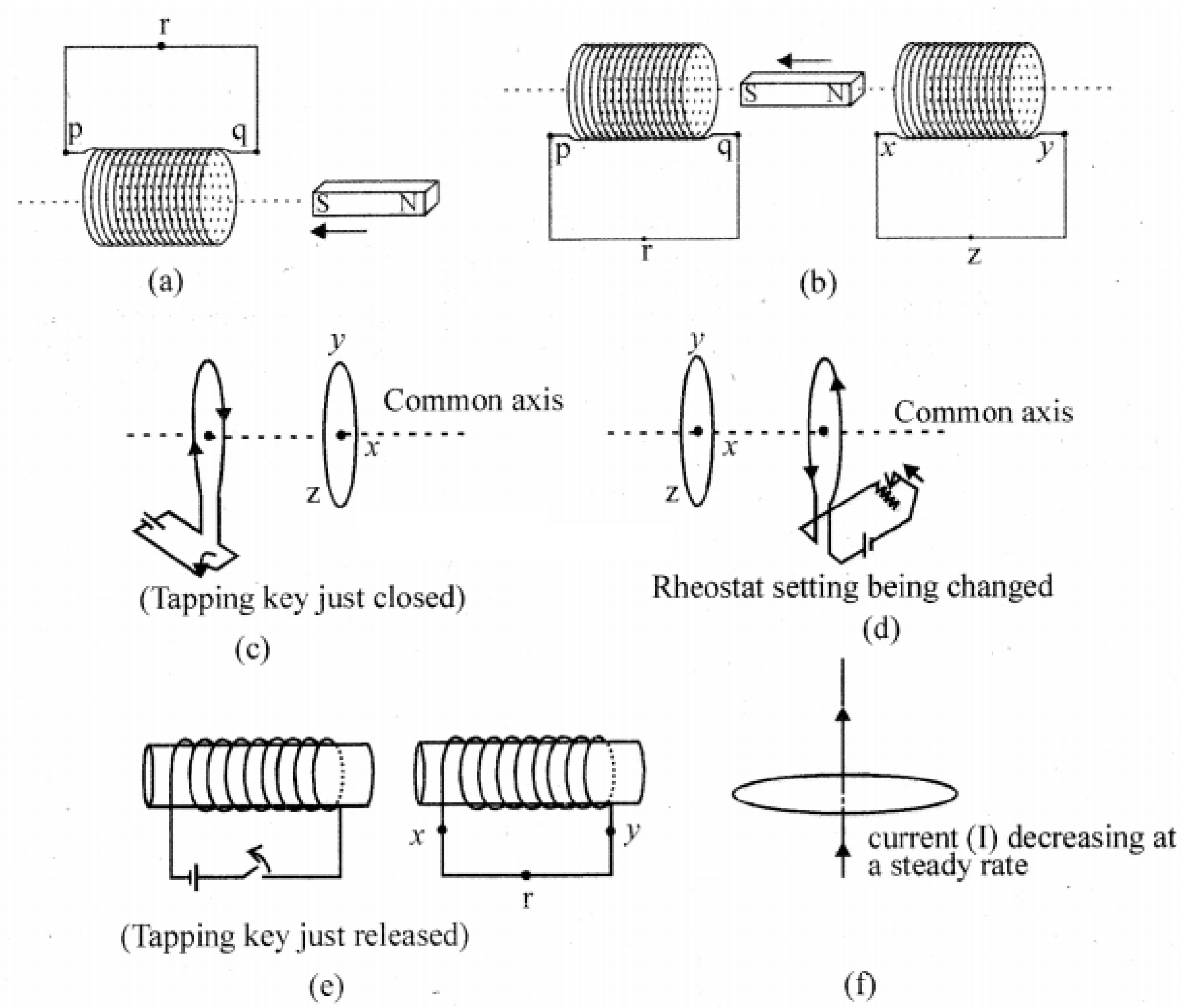
Chapter 6 Electromagnetic Induction

Question 1. Predict the direction of induced current in the situations described by the following figures (a) to (f).



