[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.1 If $f(\theta) = \min.\left(|2x-7|+|x-4|+|x-2-\sin\theta|\right)$, where $x, \theta \in R$, then maximum value of $f(\theta)$ is
(A) 2 (B) 3 (C) 4 (D) 5

Q.2 The number of real solution of the equation $\cos^{-1}x + 2\cos^{-1}x + 3\cos^{-1}x = 6\pi$ is
(A) 0 (B) 1 (C) 2 (D) infinitely many

Q.3 If $g(x) = 2\sqrt{2}\sin x + \cos x$ then maximum value of $P(x) = \sqrt{g(x)-2} + \sqrt{4-g(x)}$ is
(A) 0 (B) 1 (C) 2 (D) 3

Q.4 Domain of the function $f(x) = \dfrac{\sin^{-1}2x}{\sqrt{\cos x + \dfrac{x^2}{2} - 1}}$ is $[a, b] - \{c\}$, then $(b - a + c)$ equals
(A) 0 (B) 1 (C) 2 (D) 3

Q.5 Let $f : (-\infty, \infty) \to \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ such that $f(x) = 2\tan^{-1}(2013^x) - \dfrac{\pi}{2}$, then
(A) $f(-3) + f(3) = 0$ (B) $f(-2) - f(2) = 0$
(C) $f(x)$ is a decreasing function (D) $f^2(5) - f^2(-5)$ is non zero

Q.6 Let $f(x) = \sqrt{6 - 5^{\sin^{-1}x} - 5^{1-\sin^{-1}x}}$ and $g(x) = \sqrt{5^{\sin^{-1}x}}$ then range of the function $h(x) = f(x).g(x)$ is $[a, b]$, then $(a + b)$ equals
(A) 1 (B) 3 (C) 2 (D) 26

Q.7 If the equation $x^3 + px^2 + qx + 1 = 0$, $(p < q)$ has only one real root x_0, then value of $2\tan^{-1}(\text{cosec}\, x_0) + \tan^{-1}(2\sin x_0 \sec^2 x_0)$ is
(A) $-\pi$ (B) π (C) $\dfrac{\pi}{2}$ (D) 0

Q.8 If $x = \alpha$ satisfies the equation $\sin^{-1}x + \cos^{-1}x^2 + \dfrac{\pi}{2} = 0$, then the value of $\dfrac{\sec^{-1}\alpha - \tan^{-1}\alpha}{\cot^{-1}\alpha - \text{cosec}^{-1}\alpha}$ is equal to
(A) 0 (B) 1 (C) -1 (D) π

Q.9 If $f(x) = \sum_{r=0}^{\infty} |x|\left(\tan^{-1}\left(\tan\dfrac{1}{2^r}\right)\right)$, then number of solution(s) of the equation $f(x) + x^2 = 1$ is
(A) 0 (B) 1 (C) 2 (D) 3

Q.10 The number of solutions of the equation $\sin^{-1}x = 2\tan^{-1}x$, is
(A) 1 (B) 2 (C) 3 (D) 4

[PARAGRAPH TYPE]

Q.11 to Q.14 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Paragraph for question nos. 11 & 12

Let a_r ($a_r > 0 \ \forall\, r \in N$) be the r^{th} term of a G.P. such that $\sqrt{a_1 \cdot a_3 \cdot a_5 \cdot a_7} = (18a_1 + a_3)^2$ and $\sum_{r=1}^{\infty} \frac{1}{a_r} = \frac{3}{4}$.

Q.11 The value of $\sum_{r=1}^{10} \tan\left(\sin^{-1}\left(\frac{a_{r+1}}{\sqrt{a_r^2 + a_{r+1}^2}}\right)\right)$ equals
(A) 15 (B) 20 (C) 30 (D) 40

Q.12 The value of $\sin^{-1}\left(\sin\left(\frac{a_2}{a_1}\right)\right) + \tan^{-1}(\tan(a_1))$ is equal to
(A) – 1 (B) 0 (C) 1 (D) 2π

Paragraph for question nos. 13 & 14

Consider, $f(x) = \left|\frac{3 - 2|x|}{1 + |x|}\right|$ and $g(x) = \frac{(2|x| - 3)^2}{3}$.

