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Electric Charges and Fields

TOPIC 1 Electric Charges and Coulomb's Law

1. Three charges + Q, q, + Q are placed respectively, at distance, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at $x = 0$, is zero, then value of q is: **[9 Jan. 2019 I]**

(a) – Q/4 (b) + Q/2 (c) + Q/4 (d) – Q/2

2. Charge is distributed within a sphere of radius R with a volume charge density $p(r) = \frac{A}{r^2} e^{-2r/a}$ where A and a are constants. If Q is the total charge of this charge distribution, the radius R is: **[9 Jan. 2019, II]**

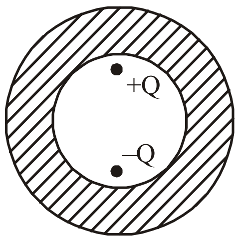
(a) $a \log\left(1 - \frac{Q}{2\pi aA}\right)$ (b) $\frac{a}{2} \log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$

(c) $a \log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$ (d) $\frac{a}{2} \log\left(1 - \frac{Q}{2\pi aA}\right)$

3. Two identical conducting spheres A and B, carry equal charge. They are separated by a distance much larger than their diameter, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to **[Online April 16, 2018]**

(a) $\frac{3F}{4}$ (b) $\frac{F}{2}$ (c) F (d) $\frac{3F}{8}$

4. Shown in the figure are two point charges +Q and –Q inside the cavity of a spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre of the shell. If σ_1 is the surface charge on the inner surface and Q_1 net charge on it and σ_2 the surface charge on the outer surface and Q_2 net charge on it then : **[Online April 10, 2015]**



(a) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$ (b) $\sigma_1 \neq 0, Q_1 = 0$; $\sigma_2 \neq 0, Q_2 = 0$

(c) $\sigma_1 = 0, Q_1 = 0$; $\sigma_2 = 0, Q_2 = 0$ (d) $\sigma_1 \neq 0, Q_1 \neq 0$; $\sigma_2 \neq 0, Q_2 \neq 0$

5. Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y<<a) along the y-axis, the net force acting on the particle is proportional to **[2013]**

(a) y (b) –y (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

6. Two balls of same mass and carrying equal charge are hung from a fixed support of length l. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to : **[Online April 9, 2013]**

(a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

7. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **[2011]**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$ (c) $v \propto x$ (d) $v \propto x^{-1/2}$

8. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[2009]**

(a) –1 (b) 1 (c) $-\frac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

9. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio **[2007]**

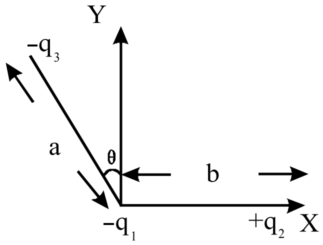
$\dfrac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

(a) g_M / g_E (b) 1 (c) 0 (d) g_E / g_M

10. Two spherical conductors *B* and *C* having equal radii and carrying equal charges on them repel each other with a force *F* when kept apart at some distance. A third spherical conductor having same radius as that *B* but **uncharged** is brought in contact with *B*, then brought in contact with *C* and finally removed away from both. The new force of repulsion between *B* and *C* is **[2004]**

(a) *F*/8 (b) 3 *F*/4 (c) *F*/4 (d) 3 *F*/8

11. Three charges $-q_1$, $+q_2$ and $-q_3$ are place as shown in the figure. The *x* - component of the force on $-q_1$ is proportional to **[2003]**



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

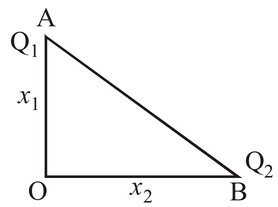
(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

12. If a charge *q* is placed at the centre of the line joining two equal charges *Q* such that the system is in equilibrium then the value of *q* is **[2002]**

(a) *Q*/2 (b) –*Q*/2 (c) *Q*/4 (d) –*Q*/4

TOPIC 2 Electric Field and Electric Field Lines

13. Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then Q_1/Q_2 is proportional to : **[Sep. 06, 2020 (I)]**



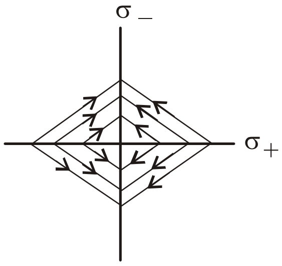
(a) $\frac{x_1^3}{x_2^3}$ (b) $\frac{x_2}{x_1}$ (c) $\frac{x_1}{x_2}$ (d) $\frac{x_2^2}{x_1^2}$

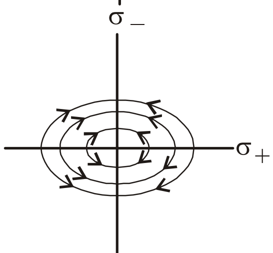
14. Consider the force F on a charge '*q*' due to a uniformly charged spherical shell of radius *R* carrying charge *Q* distributed uniformly over it. Which one of the following statements is true for F, if '*q*' is placed at distance *r* from the centre of the shell? **[Sep. 06, 2020 (II)]**

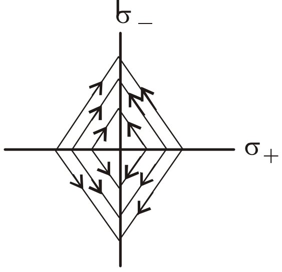
(a) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r<R$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} > F > 0$ for $r<R$

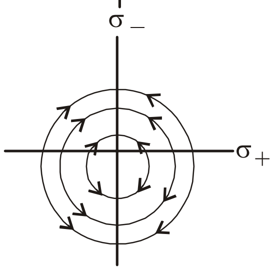
(c) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r>R$ (d) $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for all *r*

15. Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system ? **[Sep. 04, 2020 (I)]**

(a) 

(b) 

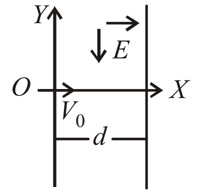
(c) 

(d) 

16. A particle of charge *q* and mass *m* is subjected to an electric field $E = E_0(1 - ax^2)$ in the *x*-direction, where *a* and E_0 are constants. Initially the particle was at rest at $x = 0$. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is : **[Sep. 04, 2020 (II)]**

(a) *a* (b) $\sqrt{\frac{2}{a}}$ (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{1}{a}}$

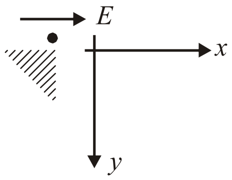
17. A charged particle (mass *m* and charge *q*) moves along *X* axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto $x = d$. Equation of path of electron in the region $x > d$ is : **[Sep. 02, 2020 (I)]**

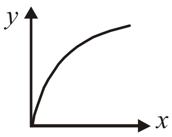


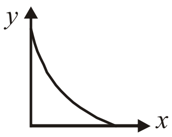
(a) $y=\frac{qEd}{mV_0^2}(x-d)$ (b) $y=\frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$

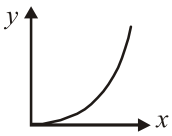
(c) $y=\frac{qEd}{mV_0^2}x$ (d) $y=\frac{qEd^2}{mV_0^2}x$

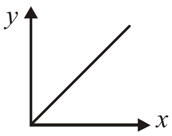
18. A small point mass carrying some positive charge on it, is released from the edge of a table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass ? (Curves are drawn schematically and are not to scale). **[Sep. 02, 2020 (II)]**



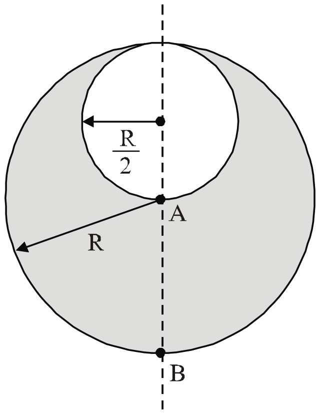
(a)


(b)


(c)


(d)


19. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is: **[9 Jan. 2020, I]**



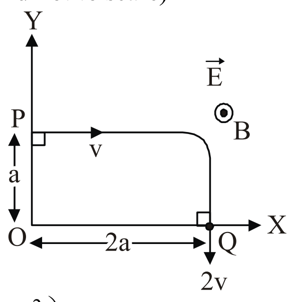
(a) $\frac{21}{34}$ (b) $\frac{18}{34}$ (c) $\frac{17}{54}$ (d) $\frac{18}{54}$

20. An electric dipole of moment $\vec{p}=(\hat{i}-3\hat{j}+2\hat{k})\times10^{-29}$ C.m is at the origin (0, 0, 0). The electric field due to this dipole at $\vec{r}=+\hat{i}+3\hat{j}+5\hat{k}$ (note that $\vec{r}.\vec{p}=0$) is parallel to: **[9 Jan. 2020, I]**

(a) $(+\hat{i}-3\hat{j}-2\hat{k})$ (b) $(-\hat{i}+3\hat{j}-2\hat{k})$

(c) $(+\hat{i}+3\hat{j}-2\hat{k})$ (d) $(-\hat{i}-3\hat{j}+2\hat{k})$

21. A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field $E\hat{i}$ and a uniform magnetic field $B\vec{k}$ follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, $v\vec{i}$ and $-2v\vec{j}$. Then which of the following statements (A, B, C, D) are the correct? (Trajectory shown is schematic and not to scale) **[9 Jan. 2020, I]**



(A) $E=\frac{3}{4}\left(\frac{mv^2}{qa}\right)$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$

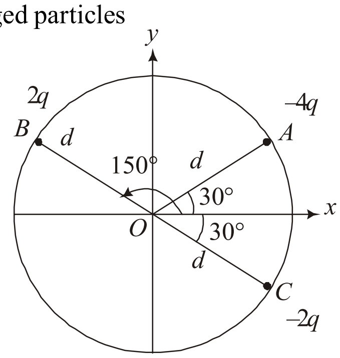
(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2 ma$v$.

