7

Permutations and Combinations

TOPIC 1 Fundamental Principle of Counting, Factorials, Permutations, Counting Formula for Permutations, Permutations in Which Things may be Repeated, Permutations in Which all Things are Different, Number of Permutations Under Certain Restricted Conditions, Circular Permutations

1. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated? **[Sep. 06, 2020 (I)]**

(a) $2!\,3!\,4!$ (b) $(3!)^3\cdot(4!)$
(c) $(3!)^2\cdot(4!)$ (d) $3!(4!)^3$

2. The value of $(2\cdot{}^1P_0 - 3\cdot{}^2P_1 + 4\cdot{}^3P_2 - \ldots$ up to 51^{th} term$)$ $+ (1! - 2! + 3! - \ldots$ up to 51^{th} term$)$ is equal to : **[Sep. 03, 2020 (I)]**

(a) $1-51(51)!$ (b) $1+(51)!$
(c) $1+(52)!$ (d) 1

3. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ________. **[NA Sep. 02, 2020 (I)]**

4. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is 336 k, then k is equal to: **[Jan. 9, 2020 (I)]**

(a) 4 (b) 6
(c) 7 (d) 8

5. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is: **[Jan. 7, 2020 (I)]**

(a) $\frac{1}{2}(6!)$ (b) $6!$
(c) 5^6 (d) $\frac{5}{2}(6!)$

6. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is: **[April 10, 2019 (I)]**

(a) 72 (b) 60
(c) 48 (d) 36

7. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is: **[April 08, 2019 (II)]**

(a) 288 (b) 360
(c) 306 (d) 310

8. Consider three boxes, each containing 10 balls labelled 1, 2,..., 10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, $(i = 1, 2, 3)$. Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is : **[Jan. 12, 2019 (I)]**

(a) 120 (b) 82
(c) 240 (d) 164

9. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repetition of digits allowed) is equal to: **[Jan. 09, 2019 (II)]**

(a) 374 (b) 372
(c) 375 (d) 250

10. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is: **[Jan. 09, 2019 (II)]**

(a) 9 (b) 18
(c) 36 (d) 32

11. The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is? **[Online April 16, 2018]**

(a) 30 (b) 48
(c) 24 (d) 36

12. n – digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is **[Online April 15, 2018]**

(a) 6 (b) 8

(c) 9 (d) 7

13. The number of ways in which 5 boys and 3 girls can be seated on a round table if a particular boy B_1 and a particular girl G_1 never sit adjacent to each other, is : **[Online April 9, 2017]**

(a) $5 \times 6!$ (b) $6 \times 6!$

(c) $7!$ (d) $5 \times 7!$

14. If all the words, with or without meaning, are written using the letters of the word QUEEN and are arranged as in English dictionary, then the position of the word QUEEN is : **[Online April 8, 2017]**

(a) 44^{th} (b) 45^{th}

(c) 46^{th} (d) 47^{th}

15. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[2016]**

(a) 52^{nd} (b) 58^{th}

(c) 46^{th} (d) 59^{th}

16. The sum $\sum_{r=1}^{10}(r^2+1) \times (r!)$ is equal to : **[Online April 10, 2016]**

(a) $11 \times (11!)$ (b) $10 \times (11!)$

(c) $(11!)$ (d) $101 \times (10!)$

17. If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is : **[Online April 9, 2016]**

(a) 110 (b) 59

(c) $\dfrac{11!}{(2!)^3}$ (d) 56

18. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : **[2015]**

(a) 820 (b) 780

(c) 901 (d) 861

19. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[2015]**

(a) 120 (b) 72

(c) 216 (d) 192

20. The number of ways of selecting 15 teams from 15 men and 15 women, such that each team consists of a man and a woman, is: **[Online April 10, 2015]**

(a) 1120 (b) 1880

(c) 1960 (d) 1240

21. Two women and some men participated in a chess tournament in which every participant played two games with each of the other participants. If the number of games that the men played between themselves exceeds the number of games that the men played with the women by 66, then the number of men who participated in the tournament lies in the interval: **[Online April 19, 2014]**

(a) [8, 9] (b) [10, 12)

(c) (11, 13] (d) (14, 17)

22. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such numbers in which the odd digits do no occupy odd places, is: **[Online April 12, 2014]**

(a) 160 (b) 120

(c) 60 (d) 48

23. An eight digit number divisible by 9 is to be formed using digits from 0 to 9 without repeating the digits. The number of ways in which this can be done is: **[Online April 11, 2014]**

(a) 72 (7!) (b) 18 (7!)

(c) 40 (7!) (d) 36 (7!)

24. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3, 4, 5 and 6, without repetition, is: **[Online April 9, 2014]**

(a) 432 (b) 108

(c) 36 (d) 18

25. 5 - digit numbers are to be formed using 2, 3, 5, 7, 9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then $p : q$ is : **[Online April 25, 2013]**

(a) 6 : 5 (b) 3 : 2

(c) 4 : 3 (d) 5 : 3

26. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is: **[2012]**

(a) 880 (b) 629

(c) 630 (d) 879

27. If seven women and seven men are to be seated around a circular table such that there is a man on either side of every woman, then the number of seating arrangements is **[Online May 26, 2012]**

(a) 6! 7! (b) $(6!)^2$

(c) $(7!)^2$ (d) 7!

28. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**

(a) 601 (b) 600

(c) 603 (d) 602

29. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**

(a) 480 (b) 240
(c) 360 (d) 120

30. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is **[2004]**

(a) $\{1, 2, 3, 4, 5\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4,\}$ (d) $\{1, 2, 3,\}$

31. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**

(a) $6! \times 5!$ (b) 6×5
(c) 30 (d) 5×4

32. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**

(a) 3000 (b) 3050
(c) 3600 (d) 3250

33. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**

(a) 125 (b) 105
(c) 374 (d) 625

34. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**

(a) 216 (b) 375
(c) 400 (d) 720

TOPIC 2 Combinations, Counting Formula for Combinations, Division and Distribution of Objects, Dearrangement Theorem, Sum of Numbers, Important Result About Point

35. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ______. **[NA Sep. 06, 2020 (II)]**

36. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is ______. **[NA Sep. 05, 2020 (I)]**

37. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is : **[Sep. 05, 2020 (II)]**

(a) 3000 (b) 1500
(c) 2255 (d) 2250

38. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is __________. **[NA Sep. 04, 2020 (II)]**

39. The total number of 3-digit numbers, whose sum of digits is 10, is _________. **[NA Sep. 03, 2020 (II)]**

40. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is : **[Sep. 02, 2020 (II)]**

(a) 201 (b) 200
(c) 101 (d) 199

41. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + ... + (101) \cdot C_{25} = 2^{25} \cdot k$, then k is equal to ________. **[NA Jan. 9, 2020 (II)]**

42. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is _________. **[NA Jan. 8, 2020 (I)]**

43. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then: **[Jan. 8, 2020 (I)]**

(a) $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$ (b) $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$
(c) $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$ (d) $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$

44. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _________. **[NA Jan. 8, 2020 (II)]**

45. The number of ordered pairs (r, k) for which $6.{}^{35}C_r = (k^2 - 3).{}^{36}C_{r+1}$, where k is an integer, is: **[Jan. 7, 2020 (II)]**

(a) 3 (b) 2
(c) 6 (d) 4

46. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct is: **[April 12, 2019 (I)]**

(a) $2^{20} - 1$ (b) 2^{21}
(c) 2^{20} (d) $2^{20}+1$

47. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to : **[April 12, 2019 (II)]**

(a) 28 (b) 27
(c) 25 (d) 24

48. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is : **[April 10, 2019 (II)]**

(a) 170 (b) 180
(c) 210 (d) 190

49. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then: **[April 9, 2019 (I)]**

(a) $m+n=68$ (b) $m=n=78$
(c) $n=m-8$ (d) $m=n=68$

50. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is : **[April 8, 2019 (I)]**

(a) 180 (b) 175
(c) 160 (d) 162

51. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is **[Jan. 12, 2019 (II)]**

(a) 12 (b) 11
(c) 9 (d) 7

52. If $\sum_{i=1}^{20}\left(\frac{^{20}C_{i-1}}{^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals: **[Jan. 10, 2019 (I)]**

(a) 400 (b) 50
(c) 200 (d) 100

53. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: **[Jan. 9, 2019 (I)]**

(a) 500 (b) 200
(c) 300 (d) 350

54. The number of four letter words that can be formed using the letters of the word BARRACK is **[Online April 15, 2018]**

(a) 144 (b) 120
(c) 264 (d) 270

55. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : **[2018]**

(a) less than 500
(b) at least 500 but less than 750
(c) at least 750 but less than 1000
(d) at least 1000

56. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[2017]**

(a) 484 (b) 485
(c) 468 (d) 469

57. If $\frac{^{n+2}C_6}{^{n-2}P_2} = 11$, then n satisfies the equation : **[Online April 10, 2016]**

(a) $n^2+n-110=0$ (b) $n^2+2n-80=0$
(c) $n^2+3n-108=0$ (d) $n^2+5n-84=0$

58. The value of $\sum_{r=1}^{15} r^2\left(\frac{^{15}C_r}{^{15}C_{r-1}}\right)$ is equal to : **[Online April 9, 2016]**

(a) 1240 (b) 560
(c) 1085 (d) 680

59. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[2015]**

(a) 275 (b) 510
(c) 219 (d) 256

60. If in a regular polygon the number of diagonals is 54, then the number of sides of this polygon is **[Online April 11, 2015]**

(a) 12 (b) 6
(c) 10 (d) 9

61. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is **[2013]**

(a) 256 (b) 220
(c) 219 (d) 211

62. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[2013]**

(a) 7 (b) 5
(c) 10 (d) 8

63. On the sides AB, BC, CA of a ΔABC, 3, 4, 5 distinct points (excluding vertices A, B, C) are respectively chosen. The number of triangles that can be constructed using these chosen points as vertices are : **[Online April 23, 2013]**

(a) 210 (b) 205
(c) 215 (d) 220

64. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is : **[Online April 22, 2013]**

(a) $^{30}C_7$ (b) $^{21}C_8$
(c) $^{21}C_7$ (d) $^{30}C_8$

65. A committee of 4 persons is to be formed from 2 ladies, 2 old men and 4 young men such that it includes at least 1 lady, at least 1 old man and at most 2 young men. Then the total number of ways in which this committee can be formed is : **[Online April 9, 2013]**

(a) 40 (b) 41
(c) 16 (d) 32

66. The number of arrangements that can be formed from the letters a, b, c, d, e, f taken 3 at a time without repetition and each arrangement containing at least one vowel, is **[Online May 19, 2012]**

(a) 96 (b) 128
(c) 24 (d) 72

67. If $n = {}^mC_2$, then the value of ${}^nC_2$ is given by **[Online May 19, 2012]**

(a) $3({}^{m+1}C_4)$ (b) ${}^{m-1}C_4$
(c) ${}^{m+1}C_4$ (d) $2({}^{m+2}C_4)$

68. **Statement 1:** If A and B be two sets having p and q elements respectively, where $q > p$. Then the total number of functions from set A to set B is q^p. **[Online May 12, 2012]**
Statement 2: The total number of selections of p different objects out of q objects is ${}^qC_p$.

