WORK, POWER & ENERGY

KEY CONCEPT

WORK

- Whenever a force acting on a body, displaces it in its direction, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the point of application, $W = \vec{F}.\vec{d}$
- Work is a scalar quantity.

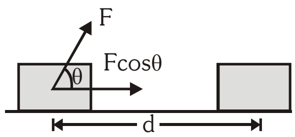
Work done by a constant force :

If the direction and magnitude of a force applied on a body is constant, the force is said to be constant. Work done by a constant force, W = Force × component of displacement along force

= displacement × component of force along displacement.

The work done will, be $W = (F\cos\theta)\, d$

$= F\,(d\cos\theta)$



In vector form, $W = \vec{F}\cdot\vec{d}$

Note : The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

Work done by a variable force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement in a body from position P_1 to position P_2. To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. If $d\vec{r}$ be small displacement of point of application and $\vec{F}$ be the force applied on the body, the work done by force is $dW = \vec{F}\cdot d\vec{r}$

The total work done in displacing body from P_1 to P_2 is given by $\int dW = \int_{P_1}^{P_2} \vec{F}.d\vec{r} \Rightarrow W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$

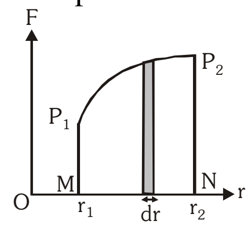
If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively, the total work done $W = \int_{r_1}^{r_2} \vec{F}.d\vec{r}$

Note : When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

Here : $W_s = \int_{x_i}^{x_f} -k\ x\ dx = \frac{1}{2}k\ \left(x_i^2 - x_f^2\right)$

Calculation of work done from force displacement graph :

Suppose a body, whose initial position is r_1, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position r_2. From position r to r + dr or for small displacement dr, the work done will be $\vec{F}.d\vec{r}$ whose value will be the area of the shaded strip of width dr. The work done on the body in displacing it from position r_1 to r_2 will be equal to the sum of areas of all such strips



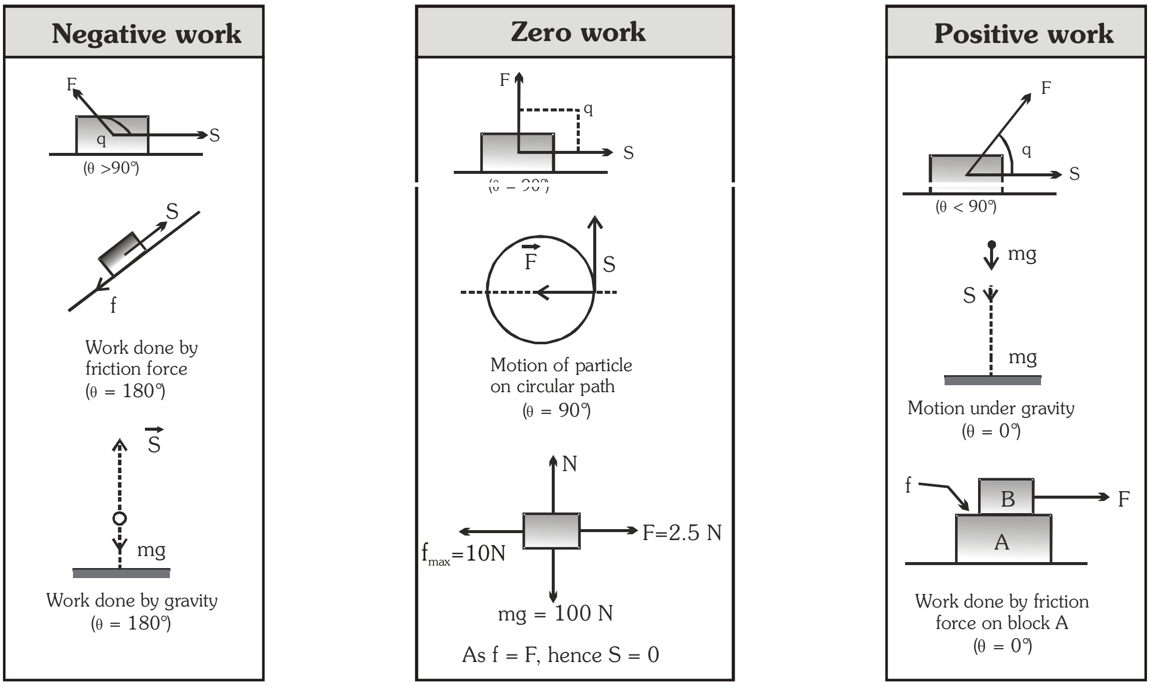
Thus, total work done, $W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} F.dr$ = Area of P_1P_2NM

The area of the graph between curve and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cos θ and r.

Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero



UNITS :

SI Unit : joule (J).

joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

1 joule = 1 newton × 1 meter = 1 kgm^2s^{-2}

erg : One erg of work is said to be done when a force of one dyne displaces a body by one cm. in the direction of force.

1 erg = 1 dyne × 1 cm. = 1 gm. $cm^2\ s^{-2}$

Other Units :

(a) 1 joule = 10^7 erg
(b) 1 erg = 10^{-7} joule
(c) 1 eV = 1.6×10^{-19} joule
(d) 1 joule = 6.25×10^{18} eV
(e) 1 MeV = 1.6×10^{-13} J
(f) 1 J = 6.25×10^{12} MeV
(g) 1 kilo watt hour (kWh) = 3.6×10^6 joule

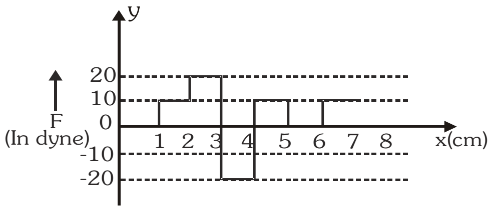
DIMENSIONS :

[Work] = [Force] [Displacement]= $[MLT^{-2}][L] = [ML^2T^{-2}]$

Ex. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joule.

Sol. $W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2) dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135 J$

Ex. For the force displacement diagram shown in adjoining diagram. Calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



Sol. Work = Area under the curve and displacement axis = 10 + 20 – 20 + 10 = 20 erg

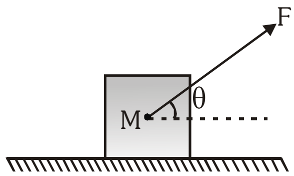
Ex. Calculate work done to move a body of mass 10 kg along a smooth inclined plane ($\theta = 30°$) with constant velocity through a distance of 10 m.

Sol. Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So

$F - Mg \sin 30° = 0 \Rightarrow F = Mg \sin 30°$ & d=10m

$W = Fd\cos\theta = (Mg\sin 30°)d \cos 0° = 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500$ J

Ex. Calculate work done in pulling an object with a constant force F as shown in figure. (given that the ground is rough with coefficient of friction μ)



Sol. From the figure $F \sin\theta + N = Mg$

$\therefore$ $N = Mg - F \sin\theta$

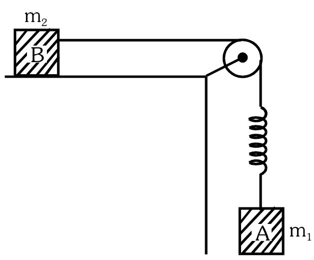
$F \cos\theta = f = \mu N = \mu[Mg - F \sin\theta]$

$F(\cos\theta + \mu \sin\theta) = \mu Mg$

$\therefore$ $F = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$ = force required to pull an object

Work done in pulling an object $W = Fd = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$

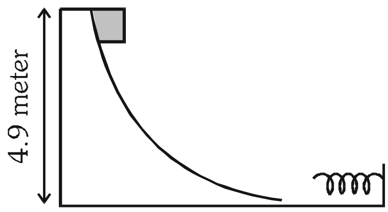
Ex. If the two blocks moves with a constant uniform speed then find coefficient of friction between the surface of the block B and the table. The spring is massless and the pulley is frictionless.



Sol. **For block B** : $T = f = \mu m_2 g$ and **For block A** : $T = m_1 g$

By solving above equations $\mu = \frac{m_1}{m_2}$

Ex. Figure shows a smooth curved track terminating in a smooth horizontal part, A spring of force constant 400 N/m is attached at one end to a wedge fixed rigidly with a horizontal part. A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Sol. From the law of conservation of mechanical energy $\Rightarrow mgh = \frac{1}{2}kx^2$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times (0.04) \times 9.8 \times 4.9}{400}} = 9.8 \text{ cm.}$$

ENERGY

- The energy of a body is defined as the capacity of doing work.
- There are various form of energy
  (i) mechanical energy (ii) chemical energy (iii) electrical energy (iv) sound energy
  (v) light energy etc (vi) magnetic energy (vii) nuclear energy
- Energy of an isolated system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

  Examples

  Electrical energy $\xrightarrow{\text{Motor}}$ Mechanical energy

  Mechanical energy $\xrightarrow{\text{Generator}}$ Electrical energy

  Light energy $\xrightarrow{\text{Photocell}}$ Electrical energy

  Electrical energy $\xrightarrow{\text{Heater}}$ Heat energy

  Electrical energy $\xrightarrow{\text{Radio / Speaker}}$ Sound energy

  Nuclear energy $\xrightarrow{\text{Nuclear Reactor}}$ Electrical energy

  Chemical energy $\xrightarrow{\text{Cell}}$ Electrical energy

  Electrical energy $\xrightarrow[\text{Chargeable}]{\text{Secondary Cell}}$ Chemical energy

- Energy is a scalar quantity
- Unit : Its unit is same as that of work or torque. In MKS : joule, watt second ; In CGS : erg
  Note : 1 eV = 1.6×10^{-19} joule; 1 kWh = 3.6×10^{6} joule; 10^{7} erg = 1 joule.
- Dimension $[M^1L^2T^{-2}]$
- According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other. Energy equivalent of mass m is, $E = mc^2$ where, m : mass of the particle, c : velocity of light , E : equivalent energy corresponding to mass m.
- In mechanics we are concerned with mechanical energy only which is of two type :
  (i) kinetic energy (ii) potential energy

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.
- If a body of mass m is moving with velocity v, its kinetic energy KE = 1/2 mv^2.
- Kinetic energy is always positive.
- If linear momentum of body is p, the kinetic energy for translatory motion is KE= $\frac{p^2}{2m} = \frac{1}{2}mv^2$.

