rightarrow m_1 m_2 = -1$

(D) $1 - |h| > 0 \Rightarrow -1 < h < 1$

2. A - r, B - q, C - s, D - t

(A) One parabola is the reflection of the other in the line $y = x$. Normal at t is $xt + y = at^3 + 2at$. Common normal must be perpendicular to $y = x$. So, slope $= -t = -1$

$\therefore$ common normal is $x + y = 3a$

(B) The tangent $y = mx + \frac{a}{m}$ passes through the point P (h,k) $\therefore m^2 h - mk + a = 0$. If its roots are m_1 and m_2

then $m_1 m_2 = 1 \Rightarrow \frac{a}{h} = 1 \Rightarrow$ locus is $x = a$

(C) The tangents are $y = m(x+a) + \frac{a}{m}$ and $y = -\frac{1}{m}(x+2a) - 2am$

Subtracting, we get $\left(m + \frac{1}{m}\right)(x+3a) = 0 \Rightarrow x + 3a = 0$

(D) If $\left(-at_1^2, 2at_1\right)$ *and* $(-at_2^2, 2at_2)$ be exteremities of the chord of contact then $t_1 t_2 = -4$, Also, point of intersection of tangents is $\left(-at_1t_2, -a(t_1+t_2)\right)$. So, the abscissa of P is $x = -at_1t_2 = 4a$

$\therefore$ locus of P is $x = 4a$

3. A - q; B - s; C - r; D - p

(A) Equation of normal is $y = -tx + 2at + at^3$

It intersect the curve again at point $Q(t_1)$ on the parabola such that

$$t_1 = -t - \frac{2}{t}$$

Again slope of OP is $\frac{2}{t} = m_{OP}$

Also, slope is OQ is $\frac{2}{t_1} = m_{OQ}$

Since, $m_{OP} . m_{OQ} = -1 = \frac{4}{tt_1} \Rightarrow tt_1 = -4$

$$t\left(-t - \frac{2}{t}\right) = -4 \Rightarrow t^2 = 2.$$

(B) The points are

$P(1,2), Q(4,4), R(16,8)$

Now ar$(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 4 & 4 & 1 \\ 16 & 8 & 1 \end{vmatrix} = 6$ sq. unit

(C) Equation of normal from any point $P(am^2, -2m)$ is

$y = mx - 2am - am^3$

It passes through $\left(\frac{11}{4}, \frac{1}{4}\right)$

$\Rightarrow \quad 4m^3 + 8m - 11m + 1 = 0$

$\Rightarrow \quad 4m^3 + 3m + 1 = 0$

Now, $f(m) = 4m^3 - 3m$

$\Rightarrow \quad f'(m) = 12m^2 - 3 = 0$

$\Rightarrow \quad m = \pm\frac{1}{2}$

Since $f\left(\frac{1}{2}\right) f\left(-\frac{1}{2}\right) < 0$ so, 3 normals are possible.

(D) Since, normal at $P(t_1)$ if meets the curve again at (t_2), then

$$t_2 = -t_1 - \frac{2}{t_1}$$

Such, that here normal at P (1) meets the curve again at $Q(t)$

$\Rightarrow \quad t = -1 - \frac{1}{2} = -3$

4. A - s; B - r; C - p; D - s

(A) $y^2 = 4x$ ..... (1)

Clearly, if $P(at^2, 2at)$ then by symmetry,

$Q(at^2, -2at)$

Equation of tangent is $ty = x + at^2$

For T, $y = 0$, $x_1 = -at^2$

and equation of normal is

$y = -tx + 2at + at^3$

For R, $y = 0$, $x_2 = (2at + at^2)$ ..... (2)

Here, $a = 1 \Rightarrow x_1 = -t^2$ and $x_2 = (2 + t^2)$

$x_2 = 3 = 2 + t^2 \Rightarrow t^2 = 1 \Rightarrow t = \pm 1$

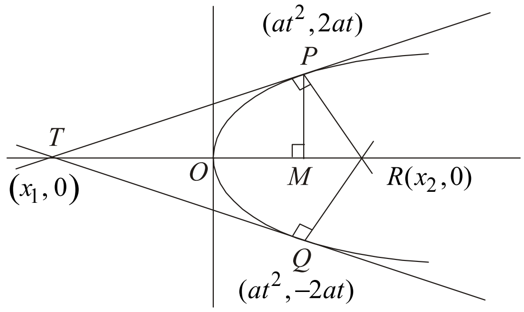
Take $t = 1$, then $x_1 = -1$,

$\therefore \quad PM = 2at = 2 \times 1 \times 1 = 2$

$RT = x_1 + x_2 = (1 + 3) = 4$

$\therefore$ Area of quadrilateral $PTQR$

$= 2 \times \left(\frac{1}{2} \times 4 \times 2\right) = 8$ sq. units



(B) Clearly, RT will be the diameter of circle

$\therefore$ Circumference $= (\pi \times \text{diameter})$

$= \pi \times RT = \frac{\pi \times 4}{4\pi} = 1.$

(C) Since in the part (1), we have found $x_2\left(2a + at^2\right) > 2a$, (if $t \neq 0$)