(a) (A), (C), (D) (b) (B), (C), (D)

(c) (A), (B), (C) (d) (A), (B), (C), (D)

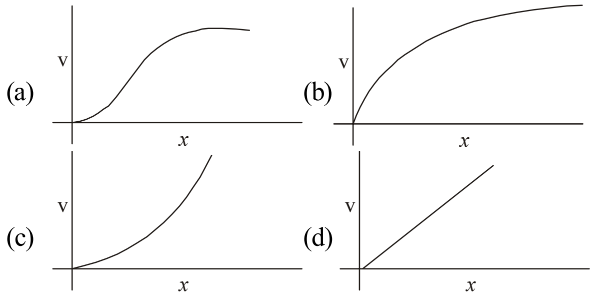
22. Three charged particles



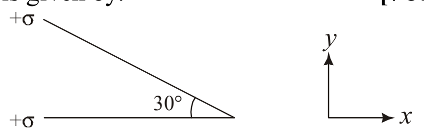
A, B and C with charges $-4q$, $2q$ and $-2q$ are present on the circumference of a circle of radius d. The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is: **[8 Jan. 2020, I]**

(a) $\frac{\sqrt{3}q}{\pi\epsilon_0 d^2}$ (b) $\frac{2\sqrt{3}q}{\pi\epsilon_0 d^2}$ (c) $\frac{\sqrt{3}q}{4\pi\epsilon_0 d^2}$ (d) $\frac{3\sqrt{3}q}{4\pi\epsilon_0 d^2}$

23. A particle of mass m and charge q is released from rest in a uniform electric field. If there is no other force on the particle, the dependence of its speed v on the distance x travelled by it is correctly given by (graphs are schematic and not drawn to scale) **[8 Jan. 2020, II]**



24. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by: **[7 Jan. 2020, I]**



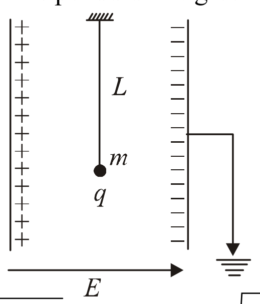
(a) $\frac{\sigma}{2\epsilon_0}\left[(1+\sqrt{3})\hat{y}-\frac{\hat{x}}{2}\right]$ (b) $\frac{\sigma}{\epsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y}+\frac{\hat{x}}{2}\right]$

(c) $\frac{\sigma}{2\epsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y}+\frac{\hat{x}}{2}\right]$ (d) $\frac{\sigma}{2\epsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y}-\frac{\hat{x}}{2}\right]$

25. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time: **[7 Jan 2020, II]**

(a) $\frac{2mv_0}{qE_0}$ (b) $\frac{3mv_0}{qE_0}$ (c) $\frac{\sqrt{3}mv_0}{qE_0}$ (d) $\frac{\sqrt{2}mv_0}{qE_0}$

26. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by : **[10 April 2019, II]**



(a) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$ (b) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$

(c) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

27. Four point charges $-q$, $+q$, $+q$ and $-q$ are placed on y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the x-axis at $x = $ D, with D>> d, will behave as: **[9 April 2019, II]**

(a) $E \propto \frac{1}{D^3}$ (b) $E \propto \frac{1}{D}$ (c) $E \propto \frac{1}{D^4}$ (d) $E \propto \frac{1}{D^2}$

28. The bob of a simple pendulum has mass 2 g and a charge of 5.0 ¼C. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is : **[8 April 2019 I]** (take $g = 10$ m/s^2)

(a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$

(c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

29. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is: **[9 Jan. 2019 I]**

(a) $\frac{R}{\sqrt{5}}$ (b) $\frac{R}{\sqrt{2}}$ (c) R (d) $R\sqrt{2}$

30. Two point charges q_1 ($\sqrt{10}$ μC) and q_2 (– 25 μC) are placed on the x-axis at $x = 1$ m and $x = 4$ m respectively. The electric field (in V/m) at a point $y = 3$ m on y-axis is, **[9 Jan 2019, II]**

$$\left[\text{take } \frac{1}{4\pi\epsilon_0} = 9\times10^9 \text{ Nm}^2\text{C}^{-2}\right]$$

(a) $(63\hat{i} - 27\hat{j}) \times 10^2$ (b) $(-63\hat{i} + 27\hat{j}) \times 10^2$

(c) $(81\hat{i} - 81\hat{j}) \times 10^2$ (d) $(-81\hat{i} + 81\hat{j}) \times 10^2$

31. A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct? **[Online April 15, 2018]**

(a) The total energy of the system is $\frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$

(b) The new equilibrium position is at a distance: $\frac{2qE}{k}$ from $x = 0$

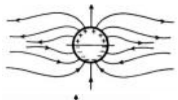
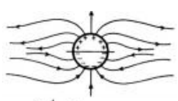
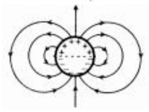
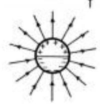
(c) The new equilibrium position is at a distance: $\frac{qE}{2k}$ from $x = 0$

(d) The total energy of the system is $\frac{1}{2}m\omega^2A^2 - \frac{1}{2}\frac{q^2E^2}{k}$

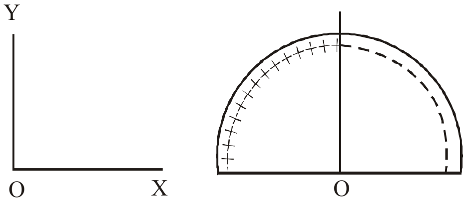
32. A solid ball of radius R has a charge density ρ given by $\rho = \rho_0\left(1-\frac{r}{R}\right)$ for $0 \le r \le R$. The electric field outside the ball is: **[Online April 15, 2018]**

(a) $\frac{\rho_0 R^3}{\varepsilon_0 r^2}$ (b) $\frac{4\rho_0 R^3}{3\varepsilon_0 r^2}$ (c) $\frac{3\rho_0 R^3}{4\varepsilon_0 r^2}$ (d) $\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$

33. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge - σ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) **[2015]**

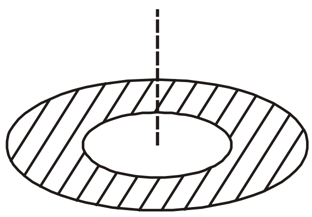
(a) 
(b) 
(c) 
(d) 

34. A wire of length L (=20 cm), is bent into a semicircular arc. If the two equal halves of the arc were each to be uniformly charged with charges $\pm$ Q, [$|Q| = 10^3 \varepsilon_0$ Coulomb where ε_0 is the permittivity (in SI units) of free space] the net electric field at the centre O of the semicircular arc would be : **[Online April 11, 2015]**



(a) $(50 \times 10^3 \text{N/C})\,\hat{j}$ (b) $(50 \times 10^3 \text{N/C})\,\hat{i}$
(c) $(25 \times 10^3 \text{N/C})\,\hat{j}$ (d) $(25 \times 10^3 \text{N/C})\,\hat{i}$

35. A thin disc of radius $b = 2a$ has a concentric hole of radius 'a' in it (see figure). It carries uniform surface charge 'σ' on it. If the electric field on its axis at height 'h' ($h << a$) from its centre is given as 'Ch' then value of 'C' is : **[Online April 10, 2015]**



(a) $\dfrac{\sigma}{4a\epsilon_0}$
(b) $\dfrac{\sigma}{8a\epsilon_0}$
(c) $\dfrac{\sigma}{a\epsilon_0}$
(d) $\dfrac{\sigma}{2a\epsilon_0}$

36. A spherically symmetric charge distribution is characterised by a charge density having the following variations:

$\rho(r) = \rho_0\left(1 - \dfrac{r}{R}\right)$ for $r < R$

$\rho(r) = 0$ for $r \geq R$

Where r is the distance from the centre of the charge distribution ρ_o is a constant. The electric field at an internal point ($r < R$) is: **[Online April 12, 2014]**

(a) $\dfrac{\rho_0}{4\varepsilon_0}\left(\dfrac{r}{3} - \dfrac{r^2}{4R}\right)$ (b) $\dfrac{\rho_0}{\varepsilon_0}\left(\dfrac{r}{3} - \dfrac{r^2}{4R}\right)$

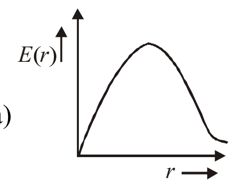
(c) $\dfrac{\rho_0}{3\varepsilon_0}\left(\dfrac{r}{3} - \dfrac{r^2}{4R}\right)$ (d) $\dfrac{\rho_0}{12\varepsilon_0}\left(\dfrac{r}{3} - \dfrac{r^2}{4R}\right)$

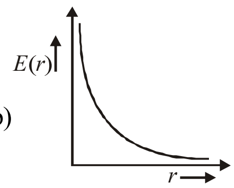
37. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about 150 N/C, directed inward towards the center of the Earth. This gives the total net surface charge carried by the Earth to be: **[Online April 9, 2014]**
[Given $\varepsilon_o = 8.85 \times 10^{-12}$ C^2/N-m^2, $R_E = 6.37 \times 10^6$ m]
(a) +670 kC (b) –670 kC
(c) –680 kC (d) +680 kC

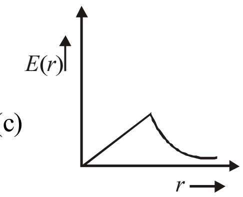
38. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\dfrac{\sigma}{2\in_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc : **[Online April 25, 2013]**
(a) reduces by 70.7% (b) reduces by 29.3%
(c) reduces by 9.7% (d) reduces by 14.6%