Two more functions $h(x)$ and $k(x)$ are defined as
$h(x) = \max.\{f(x), g(x)\}$
$k(x) = \min.\{f(x), g(x)\}$

Q.13 If $(x_i, h(x_i))$ ($i = 1, 2, 3, \ldots\ldots n$) are the points on the graph of $y = h(x)$ where no unique tangent can be drawn then $\sum_{i=1}^{n} |x_i|$ equals
(A) 3 (B) 5 (C) 7 (D) 8

Q.14 If the equation $k(x) = p$ has four solutions then number of integral values of p is(are)
(A) 0 (B) 1 (C) 2 (D) 3

[MULTIPLE CORRECT CHOICE TYPE]

Q.15 & Q.16 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct.

Q.15 If $f : R - \{-1, k\} \to R - \{\alpha, \beta\}$ is a bijective function defined by $f(x) = \frac{(2x-1)(2x^2 - 4px + p^3)}{(x+1)(x^2 - p^2x + p^2)}$
(where $p \geq 0$), then identify which of the following statement(s) is(are) correct?
(A) If $k \in (-1, 1)$ then $\alpha + \beta = 2$
(B) If $k \in (-1, 1)$ then $\alpha + \beta = 6$
(C) If $k \in (1, 3)$ then $\alpha + \beta = 4$
(D) If $k \in (1, 3)$ then $\alpha + \beta = 6$

Q.16 If equation $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x^2 - 1} + \tan^{-1}\tan y = k$ has atleast one solution, then $k \in \left(\frac{p\pi}{2}, \frac{q\pi}{2}\right)$, where $p, q \in I$, then value of $(p + q)$ is greater than or equal to

(A) 2 (B) 3 (C) 4 (D) 5

[INTEGER TYPE]

Q.17 to Q.20 are "Integer Type" questions. (The answer to each of the questions **are upto 4 digits**)

Q.17 Let $f(x)$ be a real valued function such that $|f(x) + x^2 + 1| \geq |f(x)| + |x^2 + 1|$ and $f(x) \leq 0$, then find the absolute value of $\sum_{r=1}^{5}(1 + f(r))$.

Q.18 Let $f(x) = x^2 - 3x - 4$ and $g(x) = \text{sgn } x$

then sum of all the roots of the equation $g(f(x)) = \sin\frac{\pi x}{2}$ when $x \in [-10, 20]$

(where sgn denotes signum function)

Q.19 If complete domain of the function $f(x) = \dfrac{5 + 3x + \tan^{-1} x}{\log_{2013}\left(\dfrac{1}{2 - \sin 3x}\right)} + \sqrt{10 - x} + \sqrt{x}$

is $[a, b] - A$.

Let L is the sum of all entries in set A, then $\frac{6L}{\pi} + (b - a)$ equals

Q.20 If $\sum_{n=0}^{\infty} \tan^{-1}\left(\dfrac{\cot^{-1}(n^2 + 3n + 3)}{1 + \cot^{-1}(n + 1)\cot^{-1}(n + 2)}\right) = \tan^{-1}\left(\frac{p}{4}\right)$ then find the value of $[\cos^{-1}(\cos(p - 1))]$.

[**Note:** [k] denotes greatest integer less than or equal to k.]

ANSWER KEY

Q.1	B	Q.2	B	Q.3	C	Q.4	B	Q.5	A
Q.6	C	Q.7	A	Q.8	B	Q.9	C	Q.10	C
Q.11	C	Q.12	A	Q.13	D	Q.14	B	Q.15	AD
Q.16	ABC	Q.17	[5]	Q.18	[0041]	Q.19	[0055]	Q.20	[2]

[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Q.1 (B)

$f(\theta) = \min.\left(|2x-7| + |x-4| + |x-2-\sin\theta|\right)$

Let $g(x) = |2x-7| + |x-4| + |x-2-\sin\theta|$

$g(x)|_{min} = g\left(\frac{7}{2}\right) = \frac{1}{2} + \frac{3}{2} - \sin\theta = 2 - \sin\theta = f(\theta)$

$\therefore\ f(\theta)|_{max} = 3$

Q.2 (B)