(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
(c) Statement 1 is false, Statement 2 is true
(d) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

69. If the number of 5-element subsets of the set $A = \{a_1, a_2, ..., a_{20}\}$ of 20 distinct elements is k times the number of 5-element subsets containing a_4, then k is **[Online May 7, 2012]**

(a) 5 (b) $\frac{20}{7}$
(c) 4 (d) $\frac{10}{3}$

70. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then : **[2011RS]**

(a) $N \le 100$ (b) $100 < N \le 140$
(c) $140 < N \le 190$ (d) $N > 190$

71. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

72. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**

(a) 36 (b) 66
(c) 108 (d) 3

73. From 6 different novels and 3 different dictionaries,4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**

(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000
(d) less than 500

74. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? **[2008]**

(a) $8.{}^6C_4.{}^7C_4$ (b) $6.7.{}^8C_4$
(c) $6.8.{}^7C_4$. (d) $7.{}^6C_4.{}^8C_4$

75. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size.
Thus $A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**

(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$
(c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$

76. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**

(a) 5040 (b) 6210
(c) 385 (d) 1110

77. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[2005]**

(a) ${}^{55}C_4$ (b) ${}^{55}C_3$
(c) ${}^{56}C_3$ (d) ${}^{56}C_4$

78. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**

(a) ${}^8C_3$ (b) 21
(c) 3^8 (d) 5

79. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**

(a) 346 (b) 140
(c) 196 (d) 280

80. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**

(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$
(c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$

81. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are **[2002]**

(a) 312 (b) 3125
(c) 120 (d) 216

Hints & Solutions

1. **(b)** Number of arrangement

$= (3! \times 3! \times 4!) \times 3! = (3!)^3 4!$

2. **(c)** We know, $(r+1) \cdot {}^{r}P_{r-1} = (r+1) \cdot \frac{r!}{1!} = (r+1)!$

So, $(2 \cdot {}^{1}P_0 - 3 \cdot {}^{2}P_1 + ...51 \text{ terms}) +$

$(1! - 2! + 3! - ... \text{ upto } 51 \text{ terms})$

$= [2! - 3! + 4! - ... + 52!] + [1! - 2! + 3! - ... + 51!]$

$= 52! + 1! = 52! + 1$

3. **(309)**

M	O	T	H	E	R
3	4	6	2	1	5

$\Rightarrow 2 \times 5! + 2 \times 4! + 3 \times 3! + 2! + 1$

$= 240 + 48 + 18 + 2 + 1 = 309$

4. **(d)** Number of five digit numbers with 2 at 10^{th} place $= 8 \times 8 \times 7 \times 6 = 2688$

$\because$ It is given that, number of five digit number with 2 at 10^{th} place $= 336k$

$\therefore$ $336k = 2688 \Rightarrow k = 8$

5. **(d)** Five digits numbers be 1, 3, 5, 7, 9

For selection of one digit, we have ${}^{5}C_1$ choice.

And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence, total such numbers $= \frac{5.6!}{2!} = \frac{5}{2}.6!$

6. **(b)** Given digit 0, 1, 2, 5, 7, 9

a_1	a_2	a_3	a_4	a_5	a_6

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11\text{ K}$

Therefore, (1, 2, 9) (0, 5, 7)

Number of ways to arranging them

$= 3! \times 3! + 3! \times 2 \times 2 = 6 \times 6 + 6 \times 4 = 6 \times 10 = 60$

7. **(d)** 0, 1, 2, 3, 4, 5

Number of four-digit number starting with 5 is,

5			
	6	6	6

$= 6 \times 6 \times 6 = 216$

Number of four-digit numbers starting with 45 is,

4	5		
		6	6

$= 6 \times 6 = 36$

Number of four-digit numbers starting with 44 is,

4	4		
		6	6

$= 6 \times 6 = 36$

Number of four-digit numbers starting with 43 and greater than 4321 is,

4	3		
		3 (5/4/3)	6

$= 3 \times 6 = 18$

Number of four-digit numbers starting with 432 and greater than 4321 is,

4	3	2	
			4

$= 4$

Hence, required numbers $= 216 + 36 + 36 + 18 + 4 = 310$.

8. **(a)** Collecting different labels of balls drawn $= 10 \times 9 \times 8$

$\because$ arrangement is not required.

$\therefore$ the number of ways in which the balls can be chosen is,

$\frac{10 \times 9 \times 8}{3!} = 120$

9. **(a)** Number of numbers with one digit $= 4 = 4$

Number of numbers with two digits $= 4 \times 5 = 20$

Number of numbers with three digits $= 4 \times 5 \times 5 = 100$

Number of numbers with four digits $= 2 \times 5 \times 5 \times 5 = 250$

$\therefore$ Total number of numbers $= 4 + 20 + 100 + 250 = 374$

10. **(c)** One of the possible ΔOAB is $A(a, 0)$ and $B(0, b)$.

Area of $\Delta OAB = \frac{1}{2}|ab|$.

$\therefore$ $|ab| = 100$

$|a|\,|b| = 100$

But $100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20$ or 10×10

$\therefore$ For 1×100, $a = 1$ or -1 and $b = 100$ or -100

$\therefore$ Total possible pairs are 8.

Total possible pairs for $1 \times 100, 2 \times 50, 4 \times 25$ or 5×20 are 4×8.

And for 10×10 total possible pairs are 4.

$\therefore$ Total number of possible triangles with integral coordinates are $4 \times 8 + 4 = 36$.

11. **(a)** The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.

For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.

It the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.

Case 1: If we take 2 at thousands place.

The remaining digits can be filled as:

0, 1 and 3 as $2 + 1 + 0 + 3 = 6$ is divisible by 3.

0, 3 and 4 as $2 + 3 + 0 + 4 = 9$ is divisible by 3.

In both the above combinations the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 2 \times 3! = 12$.

Case 2: If we take 3 at thousands place. The remaining digits can be filled as:

0, 1 and 2 as $3 + 1 + 0 + 2 = 6$ is divisible by 3.

0, 2 and 4 as $3 + 2 + 0 + 4 = 9$ is divisible by 3.

In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case $= 2 \times 3! = 12$.

Case 3: If we take 4 at thousands place.

The remaining digits can be filled as:

0, 2 and 3 as $4 + 2 + 0 + 3 = 9$ is divisible by 3.

In the above combination, the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 3! = 6$.

$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$.