Ex. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Sol. Initial kinetic energy, $K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 J = 1000 J$

Final kinetic energy, $K_f = \frac{10}{100} \times 1000 J = 100 J$

If v_f is emergent speed of the bullet, then $\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100$

$\Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2\ ms^{-1}$.

Note that the speed is reduced by approximately 68% and not 90%.

Work Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in it's kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Ex. A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Sol. Work done by all forces $= W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

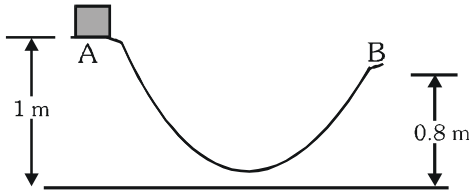
Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2 d$

Ex. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent for the first one second.

Sol. $\because x = \frac{t^3}{3}$ $\therefore v = \frac{dx}{dt} = t^2$, Velocity at t = 0, u = 0 and at t = 1s v = 1 m/s

Using work energy theorem : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}1(1)^2 = 0.5 J$

Ex. A block of mass 1kg is placed at the point A of a rough track shown in figure. If it slightly pushed towards right it stops at the point B of the track. Calculate the workdone by the frictional force on the block during its transit from A to B.



Sol. $W_c + W_{nc} + W_{ext} = \Delta K$

$mg(1 - 0.8) + W_{nc} + 0 = 0 \Rightarrow W_{nc} = -1 \times 9.8 \times 0.2 = -1.96$ J

POWER

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.

In many cases it is useful to know not just the total amount of work being done, but how fast the work is done. We define power as the rate at which work is being done.

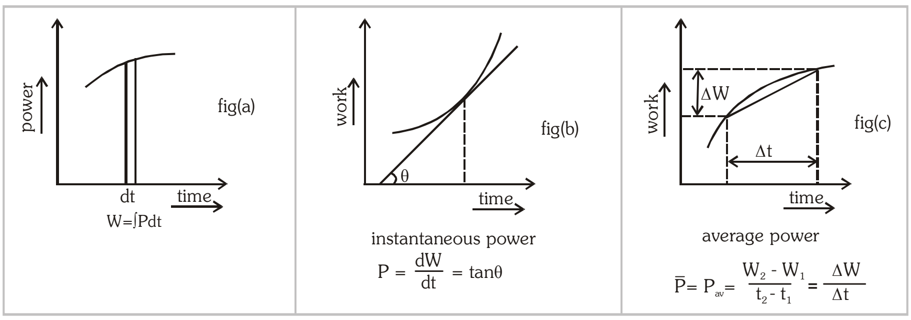
$$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

If ΔW is the amount of work done in the time interval Δt. Then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W. The definition of power is applicable to all types of work like mechanical, electrical, thermal. Instanteneous power $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt

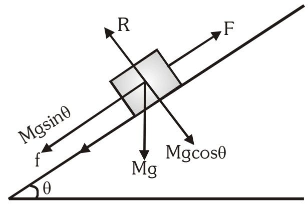


- Area under power–time graph gives the work done. $W = \int P dt$ (See Fig. a)
- The slope of tangent at a point on work time graph gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work time graph gives average power between two points (See Fig. c)

- For a system of varying mass $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$

- Ifv = constant then $F = v\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$

Ex. A truck pulls a mass of 1200 kg at constant speed of 10m/s on a level road. The tension in coupling is 1000 N. What is the power spent on the mass. Find tension when truck moves up a road with inclination 1 in 6.



Sol. Force applied by truck f = 1000 N

Power spent in pulling the mass $P = fv = 1000 \times 10 = 10^4$ W

Here $\sin\theta = 1/6$, the required force for truck to move up is

$F = f + Mg\sin\theta$

$F = 1000\text{ N} + 1200 \times 9.8 \times \frac{1}{6} = 2960\text{ N}$

CONSERVATIVE FORCE

- A force is said to be conservative if the work done by or against the force is independent of path and depends only on initial and final positions
- It does not depend on the nature of path followed between the initial and final positions.

Examples of Conservative Force

All central forces are conservative like gravitational, electrostatic, elastic force, restoring force due to spring etc.

- In presence of conservative forces mechanical energy remains constant.
- Work done along a closed path or in a cyclic process is zero. i.e. $\oint \vec{F}.d\vec{r} = 0$

CENTRAL FORCE

The force whose line of action always passes through a fixed point (which is known as centre of force) and magnitude of force depends only on the distance from this point is known as **central force.**

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

$\vec{F} = \frac{k}{r^2}\hat{r}$ is central force like Gravitational force and Coulomb force.

- All central forces are conservative forces
- Central forces are function of position

NON CONSERVATIVE FORCE

A force is said to be non–conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non–conservative force field.

The frictional forces are non–conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of non–conservative force

The velocity–dependent forces such as air resistance, viscous force etc. are non–conservative forces.

Conservative Forces	Non–conservative Forces
• Work done does not depend upon path.	• Work done depends upon path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• Central forces, spring forces etc. are conservative forces	• Force are velocity–dependent & retarding in nature e.g. friction, viscous force etc.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, doesnot change.	• Work done against a non–conservative force may be dissipated as heat energy.
• Work done is completely recoverable.	• Work done is not completely recoverable.

Potential energy

- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the workdone by an external agent against the action of conservative force in order to shift it from reference point. (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the body in moving from its present position to reference position.
- At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.
- Relationship between conservative force field and potential energy :

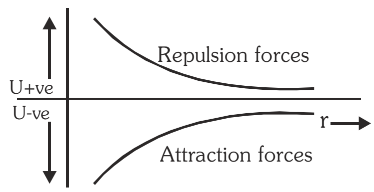
$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then $F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$

- Potential energy may be positive or negative

  i) Potential energy is positive, if force field is repulsive in nature

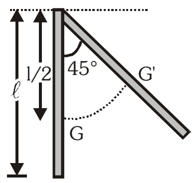
  ii) Potential energy is negative, if force field is attractive in nature



- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice–versa.

- If r ↑, U ↓, force field is repulsive in nature.

Ex. A meter scale of mass m initially vertical is displaced at 45^0 keeping the upper end fixed. Find out the change in potential energy.



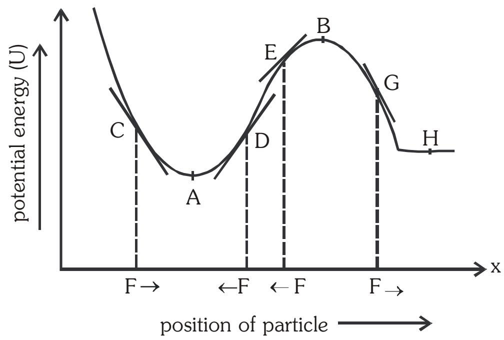
Sol. $\Delta U = mg\,\Delta h_{cm} = mg\frac{\ell}{2}(1-\cos\theta) = mg \times \frac{1}{2}\,(1-\cos 45^0) = \frac{mg}{2}\left(1-\frac{1}{\sqrt{2}}\right)$

Ex. A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. Calculate the work done in keeping it vertical with one of its ends touching the ground.

Sol. As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length

$= \ell/2$ Work done $W = mgh = mg\,\frac{\ell}{2} = (20)(9.8)\left(\frac{4}{2}\right) = 392$ J

POTENTIAL ENERGY CURVE AND EQUILIBRIUM



It is a curve which shows change in potential energy

with postion of a particle.

Stable Equilibrium :

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium

then it is said to be in stable equilibrium

At point **C :** slope $\frac{dU}{dx}$ is negative so F is positive

At point **D :** slope $\frac{dU}{dx}$ is positive so F is negative

At point **A :** it is the point of stable equilibrium.

At **A** $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = positive

Unstable equilibrium :

When a particle is slightly displaced from equilbrium and it tends to move away from equilibrium position then it is said to be in unstable equilbirum

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : it is the point of unstable equilibrium.

At **B** $U = U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative

Neutral equilibrium :

When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium

Point **H** is at neutral equilibrium $\Rightarrow U$ = constant ; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

Ex. The potential energy for a conservative force system is given by $U = ax^2 - bx$. Where a and b are constants find out

(a) The expression of force

(b) Equilibrium position

(c) Potential energy at equilibrium.

Sol. (a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$ (c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

Law of conservation of Mechanical energy

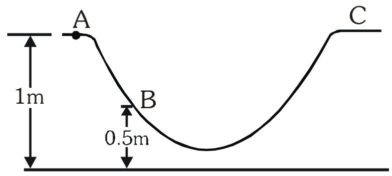
Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles and the work done by all other forces is zero.

From work energy theorem $W = \Delta KE$

For internal conservative forces $W_{int} = -\Delta U$

So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE \Rightarrow \Delta(KE + U) = 0 \Rightarrow KE + U$ = (constant)

Ex. A particle is placed at the point A of a frictionless track ABC. It is pushed slightly towards right. Find its speed when it reaches the point B. [Take $g=10\ m/s^2$]



Sol. $mg(1 - 0.5) = \frac{1}{2} \times m \times v^2 \Rightarrow v^2 = (2)(10)(0.5) \Rightarrow v = \sqrt{10}$ m/s

Ex. Determine the average force necessary to stop a bullet of mass 20 g and speed 250 ms^{-1} as it penetrates wood to a distance of 12 cm.

Sol. If F newton be the retarding force, then the work done by force $W = F \times s = F \times 0.12$ joule

Loss of kinetic energy $= \frac{1}{2} \times \frac{20}{1000} \times (250)^2 = 625$ joule

(This kinetic energy is consumed in stopping the bullet and is converted into heat energy)

Applying work–energy theorem, $F \times 0.12 = 625 \Rightarrow F = \frac{625}{0.12}\ N = 5.2 \times 10^3\ N$

It is interesting to note that the retarding force is nearly 30,000 times the weight of the bullet.