$\therefore$ For three real normals, $x_2 > 2a = 2 \times 1 = 2$

i.e. $x_2 > 2$.

(D) Equation of PT is $y = x + a \Rightarrow \angle PTM = \frac{\pi}{4}$

$\Rightarrow \sin\frac{\pi}{4} = \frac{PM}{PT} = \frac{2}{PT}$

$\therefore PT = 2\sqrt{2}$

F NUMERIC/INTEGER ANSWER TYPE

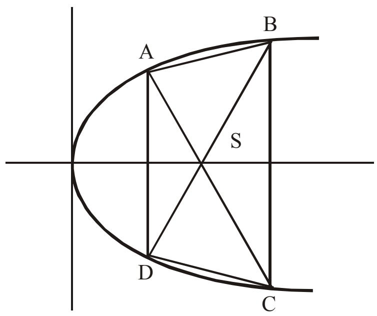
1. Ans : 75

Focus of the parabola $y^2 = 4x$ is (1, 0)

So diagonals are focal chord

$AS = 1 + t^2 = c$ (say)

$\because \frac{1}{c} + \frac{1}{\frac{25}{4} - c} = 1 \qquad \left[\because \frac{1}{AS} + \frac{1}{CS} = \frac{1}{a}\right]$



$$\frac{25}{4} = \frac{25}{4}c - c^2$$

$\Rightarrow \; 4c^2 - 25c + 25 = 0 \;\Rightarrow\; c = \frac{5}{4}, 5$

For $c = \frac{5}{4}$, $1 + t^2 = \frac{5}{4} \;\Rightarrow\; t^2 = \frac{1}{4} \;\Rightarrow\; t = \pm\frac{1}{2}$

For $c = 5$, $1 + t^2 = 5 \Rightarrow t = \pm 2$

$A \equiv \left(\frac{1}{4}, 1\right)$, $B \equiv (4, 4)$, $C \equiv (4, -4)$ and $D \equiv \left(\frac{1}{4}, -1\right)$

$AD = 2$ and $BC = 8$, distance between AD and $BC = \frac{15}{4}$

$\therefore$ Area of trapezium $ABCD = \frac{1}{2}(2 + 8) \times \frac{15}{4} = \frac{75}{4}$ Sq. units.

2. Ans : 1

Let the equation of the parabola be $y^2 = 4ax$. If the feet of the normals passing through a point be $(am_1^2, -2am_1)$, $(am_2^2, -2am_2)$ and $(am_3^2, -2am_3)$ equation of normal at $(am_1^2, -2am_1)$ is $y = m_1x - 2am_1 - a m_1^3$.

Solving with the line $x = 2a$, we get the point of intersection as $(2a, -m_1^3)$. Similarly solving with the other normals the ordinates of the points of intersection. with $x = 2a$ are $-am_2^3$, and $-am_3^3$.

These ordinates are in A.P.

So $-am_1^3 - am_3^3 = -2am_2^3$ or $m_1^3 + m_3^3 = 2m_2^3$ .... (i)

But $m_1 + m_3 = -m_2$ .... (ii)

By (i)

$2m_2^3 = (m_1 + m_3)(m_1^2 + m_3^2 - m_1 m_3) - 2m_2^2$

$= (m_1^2 + m_3^2 - m_1 m_3)$ (as $m_1 + m_3 = -m_2$).... (iii)

and by (ii) $m_2^2 = m_1^2 + m_3^2 + 2m_1 m_3$ .... (iv)

Subtracting (iii) from (iv), $3m_2^2 = 3m_1 m_3$

or $m_2^2 = m_1 m_3$.