12. **(d)** Required n digit numbers is 3^n as each place can be filled by 2, 5, 7.

So smallest value of n such that $3^n > 900$. Therefore $n = 7$.

13. **(a)** 4 boys and 2 girls in circle

$$\Rightarrow 5! \times \frac{6!}{4!\,2!} \times 2!$$

$$\Rightarrow 5 \times 6!$$

14. **(c)** E, E, N, Q, U

(i) E .... $= 4! = 24$

(ii) N .... $= \frac{4!}{2} = 12$

(iii) QE .... $= 3! = 6$

(iv) QN .... $= \frac{3!}{2!} = 3$

(v) QUEEN $= 1$

$\therefore$ Required rank

$= (24) + (12) + (6) + (c) + (a) = 46$th

15. **(b)** ALLMS

No. of words starting with

A : $\underline{A}$ _ _ _ _ $\frac{4!}{2!} = 12$

L : $\underline{L}$ _ _ _ _ $4! = 24$

M : $\underline{M}$ _ _ _ _ $\frac{4!}{2!} = 12$

S : $\underline{S}\,\underline{A}$ _ _ _ _ $\frac{3!}{2!} = 3$

: $\underline{S}\,\underline{L}$ _ _ _ $3! = 6$

SMALL $\rightarrow$ 58th word

16. **(b)** $\sum_{R-1}^{10} (r^2+1)\underline{|r}$

$T_1 = (r^2 + 1 + r - r)\underline{|r} = (r^2 + r)\underline{|r} - (r - r)\underline{|r}$

$T_1 = r\underline{|r+r} - (r-1)\underline{|r}$

$T_1 = 1\underline{|2} - 0$

$T_2 = 2\underline{|3} - 1\underline{|2}$

$T_3 = 3\underline{|4} - 2\underline{|3}$

$T_{10} = 10\underline{|11} - 9\underline{|10}$

$$\sum_{R-1}^{10} (r^2+1)\underline{|r} = 10\underline{|11}$$

17. **(b)** M, EEE, D. I, T, RR, AA, NN

R – – E

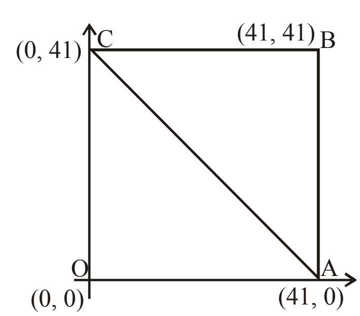
Two empty places can be filled with identical letters [EE, AA, NN] $\Rightarrow$ 3 ways

Two empty places, can be filled with distinct letters [M, E, D, I, T, R, A, N] $\Rightarrow {}^8P_2$

$\therefore$ Number of words $3 + {}^8P_2 = 59$

18. **(b)** Total number of integral points inside the square OABC $= 40 \times 40 = 1600$

No. of integral points on AC



= No. of integral points on OB

$= 40$ [namely (1, 1), (2, 2) ... (40, 40)]

$\therefore$ No. of integral points inside the ΔOAC

$= \frac{1600 - 40}{2} = 780$

19. **(d)** Four digits number can be arranged in $3 \times 4!$ ways.
Five digits number can be arranged in 5! ways.
Number of integers $= 3 \times 4! + 5! = 192$.

20. **(d)** Number of ways of selecting a man and a woman for a team from 15 men and 15 women
$= 15 \times 15 = (15)^2$
Number of ways of selecting a man and a woman for next team out of the remaining 14 men and 14 women.
$= 14 \times 14 = (14)^2$
Similarly for other teams
Hence required number of ways
$= (15)^2 + (14)^2 + ... + (1)^2 = \frac{15 \times 16 \times 31}{6} = 1240$

21. **(b)** Let no. of men $= n$
No. of women = 2
Total participants $= n + 2$
No. of games that M_1 plays with all other men
$= 2(n - 1)$
These games are played by all men
$M_2, M_3, ..., M_n$.
So, total no. of games among men $= n.2(n - 1)$.
However, we must divide it by '2', since each game is counted twice (for both players).
So, total no. of games among all men
$= n(n - 1)$ .... (i)
Now, no. of games M_1 plays with W_1 and $W_2 = 4$
(2 games with each)
Total no. of games that $M_1, M_2, ..., M_n$ play with W_1 and $W_2 = 4n$ .... (ii)
Given : $n(n - 1) - 4n = 66$
$\Rightarrow$ $n = 11, -6$
As the number of men can't be negative.
So, $n = 11$

22. **(b)** In 8 digits numbers, 4 places are odd places.
Also, in the given 8 digits, there are three odd digits 1, 1 and 3.
No. of ways three odd digits arranged at four even places $= \frac{4P_3}{2!} = \frac{4!}{2!}$
No. of ways the remaining five digits 2, 2, 2, 4 and 4 arranged at remaining five places $= \frac{5!}{3!2!}$
Hence, required number of 8 digits number
$= \frac{4!}{2!} \times \frac{5!}{3!2!} = 120$

23. **(d)** We know that any number is divisible by 9 if sum of the digits of the number is divisible by 9.
Now sum of the digits from 0 to 9
$= 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
Hence to form 8 digits numbers which are divisible by 9, a pair of digits either 0 and 9, 1 and 8, 2 and 7, 3 and 6 or 4 and 5 are not used.