Ex. A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = \frac{1}{2}t^2$ where x is in meter and t in sec. Find the work done by the force in first two seconds.

Sol. Work done = change in kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{2t}{2}\right)^2 = \frac{1}{2}\times 8\times\left[\frac{2\times 2}{2}\right]^2 = 16J$

Ex. A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then determine height upto the body would reach.

Sol. $\frac{1}{4}mgh = mgh' \quad \therefore\ h' = \frac{h}{4} = \frac{1}{4}\times 20 = 5\text{ cm}$

Ex. Calculate the stopping distance for a vehicle of mass m moving with speed v along level road. (μ is the coefficient of friction between tyres and the road)

Sol. When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = \frac{1}{2}mv^2$

and if S is the stopping distance, work done by the friction
$W = FS\cos\theta = \mu mgS\cos 180^{\circ} = -\mu mgS$

So by Work–Energy theorem, $W = \Delta K = K_f - K_i \Rightarrow -\mu mgS = 0 - 1/2\, mv^2 \Rightarrow S = \frac{v^2}{2\mu g}$

Ex. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. Calculate the total energy of the particle.

Sol. As the particle is moving in a circle, so $\frac{mv^2}{r} = \frac{k}{r^2}$. Now $K.E = \frac{1}{2}mv^2 = \frac{k}{2r}$

As $U = -\int_{\infty}^{r} F.dr = \int_{\infty}^{r} +\left(\frac{k}{r^2}\right)dr = -\frac{k}{r}$. So total energy = U + K.E. $= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$

Negative energy means that particle is in bound state.

Ex. A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save?

Sol. In first case, $W_1 = \frac{1}{2}m(v_1)^2 + mgh = \frac{1}{2}m(12)^2 + m\times 10\times 12 = 72\text{ m} + 120\text{ m} = 192m$

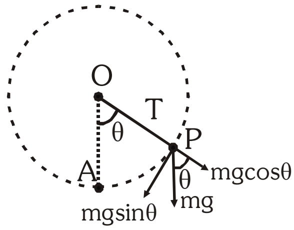
and in second case, $W_2 = mgh = 120\text{ m}$

The percentage of energy saved $= \frac{192m - 120m}{192m}\times 100 = 38\%$

CIRCULAR MOTION IN VERTICAL PLANE

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B of the circle as shown in figure. Here, $h = R(1 - \cos\theta)$ ...(i)

From conservation of mechanical energy



$\frac{1}{2}m(u^2 - v^2) = mgh \Rightarrow v^2 = u^2 - 2gh$ ....(ii)

The necessary centripetal force is provided by the resultant of tension

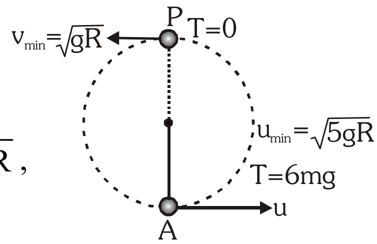
T and mg cosθ $T - mg\cos\theta = \frac{mv^2}{R}$ ...(iii)

Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^0$)

$\Rightarrow T_{max} = \frac{mv^2}{R} + mg$; $T_{min} = \frac{mv^2}{R} - mg$ at the top. Here, v' = speed of the particle at the top.

Condition of Looping the Loop $\left(u \geq \sqrt{5gR}\right)$

The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case substituting T = 0 and $\theta = \pi$ in Eq. (iii), we get $mg = \frac{mv_{min}^2}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)



Substituting $\theta = \pi$ in Eq. (i), Therefore, from Eq. (ii)

$u_{min}^2 = v_{min}^2 + 2gh = gR + 2g(2R) = 5gR \Rightarrow u_{min} = \sqrt{5gR}$

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u = \sqrt{5gR}$, velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.

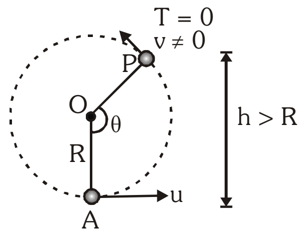
Substituting $\theta = 0^0$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T =6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

Condition of Leaving the Circle $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$

If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From Eq. (iii), tension in the string becomes zero (T = 0) where, $\cos\theta = \frac{-v^2}{Rg} \Rightarrow \cos\theta = \frac{2gh - u^2}{Rg}$

Substituting, this value of $\cos\theta$ in Eq. (i), we get $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3g} = h_1$ (say) or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2$(say)...(v) i.e., at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero. or $T = 0$ but $v \neq 0$. This is possible only when $h_1 < h_2$



$$\Rightarrow \frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

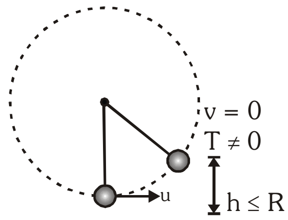
From Eq. (iv), we can see that $h > R$ if $u^2 > 2gR$. Thus, the particle, will leave the circle when $h > R$ or $90^0 < \theta < 180^0$. This situation is shown in the figure

$$\sqrt{2gR} < u < \sqrt{5gR} \text{ or } 90^0 < \theta < 180^0$$

Note : That after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $\left(0 < u \leq \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or $v = 0$, but $T \neq 0$. This is possible when $h_2 < h_1$



$$\Rightarrow \frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \Rightarrow u < \sqrt{2Rg}$$

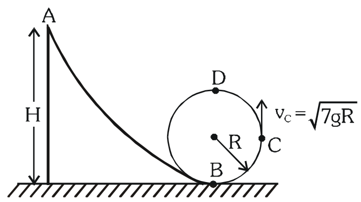
Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. (iv), we can see that $h \leq R$ if $u \leq \sqrt{2Rg}$.

Thus, for $0 < u \leq \sqrt{2gR}$, particle oscillates in lower half of the circle ($0^0 < \theta \leq 90°$)

This situation is shown in the figure. $\quad 0 < u \leq \sqrt{2gR}$ or $0° < \theta \leq 90°$

Ex. Calculate following for shown situation :–

(a) Speed at D (b) Normal reaction at D (c) Height H



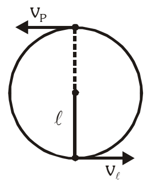
Sol. (a) $v_D^2 = v_C^2 - 2gR = 5gR \Rightarrow v_D = \sqrt{5gR}$

(b) $mg + N_D = \frac{mv_D^2}{R} \Rightarrow N_D = \frac{m(5gR)}{R} - mg = 4mg$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_C^2 + mgR = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \Rightarrow H = \frac{9}{2}R$$

Ex. A stone of mass 1 kg tied to a light string of length $\ell = \frac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest points

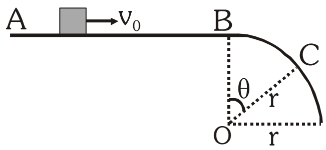


Sol. $\because \frac{T_{max}}{T_{min}} = 4 \therefore \frac{\frac{mv_\ell^2}{\ell}+mg}{\frac{mv_p^2}{\ell}-mg} = 4 \Rightarrow \frac{v_\ell^2+g\ell}{v_p^2-g\ell} = 4$

We know $v_\ell^2 = v_p^2 + 4g\ell \Rightarrow \frac{v_P^2+5g\ell}{v_P^2-g\ell} = 4 \Rightarrow 3v_P{}^2 = 9g\ell$

$\Rightarrow v_P{}^2 = \sqrt{3g\ell} = \sqrt{3\times10\times\frac{10}{3}} = 10\ ms^{-1} \Rightarrow v_\ell = \sqrt{7g\ell} = \sqrt{7\times10\times\frac{10}{3}} = 15.2\ ms^{-1}$

Ex. A small block slides with velocity $0.5\sqrt{gr}$ on the horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. Calculate angle θ in the figure.

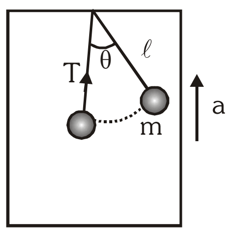


Sol. As block leaves the surface at C so at C, normal reaction = 0 $\Rightarrow$ mgcos$\theta = \frac{mv_C^2}{r}$

By energy conservation at point B & C $\frac{1}{2}mv_C{}^2 - \frac{1}{2}mv_0{}^2 = mgr\,(1-\cos\theta)$

$\Rightarrow \frac{1}{2}m\,(rg\cos\theta) - \frac{1}{2}m\left(0.5\sqrt{gr}\right)^2 = mgr\left(1-\cos\theta\right) \Rightarrow \cos\theta = \frac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{3}{4}\right)$

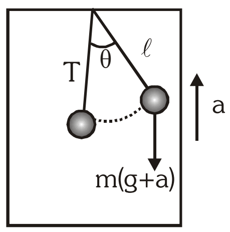
Ex. A particle of mass m is attached to the ceiling of a cabin with an inextensible light string of length ℓ. The cabin is moving upward with an acceleration 'a'. The particle is taken to a position such that the string makes an angle θ with vertical. When string becomes vertical, find the tension in the string.