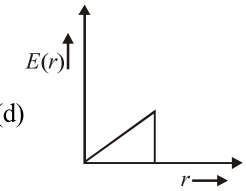
39. A liquid drop having 6 excess electrons is kept stationary under a uniform electric field of 25.5 kVm^{-1}. The density of liquid is 1.26×10^3 kg m^{-3}. The radius of the drop is (neglect buoyancy). **[Online April 23, 2013]**
(a) 4.3×10^{-7}m (b) 7.8×10^{-7}m
(c) 0.078×10^{-7}m (d) 3.4×10^{-7}m

40. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre, The graph which would correspond to the above will be: **[2012]**

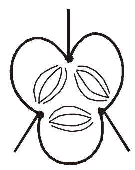
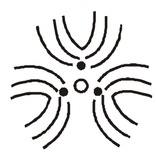
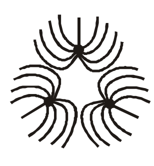
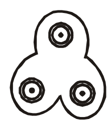
(a) 

(b) 

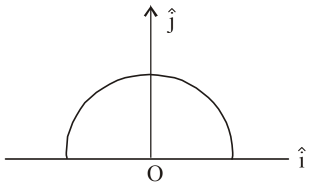
(c) 

(d) 

41. Three positive charges of equal value q are placed at vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[Online May 26, 2012]**

(a) 
(b) 
(c) 
(d) 

42. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is **[2010]**



(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

43. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r (r < R)$ from the origin is given by **[2010]**

(a) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(c) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

44. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2 : The net work done by a conservative force on an object moving along a closed loop is zero. **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is false, Statement-2 is true.

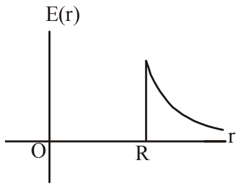
(d) Statement-1 is true, Statement-2 is false.

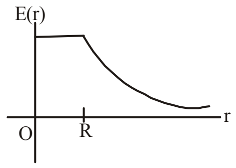
45. Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'P' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is : **[2009]**

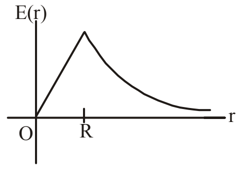
(a) $\frac{Q}{4\pi \in_0 r_1^2}$ (b) $\frac{Q r_1^2}{4\pi \in_0 R^4}$

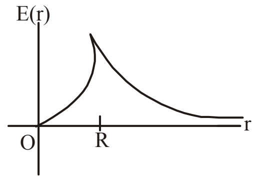
(c) $\frac{Q r_1^2}{3\pi \in_0 R^4}$ (d) 0

46. A thin spherical shell of radus R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[2008]**

(a)


(b)


(c)


(d)


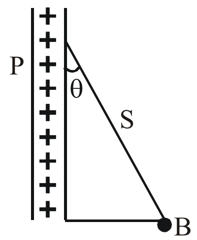
47. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **[2006]**

(a) 4 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

48. Two point charges + $8q$ and – $2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is **[2005]**

(a) $\frac{L}{4}$ (b) $2L$ (c) $4L$ (d) $8L$

49. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to **[2005]**



(a) cot θ (b) cos θ (c) tan θ (d) sin θ

50. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is **[2004]**

(a) $-\frac{Q}{2}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{4}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

51. A charged oil drop is suspended in a uniform field of 3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass of the charge = 9.9×10^{-15} kg and $g = 10$ m/s^2) **[2004]**

(a) 1.6×10^{-18}C (b) 3.2×10^{-18}C

(c) 3.3×10^{-18}C (d) 4.8×10^{-18}C

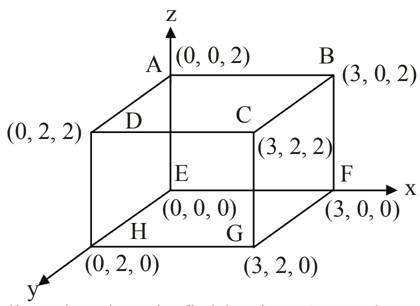
TOPIC 3 Electric Dipole, Electric Flux and Gauss's Law

52. Two identical electric point dipoles have dipole moments $\vec{P_1} = P\hat{i}$ and $\vec{P_2} = -P\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is : **[Sep. 06, 2020 (II)]**

(a) $\frac{P}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (b) $\frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$

(c) $\frac{P}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$ (d) $\frac{P}{a}\sqrt{\frac{2}{2\pi\varepsilon_0 ma}}$

53. An electric field $\vec{E} = 4x\hat{i} - (y^2+1)\hat{j}$ N/C passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as ϕ_1 and ϕ_{11} respectively. The difference between $(\phi_1 - \phi_{11})$ is (in Nm^2/C) _______. **[9 Jan 2020, II]**



54. In finding the electric field using Gauss law the formula $|\vec{E}| = \frac{q_{enc}}{\epsilon_0 |A|}$ is applicable. In the formula ϵ_0 is permittivity of free space, A is the area of Gaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation? **[8 Jan 2020, I]**

(a) Only when the Gaussian surface is an equipotential surface.

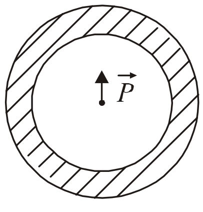
Only when the Gaussian surface is an

(b) equipotential surface and $|\vec{E}|$ is constant on the surface.

(c) Only when $|\vec{E}|$ = constant on the surface.

(d) For any choice of Gaussian surface.

55. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case : **[12 April 2019, I]**



(a) surface change density on the inner surface is uniform and equal to $\frac{Q/2}{4\pi a^2}$

(b) electric field outside the shell is the same as that of a point charge at the centre of the shell.

(c) surface charge density on the outer surface depends on $|\vec{P}|$

(d) surface charge density on the inner surface of the shell is zero everywhere.-

56. Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of – Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then. **[12 April 2019, II]**

(a) $a = 8^{-1/4}R$ (b) $a = \frac{3R}{2^{1/4}}$

(c) $a = 2^{-1/4}R$ (d) $a = R/\sqrt{3}$

57. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is : **[8 April 2019, II]**

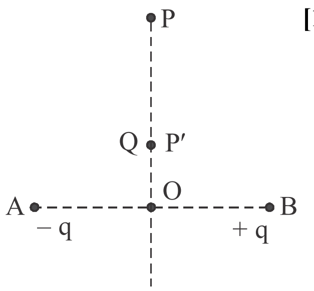
(a) $\sqrt{\frac{qE}{md}}$ (b) $\sqrt{\frac{2qE}{md}}$ (c) $2\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

58. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole? **[11 Jan 2019, II]**

(a) -20×10^{-18} J (b) -7×10^{-27} J

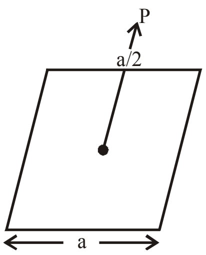
(c) -10×10^{-29} J (d) -9×10^{-20} J

59. Charges – q and + q located at A and B, respectively, constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P where OP = y and $y >> 2a$. The charge Q experiences an electrostatic force F. If Q is now moved along the equatorial line to P′ such that OP′ $= \left(\frac{y}{3}\right)$, the force on Q will be close to: $\left(\frac{y}{3} >> 2a\right)$ **[10 Jan 2019, II]**



(a) 3 F (b) $\frac{F}{3}$ (c) 9 F (d) 27 F

60. A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is: **[Online April 15, 2018]**



(a) $\frac{Q}{3\varepsilon_0}$

(b) $\frac{Q}{6\varepsilon_0}$

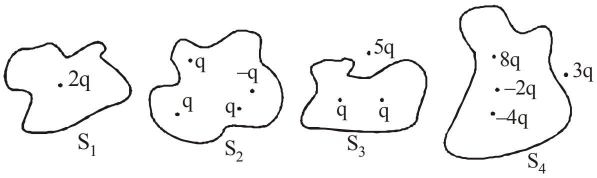
(c) $\frac{Q}{2\varepsilon_0}$

(d) $\frac{Q}{\varepsilon_0}$

61. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E_1} = E\hat{i}$, it experiences a torque $\vec{T_1} = \tau\hat{i}$. When subjected to another electric field $\vec{E_2} = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T_2} = -\vec{T_1}$. The angle θ is: **[2017]**

(a) 60° (b) 90° (c) 30° (d) 45°

62. Four closed surfaces and corresponding charge distributions are shown below. **[Online April 9, 2017]**

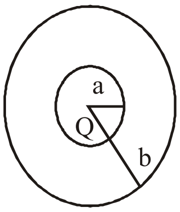


Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3, and Φ_4. Then :

(a) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$ (b) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$

(c) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ (d) $\Phi_1 > \Phi_3 ; \Phi_2 < \Phi_4$

63. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[2016]**



(a) $\frac{2Q}{\pi(a^2 - b^2)}$ (b) $\frac{2Q}{\pi a^2}$

(c) $\frac{Q}{2\pi a^2}$ (d) $\frac{Q}{2\pi(b^2 - a^2)}$

64. The electric field in a region of space is given by, $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$ where $E_o = 100$ N/C. The flux of the field through a circular surface of radius 0.02 m parallel to the Y-Z plane is nearly: **[Online April 19, 2014]**