Digits which are not used to form 8 digits number divisible by 9	**Number of 8 digits numbers which are divisible by 9**
0 and 9	$8 \times 7!$
1 and 8	$7 \times 7!$
2 and 7	$7 \times 7!$
3 and 6	$7 \times 7!$
4 and 5	$7 \times 7!$

Hence total number of 8 digits numbers which are divisible by 9
$= 8 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!) + 7 \times (7!)$
$= 36 \times (7!)$

24. **(b)** With 3 at unit place,
total possible four digit number (without repetition) will be 3 ! = 6
With 4 at unit place,
total possible four digit numbers will be 3! = 6
With 5 at unit place,
total possible four digit numbers will be 3! = 6
With 6 at unit place,
total possible four digit numbers will be 3! = 6
Sum of unit digits of all possible numbers
$= 6 \times 3 + 6 \times 4 + 6 \times 5 + 6 \times 6$
$= 6 [3 + 4 + 5 + 6]$
$= 6 [18] = 108$

25. **(d)** p :

0	0	0	0	0	place
5	4	3	2	1	ways

Total no. of ways $= 5 ! = 120$
Since all numbers are $> 20,000$
$\therefore$ all numbers 2, 3, 5, 7, 9 can come at first place.

q :

0	0	0	0	0	place
3	4	3	2	1	ways

Total no. of ways $= 3 \times 4 ! = 72$
($\because$ 2 and 9 can not be put at first place)
So, $p : q = 120 : 72 = 5 : 3$

26. **(d)** Given that number of white balls = 10
Number of green balls = 9
and Number of black balls = 7
$\therefore$ Required probability

$= (10+1)(9+1)(7+1) - 1$

$= 11.10.8 - 1 = 879$

[$\because$ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is

$(p+1)(q+1)(r+1) - 1$]

27. **(a)** 7 women can be arranged around a circular table in 6! ways.

Among these 7 men can sit in 7! ways.

Hence, number of seating arrangement

$= 7! \times 6!$

28. **(a)** Alphabetical order is

A, C, H, I, N, S

No. of words starting with A = 5! = 120

No. of words starting with C = 5! = 120

No. of words starting with H = 5! = 120

No. of words starting with I = 5! = 120

No. of words starting with N = 5! = 120

SACHIN – 1

$\therefore$ Sachin appears at serial no. 601

29. **(c)** Total number of arrangements of letters in the word GARDEN = 6 ! = 720 there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E. So, numbers of word with vowels in alphabetical order in

$\frac{1}{2} \times 720 = 360$

30. **(d)** ${}^{7-x}P_{x-3}$ is defined if

$7 - x \ge 0, x - 3 \ge 0$ and $7 - x \ge x - 3$

$\Rightarrow 3 \le x \le 5$ and $x \in \mathbf{I}$

$\therefore x = 3, 4, 5$

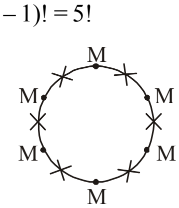
$\therefore f(3) = {}^{7-3}P_{3-3} = {}^{4}P_0 = 1$

$\therefore f(4) = {}^{7-4}P_{4-3} = {}^{3}P_1 = 3$

$\therefore f(5) = {}^{7-5}P_{5-3} = {}^{2}P_2 = 2$

Hence range = {1, 2, 3}

31. **(a)** No. of ways in which 6 men can be arranged at a round table = (6 – 1)! = 5!



Now women can be arranged in ${}^{6}P_5$

= 6! ways.

Total Number of ways = 6! × 5!

32. **(b)** Required sum

$= (2+4+6+...+100) + (5+10+15+...+100)$
$- (10+20+...+100)$

$= 2(1+2+3...+50) + 5(1+2+3+...+50)$
$-10(1+2+3+....+10)$

$= 2550 + 1050 - 530 = 3050.$

33. **(c)** Total number of numbers

$= 3 \times 5 \times 5 \times 5 - 1 = 374$

34. **(d)** Total number of numbers formed using 0, 1, 2, 3, 5, 7

$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

35. **(120.00)**

For vowels not together

Number of ways to arrange L, T, T, R $= \frac{4!}{2!}$

Then put both E in 5 gaps formed in ${}^{5}C_2$ ways.

$\therefore$ No. of ways $= \frac{4!}{2!} \cdot {}^{5}C_2 = 120$

36. **(240)**

$S \to 2,\ L \to 2,\ A, B, Y, U.$

$\therefore$ Required number of ways $= {}^{2}C_1 \times {}^{5}C_2 \times \frac{4!}{2!} = 240.$

37. **(d)** Since, each section has 5 questions.

$\therefore$ Total number of selection of 5 questions

$= 3 \times {}^{5}C_1 \times {}^{5}C_1 \times {}^{5}C_3 + 3 \times {}^{5}C_1 \times {}^{5}C_2 \times {}^{5}C_2$

$= 3 \times 5 \times 5 \times 10 + 3 \times 5 \times 10 \times 10$

$= 750 + 1500 = 2250.$

38. **(135)**

Select any 4 correct questions in ${}^{6}C_4$ ways.

Number of ways of answering wrong question = 3

$\therefore$ Required number of ways $= {}^{6}C_4 (1)^4 \times 3^2 = 135.$

39. **(54)**

Let xyz be the three digit number

$x + y + z = 10,\ x \le 1,\ y \ge 0,\ z \ge 0$

$x - 1 = t \Rightarrow x = 1 + t$ $\qquad x - 1 \ge 0,\ t \ge 0$

$t + y + z = 10 - 1 = 9$ $\qquad 0 \le t, z, z \le 9$

$\therefore$ Total number of non-negative integral solution

$= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = \frac{11 \cdot 10}{2} = 55$

But for $t = 9$, $x = 10$, so required number of integers

$= 55 - 1 = 54.$

40. **(a)** Number of two consecutive stations (Blue lines) = n

Number of two non-consecutive stations (Red lines)

$= {}^{n}C_2 - n$

Now, according to the question, ${}^{n}C_2 - n = 99n$

$\Rightarrow \dfrac{n(n-1)}{2}-100n=0 \;\Rightarrow n(n-1-200)=0$

$\Rightarrow n-1-200=0 \Rightarrow n=201$

41. **(51)** $\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r.\,{}^{25}C_r + \sum_{r=0}^{25}{}^{25}C_r$

$=4\sum_{r=1}^{25} r\times\dfrac{25}{r}\,{}^{24}C_{r-1}+2^{25} = 100\sum_{r=1}^{25}{}^{24}C_{r-1}+2^{25}$

$=100.2^{24}+2^{25}=2^{25}(50+1)=51.2^{25}$

Hence, by comparison $k=51$

42. **(490)**

0 Red, 1 Red, 2 Red, 3 Red

Number of ways of selecting atmost three red balls

$={}^7C_4+{}^5C_1\cdot{}^7C_3+{}^5C_2\cdot{}^7C_2+{}^5C_3\cdot{}^7C_1$

$=35+175+210+70=490$

43. **(c)** We know ${}^nC_r$ is greatest at middle term.