Solution: (a) According to Lenz's law, the end of the coil facing the S-pole of the magnet coil becomes an S-pole so as to repel the approaching magnet. For this to happen, the current in the coil will flow in the direction QRPQ.

(b) For the coil on left: For the reason given in (a), the current in the left coil will flow in the direction of PRQP.
For the coil on right: According to Lenz's law, the end of the coil facing the N-pole of the magnet will become S-pole so as to attract the magnet moving

away from it. For this to happen, the current in the coil will flow in the direction YZXY.

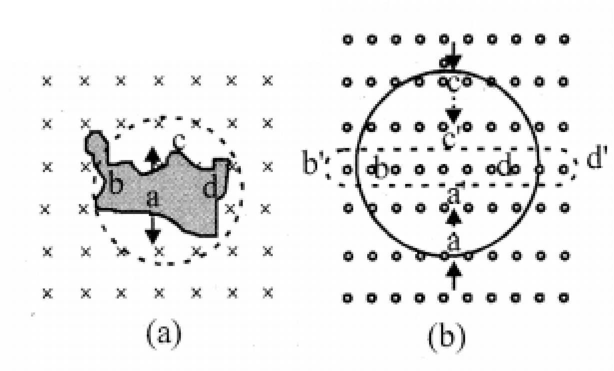
(c) When the tapping key is just closed, the current flow's in the coil and hence magnetic flux linked with it starts growing. According to Lenz's law, the induced current in the adjoining coil should set up in a direction so as to oppose the growth of magnetic flux linked with the coil. It will happen so if the direction of the magnetic field produced by the induced current in the adjoining coil is from right to left i.e. the induced current is produced in the direction YZX.

(d) Due to change in rheostat setting (resistance decreased), the current in the coil and hence magnetic flux linked with it, will start increasing. According to Lenz's law, the induced current in the adjoining coil should set up in a direction. so as to oppose the increase in magnetic flux linked with the coil. It will happen. So, if the direction of the magnetic field produced by the induced current in the adjoining coil is from right to left i.e. the induced current is produced in the direction ZYX.

(e) Before releasing the tapping keg, the right end of the core is N-pole and the magnetic field linked with the coil is from left to right. As the tapping keg is released, the magnetic field linked with the coil will start collapsing. According to Lenz's Law, the induced current in the adjoining coil should set up in a direction so as to reinforce the collapsing magnetic field. It will happen so if the direction of the magnetic field produced by the induced current in the adjoining coil is from left to right i.e. the induced current is produced in the direction XRY.

(f) The current flowing through the straight conductor produces a magnetic field in the plane of the coil. Since no magnetic flux is linked with the coil, no induced current will be set up in the coil.

Question 2. Use Lenz's law to determine the direction of induced current in the situations described by the following figures.



(a) A wire of irregular shape turning into a circular shape;
(b) A circular loop being deformed into a narrow straight wire.
Solution:
(a) By right hand thumb rule, the current direction is along adcba.
(b) Here current direction is opposite to that in fig. (a) i.e., along a'd'c'b'a'.

Question 3. A long solenoid with 15 turns per cm has a small loop area 2.0 cm² placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?
Solution:

$$\mathcal{E} = NA\frac{dB}{dt} \qquad [\because \phi = BAN]$$

$$\text{But } B = \frac{\mu_0 N'}{l}.I \qquad \frac{N'}{l} = 15 \text{ cm} = \frac{15}{10^{-2}} \text{ m}^{-1} = 1500 \text{ m}^{-1}$$

$$dB = \frac{\mu_0 N'}{l}(I_2 - I_1) = \frac{4\pi \times 10^{-7}}{1} \times 1500(4-2)$$

$$dt = 0.1 \text{ s}, \quad A = 2.0 \text{ cm}^2 = 2.0 \times 10^{-4} \text{m}^2$$

$$\therefore \mathcal{E} = 1 \times 2 \times 10^{-4} \times \frac{4\pi \times 10^{-7} \times 1500(2)}{1 \times 0.1}$$

$$\mathcal{E} = 7.51 \times 10^{-6} \text{V}$$

Question 4. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of the uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1cm s^{-1} in a direction normal to the (i) longer side, (ii) shorter side of the loop? For how long does the induced voltage last in each case?