Sol. In a frame associated with cabin work done on the particle when it comes in the vertical position = $mg\ell\left(1-\cos\theta\right) + ma\left(1-\cos\theta\right)$

By work energy theorem,

$\frac{mv^2}{2} = \left(mg\ell + ma\ell\right)\left(1-\cos\theta\right) \Rightarrow \frac{v^2}{2} = (g+a)\,l\,(1-\cos\theta)$



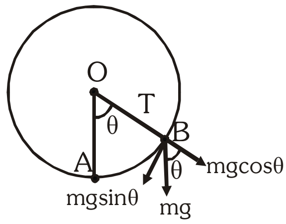
At vertical position, $T = (mg + ma) = \frac{mv^2}{\ell}$

$\Rightarrow T = (mg + ma) + 2m(g+a)(1-\cos\theta) = mg\,(g+a)\,(3-2\cos\theta)$

Ex. A heavy particle hanging from a fixed point by a light inextensible string of length ℓ, is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Sol. Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$$F_R = T - mg\cos\theta \quad ....(i)$$



If v_B be the speed of the particle at B, then

$$F_R = \frac{mv_B^2}{\ell} \quad ....(ii)$$

From (i) and (ii), we get, $T - mg\cos\theta = \dfrac{mv_B^2}{\ell}$ ....(iii)

Since at B, T = mg $\Rightarrow mg\left(1-\cos\theta\right) = \dfrac{mv_B^2}{\ell} \Rightarrow v_B^2 = g\ell\left(1-\cos\theta\right)$ ...(iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$$\frac{1}{2}mv_A{}^2 = mg\ell\,(1-\cos\theta) + \frac{1}{2}mv_B^2\text{; where } v_A = \sqrt{g\ell} \text{ and } v_B = \sqrt{g\ell\left(1-\cos\theta\right)}$$

$$\Rightarrow g\ell = 2g\ell\left(1-\cos\theta\right) + g\ell\,(1-\cos\theta) \Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Putting the value of $\cos\theta$ in equation (iv), we get : $v = \sqrt{\dfrac{g\ell}{3}}$

EXERCISE (S-1)

Work

1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: **(NCERT)**

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

WE0001

2. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by **(NCERT)**

$$F = -\hat{i} + 2\hat{j} + 3\hat{k}N$$

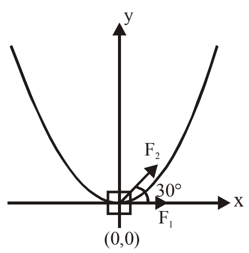
where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

WE0002

3. A point mass of 0.5 kg is moving along x-axis as $x = t^2 + 2t$, where, x is in meters and t is in seconds. Find the work done (in J) by all the forces acting on the body during the time interval [0, 2s].

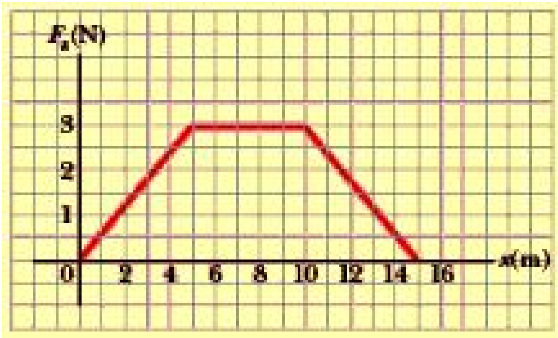
WE0003

4. A sleeve of mass 2 kg at origin can move on wire of parabolic shape $x^2 = 4y$. Two forces $F_1 = 6N$ and $F_2 = 8N$ are applied on the sleeve. F_1 is constant and is in x-direction. F_2 is constant in direction and magnitude. Body is displaced from origin to x = 4, then net work done by F_1 and F_2 is



WE0004

5. A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the body as it moves (a) from x = 0 to x = 5.00 m, (b) from x = 5.00 m to x = 10.0 m, and (c) from x = 10.0 m to x = 15.0 m. (d) What is the total work done by the force over the distance x = 0 to x = 15.0 m?



WE0005

6. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2. Find $\frac{W_2}{W_1}$.

WE0006

Kinetic energy, Work energy theorem, Power

7. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the **(NCERT)**

(a) work done by the applied force in 10 s,

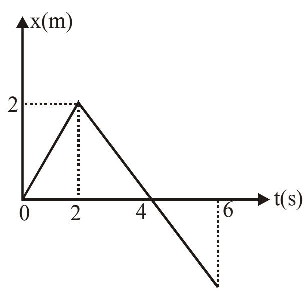
(b) work done by friction in 10 s,

(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s, and interpret your results.

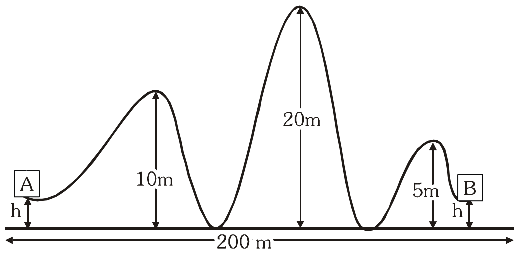
WE0007

8. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from t = 0 to t = 4s is



WE0008

9. A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$. [$g = 10\ m/s^2$]



WE0009

10. A 4 kg particle moves along the X-axis. It's position x varies with time according to $x(t) = t + 2t^3$, where x is in m and t is in seconds. Compute:

(i) The kinetic energy at time t.

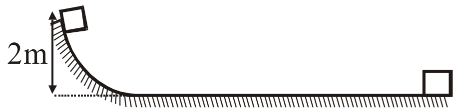
(ii) The force acting on the particle at time t.

(iii) The power delivered to the particle at time t.

(iv) The work done on the particle from t = 0 to t = 2 seconds.

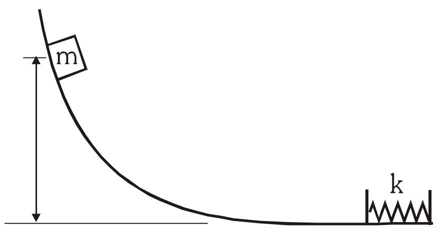
WE0010

11. A block is released from rest from top of a rough curved track as shown in figure. It comes to rest at some point on the horizontal part. If its mass is 200 gm, calculate negative of work by friction in joules.



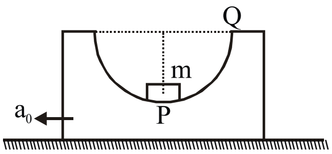
WE0011

12. A mass m slides from rest at height h down a smooth curved surface which becomes horizontal at zero height (see figure). A spring is fixed horizontally on the level part of the surface. The spring constant is k N/m. When the mass encounters the spring it compresses it by an amount x = h/10. If m = 1 kg, h = 5m then find k /100.



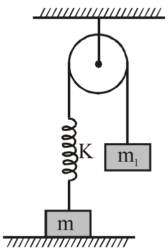
WE0012

13. A small block of mass m is lying at rest at point P of a wedge having a smooth semi circular track of radius R. The minimum value of horizontal acceleration a_0 (in m/s^2) of wedge so that mass can just reach the point Q, is



WE0013

14. Initially spring is relaxed when m_1 is released from rest. Calculate for what minimum value of m_1 the block of mass m will just leave the contact with surface ? Give answer in terms of $\frac{m}{m_1}$.

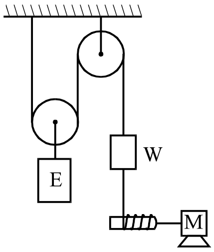


WE0014

15. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor

(a) when the elevator is moving down at a constant speed of 3 m/s,

(b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2.



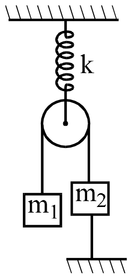
WE0015

16. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time $t = 2$ s and $t = 4$ s.

WE0016

Conservative & non conservation forces, Potential energy, Conservation of energy

17. In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring $(m_1 > m_2)$.



WE0017

18. The potential energy (in joules) function of a particle in a region of space is given as :

$$U = (2x^2 + 3y^3 + 2z)$$

Here x, y and z are in metres. Find the magnitude of x component of force (in newton) acting on the particle at point P (1m, 2m, 3m).

WE0018

19. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

(i) Find the mean position (ii) Find the max. K.E. of the particle

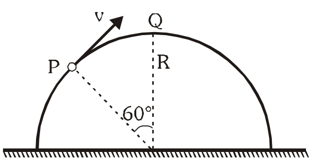
WE0019

20. The potential energy of a 2 kg particle moving along the x axis is given by $U(x) = (4.0 J/m^2)x^2 + (1.0 J/m^4)x^4$. When the particle is at x = 1.0m, find its acceleration. [only conservative forces are acting]

WE0020

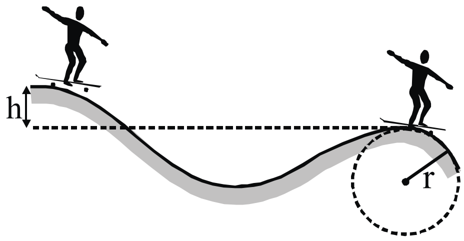
Potential energy diagram, Equilibrium, Vertical circular motion

21. A particle is given a certain veloicty v at point P as shown on a hemispherical smooth surface. Find the value of v (in m/s), such that when particle reaches Q, the normal reaction of surface becomes equal to particle's weight. [$R = 1.6$ m, $g = 10$ m/s^2]



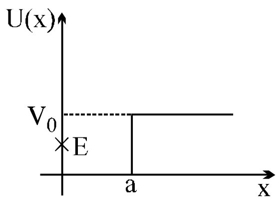
WE0021

22. A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of r = 36 m. Neglect friction and air resistance. What must be the height h (in m) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?

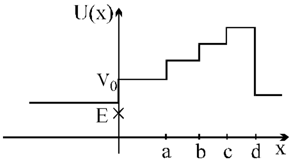


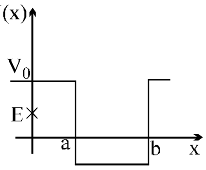
WE0022

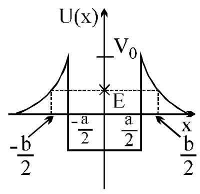
23. The given graph is a potential energy function in one dimension. The total energy of particle is indicated by cross on the ordinate axis. The graph of figure-1 is given as an example.From the figure-1, it can be interpreted that for the given total energy indicated by cross on the ordinate axis the particle cannot be found in the Region : x > a. Now, for the following potential functions in one dimensions, specify the regions, in which the particle cannot be found for the energy marked as E on graphs. Give your answer in the blocks shown.



(Figure -1)

(Graph-1) 

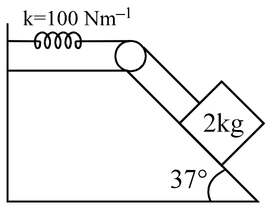
(Graph-2) 

(Graph-3) 

WE0023

24. (a) A 2 kg block situated on a smooth fixed incline is connected to a spring of negligible mass, with spring constant $k = 100\ Nm^{-1}$, via a frictionless pulley. The block is released from rest when the spring is unstretched. How far does the block move down the incline before coming (momentarily) to rest? What is its acceleration at its lowest point?