$\cos^{-1}x + 2\cos^{-1}x + 3\cos^{-1}x = \pi + 2\pi + 3\pi$

$\Rightarrow x = -1.$

Q.3 (C) 2

$g(x) = 2\sqrt{2}\sin x + \cos x$

$g(x) \in [-3, 3]$

$y = \sqrt{g(x)-2} + \sqrt{4-g(x)}$

$y^2 = g(x) - 2 + 4 - g(x) + 2\sqrt{g(x)-2}\sqrt{4-g(x)}$

$y^2 = 2 + 2\sqrt{4g - g^2 - 8 + 2g} = 2 + 2\sqrt{-[g(x)]^2 + 6(g(x)) - 8} = 2 + 2\sqrt{1 - [g(x)-3]^2}$

where $g(x) = 3$ then $y^2{}_{max} = 4$

$y_{max} = 2$ **Ans.**

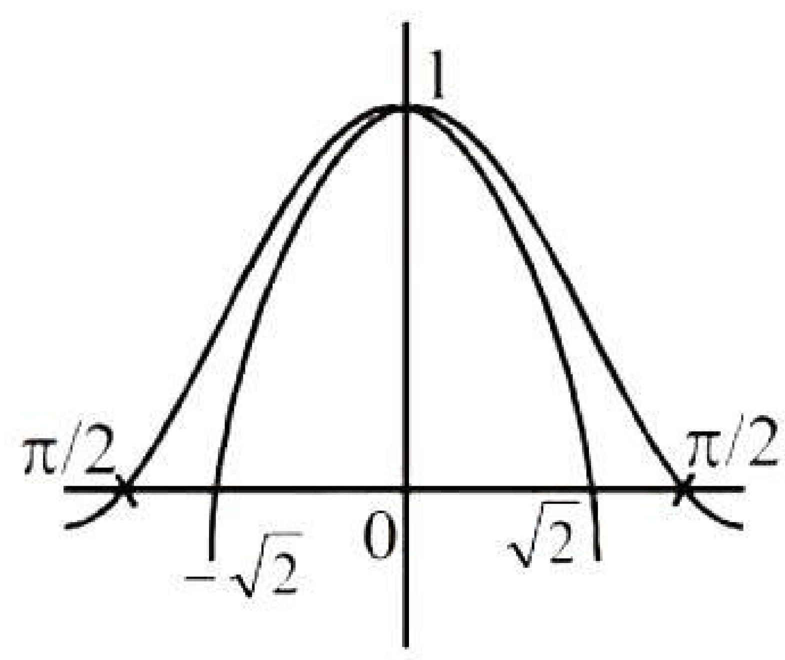
Q.4 (B)

Sol. $2x \in [-1, 1]$

$x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

and $\cos x + \frac{x^2}{2} - 1 > 0$

$\cos x > 1 - \frac{x^2}{2}$ from graph



$x \in R - \{0\}$

Domain is $\left[\frac{-1}{2}, \frac{1}{2}\right] - \{0\}$

Q.5 (A)

Sol. $f(x) = 2\tan^{-1}(2013^x) - \frac{\pi}{2}$

$f(-x) = 2\tan^{-1}(2013^{-x}) - \frac{\pi}{2}$

$= 2\tan^{-1}\left(\frac{1}{2013^x}\right) - \frac{\pi}{2} = 2\cot^{-1}(2013^x) - \frac{\pi}{2} = 2\left[\frac{\pi}{2} - \tan^{-1}(2013^x)\right] - \frac{\pi}{2}$

$= \frac{\pi}{2} - 2\tan^{-1}(2013^x) = -\left[2\tan^{-1}(2013^x) - \frac{\pi}{2}\right] \Rightarrow f(-x) = -f(x) \Rightarrow f(x)$ is odd.

As, $f(-5) = -f(5) \Rightarrow f^2(-5) = f^2(5) \Rightarrow f^2(5) - f^2(-5) = 0$ **Ans.**

Q.6 (C)

Sol. $f(x) = \sqrt{6 - 5^{\sin^{-1}x} - 5^{1-\sin^{-1}x}}$ and $g(x) = \sqrt{5^{\sin^{-1}x}}$

$x \in [-1, 1]$

$6 - 5^{\sin^{-1}x} - 5^{1-\sin^{-1}x} = 6 - 5^{\sin^{-1}x} - \frac{5}{5^{\sin^{-1}x}} = -\left[\frac{(5^{\sin^{-1}x})^2 - 6(5^{\sin^{-1}x}) + 5}{5^{\sin^{-1}x}}\right]$