So, $a=({}^{19}C_p)_{\max}={}^{19}C_{10}={}^{19}C_9$

$b=({}^{20}C_q)_{\max}={}^{20}C_{10}$

$c=({}^{21}C_6)_{\max}={}^{21}C_{10}={}^{21}C_{11}$

Now, $\dfrac{a}{{}^{19}C_9}=\dfrac{b}{\frac{20}{10}.\,{}^{19}C_9}=\dfrac{c}{\frac{21}{11}.\frac{20}{10}\,{}^{19}C_9}$

$\Rightarrow \dfrac{a}{1}=\dfrac{b}{2}=\dfrac{c}{42/11}$ $\quad\therefore\; \dfrac{a}{11}=\dfrac{b}{22}=\dfrac{c}{42}$

44. **(2454)** EXAMINATION

2N, 2A, 2I, E, X, M, T, O

Case I : If all are different, then

${}^8p_4=\dfrac{8!}{4!}=8.7.6.5=1680$

Case II : If two are same and two are different, then

${}^3C_1\,.\,{}^7C_2\,.\,\dfrac{4!}{2!}=3.21.12=756$

Case III : If two are same and other two are same, then

${}^3C_2\,.\,\dfrac{4!}{2!2!}=3.6=18$

$\therefore$ Total cases $=1680+756+18=2454$

45. **(1)** $\dfrac{36}{r+1}\times{}^{35}C_r(k^2-3)={}^{35}C_r.6$

$\Rightarrow k^2-3=\dfrac{r+1}{6}$

$\Rightarrow k^2=3+\dfrac{r+1}{6}$

r can be 5, 35 for $k\in I$

$r=5, k=\pm 2$

$r=35, k=\pm 3$

Hence, number of ordered pairs = 4.

46. **(c)** Number of ways of selecting 10 objects

$=(10I, 0D)$ or $(9I, 1D)$ or $(8I, 1D)$ or ... $(0I, 10D)$

Here, D signifies distinct object and I indicates identical object

$=1+{}^{21}C_1+{}^{21}C_2+\ldots+{}^{21}C_{10}=\dfrac{2^{21}}{2}=2^{20}$

47. **(c)** Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$={}^{n+5}C_3-{}^nC_3-{}^5C_3=1750$

$\Rightarrow \dfrac{(n+5)!}{3!(n+2)!}-\dfrac{n!}{3!(n-3)!}-\dfrac{5!}{3!\,2!}=1750$

$\Rightarrow \dfrac{(n+5)(n+4)(n+3)}{6}-\dfrac{n(n-1)(n-2)}{6}=1760$

$\Rightarrow n^2+3n-700=0 \Rightarrow \quad n=25 \quad [n=-28 \text{ rejected}]$

48. **(a)** Total number of beams $={}^{20}C_2-20=190-20=170$

49. **(b)** Since, m = number of ways the committee is formed with at least 6 males

$={}^8C_6\,.\,{}^5C_5+{}^8C_7\,.\,{}^5C_4+{}^8C_8\,.\,{}^5C_3=78$

and n = number of ways the committee is formed with at least 3 females

$={}^5C_3\,.\,{}^8C_8+{}^5C_4\,.\,{}^8C_7+{}^5C_5\,.\,{}^8C_6=78$

Hence, m = n = 78

50. **(a)** $\because$ There are total 9 digits and out of which only 3 digits are odd.

$\therefore$ Number of ways to arrange odd digits first $={}^4C_3.\dfrac{3!}{2!}$

Hence, total number of 9 digit numbers

$=\left({}^4C_3.\dfrac{3!}{2!}\right).\dfrac{6!}{2!\,4!}=180$

51. **(a)** ${}^mC_2\times 2={}^mC_1\cdot{}^2C_1\times 2+84$

$m(m-1)=4m+84$

$m^2-5m-84=0$

$m^2-12m-7m-84=0$

$m(m-12)+7(m-12)=0$

$m=12, m=-7$

$\because \quad m>0$

$m=12$

52. **(d)** Consider the expression,

$\dfrac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}=\dfrac{{}^{20}C_{i-1}}{{}^{21}C_1}$

$$=\frac{20!}{(i-1)!(21-i)!}\times\frac{i!(21-i)!}{21!}=\frac{i}{21}$$

$$\therefore\quad \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\sum_{i=1}^{20}\left(\frac{i}{21}\right)^3=\frac{(1)}{(21)^3}\sum_{i=1}^{20}i^3$$

$$=\frac{1}{(21)^3}\times\left(\frac{20\times 21}{2}\right)^2=\frac{100}{21}$$

$$\because\quad \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$$

$\therefore\quad k=100$

53. **(c)** Since, the number of ways to select 2 girls is ${}^5C_2$.
Now, 3 boys can be selected in 3 ways.
(a) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways
(b) Selection of B and selection of any 2 two other boys (except A) in ${}^5C_2$ ways
(c) Selection of 3 boys (except A and B) in ${}^5C_3$ ways
Hence, required number of different teams
$= {}^5C_2({}^5C_2+{}^5C_2+{}^5C_3)=300$

54. **(d)** If all four letters are different then the number of words ${}^5C_4\times 4!=120$
If two letters are R and other two different letters are chosen from B, A, C, K then the number of words

$$={}^4C_2\times\frac{4!}{2!}=72$$

If two letters are A and other two different letters are chosen from B, R, C, K then the number of words

$$={}^4C_2\times\frac{4!}{2!}=72$$

If word is formed using two R's and two A's then the number of words $=\dfrac{4!}{2!\,2!}=6$