(b) The experiment is repeated on a rough incline. If the block is observed to move 0.20 m down along the incline before it comes to instantaneous rest, calculate the coefficient of kinetic friciton.

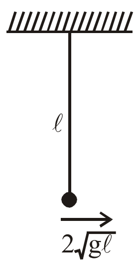


WE0024

25. A ball is attached to a horizontal cord of length L whose other end is fixed, (a) If the ball is released, what will be its speed at the lowest point of its path ? (b) A peg is located a distance h directly below the point of attachment of the cord. If h = 0.75 L, what will be the speed of the ball when it reaches the top of its circular path about the peg ?

WE0025

26. One end of a string of length $\ell = \frac{14}{9}$ m is fixed and a mass of 1 kg is tied to the other end. The ball is given a velocity $2\sqrt{g\ell}$ at the bottom most point as shown in figure. The string is cut when the ball becomes horizontal. Find the distance (in m) travelled till it stop for the 1st time (Take $\pi = \frac{22}{7}$).



WE0026

EXERCISE (S-2)

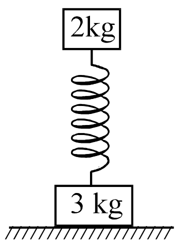
1. A ring of mass m can slide over a smooth vertical rod. The ring is connected to a spring of force constant $K = \frac{4mg}{R}$ where 2R is the natural length of the spring. The other end of the spring is fixed to the ground at a horizontal distance 2R from the base of the rod. The mass is released at a height of 1.5R from ground

(i) calculate the work done by the spring.

(ii) calculate the velocity of the ring as it reaches the ground.

WE0027

2. The ends of spring are attached to blocks of mass 3kg and 2kg. The 3kg block rests on a horizontal surface and the 2kg block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2kg mass must be compressed further by at least _______, so that when it is released, the 3 kg block may be lifted off the ground.



WE0028

3. A uniform rod of mass *m* length *L* is sliding along its length on a horizontal table whose top is partly smooth & rest rough with friction coefficient μ. If the rod after moving through smooth part, enters the rough with velocity v_0.

(i) What will be the magnitude of the friction force when its x length (< L) lies in the rough part during sliding.

(ii) Determine the minimum velocity v_0 with which it must enter so that it lies completely in rough region before coming to rest.

(iii) If the velocity is double the minimum velocity as calculated in part (a) then what distance does its front end A would have travelled in rough region before rod comes to rest.

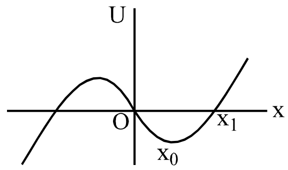
WE0029

4. A particle is confined to move along the +x axis under the action of a force F(x) that is derivable from the potential $U(x) = ax^3 - bx$.

(i) Find the expression for F(x)

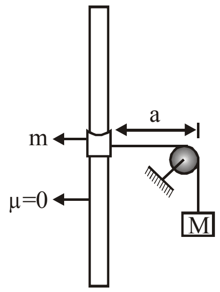
(ii) When the total energy of the particle is zero, the particle can be trapped with in the interval x = 0 to $x = x_1$. For this case find the values of x_1.

(iii) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b. At what value of x will the kinetic energy be maximum ?



WE0030

5. A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M (M>m). The ring is held on a level with the peg and released. Show that it first comes to rest after falling a distance $\frac{2mMa}{M^2 - m^2}$.



WE0031

6. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.

(i) What is the average power of the elevator motor during this period?

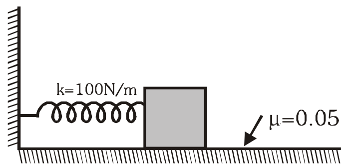
(ii) How does this power compare with its power when it moves at its cruising speed?

WE0032

7. A particle is moving with kinetic energy E, straight up an inclined plane with angle α, the coefficient of friction being μ. The work done against friction before the particle comes down to rest is

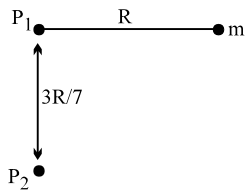
WE0033

8. A block of mass 1 kg is attached to a spring with a force constant 100 N/m and rests on a rough horizontal ground as shown in the figure. Initial displacement of block from natural length is 50 cm. The total distance covered by the block if coefficient of friction between block & ground is 0.05. [$g = 10 m/s^2$]



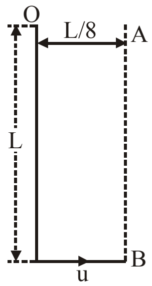
WE0034

9. A simple pendulum consists of a bob of mass m and a string of length R suspended from a peg P_1 on the wall. A second peg P_2 is fixed vertically below the first one at a distance $3R/7$ from it. The pendulum is drawn aside such that the string is horizontal and released. Calculate the maximum height (with respect to the lowest point) to which it rises



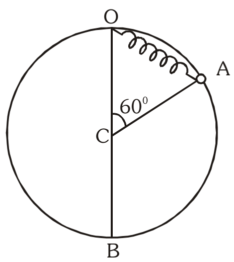
WE0035

10. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $\frac{L}{8}$ from O as shown in figure. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.



WE0036

11. A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle OCA = 60^0$, C being the centre of the ring. The natural length of the spring is also equal to r = 20 cm. After the particle is released and slides down the ring the contact force between the particle & the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



WE0037

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Work

1. A force of magnitude of 30 N acting along $\hat{i}+\hat{j}+\hat{k}$, displaces a particle from point (2, 4, 1) to (3, 5, 2). The work done during this displacement is :-

(A) 90 J (B) 30 J (C) $30\sqrt{3}$ J (D) $30/\sqrt{3}$ J.

WE0038

2. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5cm to 15 cm is- **[AIEEE - 2002]**

(A) 16 J (B) 8 J (C) 32 J (D) 24 J

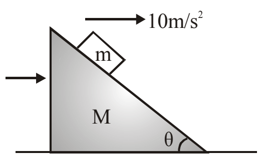
WE0039

3. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

(A) Mgx (B) ½ Mgx^2 (C) – ½ Mgx (D) Mgx^2

WE0040

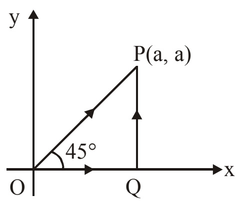
4. In the figure shown all the surfaces are frictionless, and mass of the block, m = 1 kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is :



(A) 30J (B) 60 J (C) 150 J (D) $100\sqrt{3}$ J

WE0041

5. A particle is moved from (0, 0) to (a, a) under a force $\vec{F}=(3\hat{i}+4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :

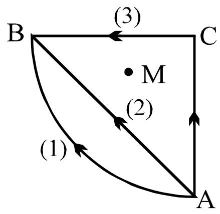


(A) $W_1 = W_2$ (B) $W_1 = 2W_2$ (C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

WE0042

6. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then

[IIT-JEE (Scr.)'2003]



(A) $W_1 = W_2 = W_3$ (B) $W_1 = W_2 > W_3$ (C) $W_1 > W_2 > W_3$ (D) $W_1 < W_2 < W_3$

WE0043

Kinetic energy, Work energy theorem, Power

7. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

(A) 1.5 J (B) 50 J (C) 10 J (D) 100 J

WE0044

8. A man who is running has half the kinetic energy of the boy of half his mass. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. The original speed of the man was

(A) $\sqrt{2}$ m/s (B) $(\sqrt{2} - 1)$ m/s (C) 2 m/s (D) $(\sqrt{2} + 1)$ m/s

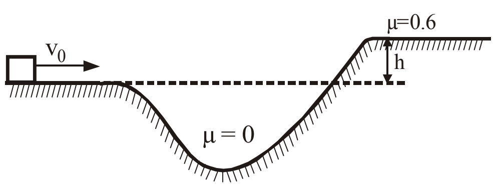
WE0045

9. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be

(A) $250\ \pi^2$ joule (B) $100\pi^2$ joule (C) $5\pi^2$ joule (D) 0 joule

WE0046

10. In the figure, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless untill the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6 m/s, the height difference h is 1.1m and the coefficient of kinetic friction μ is 0.6. The value of d is



(A) 1.17 m (B) 1.71 m (C) 7.11 m (D) 11.7 m

11. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m/s at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2, the kinetic energy of the shotput when it just reaches the ground will be

(A) 2.5 J (B) 5.0 J (C) 52.5 J (D) 155.0 J

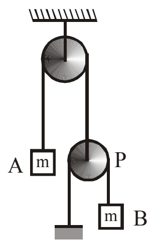
WE0048

12. A particle moves on a rough horizontal ground with some initial velocity say v_0. If 3/4 of its kinetic energy is lost in friction in time t_0 then coefficient of friction between the particle and the ground is :-

(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{gt_0}$

WE0049

13. In the figure shown, the system is released from rest. Find the velocity of block A when block B has fallen a distance 'ℓ'. Assume all pulleys to be massless and frictionless.



(A) $\sqrt{\frac{g\ell}{5}}$ (B) $\sqrt{g\ell}$ (C) $\sqrt{5g\ell}$ (D) None of these

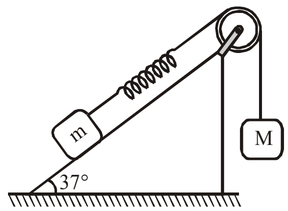
WE0050

14. A block of mass m is hung vertically from an elastic thread of force constant mg/a. Initially the thread was at its natural length and the block is allowed to fall freely. The kinetic energy of the block when it passes through the equilibrium position will be :

(A) mga (B) mga/2 (C) zero (D) 2mga

WE0051

15. A block of mass m is attached with a massless spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is : (Neglect mass of string and pulley and friction in pulley)



(A) $\frac{3}{5}$ m (B) $\frac{4}{5}$ m (C) 2 m (D) $\frac{3}{2}$ m

WE0052

16. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to- **[AIEEE - 2003]**

(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

WE0053

17. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to- **[AIEEE - 2004]**

(A) x^2 (B) e^x (C) x (D) $\log_e x$

WE0054

18. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is- **[AIEEE - 2004]**

(A) $\frac{mv_1 t}{t_1}$ (B) $\frac{mv_1^2 t}{t_1^2}$ (C) $\frac{mv_1 t^2}{t_1}$ (D) $\frac{mv_1^2 t}{t_1}$

WE0055

19. Assume the aerodynamic drag force on a car is proportional to its speed. If the power output from the engine is doubled, then the maximum speed of the car.