(a) 0.125 Nm²/C (b) 0.02 Nm²/C

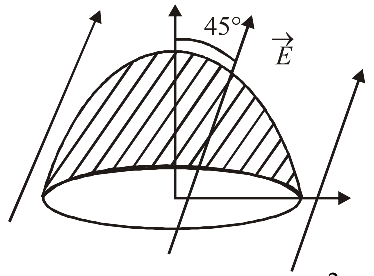
(c) 0.005 Nm²/C (d) 3.14 Nm²/C

65. Two point dipoles of dipole moment $\vec{p_1}$ and $\vec{p_2}$ are at a distance x from each other and $\vec{p_1} \parallel \vec{p_2}$. The force between the dipoles is : **[Online April 9, 2013]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{4p_1p_2}{x^4}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{x^3}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{x^4}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{8p_1p_2}{x^4}$

66. The flat base of a hemisphere of radius a with no charge inside it lies in a horizontal plane. A uniform electric field $\vec{E}$ is applied at an angle $\frac{\pi}{4}$ with the vertical direction. The electric flux through the curved surface of the hemisphere is **[Online May 19, 2012]**



(a) $\pi a^2 E$ (b) $\frac{\pi a^2 E}{\sqrt{2}}$

(c) $\frac{\pi a^2 E}{2\sqrt{2}}$ (d) $\frac{(\pi + 2)\pi a^2 E}{(2\sqrt{2})^2}$

67. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[2006]**

(a) a translational force only in the direction of the field

(b) a translational force only in a direction normal to the direction of the field

(c) a torque as well as a translational force

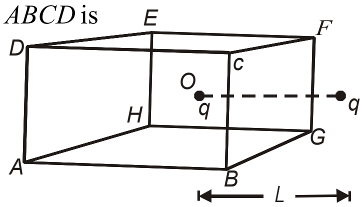
(d) a torque only

68. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[2003]**

(a) $(\phi_2 - \phi_1)\varepsilon_o$ (b) $(\phi_1 - \phi_2)/\varepsilon_o$

(c) $(\phi_2 - \phi_1)/\varepsilon_o$ (d) $(\phi_1 - \phi_2)\varepsilon_o$

69. A charged particle q is placed at the centre O of cube of length L ($A\ B\ C\ D\ E\ F\ G\ H$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is **[2002]**

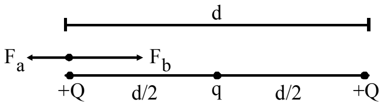


(a) $q/4\pi\in_0 L$ (b) zero

(c) $q/2\pi\in_0 L$ (d) $q/3\pi\in_0 L$

Hints & Solutions

1. **(a)**



Force due to charge + Q,

$$F_a = \frac{KQQ}{d^2}$$

Force due to charge q,

$$F_b = \frac{KQq}{\left(\frac{d}{2}\right)^2}$$

For equilibrium,

$$\vec{F}_a + \vec{F}_b = 0$$

$$\Rightarrow \frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \quad \therefore q = -\frac{Q}{4}$$

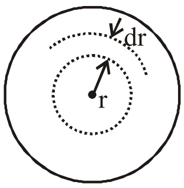
2. **(b)** $Q = \int \rho dv = \int_0^R \frac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right)$

$$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A \left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R$$

$$= 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$$

$$Q = 2\pi a A(1 - e^{-2R/a})$$

$$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi a A}}\right)$$



3. **(d)** Spheres A and B carry equal charge say 'q'

$\therefore$ Force between them, $F = \frac{kqq}{r^2}$

When A and C are touched, charge on both $q_A = q_C = \frac{q}{2}$

Then when B and C are touched, charge on B

$$q_B = \frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$$

Now, the force between charge q_A and q_B

$$F' = \frac{kq_A q_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$$

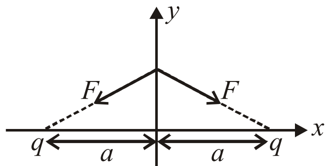
4. **(c)** Inside the cavity net charge is zero.

$\therefore$ $Q_1 = 0$ and $\sigma_1 = 0$

There is no effect of point charges $+Q$, $-Q$ and induced charge on inner surface on the outer surface.

$\therefore$ $Q_2 = 0$ and $\sigma_2 = 0$

5. **(a)**



$\Rightarrow F \sin\theta \leftarrow \quad \rightarrow F \sin\theta$

$2F\cos\theta$

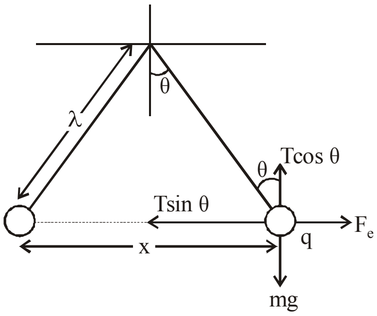
$\Rightarrow F_{net} = 2F\cos\theta$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2 + a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2 + a^2)^{3/2}} \qquad (\because y << a)$$

$$\Rightarrow \frac{kq^2 y}{a^3} \qquad \text{So, } F \propto y$$

6. **(d)**



In equilibrium, $F_e = T\sin\theta$

$mg = T\cos\theta$

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi \in_0 x^2 \times mg}$$

also $\tan\theta \approx \sin = \frac{x/2}{\ell}$

Hence, $\frac{x}{2\ell} = \frac{q^2}{4\pi \in_0 x^2 \times mg}$

$$\Rightarrow x^3 = \frac{2q^2\ell}{4\pi \in_0 mg}$$

$$\therefore x = \left(\frac{q^2\ell}{2\pi \in_0 mg}\right)^{1/3}$$

Therefore $x \propto \ell^{1/3}$

7. **(d)** From figure

$T\cos\theta = mg$ ....(i)

$T\sin\theta = F_e$ ....(ii)

Dividing equation (ii) by (i), we get

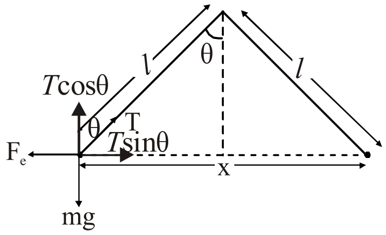
$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{F_e}{mg}$ $\Rightarrow F_e = mg\tan\theta$

$\Rightarrow \frac{kq^2}{x^2} = mg\tan\theta$ $\Rightarrow q^2 = \frac{x^2 mg\tan\theta}{k}$

Since θ is small

$\therefore \tan\theta \approx \sin\theta = \frac{x}{2l}$

$\therefore q^2 = \frac{x^3 mg}{2kl} \Rightarrow q^2 \propto x^{3/2}$

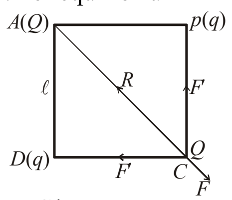


$\Rightarrow \frac{dq}{dt} \alpha \frac{3}{2}\sqrt{x}\frac{dx}{dt} = \frac{3}{2}\sqrt{x}V$

Since $\frac{dq}{dt}$ = const.

$\Rightarrow v \propto x^{-1/2}$ $[\because q^2 \propto x^3]$

8. **(d)** Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. The resultant of F' and F' is R. For equilibrium



Net force on Q at C is zero.

$\therefore \vec{R} + \vec{F} = 0 \Rightarrow \sqrt{2}F' = -F$

$\Rightarrow \sqrt{2} \times k\frac{Qq}{\ell^2} = -k\frac{Q^2}{(\sqrt{2}\,\ell)^2}$

$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$

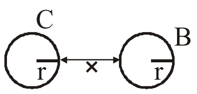
9. **(b)** It is obvious that by charge conservaiton law, electronic charge does not depend on acceleration due to gravity as it is a universal constant.

So, electronic charge on earth

= electronic charge on moon

∴ Required ratio = 1.

10. **(d)**



Initial force, $F = K\frac{Q_B Q_C}{x^2}$

x is distance between the spheres. When third spherical conductor comes in contact with B charge on B is halved *i.e.*, $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$. Now it is touched to C, charge then equally distributes themselves to make potential same, hence charge on C becomes $\left(Q + \frac{Q}{2}\right)\frac{1}{2} = \frac{3Q}{4}$.

$\therefore F_{new} = k\frac{Q'_C Q'_B}{x^2} = k\frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2} = k\frac{3}{8}\frac{Q^2}{x^2}$

or $F_{new} = \frac{3}{8}F$

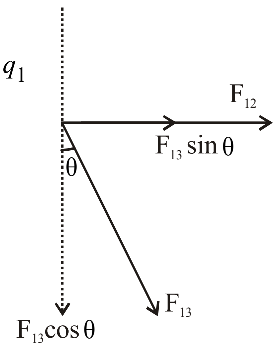
11. **(b)** Force applied by charge q_2 on q_1

$F_{12} = k\frac{q_1 q_2}{b^2}$

Force applied by charge q_3 on q_1

$F_{13} = k\frac{q_1 q_3}{a^2}$

The X-component of net force (F_x) on q_1 is $F_{12} + F_{13}\sin\theta$

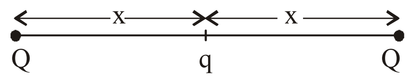


$\therefore F_x = k\frac{q_1 q_2}{b^2} + k\frac{q_1 q_2}{a^2}\sin\theta$

$\therefore F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

12. **(d)** At equilibrium net force is zero,

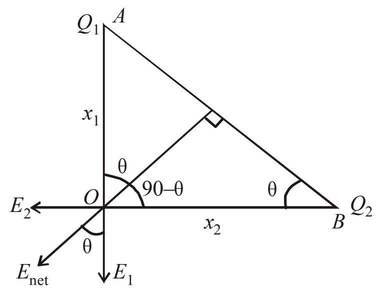
$\therefore k\frac{Q \times Q}{(2x)^2} + k\frac{Qq}{x^2} = 0$