Therefore, the number of four-letter words that can be formed $=120+72+72+6=270$

55. **(d)** $\therefore$ Required number of ways $={}^6C_4\times{}^3C_1\times 4!$
$=15\times 3\times 24=1080$

56. **(b)**
X → 4 ladies, 3 men
Y → 3 ladies, 4 men

Possible cases for X are
(1) 3 ladies, 0 man
(2) 2 ladies, 1 man
(3) 1 lady, 2 men
(4) 0 ladies, 3 men
Possible cases for Y are
(1) 0 ladies, 3 men
(2) 1 lady, 2 men
(3) 2 ladies, 1 man
(4) 3 ladies, 0 man
No. of ways $={}^4C_3\,.\,{}^4C_3+({}^4C_2\,.\,{}^3C_1)^2+({}^4C_1\,.\,{}^3C_2)^2+({}^3C_3)^2$
$=16+324+144+1=485$

57. **(c)** $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2}=11$

$$\Rightarrow\quad \frac{\dfrac{(n+2)(n+1)n(n-1)(n-2)(n-3)}{6.5.4.3.2.1}}{\dfrac{(n-2)(n-3)}{2.1}}=11$$

$\Rightarrow\quad (n+2)(n+1)n(n-1)=11\,.\,10.\,9.\,4$
$\Rightarrow\quad n=9$
$n^2+3n-108=(9)^2+3(9)-108$
$=81+27-108$
$=108-108=0$

58. **(d)** $\displaystyle\sum_{r=1}^{15}r^2\left(\frac{{}^{15}C_r}{{}^{15}C_{r-1}}\right)$

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}}=\frac{\dfrac{\lfloor 15}{\lfloor 15-r\lfloor r}}{\dfrac{\lfloor 15}{\lfloor r-1\lfloor 16-r}}=\frac{\dfrac{1}{\lfloor 15-r\lfloor r-1}}{\dfrac{1}{\lfloor r-1\,|\lfloor 15-r.(16-r)}}$$

$$=\frac{16-r}{r}$$

$$=\sum_{r=1}^{15}r^2\left(\frac{16-r}{r}\right)=\sum_{r=1}^{15}r(16-r)$$

$$=16\sum_{r=1}^{15}r-\sum_{r=1}^{15}r^2$$

$$=\frac{16\times 15\times 16}{2}-\frac{15\times 31\times 16}{6}$$

$=8\times 15\times 16-5\times 8\times 31=1920-1240=680$

59. **(c)** Given
$n(\text{A})=4,\ n(\text{B})=2,\ n(\text{A}\times\text{B})=8$
Required number of subsets
$={}^8C_3+{}^8C_4+....+{}^8C_8=2^8-{}^8C_0-{}^8C_1-{}^8C_2$
$=256-1-8-28=219$

60. **(a)** Number of diagonal = 54

$\Rightarrow \dfrac{n(n-3)}{2} = 54$

$\Rightarrow n^2 - 3n - 108 = 0 \Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow n(n-12) + 9(n-12) = 0$

$\Rightarrow n = 12, -9 \Rightarrow n = 12\ (\because n \neq -9)$

61. **(c)** Given

$n(A) = 2, n(B) = 4, n(A \times B) = 8$

Required number of subsets =

${}^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 1 - 8 - 28 = 219$

62. **(b)** We know,

$T_n = {}^nC_3,\ T_{n+1} = {}^{n+1}C_3$

ATQ, $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$

$\Rightarrow {}^nC_2 = 10$

$\Rightarrow n = 5.$

63. **(b)** Required number of triangles

$= {}^{12}C_3 - ({}^3C_3 + {}^4C_3 + {}^5C_3) = 205$

64. **(c)** 30 marks to be alloted to 8 questions. Each question has to be given ≥ 2 marks

Let questions be a, b, c, d, e, f, g, h

and $a + b + c + d + e + f + g + h = 30$

Let $a = a_1 + 2$ so, $a_1 \geq 0$

$b = a_2 + 2$ so, $a_2 \geq 0, ..., a_8 \geq 0$

So, $\left.\begin{matrix} a_1 + a_2 + ... + a_8 \\ +2 + 2 + ... + 2 \end{matrix}\right\} = 30$

$\Rightarrow a_1 + a_2 + ... + a_8 = 30 - 16 = 14$

So, this is a problem of distributing 14 articles in 8 groups.

Number of ways = ${}^{14+8-1}C_{8-1} = {}^{21}C_7$

65. **(b)**

L	O	Y
2	2	4
≥ 1	≥ 1	2 ≤

$\Rightarrow$

L	O	Y
1	1	2
1	2	1
2	1	1
2	2	0

Required number of ways

$= {}^2C_1 \times {}^2C_1 \times {}^2C_2 + {}^2C_1 \times {}^2C_2 \times {}^4C_1 + {}^2C_2 \times {}^2C_1 \times {}^4C_1 + {}^2C_2 \times {}^2C_2 \times {}^4C_0$

$= 2 \times 2 \times \dfrac{4 \times 3}{2} + 2 \times 1 \times 4 + 1 \times 2 \times 4 + 1 \times 1 \times 1$

$= 24 + 8 + 8 + 1 = 41$

66. **(a)** There are 2 vowels and 4 consonants in the letters a, b, c, d, e, f.

If we select one vowel, then number of arrangements

$= {}^2C_1 \times {}^4C_2 \times 3! = 2 \times \dfrac{4 \times 3}{2} \times 3 \times 2 = 72$

If we select two vowels, then number of arrangements

$= {}^2C_2 \times {}^4C_1 \times 3! = 1 \times 4 \times 6 = 24$

Hence, total number of arrangements

$= 72 + 24 = 96$

67. **(a)** $n = {}^mC_2 = \dfrac{m(m-1)}{2}$

Since m and $(m-1)$ are two consecutive natural numbers, therefore their product is an even natural number. So $\dfrac{m(m-1)}{2}$ is also a natural number.