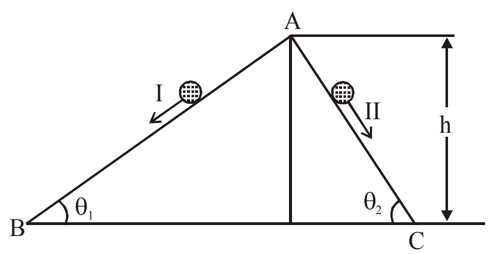
(A) is unchanged (B) increases by a factor of $\sqrt{2}$

(C) is also doubled (D) increases by a factor of four.

WE0056

Conservative & non conservative forces, Potential energy, Conservation of energy

20. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?



(A) Both the stones reach the bottom at the same time but not with the same speed.

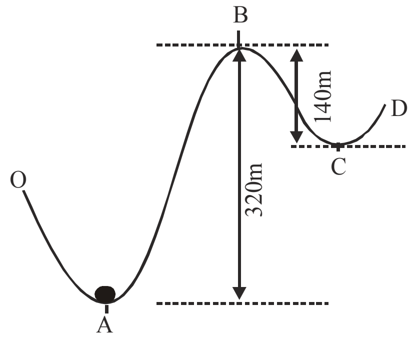
(B) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

(C) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(D) Both the stones reach the bottom at different times and with different speeds.

WE0057

21. Track OABCD (as shown is figure) is smooth and fixed in vertical plane. What minimum speed has to be given to a particle lying at point A, so that it can reach point C?

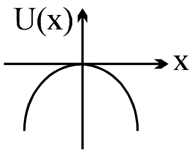


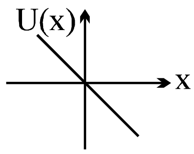
(A) 60 m/s (B) 100 m/s (C) 70 m/s (D) 80 m/s

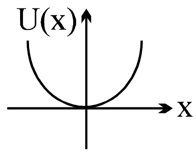
WE0058

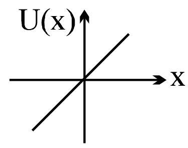
22. A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function)

[IIT-JEE' 2004(Scr)]

(A) 

(B) 

(C) 

(D) 

WE0059

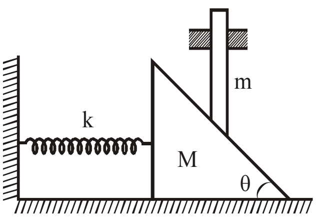
23. When a conservative force does positive work on a body

(A) the potential energy increases (B) the potential energy decreases

(C) total energy increases (D) total energy decreases

WE0060

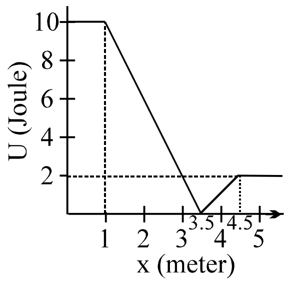
24. A wedge of mass M fitted with a spring of stiffness 'k' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is:



(A) $\dfrac{mg^2\tan^2\theta}{2k}$ (B) $\dfrac{m^2g\tan^2\theta}{2k}$ (C) $\dfrac{m^2g^2\tan^2\theta}{2k}$ (D) $\dfrac{m^2g^2\tan^2\theta}{k}$

Potential energy diagram, Equilibrium, Vertical circular motion

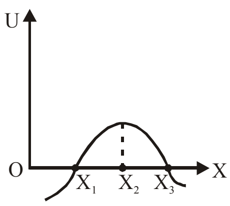
25. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at x = 2m, then its speed when it crosses x = 5 m is



(A) zero (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 3 ms^{-1}

WE0062

26. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at :

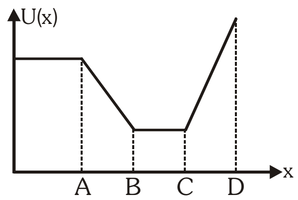


(A) x_1 is in stable equilibrium (B) x_2 is in stable equilibrium

(C) x_3 is in stable equilibrium (D) None of these

WE0063

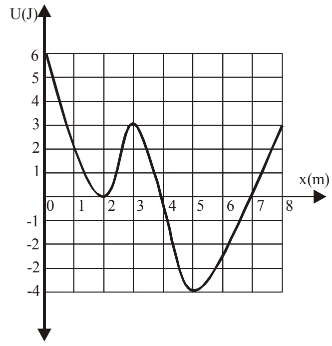
27. As a particle moves along the x–axis it is acted upon by a conservative force. The potential energy is shown below as a function of the coordinate x of the particle. Rank the labelled regions according to the magnitude of the force, least to greatest.



(A) AB, BC, CD (B) AB, CD, BC (C) BC, CD, AB (D) BC, AB, CD

WE0064

28. Potential energy curve U of a particle as function of the position of a particle is shown. The particle has total mechanical energy E of 3.0 joules.

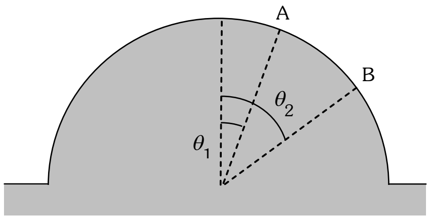


(A) It can never be present at x= 0 m. (B) It can never be present at x=5 m

(C) At x=2 its kinetic energy is 0 J (D) At x =1 its kinetic energy 3 J

WE0065

29. A small block slides down from rest at point A on the surface of a smooth circular cylinder, as shown. At point B, the block falls off (leaves) the cylinder. The equation relating the angles θ_1 and θ_2 is given by



(A) $\sin\theta_2 = \frac{2}{3}\sin\theta_1$ (B) $\sin\theta_2 = \frac{3}{2}\sin\theta_1$ (C) $\cos\theta_2 = \frac{2}{3}\cos\theta_1$ (D) $\cos\theta_2 = \frac{3}{2}\cos\theta_1$

WE0066

MULTIPLE CORRECT TYPE QUESTIONS

30. Which of the following statements is TRUE for a system comprising of two bodies in contact exerting frictional force on each other :

(A) total work done by static friction on whole system is always zero.

(B) work done by static friction on a body is always zero

(C) work done by kinetic friction on a body is always negative

(D) total work done by internal kinetic friction on whole system is always negative

WE0067

31. A particle of mass *m* is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion of train for time t_0. A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **INCORRECT**?

(A) Both will measure the same work

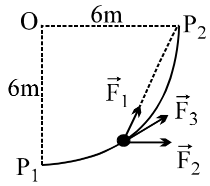
(B) Boy will measure higher value than the girl

(C) Girl will measure higher value than the boy

(D) Data are insufficient for the measurement of work done by the force F_0

WE0068

32. A smooth track in the form of a quarter circle of radius 6 m lies in the vertical plane. A particle moves from P_1 to P_2 under the action of forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Force $\vec{F}_1$ is always toward P_2 and is always 20N in magnitude. Force $\vec{F}_2$ always acts horizontally and is always 30 N in magnitude. Force $\vec{F}_3$ always acts tangentially to the track and is of magnitude 15 N. Select the correct alternative(s)

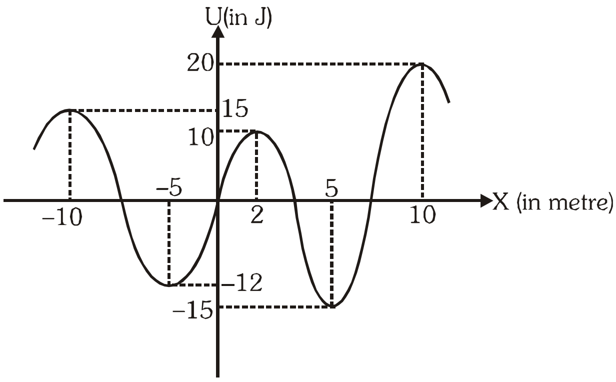


(A) work done by $\vec{F}_1$ is 120 J
(B) work done by $\vec{F}_2$ is 180 J
(C) work done by $\vec{F}_3$ is 45 π
(D) $\vec{F}_1$ is conservative in nature

WE0069

COMPREHENSION TYPE QUESTIONS
Paragraph for Question No. 33 to 35

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



33. If the particle is released at the origin then :
(A) It will move towards positive x-axis.
(B) It will move towards negative x-axis.
(C) It will remain stationary at the origin.
(D) Its subsequent motion cannot be decided due to lack of information.

WE0070

34. If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be-
(A) $\sqrt{22}$ m/s
(B) $\sqrt{25}$ m/s
(C) $\sqrt{24}$ m/s
(D) $\sqrt{23}$ m/s

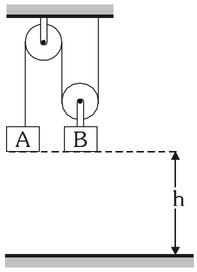
WE0070

35. x = –5 m and x = 10 m positions of the particle are respectively of-
(A) Neutral and stable equilibrium
(B) Neutral and unstable equilibrium
(C) Unstable and stable equilibrium
(D) Stable and unstable equilibrium.

WE0070

MATRIX MATCH TYPE QUESTIONS

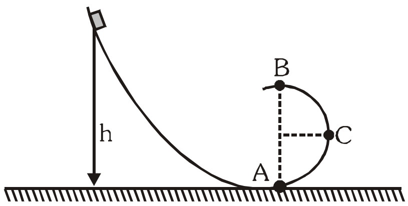
36. In the figure shown are two blocks A and B of same mass connected with pulley and string to each other. Initially both of them are at a height of h = 0.5 m from ground. After they are released they move in either direction and one of them strike the ground. For, the interval from releasing to when one of them strike, some physical quantities are in column I and their modulus values in SI units are in column II.