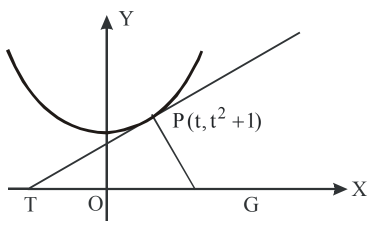
3. Ans : 930

Any point on parabola $y = x^2 + 1$ is $P(t, t^2 + 1)$

Equation of tangent at P,

$y - t^2 - 1 = 2t(x - t)$ Put $y = 0$, then $x = t - \frac{t^2+1}{2t}$

Hence $T\left(\frac{t^2-1}{2t}, 0\right)$



Equation of normal at P, $y - t^2 - 1 = -\frac{1}{2t}(x - t)$

Put $y = 0$, then $x = t + 2t(t^2 + 1)$.

Hence $G(2t(t^2 + 1) + t, 0)$

$\therefore \; TG = 2t(t^2 + 1) + t - t + \frac{t^2+1}{2t} = 2t(t^2 + 1) + \frac{t^2+1}{2t}$

$\therefore$ Area $A = \frac{1}{2}(t^2 + 1)\left[2t(t^2 + 1) + \frac{t^2+1}{2t}\right]$

$= \frac{(t^2+1)^2(4t^2+1)}{4t}$

$\therefore \; A' = \frac{(t^2+1)(20t^4 + 7t^2 - 1)}{4t^2} > 0$

$[\because 1 \le x \le 3 \Rightarrow 1 \le t \le 3]$

$\therefore A$ is increasing in $1 \le t \le 3$

$\therefore$ Maximum area $= f(3) = \frac{925}{3}$ and minimum area $= f(1) = 5$.

4. Ans : 4

Let equation of line passing through $P(1, t)$ be

$\frac{x-1}{\cos\theta} = \frac{y-t}{\sin\theta} = r$

$\Rightarrow \; x = r\cos\theta + 1$ and $y = r\sin\theta + t$.

Line meets the parabola at A and B

$\Rightarrow \; (r\sin\theta + t)^2 = 4(r\cos\theta + 1)$

$\Rightarrow \; r^2\sin^2\theta + 2r(t\sin\theta - 2\cos\theta) + t^2 - 4 = 0$

$\therefore \; PA.PB = \left|\frac{t^2-4}{\sin^2\theta}\right| = 3|t|$

$\Rightarrow \; \frac{|t^2-4|}{3|t|} = \sin^2\theta \le 1$

$\Rightarrow \; t^2 - 3|t| - 4 \le 0$

$\Rightarrow \; (|t| + 1)(|t| - 4) \le 0$

$\Rightarrow \; |t| \le 4$

Hence the maximum value of t is 4.

5. Ans : 3

Let the parameters of the points D, E, F on the parabola be t_1, t_2, t_3 respectively. Tangents at E and F meet at A, so the coordinates of A are $(at_2t_3, a(t_2 + t_3))$

Now AD passes through focus S, so

$\frac{a(t_2+t_3) - 0}{at_2t_3 - a} = \frac{2at_1 - 0}{at_1^2 - a}$

$\Rightarrow \; 2t_1t_2t_3 - 2t_1 = (t_2 + t_3)t_1^2 - (t_2 + t_3)$

$\Rightarrow \; t_1^3 - (t_1 + t_2 + t_3)t_1^2 - 3t_1 + 2t_1t_2t_3$

$+ (t_1 + t_2 + t_3) = 0$

$\Rightarrow t_1, t_2, t_3$ are roots of the equation

$$t^3 - (t_1 + t_2 + t_3)t^2 - 3t + 2t_1t_2t_3 + (t_1 + t_2 + t_3) = 0 \quad ...(1)$$

$\Rightarrow \ t_1t_2 + t_2t_3 + t_3t_1 = -3$ and

$$t_1t_2t_3 = -2t_1t_2t_3 - (t_1 + t_2 + t_3)$$

$$\Rightarrow t_1t_2t_3 = -\frac{1}{3}(t_1 + t_2 + t_3)$$

So, the equation (1) becomes

$$t^3 + 3t_1t_2t_3\ t^2 - 3t - t_1t_2t_3 = 0$$

$$\Rightarrow -t_1t_2t_3 = \frac{3t - t^3}{1 - 3t^2}$$

$\because$ The slopes of the sides of triangle are

$$\frac{1}{t_1} = \tan\theta_1, \frac{1}{t_2} = \tan\theta_2, \frac{1}{t_3} = \tan\theta_3,$$

so $\cot\theta_1, \cot\theta_2, \cot\theta_3$ are roots of the equation

$$\frac{3\cot\theta - \cot^3\theta}{1 - 3\cot^2\theta} = \cot\alpha, \text{ where } \cot\alpha = -t_1t_2t_3$$

$$\therefore \cot 3\theta = \cot\alpha \Rightarrow 3\theta = n\pi + \alpha \Rightarrow \theta = \frac{n\pi + \alpha}{3}$$

$$\therefore \theta_1 = \frac{\alpha}{3}, \theta_2 = \frac{\pi + \alpha}{3} \text{ and } \theta_3 = \frac{2\pi + \alpha}{3}$$

$$\therefore \theta_2 - \theta_1 = \theta_3 - \theta_2 = \frac{\pi}{3}$$

6. Ans : 2

Axis of the parabola $y^2 = 4x$ is $y = 0$, and the point which is at shortest distance from the circle