$\Rightarrow q = -\frac{Q}{4}$

13. **(c)** Electric field due charge Q_2, $E_2 = \frac{kQ_2}{x_2^2}$

Electric field due charge Q_1, $E_1 = \frac{kQ_1}{x_1^2}$



From figure,

$\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2} \Rightarrow \frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$

$$\Rightarrow \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1} \text{ or, } \frac{Q_1}{Q_2} = \frac{x_1}{x_2}.$$

14. **(c)** For spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (\text{if } r \geq R)$$

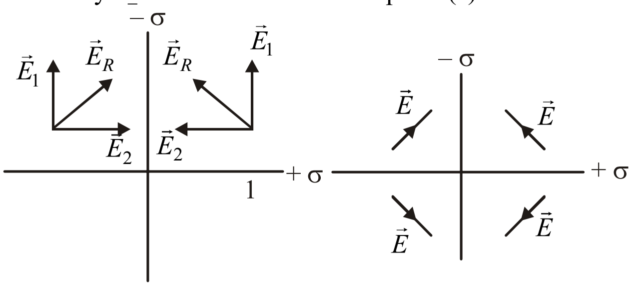
$$= 0 \qquad (\text{if } r < R)$$

Force on charge in electried field, $F = qE$

$$\therefore F = 0 \qquad (\text{For } r < R)$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} \qquad (\text{For } r > R)$$

15. **(c)** The electric field produced due to uniformly charged infinite plane is uniform. So option (b) and (d) are wrong. And +ve charge density σ_+ is bigger in magnitude so its field along Y direction will be bigger than field of –ve charge density $\sigma\_$ in X direction. Hence option (c) is correct.



16. **(c)** Given,

Electric field, $E = E_0(1 - x^2)$

$\therefore$ Force, $F = qE = qE_0(1 - x^2)$

Also, $F = ma = mv\dfrac{dv}{dx}$ $\qquad \left(\because a = v\dfrac{dv}{dx}\right)$

$$\therefore mv\frac{dv}{dx} = qE_0(1 - x^2)$$

$$\Rightarrow v\,dv = \frac{qE_0(1 - x^2)dx}{m}$$

Integrating both sides we get,

$$\Rightarrow \int_0^v v\,dv = \int_0^x \frac{qE_0(1 - x^2)dx}{m}$$

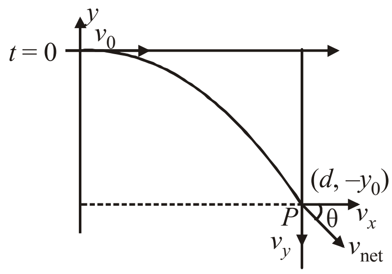
$$\Rightarrow \frac{v^2}{2} = \frac{qE_0}{m}\left(x - \frac{9x^3}{3}\right) = 0$$

$$\Rightarrow x = \sqrt{\frac{3}{a}}$$

17. **(b)** $F_x = 0, a_x = 0, (v)_x =$ constant

Time taken to reach at $'P' = \dfrac{d}{v_0} = t_0$ (let) $\qquad$ ...(i)

(Along – y), $y_0 = 0 + \dfrac{1}{2}\cdot\dfrac{qE}{m}\cdot t_0^2$ $\qquad$ ...(ii)



$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m \cdot v_0}, \left(t = \frac{d}{v_0}\right)$$

$$\tan\theta = \frac{qEd}{m \cdot v_0^2}, \text{ Slope } = \frac{-qEd}{mv_0^2}$$

No electric field $\Rightarrow F_{\text{net}} = 0, \vec{v} =$ const.

$$y = mx + c, \left\{\begin{matrix} m = \dfrac{qEd}{mv_0^2} \\ (d, -y_0) \end{matrix}\right\}$$

$$-y_0 = \frac{-qEd}{mv_0^2}, d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEd}{mv_0^2}x - y_0 + \frac{qEd^2}{mv_0^2}$$

$$y_0 = \frac{1}{2}\cdot\frac{qE}{m}\left(\frac{d}{v_0}\right)^2 = \frac{1}{2}\frac{qEd^2}{mv_0^2}$$

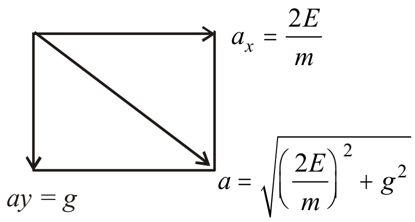
$$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2}\frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEd}{mv_0^2} + \frac{1}{2}\frac{qEd^2}{mv_0^2} \Rightarrow y = \frac{\text{qEd}}{\text{mv}_0^2}\left(\frac{d}{2} - x\right)$$

18. **(d)** Net force acting on the particle,

$$\vec{F} = qE\hat{i} + mg\hat{j}$$

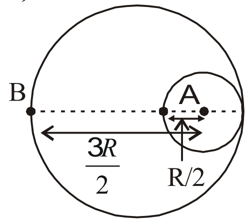
Net acceleration of particle is constant, initial velocity is zero therefore path is straight line.



19. **(b)** Electric field at A $\left(R' = \dfrac{R}{2}\right)$

$$E_A.ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \vec{E}_A = \frac{\rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0 \cdot 4\pi\left(\frac{R}{2}\right)^2}$$



$$\Rightarrow \vec{E}_A = \frac{\sigma(R/2)}{3\varepsilon_0} = \left(\frac{\sigma R}{6\varepsilon_0}\right)$$

Electric fields at 'B'

$$\vec{E}_B = \frac{k\times\rho\times\frac{4}{3}\pi R^3}{R^2} - \frac{k\times\rho\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \frac{\sigma R}{54\varepsilon_0}$$

$$\Rightarrow E_B = \frac{17}{54}\left(\frac{\sigma R}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right| = \frac{1\times 54}{6\times 17} = \left(\frac{9}{17}\right) = \frac{9}{17}\times\frac{2}{2} = \frac{18}{34}$$

20. **(c)** Since $\vec{r}\cdot\vec{p} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$

$\therefore\ \hat{E}$ is parallel to $\left(\hat{i}+3\hat{j}-2\hat{k}\right)$

21. **(c)** (A) By work energy theorem

$$W_{mg} + W_{ele} = \frac{1}{2}m(2v)^2 - \frac{1}{2}m(v)^2$$

$$0 + qE_0 2a = \frac{3}{2}mv^2 \quad \Rightarrow E_0 = \frac{3}{4}\frac{mv^2}{qa}$$

(B) Rate of work done at P = power of electric force

$$= qE_0 V = \frac{3}{4}\frac{mv^3}{a}$$

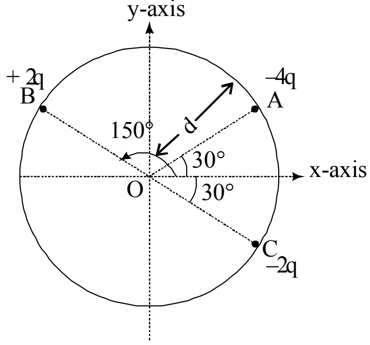
(C) At, Q, $\frac{dw}{dt} = 0$ for both the fields

(D) The difference of magnitude of angular momentum of the particle at P and Q,

$$\Delta\vec{L} = \left(-m2v2a\hat{k}\right) - \left(-mva\hat{k}\right)$$

$$\left|\Delta\vec{L}\right| = 3mva$$

22. **(a)**



Electric field due to charge +2q at centre O

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\times\frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i}-\hat{j}}{2}\right]$$

Electric field due to charge $-2q$ at centre O

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\times\frac{2q}{d^2}\left[\frac{\sqrt{3}\hat{i}-\hat{j}}{2}\right]$$

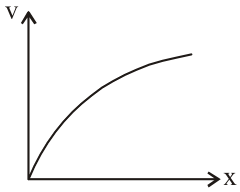
Electric field due to charge $-4q$ at centre O

$$\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\times\frac{4q}{d^2}\left[\frac{\sqrt{3}\hat{i}+\hat{j}}{2}\right]$$

$\therefore$ Net electric field at point O

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}\hat{i}$$

23. **(b)**



Using

$v^2 - u^2 = 2aS$ ...(i)

Here, $u = 0, s = x$

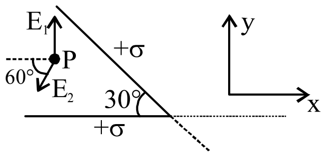
Also, $F_{electric} = ma$

$$\Rightarrow qE = ma \quad \Rightarrow \quad a = \frac{qE}{m} \Rightarrow \quad a = \frac{qE}{m}$$

Substituting the values in (i) we get

$$v^2 = \frac{2qE}{m}.x$$

24. **(d)**



From figure,

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{y} \quad \text{and} \quad \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x} - \sin 60°\hat{y})$$

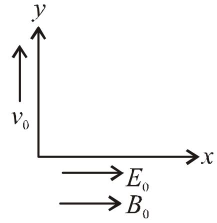
$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

Electric field in the region shown in figure (P)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x} + \left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

$$\text{or, } \vec{E}_P = \frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$$

25. **(c)** In the x direction



$F_x = qE$

$\Rightarrow \quad ma_x = qE$

$$\Rightarrow \quad a_x = \frac{E_0 q}{m}$$

For speed to be double,

$$v_0^2 + v_x^2 = (2v_0)^2$$

$$\Rightarrow \quad v_x = \sqrt{3}\ v_0 = a_x t$$

$$\Rightarrow \sqrt{3}v_0 = 0 + \frac{qE_0 t}{m} \quad \Rightarrow \quad t = \frac{\sqrt{3}v_0 m}{E_0 q}$$