	Column I		**Column II**
(A)	Velocity of A immediately before any one of them strike ground.	(P)	1
(B)	Velocity of B immediately before any one of them strike ground.	(Q)	2
(C)	Ratio of work done by gravity on A to workdone by gravity on B.	(R)	3
(D)	Acceleration of block A before any one of them strike ground.	(S)	4
		(T)	5

WE0071

37. A block of mass m is released from top of a smooth track as shown in the figure. The end part of the track is a circle in vertical plane of radius R. N is normal reaction of the track at any point of the track. Match the entries of column I with entries of column–II.



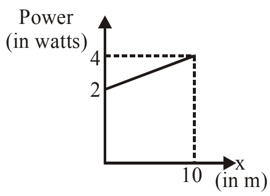
	Column I		**Column II**
(A)	$h = \frac{5}{2}R$	(P)	Net force on the block at C is mg
(B)	$h = \frac{9}{2}R$	(Q)	$N_A - N_B = 6$ mg
(C)	h = R	(R)	Block leaves contact before B
(D)	h = 2R	(S)	Block will keep contact with the track is region between A & B.
		(T)	$N_C > mg$

WE0072

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

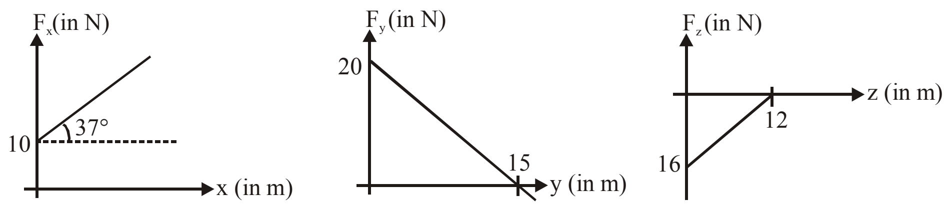
1. A particle 'A' of mass $\frac{10}{7}$ kg is moving in the positive x–direction. Its initial position is x = 0 & initial velocity is 1 m/s. The velocity at x = 10m is : (use the graph given)



(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}\ m/s$ (D) 100/3 m/s

WE0073

2. The components of a force acting on a particle are varying according to the graphs shown. To reach at point B (8, 20, 0) from point A(0, 5, 12) the particle moves on paths parallel to x-axis then y-axis and then z-axis, then work done by this force is :-



(A) 192 J (B) 58 J (C) 250 J (D) 125 J

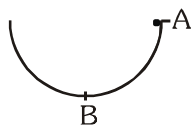
WE0074

3. A light spring of length 20 cm and force constant 2 N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the block will have the maximum velocity is :

(A) 20 cm (B) 15 cm (C) 10 cm (D) 5cm

WE0075

4. A small ball can move in a vertical plane along a semi–circle of radius r without friction. At what speed is the ball to launch from point A so that its acceleration is 3g at point B ?



(A) $(3gr)^{1/2}$ (B) $(2gr)^{1/2}$ (C) $(gr)^{1/2}$ (D) $2(gr)^{1/2}$

WE0076

MULTIPLE CORRECT TYPE QUESTIONS

5. A particle of mass 2 kg is projected with an initial speed u = 10 m/sec at an angle $\theta = 30°$ with the horizontal

(A) Total work done on the particle during the first half of the total time of flight of the particle is (–25) J.

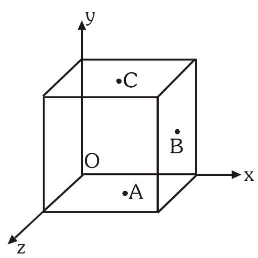
(B) Total work done on the particle during the total time of flight of the particle is 0 J.

(C) Average power delivered to the particle during the first half of the flight is (–50) watt.

(D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

WE0077

6. A particle is shifted from A to B and then from B to C where A, B and C are the midpoints of the corresponding faces of a cube of side 2m. If a force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})N$ is continuously acting on the particle, then select correct alternative :-



(A) work done from A to B is 7 J

(B) work done from B to C is 1 J

(C) work done A to C is 8 J

(D) $\vec{F}$ is a conservative force

WE0078

7. Which of the following is/are conservative force(s)?

(A) $\vec{F} = 2r^3\hat{r}$

(B) $\vec{F} = -\frac{5}{r}\hat{r}$

(C) $\vec{F} = \frac{3(xi + y\hat{j})}{(x^2+y^2)^{3/2}}$

(D) $\vec{F} = \frac{3(yi + x\hat{j})}{(x^2+y^2)^{3/2}}$

WE0079

8. If one of the forces acting on a particle is conservative then :

(A) Work done by this force is zero when the particle moves exactly once around any closed path.

(B) Work done by this force equals the change in the kinetic energy of the particle.

(C) It obeys Newton's second law.

(D) Work done by this force depends on the end points of the motion, not on the path in between.

WE0080

9. A particle of mass m = 1 kg lying on x–axis experiences a force given by law F=x(3x–2) Newton, where x is the x–coordinate of the particle in meters. The points on x–axis where the particle is in equilibrium are :

(A) x = 0 (B) x = 1/3 (C) x = 2/3 (D) x = 1

WE0081

10. A particle is given a velocity at the bottom most position in a smooth spherical shell of radius 2m. It just complete a vertical circular motion. Then

(A) acceleration of particle when the velocity of the particle is in vertically upward direction is $g\sqrt{10}$

(B) acceleration of particle when the velocity of the particle is in vertically downward direction is $g\sqrt{10}$

(C) acceleration of the particle at the top most point of path is g

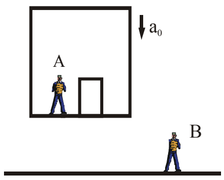
(D) acceleration of the particle at the bottom most point is 5 g

WE0082

COMPREHENSION TYPE QUESTIONS

Comprehension for Question no. 11 to 14

A block of mass m is kept in an elevator which starts moving downward with an acceleration a_0 as shown in figure. The block is observed by two observers A and B for a time interval t_0.



11. The observer B finds that the work done by gravity is

(A) $\frac{1}{2}mg^2t_0^2$ (B) $-\frac{1}{2}mg^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

WE0083

12. The observer B finds that work done by normal reaction N is :-

(A) zero (B) $-Na_0t_0^2$ (C) $\frac{Na_0t^2}{2}$ (D) None of these

WE0083

13. According to observer B, the net work done on the block is

(A) $-\frac{1}{2}ma_0t_0^2$ (B) $\frac{1}{2}ma_0^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

WE0083

14 According to the observer A

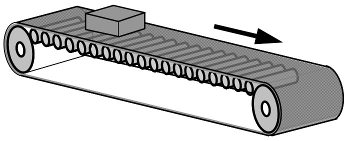
(A) the work done by gravity is zero (B) the work done by normal reaction is zero

(C) the work done by pseudo force is zero (D) all the above

WE0083

Paragraph for Question 15 & 16

An object of mass M is gently placed on a horizontal conveyor belt, which is moving with uniform velocity v_o as shown in the figure. The coefficient of static friction is μ_s, the coefficient of kinetic friction is μ_k, and the acceleration of gravity is g. Initially the object slips for a while but finally moves without slipping together with the belt.



15. How far the conveyor belt moves while the object is slipping?

(A) $\frac{v_o^2}{\mu_k g}$ (B) $\frac{v_o^2}{2\mu_k g}$ (C) $\frac{v_o^2}{\mu_s g}$ (D) $\frac{v_o^2}{2\mu_s g}$

WE0084

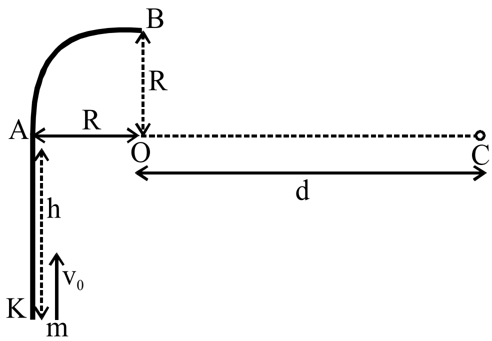
16. Work done on the object by friction force relative to the reference frame moving with the conveyer belt is

(A) $\frac{1}{2}Mv_o^2$ (B) $-\frac{1}{2}Mv_o^2$ (C) Mv_o^2 (D) zero

WE0084

Paragraph for Question Nos.17 to 19

A point like object of mass m starts from point K as shown in the figure. It slides inside along the full length of the smooth track of radius R, and then moves freely and travels to point C. [The track is kept in vertical plane]



17. Determine the vertical initial velocity of the pointlike object.

(A) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{R}}$ (B) $v_0 = \sqrt{2g(h-R)+\frac{d^2g}{2R}}$

(C) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{2R}}$ (D) $v_0 = \sqrt{2g(h+R)+\frac{dg}{2R}}$

WE0085

18. What is the minimum possible distance OC = d, necessary for the object to slide along the entire length of the track ?

(A) $d_{min} = R\sqrt{2}$ (B) $d_{min} = \sqrt{3}R$ (C) $d_{min} = \frac{R}{\sqrt{2}}$ (D) $d_{min} = \frac{R}{\sqrt{3}}$

WE0085

19. Find the normal force exerted by the track at point A.

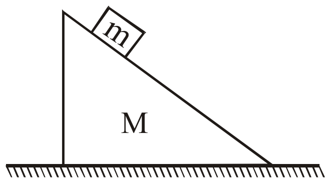
(A) $F_A = mg\left(\frac{d^2}{2R^2} - 2\right)$ (B) $F_A = mg\left(\frac{d^2}{2R^2} + 2\right)$

(C) $F_A = mg\left(\frac{3d^2}{R^2} + 2\right)$ (D) $F_A = mg\left(\frac{d}{3R^2} + 2\right)$

WE0085

MATRIX MATCH TYPE QUESTIONS

20. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column–I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge, g is acceleration due to gravity).