$= -\frac{(5^{\sin^{-1}x} - 5)(5^{\sin^{-1}x} - 1)}{5^{\sin^{-1}x}} = \frac{4 - (5^{\sin^{-1}x} - 3)^2}{5^{\sin^{-1}x}}$

$\Rightarrow y = h(x) = f(x).g(x) = \sqrt{-(5^{\sin^{-1}x} - 5)(5^{\sin^{-1}x} - 1)} = \sqrt{4 - (5^{\sin^{-1}x} - 3)^2}$

for domain $1 \le 5^{\sin^{-1}x} \le 5 \Rightarrow \sin^{-1}x \in [0, 1]$

$\Rightarrow 0 \le 4 - (5^{\sin^{-1}x} - 3)^2 \le 4 \Rightarrow y \in [0, 2]$

Q.7 (A)

$f(x) = x^3 + px^2 + qx + 1$

$f(0) = 1 > 0$; $f(-1) = p - q < 0$

$\therefore \quad x_0 \in (-1, 0)$

$2\tan^{-1}\left(\frac{1}{\sin x_0}\right) + \tan^{-1}\left(\frac{2\sin x_0}{1 - \sin^2 x_0}\right)$

$2\left(\tan^{-1}\left(\frac{1}{\sin x_0}\right) + \tan^{-1}(\sin x_0)\right)$

$2\left(-\frac{\pi}{2}\right) = -\pi \quad (\because \sin x_0 < 0)$

Q.8 (B)

$\because \quad \sin^{-1}x + \cos^{-1}x^2 + \frac{\pi}{2} = 0$

$\Rightarrow \quad \frac{\pi}{2} + \sin^{-1}x = -\cos^{-1}x^2$

$$\Rightarrow \quad \cos\left(\frac{\pi}{2}+\sin^{-1}x\right)=\cos\left(-\cos^{-1}x^2\right)$$