26. (d) Time period of the pendulum (T) is given by

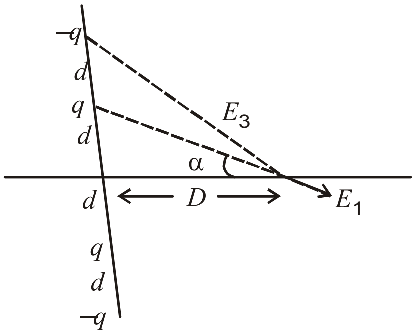
$$T = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

$$g_{eff} = \frac{\sqrt{(mg)^2 + (qE)^2}}{m}$$

$$\Rightarrow g_{eff} = \sqrt{g^2 + \left(\frac{gE}{m}\right)^2} \Rightarrow T = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

27. (d) $\vec{E} = (\vec{E_1} + \vec{E_2}) + (\vec{E_3} + \vec{E_4})$

or $E = 2E\cos\alpha - 2E\cos\beta$



$$= \frac{2kq}{(D^2 + d^2)} \times \frac{D}{\sqrt{D^2 + d^2}} - \frac{2kq}{(D^2 + (2d)^2} \times \frac{D}{\sqrt{D^2 + (2d)^2}}$$

$$= \frac{2kqD}{(D^2 + d^2)^{3/2}} - \frac{2kqD}{[D^2 + (2d)^2]^{3/2}}$$

For d < < D

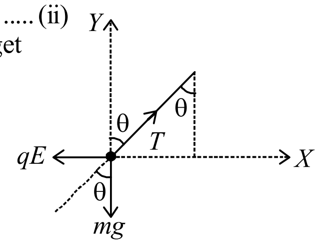
$$E \propto \frac{D}{D^3} \propto \frac{1}{D^2}$$

28. (d) At equilibrium resultant force on bob must be zero, so

$T\cos\theta = mg$ ..... (i)

$T\sin\theta = qE$ ..... (ii)

Solving (i) and (ii) we get



$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10} = \frac{1}{2}$$

[Here, $q = 5 \times 10^{-6}$C, E = 2000 v/m, m = 2×10^{-3} kg]

$$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right)$$

29. (b) Electric field on the axis of a ring of radius R at a distance h from the centre,

$$E = \frac{kQh}{\left(h^2 + R^2\right)^{3/2}}$$

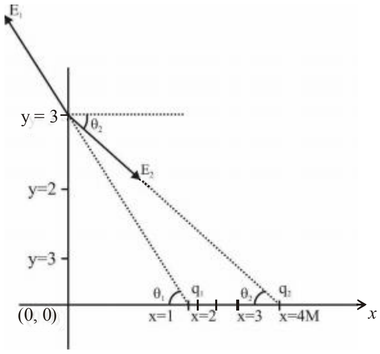
Condition: for maximum electric field $\frac{dE}{dh} = 0$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2 + h^2\right)^{3/2}}\right] = 0$$

By using the concept of maxima and minima we get,

$$h = \frac{R}{\sqrt{2}}$$

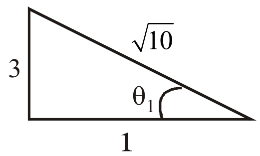
30. (a)



Let $\vec{E}_1$ and $\vec{E}_2$ are the vaues of electric field due to charge, q_1 and q_2 respectively

magnitude of $E_1 = \frac{1}{4\pi \in_0}\frac{q_1}{r_1^2}$

$$= \frac{1}{4\pi \in_0}\frac{\sqrt{10} \times 10^{-6}}{\left(1^2 + 3^2\right)}$$



$$= \left(9 \times 10^9\right) \times \sqrt{10} \times 10^{-7}$$

$$= 9\sqrt{10} \times 10^2$$

$$\therefore \vec{E}_1 = 9\sqrt{10} \times 10^2\left[\cos\theta_1\left(-\vec{i}\right) + \sin\theta_1\vec{j}\right]$$

$$\Rightarrow E_1 = 9 \times \sqrt{10} \times 10^2\left[\frac{1}{\sqrt{10}}\left(-\hat{i}\right) + \frac{3}{\sqrt{10}}\hat{j}\right]$$

$$\Rightarrow E_1 = 9 \times 10^2\left[-\hat{i} + 3\hat{j}\right] = \left[-9\hat{i} + 27\hat{j}\right]10^2$$

Similarly, $E_2 = \frac{1}{4\pi \in_0}\frac{q_2}{r^2}$

$$E_2 = \frac{9 \times 10^9 \times (25) \times 10^{-6}}{\left(4^2 + 3^2\right)} \quad E_2 = 9 \times 10^3 \text{ V/m}$$

$$\therefore \vec{E}_2 = 9 \times 10^3\left(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}\right) \quad \because \tan\theta_2 = \frac{3}{4}$$

$$\therefore \vec{E}_2 = 9 \times 10^3\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right) = \left(72\hat{i} - 54\hat{j}\right) \times 10^2$$

$$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2 = \left(63\hat{i} - 27\hat{j}\right) \times 10^2\ V/m$$

31. **(a)** Equilibrium position will shift to point where resultant force = 0

$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

$$\text{Total energy } = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}kx_{eq}^2$$

$$\text{Total energy } = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$$

32. **(d)** Charge density, $\rho = \rho_0\left(1-\frac{r}{R}\right)$

$$dq = \rho dv$$

$$q_{in} = \int dq = \rho dv$$

$$= \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr \quad (\because dv = 4\pi r^2 dr)$$

$$= 4\pi\rho_0\int_0^R\left(1-\frac{r}{R}\right)r^2 dr$$

$$= 4\pi\rho_0\int_0^R r^2 dr - \frac{r^2}{R}dr$$

$$= 4\pi\rho_0\left[\left[\frac{r^3}{3}\right]_0^R - \left[\frac{r^4}{4R}\right]_0^R\right] = 4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^4}{4R}\right]$$

$$= 4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^3}{4}\right] = 4\pi\rho_0\left[\frac{R^3}{12}\right]$$

$$q = \frac{\pi\rho_0 R^3}{3}$$

$$E.4\pi r^2 = \left(\frac{\pi\rho_0 R^3}{3\epsilon_0}\right)$$

$\therefore$ Electric field outside the ball, $E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$

33. **(c)** Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

34. **(d)** Given: Length of wire L = 20 cm

charge $Q = 10^3\varepsilon_0$

We know, electric field at the centre of the semicircular arc

$$E = \frac{2K\lambda}{r}$$

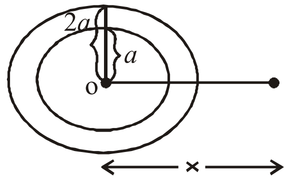
or, $$E = \frac{2K\left(\frac{2Q}{\pi r}\right)}{r}\left[As\lambda = \frac{2Q}{\pi r}\right]$$

$$= \frac{4KQ}{\pi r^2} = \frac{4KQ\pi^2}{\pi L^2} = \frac{4\pi KQ}{L^2} = 25\times10^3 N/C\hat{i}$$

35. **(a)** Electric field due to complete disc ($R = 2a$) at a distance x and on its axis

$$E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right] \quad E_1 = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{\sqrt{4a^2+h^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right] \quad \left[\begin{array}{l}\text{here } x = h\\ \text{and, } R = 2a\end{array}\right]$$



Similarly, electric field due to disc ($R = a$)

$$E_2 = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{h}{a}\right)$$

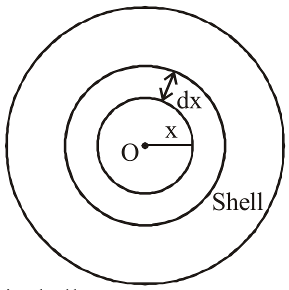
Electric field due to given disc

$E = E_1 - E_2$

$$\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{2a}\right]-\frac{\sigma}{2\varepsilon_0}\left[1-\frac{h}{a}\right] = \frac{\sigma h}{4\varepsilon_0 a}$$

Hence, $c = \frac{\sigma}{4a\varepsilon_0}$

36. **(b)** Let us consider a spherical shell of radius x and thickness dx.



Charge on this shell

$$dq = \rho.4\pi x^2 dx = \rho_0\left(1-\frac{x}{R}\right).4\pi x^2 dx$$

$\therefore$ Total charge in the spherical region from centre to r (r < R) is

$$q = \int dq = 4\pi\rho_0\int_0^r\left(1-\frac{x}{R}\right)x^2 dx$$

$$= 4\pi\rho_0\left[\frac{x^3}{3}-\frac{x^4}{4R}\right]_0^r = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right] = 4\pi\rho_0 r^3\left[\frac{1}{3}-\frac{r}{4R}\right]$$

$\therefore$ Electric field at r, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{4\pi\rho_0 r^3}{r^2}\left[\frac{1}{3}-\frac{r}{4R}\right] = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^2}{4R}\right]$$

37. (c) Given,
Electric field E = 150 N/C
Total surface charge carried by earth q = ?
or, $q = \epsilon_0 E A$
$= \epsilon_0 E \pi r^2$.
$= 8.85 \times 10^{-12} \times 150 \times (6.37 \times 10^6)^2$.
$\simeq 680$ Kc
As electric field directed inward hence
q = – 680 Kc

38. (a) Electric field intensity at the centre of the disc.

$$E = \frac{\sigma}{2\epsilon_0} \text{ (given)}$$

Electric field along the axis at any distance x from the centre of the disc

$$E' = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

From question, x = R (radius of disc)

$$\therefore E' = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{R}{\sqrt{R^2 + R^2}}\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(\frac{\sqrt{2}R - R}{\sqrt{2}R}\right)$$

$$= \frac{4}{14}E$$

∴ % reduction in the value of electric field

$$= \frac{\left(E - \frac{4}{14}E\right) \times 100}{E} = \frac{1000}{14}\% \simeq 70.7\%$$