$x^2 + y^2 + 2x - 2\sqrt{2}y + 2 = 0$ is $(-1, 0)$.

Since (– 1, 0) lies on the directrix of the parabola hence tangents to the parabola are $y = x + 1$ and $y = -x - 1$ and these lines are also the tangents to the given circle. Clearly the ΔPAB is possible in two ways one shown by using PAB and other by using PA_1B_1.

Length of $PA = PA_1 = 1$ also $B(1, 2)$ and B_1 (1, – 2) as B and B_1 lies on the latus rectum of the parabola.

Also, ΔPAB and ΔPA_1B_1 are right angle triangles.

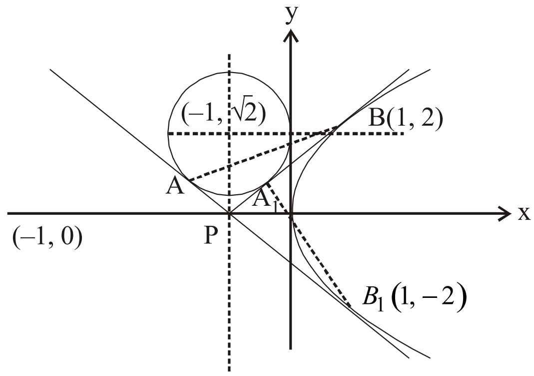
$$\text{So, area } \Delta PAB = \frac{1}{2} \times PA \times PB$$

$$= \frac{1}{2} \times 1 \times 2\sqrt{2} = \sqrt{2} \text{ sq}$$

$$\text{Area } (\Delta PA_1B_1) = \frac{1}{2} \times PA_1 \times PB_1$$

$$= \frac{1}{2} \times 1 \times 2\sqrt{2} = \sqrt{2} \text{ sq.}$$

Hence area of $\Delta PAB = \sqrt{2}$ square units.



7. Ans : 5

Let coordinates be $P(t_1^2, 2t_1)$ and $Q(t_2^2, 2t_2)$.

Coordinates of points of intersection R is $(t_1t_2, (t_1 + t_2))$

R lies on $y^2 = -x$, then $(t_1 + t_2)^2 = -t_1t_2$ ...(i)

Coordinates of the point of intersection of normal at P and Q are $S(2 + (t_1^2 + t_2^2 + t_1t_2), -t_1t_2(t_1 + t_2))$

From equation (i), coordinates of S are

$$(2 - 2t_1t_2, -t_1t_2(t_1 + t_2))$$

P, Q, R, S are concyclic points. Hence centre of circle is the point of intersection of PQ and RS.

Co-ordinates of mid-point of PQ $\left(\frac{t_1^2 + t_2^2}{2}, \frac{2(t_1 + t_2)}{2}\right)$

and the co-ordinates of mid-point of RS

$$\left(\frac{t_1t_2 + 2 - 2t_1t_2}{2}, \frac{(t_1 + t_2) - t_1t_2(t_1 + t_2)}{2}\right)$$

$$\text{So, } \frac{(t_1^2 + t_2^2)}{2} = \frac{2 - t_1t_2}{2} \text{ and } \frac{2(t_1 + t_2)}{2} = \frac{(t_1 + t_2)[1 - t_1t_2]}{2}$$

$\Rightarrow \ -3t_1t_2 = 2 - t_1t_2$ and $(t_1 + t_2)[1 - t_1t_2 - 2] = 0$

$\Rightarrow \ t_1t_2 = -1$ ...(ii)

$\Rightarrow$ From equation (i), $t_1 + t_2 = \pm 1$ ...(iii)

Hence coordinates of centre are $\left(\frac{3}{2}, \pm 1\right)$ and

coordinates of R are $(-1, \pm 1)$.

The radius of required circle is $\sqrt{\left(\frac{3}{2} + 1\right)^2 + (\pm 1 \mp 1)^2} = \frac{5}{2}$.