$\Rightarrow \quad -x = x^2$

$\Rightarrow \quad x = 0, -1 \qquad (x = 0 \text{ rejected})$

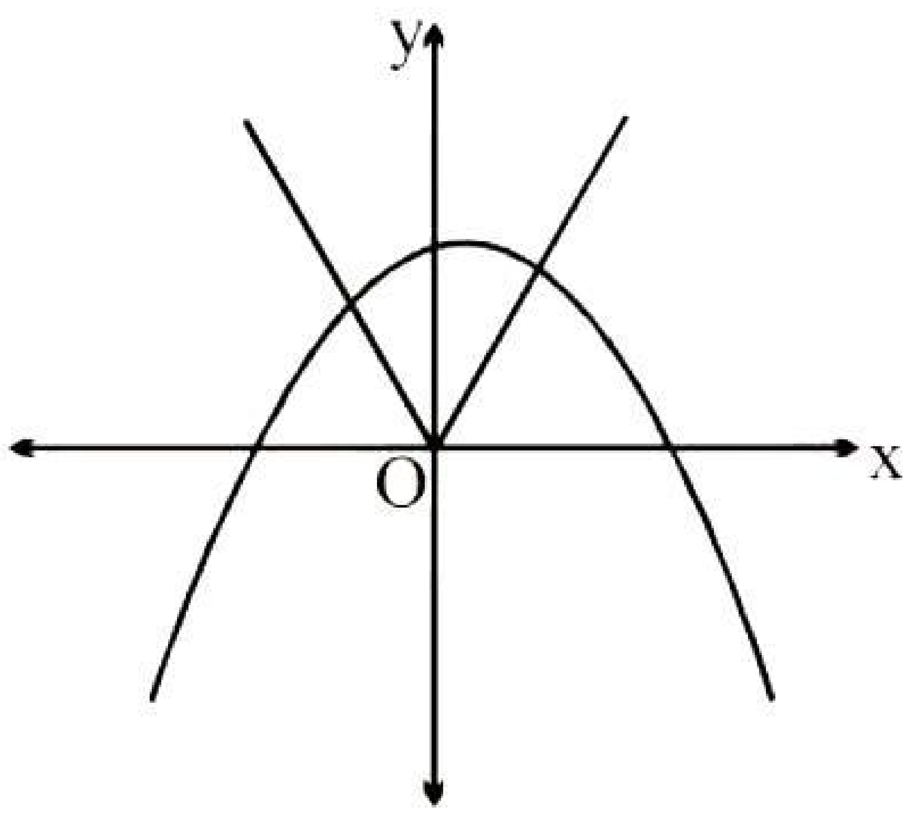
$\therefore \quad x = -1 = \alpha$

$$\therefore \quad \text{Given expression} = \frac{\sec^{-1}(-1)-\tan^{-1}(-1)}{\cot^{-1}(-1)-\operatorname{cosec}^{-1}(-1)} = \frac{\pi-\left(\frac{-\pi}{4}\right)}{\frac{3\pi}{4}-\left(\frac{-\pi}{2}\right)} = \frac{\frac{5\pi}{4}}{\frac{5\pi}{4}} = 1$$

Q.9 (C)

$$f(x)=\sum_{r=0}^{\infty}|x|\left(\tan^{-1}\left(\tan\frac{1}{2^r}\right)\right)=|x|\sum_{r=0}^{\infty}\frac{1}{2^r}$$

$$=|x|\left(1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\infty\right)=|x|\left(\frac{1}{1-\frac{1}{2}}\right)=2|x|$$



$\therefore \; f(x) = 1 - x^2$

$\Rightarrow 2|x| = 1 - x^2$

$\therefore$ Number of solutions = 2. **Ans.**

Q.10 (C)

$(\sin^{-1}x) = (2\tan^{-1}x)$

Take sine on both sides, we get

$x = \sin(2\tan^{-1}x) \Rightarrow x = 0, \pm 1$ **Ans.**

Q.11 (C)

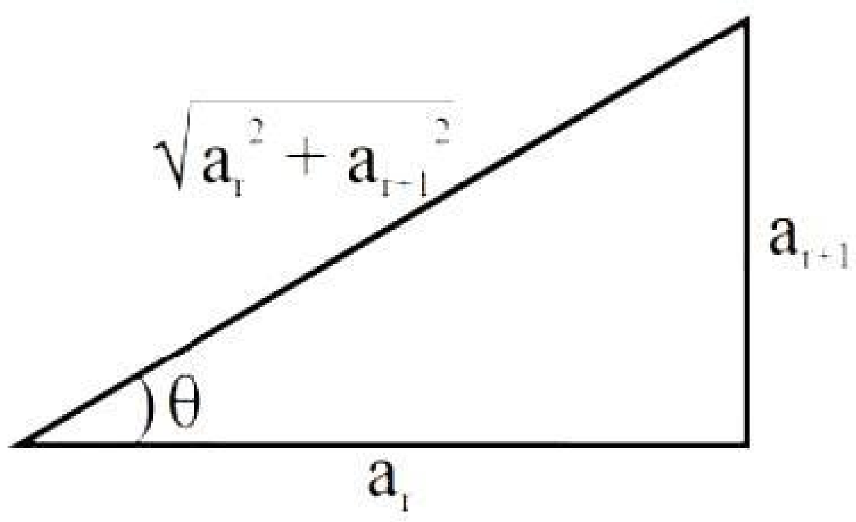
Q.12 (A)