39. (b) $F = qE = mg$ $(q = 6e = 6 \times 1.6 \times 10^{-19})$

$$\text{Density (d)} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\text{or } r^3 = \frac{m}{\frac{4}{3}\pi d}$$

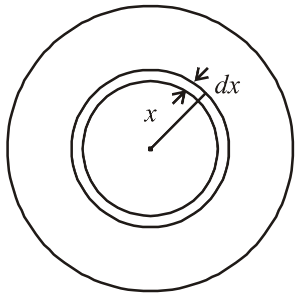
Putting the value of d and m $\left(= \frac{qE}{g}\right)$ and solving we get r $= 7.8 \times 10^{-7}$ m

40. (a) Let us consider a spherical shell of radius x and thickness dx.
Charge on this shell

$$dq = \rho . 4\pi x^2 dx = \rho_0\left(\frac{5}{4} - \frac{x}{R}\right).4\pi x^2 dx$$

∴ Total charge in the spherical region from centre to $r (r < R)$ is

$$q = \int dq = 4\pi\rho_0 \int_0^r \left(\frac{5}{4} - \frac{x}{R}\right) x^2 dx$$



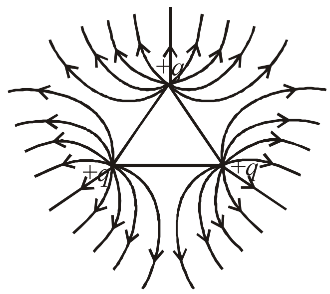
$$= 4\pi\rho_0\left[\frac{5}{4}.\frac{r^3}{3} - \frac{1}{R}.\frac{r^4}{4}\right] = \pi\rho_0 r^3\left(\frac{5}{3} - \frac{r}{R}\right)$$

∴ Electric field at r,

$$E = \frac{1}{4\pi\epsilon_0}.\frac{q}{r^2}$$

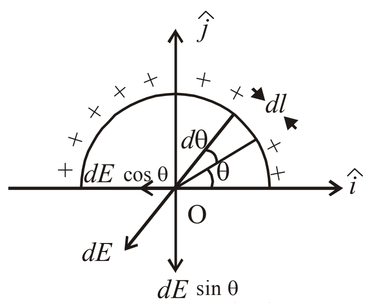
$$= \frac{1}{4\pi\epsilon_0}.\frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4\epsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

41. (c) Electric lines of force due to a positive charge is spherically symmetric.
All the charges are positive and equal in magnitude. So repulsion takes place. Due to which no lines of force are present inside the equilateral triangle and the resulting lines of force obtained as shown:



42. (c) Let us consider a differential element dl subtending at angle dQ at the centre Q as shown in the figure. Linear charge density

$$\lambda = \frac{q}{Qr}$$



Charge on the element, $dq = \left(\frac{q}{\pi r}\right) dl$

$$= \frac{q}{\pi r}(r d\theta) \qquad (\because dl = r d\theta)$$

$$= \left(\frac{q}{\pi}\right) d\theta$$

Electric field at the center O due to dq is

$$dE = \frac{1}{4\pi \epsilon_0} \cdot \frac{dq}{r^2} = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{\pi r^2} d\theta$$

Resolving dE into two rectangular component, we find the component $dE \cos \theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant of the component $dE \sin \theta$ only.

$$\therefore\ E = \int dE \sin\theta = \int_0^{\pi} \frac{q}{4\pi^2 r^2 \epsilon_0} \sin\theta d\theta$$

$$= \frac{q}{4\pi^2 r^2 \epsilon_0}[-\cos\theta]_0^{\pi}$$

$$= \frac{q}{4\pi^2 r^2 \epsilon_0}(+1+1) = \frac{q}{2\pi^2 r^2 \epsilon_0}$$

The direction of E is towards negative y-axis.

$$\therefore\ \vec{E} = -\frac{q}{2\pi^2 r^2 \epsilon_0}\hat{j}$$

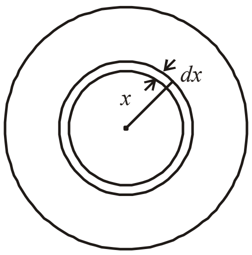
43. (a) Let us consider a spherical shell of radius x and thickness dx.

Due to shpherically symmetric charge distribution, the chrge on the spherical surface of radius x is

$$dq = dV\rho \cdot 4\pi x^2 dx = \rho_0\left(\frac{5}{4} - \frac{x}{R}\right) \cdot 4\pi x^2 dx$$

$\therefore$ Total charge in the spherical region from centre to r $(r < R)$ is

$$q = \int dq = 4\pi\rho_0 \int_0^r \left(\frac{5}{4} - \frac{x}{R}\right) x^2 dx$$



$$= 4\pi\rho_0\left[\frac{5}{4} \cdot \frac{r^3}{3} - \frac{1}{R} \cdot \frac{r^4}{4}\right] = \pi\rho_0 r^3\left(\frac{5}{3} - \frac{r}{R}\right)$$

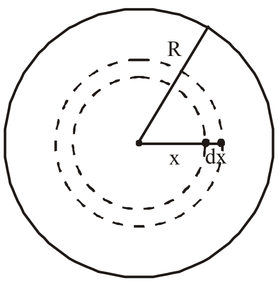
$\therefore$ Electric field intensity at a point on this spherical surface

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{r^2}$$

$$= \frac{1}{4\pi \epsilon_0} \cdot \frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4 \epsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

44. (a)

45. (b)



Let us consider a spherical shell of thickness dx and radius x. The area of this spherical shell = $4\pi x^2$.

The volume of this spherical shell = $4\pi x^2 dx$. The charge enclosed within shell

$$dq = \left[\frac{Qx}{\pi R^4}\right][4\pi x^2 dx] = \frac{4Q}{R^4} x^3 dx$$

The charge enclosed in a sphere of radius r_1 can be calculated by

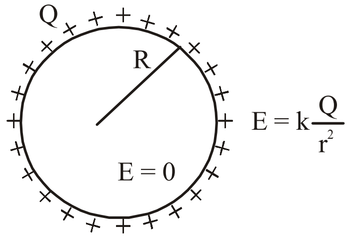
$$Q = \int dq = \frac{4Q}{R^4}\int_0^{r_1} x^3 dx = \frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1} = \frac{Q}{R^4} r_1^4$$

$\therefore$ The electric field at point P inside the sphere at a distance r_1 from the centre of the sphere is

$$E = \frac{1}{4\pi E}\frac{Q}{r_1^2}$$

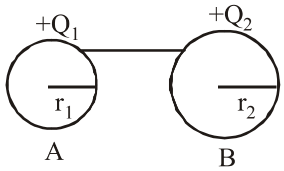
$$\Rightarrow E = \frac{1}{4\pi \epsilon_0}\frac{\left[\frac{Q}{R^4} r_1^4\right]}{r_1^2} = \frac{1}{4\pi \epsilon_0}\frac{Q}{R^4} r_1^2$$

46. (a) The electric field inside a thin spherical shell of radius R has charge Q spread uniformly over its surface is zero.



Outside the shell the electric field is $E = k\frac{Q}{r^2}$. These characteristics are represented by graph (a).

47. (c)



When the two conducting spheres are connected by a conducting wire, charge will flow from one to other till both acquire same potential.

$\therefore$ After connection, $V_1 = V_2$

$$\Rightarrow k\frac{Q_1}{r_1} = k\frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{r_1} = \frac{Q_2}{r_2}$$

The ratio of electric fields

$$\frac{E_1}{E_2} = \frac{k\frac{Q_1}{r_1^2}}{k\frac{Q_2}{r_2^2}} \Rightarrow \frac{E_1}{E_2} = \frac{Q_1}{r_1^2} \times \frac{r_2^2}{Q_2}$$

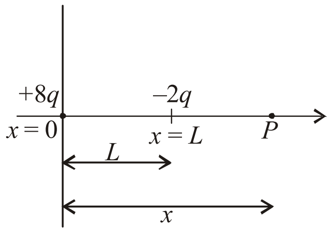
$$\Rightarrow \frac{E_1}{E_2} = \frac{r_1 \times r_2^2}{r_1^2 \times r_2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2}{1}$$

48. (b) At $P \frac{-K2q}{(x-L)^2}+\frac{K8q}{x^2}=0$

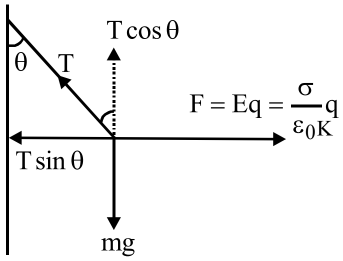
$\Rightarrow \frac{1}{(x-L)^2}=\frac{4}{x^2}$

or $\frac{1}{x-L}=\frac{2}{x}$

$\Rightarrow x=2x-2L$ or $x=2L$



49. (c)



$T\sin\theta = qE$ .... (i)

$T\cos\theta = mg$ .... (ii)

Dividing (i) by (ii),

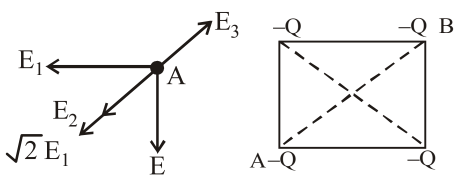
$\tan\theta=\frac{qE}{mg}=\frac{q}{mg}\left(\frac{\sigma}{\varepsilon_0 K}\right)\frac{\sigma q}{\varepsilon_0 K.mg}$

$\therefore \sigma \propto \tan\theta$

50. (b) For the system to be equilibrium, net field at A should be zero

$\sqrt{2}\,E_1+E_2=E_3$

$\therefore \frac{kQ\times\sqrt{2}}{a^2}+\frac{kQ}{(\sqrt{2}\,a)^2}=\frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$



$\Rightarrow \frac{Q\sqrt{2}}{1}+\frac{Q}{2}=2q \;\Rightarrow q=\frac{Q}{4}(2\sqrt{2}+1)$

51. (c) Given, Electric field, $E=3\times10^4$

Mass of the drop, $m=9.9\times10^{-15}$ kg

At equilibrium, coulomb force on drop balances weight of drop.