Solution: (i) Direction of motion normal to longer side: Length of the longer side,

1 = 8 cm = 8 x 10^{-2} m

∴ e = BlV = 0.3 x 8 x 10^{-2} ×10^{-2} = 0.24 x 13^{-3}V = 0.24 mV

The.e.m.f. will last in the loop, till it does not get out of the magnetic field i.e. for the time the loop takes to travel a distance equal to the length of the shorter arm. If t is the time for which the e.m.f lasts in the loop, then

$$t = \frac{length \quad of \quad the \quad shorter \quad arm}{V} = \frac{2\times10^{-2}}{10^{2}} = 2S$$

(ii) Direction of motion normal to the shorter side:

Length of the shorter side, 1 = 2 cm = 2 x 10^{-2}m

∴ e = Blv = 0.3 x 2 x 10^{2} x 10^{-2} = 0.06mV

The time for which the e.m.f. lasts in the loop,

$$t = \frac{length \quad of \quad the \quad longer \quad arm}{V} = \frac{8\times10^{-2}}{10^{-4}} = 8S$$

Question 5. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Solution:

$$\mathscr{E} = \frac{Blv}{2} = \frac{Bl^2 \cdot \omega}{2} \ [\therefore v = l\omega] = \frac{0.5\times1\times1\times400}{2} = 100 \text{ V}$$

Question 6. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0 x 10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Solution: For a coil rotating in a magnetic field, E = – BANωinωt

E = – BANω (numerically)

Here B = 3 x 10^{-2} T, A= π^2= π x (8 x $10^{-2})^2$

N = 20, ω = 50 rad s^{-1}

$$\mathcal{E}_{max} = 20 \times 50 \times \pi \times 64 \times 10^{-4} \times 3 \times 10^{-2} = 0.603 \text{ V}$$

$\mathcal{E}_{average}$ over a cycle is zero.

$$I_{max} = \frac{\mathcal{E}_{max}}{R} = \frac{0.603}{10} = 0.0603 \text{ A}$$

$$P_{average} = \frac{1}{2} \mathcal{E}_{max} I_{max} = \frac{1}{2} \times 0.603 \times 0.0603 = 0.018 \text{ W}$$

The Source of power loss is the external rotor which provides the necessary torque.

Question 7. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0. 30 x 10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Solution: Here, length of the wire, 1 = 10m;

Velocity of the wire, V = 5.0 ms^1

Horizontal component of earth's magnetic field,

B_H = 0.30 x 10^{-4} Wb m^{-2}

(a) Now,e = $B_H I_v$ = 0.30 x $l0^{-4}$x 10 x 5.0 = 1.5 x 10^{-3} V

(b) The induced e.m.f. will be set up from west to east end.

(c) The eastern end will be at higher potential.

Question 8. Current in a circuit falls from 5.0 A to 0.0 A in 0.1s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit.

Solution:

$$\mathcal{E} = \left|L.\frac{dI}{dt}\right|, \qquad \mathcal{E} = 200\text{ V}, dI = (5-0)\text{A}, dt = 0.1\text{ sec}$$

$$L = \frac{\mathcal{E}}{\left(\frac{dI}{dt}\right)} = \frac{200 \times 0.1}{5} = 4\text{ H}$$

Question 9. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?
Solution:
Hers, M = 1.5H; dl = 20-0 = 20A;
dt = 0.5 s
Now φ = MI
Or dφ= Mdl=1.5 x 20 = 30Wb.