	Column I		Column II
(A)	Work done by normal reaction acting on the block is	(p)	Positive
(B)	Work done by normal reaction (exerted by block) acting on wedge is	(q)	Negative
(C)	The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r)	Zero
(D)	Net work done by all forces on block is	(s)	Less than mgh in magnitude

WE0086

EXERCISE (JM)

1. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constant and x is the distance between the atoms. if the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is : **[AIEEE-2010]**

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

WE0087

2. At time t = 0s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :- **[AIEEE-2011]**

(1) $\sqrt{t}$ (2) constant (3) t (4) $\frac{1}{\sqrt{t}}$

WE0088

3. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE-2012]**

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement-2 : $k_1 < k_2$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

WE0089

4. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:- **[JEE-Main-2014]**

(1) $\frac{aL^2}{2}+\frac{bL^3}{3}$ (2) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (3) $aL^2 + bL^3$ (4) $\frac{1}{2}(aL^2+bL^3)$

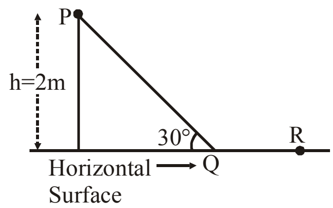
WE0090

5. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:- **[JEE-Main-2016]**

(1) 12.89×10^{-3} kg (2) 2.45×10^{-3} kg (3) 6.45×10^{-3} kg (4) 9.89×10^{-3} kg

WE0091

6. A point particle of mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and PR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(=QR) are, respecitvely close to :- **[JEE-Main-2016]**



(1) 0.29 and 6.5 m (2) 0.2 and 6.5 m (3) 0.2 and 3.5 m (4) 0.29 and 3.5 m

WE0092

7. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :- **[JEE-Main-2017]**

(1) 10^{-4} kg m^{-1} (2) 10^{-1} kg m^{-1} s^{-1} (3) 10^{-3} kg m^{-1} (4) 10^{-3} kg s^{-1}

WE0093

8. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : **[JEE-Main-2017]**

(1) 9 J (2) 18 J (3) 4.5 J (4) 22 J

WE0094

9. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is :- **[JEE-Main-2018]**

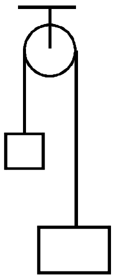
(1) $\frac{k}{2a^2}$ (2) Zero (3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

WE0095

EXERCISE (JA)

1. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (in **joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

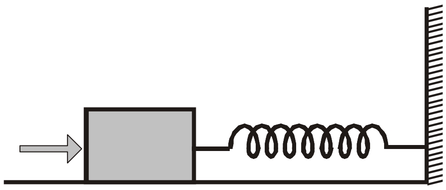
[IIT-JEE-2009]



WE0096

2. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is

[IIT-JEE-2011]



WE0097

3. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is :- **[JEE-Advance-2013]**

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

WE0098

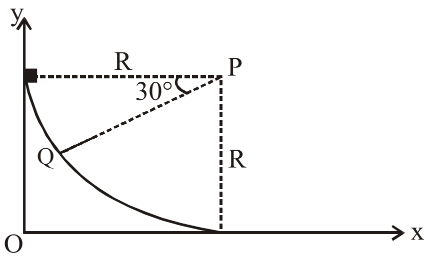
4. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is.

[JEE-Advance-2013]

WE0099

Paragraph for Questions 5 and 6

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, $g = 10\ m\ s^{-2}$) **[JEE-Advance-2013]**



5. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

WE0100

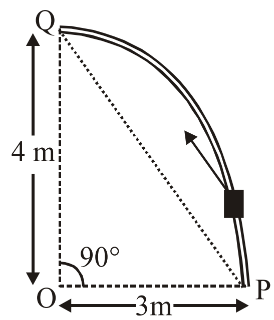
6. The speed of the block when it reaches the point Q is

(A) $5\ ms^{-1}$ (B) $10\ ms^{-1}$ (C) $10\sqrt{3}ms^{-1}$ (D) $20\ ms^{-1}$

WE0100

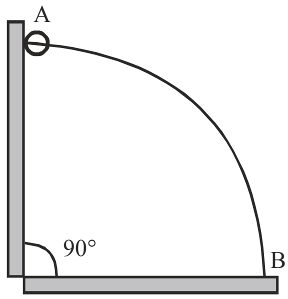
7. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = $10\ ms^{-2}$)

[JEE-Advance-2014]



WE0101

8. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[JEE Advanced-2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

WE0102

9. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ? **[JEE Advanced-2018]**

(A) The force applied on the particle is constant

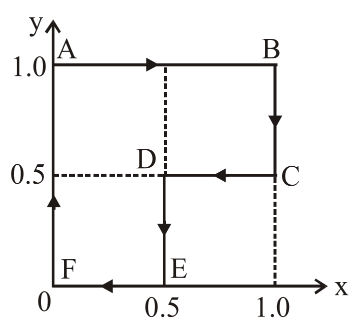
(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

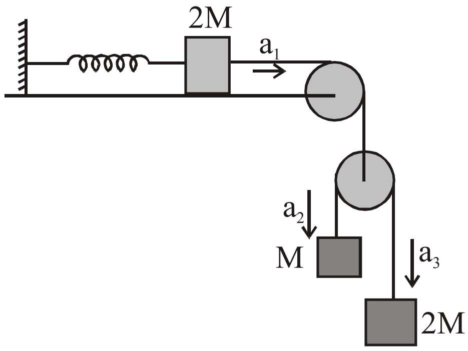
WE0103

10. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)$ N, where x and y are in meter and $\alpha = -1\ N/m^{-1}$. The work done on the particle by this force $\vec{F}$ will be ____ Joule. **[JEE Advanced-2019]**



WE0104

11. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction] **[JEE Advanced-2019]**



(1) $x_0 = \frac{4Mg}{k}$

(2) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(3) $a_2 - a_1 = a_1 - a_3$

(4) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

WE0105

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) +ve (b) –ve (c) –ve (d) +ve (e) –ve
2. Ans. 12 J
3. Ans. 8
4. Ans. 67.7 J
5. Ans. (a) 7.5 J; (b) 15 J; (c) 7.5 J; (d) 30 J
6. Ans. 3
7. Ans. (a) 875 J; (b) –250 J; (c) 625 J ; (d) 625 J;
8. Ans. 0
9. Ans. 400 J
10. Ans. (i) $2 + 24t^2 + 72t^4$ J, (ii) 48 t N, (iii) $48t + 288t^3$ W, (iv) 1248 J
11. Ans. 4
12. Ans. 4
13. Ans. 10
14. Ans. $m_1 = m/2$
15. Ans. (a) –30 kW, 19.5 kW
16. Ans. 46 J
18. Ans. 4
19. Ans. (i) x =2, (ii) 16 J
20. Ans. $-6\ m/s^2$
21. Ans. 4
22. Ans. 018
23. Ans. Graph-1 : For all x, Graph-2 : x < a & x > b, Graph-3 : $-\frac{b}{2} < x < -\frac{a}{2}$ & $\frac{a}{2} < x < \frac{b}{2}$
24. Ans. (a) s = 0.24 m, a = 6 m/s², (b) μ = 1/8
25. Ans. $\sqrt{2gL}$, $\sqrt{gL}$
26. Ans. 4

EXERCISE (S-2)

1. Ans. (i) mgR/2, (ii) $2\sqrt{gR}$
2. Ans. 2.5cm
3. Ans. (i) $f = \frac{\mu m}{\ell} xg$; (ii) $\sqrt{\mu g \ell}$; (iii) $\frac{5\ell}{2}$
4. Ans. $F = -3ax^2 + b$, $x = \sqrt{\frac{b}{a}}$, $K_{max} = \frac{2b}{3}\sqrt{\frac{b}{3a}}$
6. Ans. (i) 6×10^3 W (ii) 1×10^4 W
7. Ans. $\frac{E\mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}$
8. Ans. 25 m
9. Ans. 27R/28
10. Ans. $u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$
11. Ans. 500 N/m

EXERCISE (O-1)

1. Ans. (C)	2. Ans. (B)	3. Ans. (C)	4. Ans. (C)	5. Ans. (A)
6. Ans. (A)	7. Ans. (B)	8. Ans. (D)	9. Ans. (A)	10. Ans. (A)
11. Ans. (D)	12. Ans. (A)	13. Ans. (A)	14. Ans. (B)	15. Ans. (A)
16. Ans. (B)	17. Ans. (A)	18. Ans. (B)	19. Ans. (B)	20. Ans. (C)
21. Ans. (D)	22. Ans. (A)	23. Ans. (B)	24. Ans. (C)	25. Ans. (C)
26. Ans. (D)	27. Ans. (D)	28. Ans. (A)	29. Ans. (C)	30. Ans. (A,D)
31. Ans. (A,C)	32. Ans. (B,C,D)	33. Ans. (B)	34. Ans. (B)	35. Ans. (D)

36. Ans. (A) Q; (B) P; (C) Q; (D) S
37. Ans. (A) Q,S,T; (B) Q,S,T; (C) P; (D) R,T

EXERCISE (O-2)

1. Ans. (A)	2. Ans. (C)	3. Ans. (B)	4. Ans. (C)	5. Ans. (A,B,C,D)
6. Ans. (ABCD)	7. Ans. (A,B,C)	8. Ans. (A,C,D)	9. Ans. (A,C)	10. Ans. (A,B,C,D)
11. Ans. (C)	12. Ans. (D)	13. Ans. (B)	14. Ans. (D)	15. Ans. (A)
16. Ans. (B)	17. Ans. (C)	18. Ans. (A)	19. Ans. (B)	

20. Ans. (A) Q,S; (B) P,S; (C) R, S; (D) P,S

EXERCISE (J-M)

1. Ans. (4)	2. Ans. (4)	3. Ans. (2)	4. Ans. (1)	5. Ans. (1)
6. Ans. (4)	7. Ans. (1)	8. Ans. (3)	9. Ans. (2)	

EXERCISE (J-A)

1. Ans. 8	2. Ans. 4	3. Ans. (D)	4. Ans. 5	5. Ans. (A)	6. Ans. (B)
7. Ans. 5	8. Ans. (D)	9. Ans. (A,B,D)	10. Ans. 0.75	11. Ans. (3)	