$qE=mg$

$\Rightarrow q=\frac{mg}{E} \Rightarrow q=\frac{9.9\times10^{-15}\times10}{3\times10^4}=3.3\times10^{-18}\text{C}$

52. (b) Let v be the speed of dipole.

Using energy conservation

$K_i+U_i=K_f+U_f$

$\Rightarrow 0-\frac{2k\cdot p_1}{r^3}p_2\cos(180°)=\frac{1}{2}mv^2+\frac{1}{2}mv^2+0$

$\left(\because \text{ Potential energy of interaction between dipole } =\frac{-2p_1p_2\cos\theta}{4\pi\epsilon_0 r^3}\right)$

$\Rightarrow mv^2=\frac{2kp_1p_2}{r^3} \Rightarrow v=\sqrt{\frac{2kp_1p_2}{mr^3}}$

When $p_1=p_2=p$ and $r=a$

$v=\frac{p}{a}\sqrt{\frac{1}{2\pi\epsilon_0 ma}}$

53. (–48)

Flux of electric field $\vec{E}$ through any area $\vec{A}$ is defined as

$\phi=\int E.A\cos\theta$

Here, θ = angle between electric field and area vector of a surface

For surface $ABCD$ Angle, $\theta=90°$

$\therefore \quad \phi_1=\int E.A\cos 90°=0$

For surface $BCGF$ $\phi_n=\int \vec{E}\,.\,\overrightarrow{dA}$

$\therefore \quad \phi_{11}=\left[4\times\hat{i}-(y^2+1)\hat{j}\right].4\hat{i}=16x$

$\phi_{11}=48\frac{Nm^2}{C}$

$\phi_1-\phi_{11}=-48$

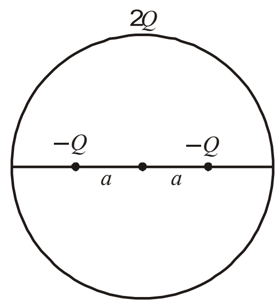
54. (a)

55. (b) Surface charge density depends only due to Q. Also

$\oint \vec{E}.d\vec{A}=\frac{q_1\lambda}{\varepsilon_0}$

or $E\times4\pi r^2=\frac{Q}{\varepsilon_0}\Rightarrow E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},\ r\geq R$

56. (a) $\oint \vec{E}.d\vec{A}=\frac{q_{in}}{\varepsilon_0}$



or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int S(4\pi r^2)dr$

or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r (kr)(4\pi r^2)dr$

or $E \times 4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left(\frac{r^4}{4}\right)$

$\therefore \quad E = \frac{k}{4\varepsilon_0}r^2 \qquad ...(i)$

Also $2Q = \int_0^R (kr)\ (4\pi r^2)dr \ = 4\pi k\left|\frac{r^4}{4}\right|_0^R$

$Q = \frac{\pi k R^4}{2} \qquad ....(ii)$

From above equations,

$E = \frac{Qr^2}{2\pi\varepsilon_0 R^4} \qquad ....(iii)$

According to given condition

$= EQ\ \frac{Q^4}{4\pi\varepsilon_0 (20)^2} \qquad ....(iv)$

From equations (iii) and (iv), we have

$a = 8^{-1/4}$ R.

57. **(b)** $\tau = -PE\sin\theta$

or $I\alpha = -PE(\theta)$

$\alpha = \frac{PE}{I}(-\theta)$

On comparing with

$\alpha = -\omega^2\theta$

$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$

58. **(b)** Potential energy of a dipole is given by

$U = -\vec{P}.\vec{E}$

$= -$ PE cos θ

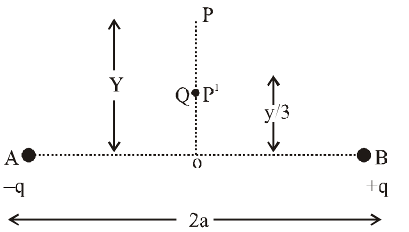
[Whereθ = angle between dipole and perpendicular to the field]

$= -(10^{-29})(10^3)\cos 45°$

$= -0.707 \times 10^{-26}$ J $= -7 \times 10^{-27}$ J

59. **(d)** Electric field of equitorial plane of dipole

$= -\frac{K\vec{P}}{r^3}$



$\therefore$ At point P, $= +\frac{KP}{y^3}Q$

At Point P^1, $F^1 = +\frac{KPQ}{(y/3)^3} = 27F.$

60. **(b)** When cube is of side a and point charge Q is at the center of the cube then the total electric flux due to this charge will pass evenly through the six faces of the cube. So, the electric flux through one face will be equal to 1/6 of the total electric flux due to this charge.

Flux through 6 faces $= \frac{Q}{\epsilon_0}$

$\therefore$ Flux through 1 face, $= \frac{Q}{6\epsilon_0}$

61. **(a)** T = PE sin θ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\ \hat{i} + p\sin\theta\ \hat{j}$

$\vec{E}_1 = E\vec{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\,\hat{k} = pE\sin\theta(-\hat{k}) \qquad ...(i)$

$\vec{E}_2 = \sqrt{3}\,E_1\hat{j}$

$\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}\,E_1\hat{j}$

$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k} \qquad ...(ii)$

From eqns. (*i*) and (*ii*)

$pE\sin\theta = \sqrt{3}\,pE\cos\theta$

$\tan\theta = \sqrt{3} \quad \therefore \quad \theta = 60°$

62. **(c)** The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1, $\phi_1 = \frac{1}{\varepsilon_0}(2q)$

For surface S_2, $\phi_2 = \frac{1}{\varepsilon_0}(q+q+q-q) = \frac{1}{\varepsilon_0}2q$

For surface S_3, $\phi_3 = \frac{1}{\varepsilon_0}(q+q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\phi_4 = \frac{1}{\varepsilon_0}(8q-2q-4q) = \frac{1}{\varepsilon_0}(2q)$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4$ i.e. net electric flux is same for all surfaces.

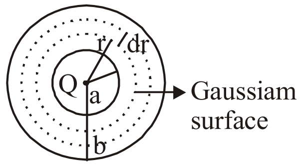
Keep in mind, the electric field due to a charge outside (S_3 and S_4), the Gaussian surface contributes zero net flux through the surface, because as many lines due to that charge enter the surface as leave it.

63. **(c)** Applying Gauss's law

$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\epsilon_0}$

$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi A r^2 - 2\pi A a^2}{\epsilon_0}$

$\rho = \frac{dr}{dV}$



$Q = \rho 4\pi r^2$

$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$

$E = \frac{1}{4\pi \in_0}\left[\frac{Q - 2\pi A a^2}{r^2} + 2\pi A\right]$

For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$\therefore\ A = \frac{Q}{2\pi a^2}$

64. **(a)** $\vec{E} = E_0\hat{i} + 2E_0\hat{j}$

Given, $E_0 = 100\,N/c$

So, $\vec{E} = 100\hat{i} + 200\hat{j}$

Radius of circular surface = 0.02 m

Area = $\pi r^2 = \frac{22}{7} \times 0.02 \times 0.02$

$= 1.25 \times 10^{-3}\,\hat{i}\,m^2$ [Loop is parallel to Y-Z plane]

Now, flux (ϕ) = EA cosθ

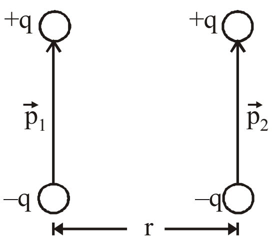
$= \left(100\hat{i} + 200\hat{j}\right).1.25 \times 10^{-3}\,\hat{i}\cos\theta°$ [$\theta = 0°$]

= 125×10^{-3} Nm^2/c

= 0.125 Nm^2/c

65. **(c)** Force of interaction

$F = \frac{1}{4\pi \in_0}.\frac{6p_1p_2}{r^4}$



66. **(b)** We know that,

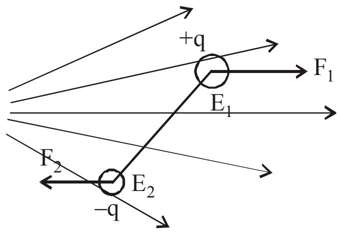
$\phi = \oint E.dS = E\oint dS\ \cos 45°$

In case of hemisphere

$\phi_{curved} = \phi_{circular}$

Therefore, $\phi_{curved} = E\pi a^2.\frac{1}{\sqrt{2}} = \frac{E\pi a^2}{\sqrt{2}}$

67. **(c)**



As the dipole is placed in non-uniform field, so the force acting on the dipole will not cancel each other. This will result in a force as well as torque.

68. **(a)** The electric flux ϕ_1 entering an enclosed surface is taken as negative and the electric flux ϕ_2 leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface, $\phi = \phi_2 - \phi_1$

According to Gauss theorem

$\phi = \frac{q}{\in_0} \quad \Rightarrow q = \in_0 \phi = \in_0(\phi_2 - \phi_1)$

69. **(None)** Electric flux due to charge placed outside is zero. But for the charge inside the cube, flux due to each face is $\frac{1}{6}\left[\frac{q}{\in_0}\right]$ which is not in option.