Question 10. A jet plane is traveling towards the west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the earth's magnetic field at the location has a magnitude of 5 x 10^{-4} T and the dip angle is 30°?
Solution: Here, v = 1,800 Km h^{-4} = 1,800 x [1,000 m] x $[60 \times 60\text{ s}]^{-1}$ = 500 ms^{-1}
Earth's magnetic field, B = 5.0 x 10^{-4} t;
angle of dip, 5 = 30°; length of the wing = 25 m
Now, B_v = B sin5 = 5.0x 10^{-4} sin 30° = 2.5×10^{-4}T
The vertical component [B_v] of the earth's field is normal to both the wings and the direction of motion. Therefore, induced e.m.f. produced,
e = B_v x l x V = 2.5 x 500 = 3.125V.

Question 11. Suppose the loop in Exercise.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 T s^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat? What is the source of this power?
Solution:

$\mathscr{E} = \frac{d\phi_B}{dt} = A.\frac{dB}{dt}$

$= 8 \times 2 \times 10^{-4} \times 0.02 = 3.2 \times 10^{-5}$ V

Induced current, $I = \frac{\mathscr{E}}{R}$

$= \frac{3.2 \times 10^{-5}}{1.6}$

$= 2 \times 10^{-5}$ A

Power loss $= P = \mathscr{E}I = 3.2 \times 10^{-5} \times 2 \times 10^{-5}$

$= 6.4 \times 10^{-10}$ W

Source of this power is the external agency which brings the change in the magnetic field.

Question 12. A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cm s^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction (that is it increases by 10^{-3} T cm^{-1} as one move in the negative x-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 m Ω.

Solution: Rate of change of flux due to time variation in B = $144 \times 10^{-4} m^2 \times 10^{-3}$ $Ts^{-1} = 1.44 \times 10^{-5}$ Wb/s

Rate of change of flux due to motion of the loop in nonuniform B = 144×10^{-4} $m^2 \times 10^{-3}$ $Tcm^{-1} \times 8$ cm/s $= 11.52 \times 10^{-5}$ Wb/s

Net change in flux $= \mathscr{E} = (1.44 + 11.52) \times 10^{-5} = 12.96 \times 10^{-5}$ V

Current $I = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}} = 2.88 \times 10^{-2}$ A

The direction of the induced current is such that it increases the magnetic flux linked with the loop in the positive direction.

Question 13. It is desired to measure the magnitude of the field between the poles of a powerful loudspeaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flew in the coil (measured by a ballistic galvanometer connected to the coil) is 7.5mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of the magnet.

Solution:

$$\text{Charge } Q = \frac{N}{R}(\phi_i - \phi_f)$$

$$B = \frac{\phi_i}{A},\ \phi_i = BA$$

$\phi_i = 1.5 \times 10^{-4},\ \phi_f = 0,\ N = 25,\ R = 0.5$

$Q = 7.5 \times 10^{-3}C,\ A = 2 \times 10^{-4}m^2$

$$\therefore B = \frac{RQ}{NA} = \frac{0.5 \times 7.5 \times 10^{-3}}{25 \times 2 \times 10^{-4}} = \frac{5 \times 7.5}{50}$$

i.e., $B = 0.75$ T

Question 14.

An air-cored solenoid with a length of 30 cm, area of cross-section 25 cm^2, and a number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in the magnetic field near the ends of the solenoid.

Solution:

$$\mathcal{E} = \left|\frac{d\phi_B}{dt}\right|$$

$$B = \frac{\mu_0 NI}{l} = 4\pi \times 10^{-7} \times \frac{500}{30 \times 10^{-2}} \times 2.5$$

$$\phi_B = BAN = \frac{4\pi \times 10^{-7} \times 500 \times 2.5}{30 \times 10^{-2}} \times 25 \times 10^{-4} \times 500 = 65.45 \times 10^{-4}\ \text{Wb}$$

i.e., Initial flux linkage, $\phi_1 = 65.45 \times 10^{-4}$ Wb, Final flux linkage, $\phi_2 = 0$

$\therefore d\phi_B = -65.45 \times 10^{-4}$Wb

$$\therefore \mathcal{E} = \left|\frac{d\phi_B}{dt}\right| = \frac{65.45 \times 10^{-4}}{10^{-3}} = 6.545\ \text{V}$$