Now $\dfrac{m(m-1)}{2} = \dfrac{m^2 - m}{2}$

$\therefore {}^{\frac{m(m-1)}{2}}C_2 = \dfrac{\left(\dfrac{m^2-m}{2}\right)\left(\dfrac{m^2-m}{2} - 1\right)}{2}$

$= \dfrac{m(m-1)(m^2 - m - 2)}{8}$

$= \dfrac{m(m-1)[m^2 - 2m + m - 2]}{8}$

$= \dfrac{m(m-1)[m(m-2) + 1(m-2)]}{8}$

$= \dfrac{m(m-1)(m-2)(m+1)}{8}$

$= \dfrac{3 \times (m+1)m(m-1)(m-2)}{4 \times 3 \times 2 \times 1} = 3\left({}^{m+1}C_4\right)$

68. **(d)** **Statement - 1 :** $n(A) = p, n(B) = q, q > p$

Total number of functions from $A \to B = q^p$

It is a true statement.

Statement - 2 : The total number of selections of p different objects out of q objects is ${}^qC_p$.

It is also a true statement and it is a correct explanation for statement - 1 also.

69. **(c)** Set $A = \{a_1, a_2, ..., a_{20}\}$ has 20 distinct elements.

We have to select 5-element subset.

$\therefore$ Number of 5-element subsets = ${}^{20}C_5$

According to question

${}^{20}C_5 = \left({}^{19}C_4\right).k$

$\Rightarrow \dfrac{20!}{5!\ 15!} = k.\left(\dfrac{19!}{4!\ 15!}\right)$

$\Rightarrow \dfrac{20}{5} = k \Rightarrow k = 4$

70. **(a)** Number of required triangles = ${}^{10}C_3 - {}^{6}C_3$

$= \frac{10\times9\times8}{6} - \frac{6\times5\times4}{6} = 120 - 20 = 100$

71. **(a)** The number of ways of distributing 10 identical balls in 4 distinct boxes

$= {}^{10-1}C_{4-1} = {}^{9}C_3$

72. **(c)** Two balls are taken from each urn

Total number of ways = ${}^{3}C_2 \times {}^{9}C_2$

$= 3 \times \frac{9\times8}{2} = 3\times36 = 108$

73. **(c)** 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in ${}^{6}C_4 \times {}^{3}C_1$ ways

Then 4 novels with one dictionary in the middle can be arranged in 4! ways.

$\therefore$ Total ways of arrangement

$= {}^{6}C_4 \times {}^{3}C_1 \times 4! = 1080$

74. **(d)** First let us arrange M, I, I, I, P, P

Which can be done in $\frac{7!}{4!2!}$ ways

* M * I * I * I * P * P *

Now 4 S can be kept at any of the * places in ${}^{8}C_4$ ways so that no two *S* are adjacent.

Total required ways

$= \frac{7!}{4!2!}\,{}^{8}C_4 = \frac{7!}{4!2!}\,{}^{8}C_4 = 7\times {}^{6}C_4 \times {}^{8}C_4$

75. **(a)** Set S = {1, 2, 3, ... 12}

$A\cup B\cup C = S,\ A\cap B = B\cap C = A\cap C = \phi$

$\therefore$ Each sets contain 4 elements.

$\therefore$ The number of ways to partition

$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$= \frac{12!}{4!8!}\times\frac{8!}{4!4!}\times\frac{4!}{4!0!} = \frac{12!}{(4!)^3}$

76. **(c)** The number of ways can vote

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

77. **(d)** ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$

$= {}^{50}C_4 + \left[{}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3 \right]$

We know $\left[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r \right]$

$= ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

$= ({}^{51}C_4 + {}^{51}C_3) + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$

Proceeding in the same way, we get

$= {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$.

78. **(b)** We know that the number of ways of distributing n identical items among *r* persons, when each one of them receives at least one item is ${}^{n-1}C_{r-1}$

$\therefore$ The required number of ways

$= {}^{8-1}C_{3-1} = {}^{7}C_2 = \frac{7!}{2!5!} \quad = \frac{7\times6}{2\times1} = 21$

79. **(c)** According to given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining question

$= {}^{5}C_4 \times {}^{8}C_6 = 140$ ways

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions $= {}^{5}C_5 \times {}^{8}C_5 = 56$ ways

Therefore, total number of choices

$= 140 + 56 = 196$.

80. **(c)** ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2\,{}^{n}C_r$

$= {}^{n}C_{r-1} + {}^{n}C_r + {}^{n}C_r + {}^{n}C_{r+1}$

$\left[\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right]$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

81. **(d)** We know that a number is divisible by 3 only when the sum of the digits is divisible by 3. The given digits are 0, 1, 2, 3, 4, 5.

Here the possible number of combinations of 5 digits out of 6 are ${}^{5}C_4 = 5$, which are as follows–

$1 + 2 + 3 + 4 + 5 = 15 = 3 \times 5$ (divisible by 3)

$0 + 2 + 3 + 4 + 5 = 14$ (not divisible by 3)

$0 + 1 + 3 + 4 + 5 = 13$ (not divisible by 3)

$0 + 1 + 2 + 4 + 5 = 12 = 3 \times 4$ (divisible by 3)

$0 + 1 + 2 + 3 + 5 = 11$ (not divisible by 3)

$0 + 1 + 2 + 3 + 4 = 10$ (not divisible by 3)

Thus the number should contain the digits 1, 2, 3, 4, 5 or the digits 0, 1, 2, 4, 5.

Taking 1, 2, 3, 4, 5, the 5 digit numbers are

$= 5! = 120$

Taking 0, 1, 2, 4, 5, the 5 digit numbers are

$= 5! - 4! = 96$

$\therefore$ Total number of numbers $= 120 + 96 = 216$