Let GP is a, ar, ar^2, ... from given information

$a = 2, r = 3$

(i) $$\sum_{r=1}^{10}\tan\tan^{-1}\left(\frac{a_{r+1}}{a_r}\right)$$

$$\sum_{r=1}^{10}\frac{a_{r+1}}{a_r}$$



$$=\frac{a_2}{a_1}+\frac{a_3}{a_2}+\ldots\ldots+\frac{a_{11}}{a_{10}}$$

$= 10 \times r = 30.$

(ii) $$\sin^{-1}\sin\left(\frac{ar}{a}\right)+\tan^{-1}\tan a$$

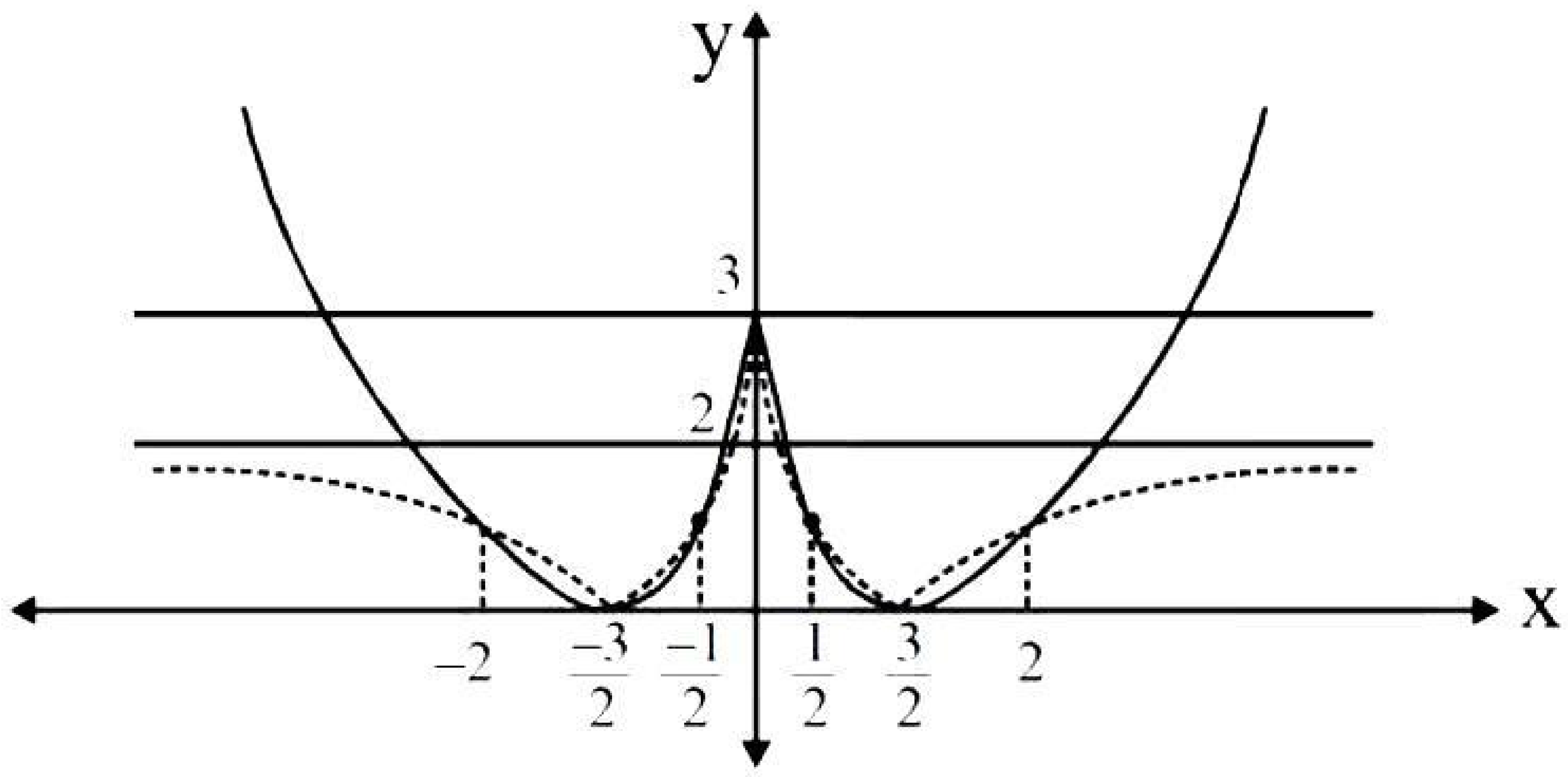
$\sin^{-1}\sin 3 + \tan^{-1}\tan 2$

$= \pi - 3 + 2 - \pi = -1.$ **Ans.**

Q.13 (D)

Q.14 (B)

$$f(x) = \left|\frac{3-2|x|}{1+|x|}\right|$$



(i) $x = \pm 2, \pm\frac{3}{2}, \pm\frac{1}{2}, 0$

$$\therefore \sum_{i=1}^{5}|x_i| = 7$$

(ii) $k(x) = p$ has four solutions.

$\therefore\ p \in (0, 2)$

Q.15 (AD)

$f : R - \{-1, k\} \to R - \{\alpha, \beta\}$

$$f(x) = \frac{(2x-1)(2x^2-4px+p^3)}{(x+1)(x^2-p^2x+p^2)}$$

For the domain to be $R - \{-1, k\}$, k must be a repeated root of $x^2 - p^2x + p^2 = 0$

$\therefore p^4 - 4p^2 = 0 \Rightarrow p^2(p^2-4) = 0, p = 0, \pm 2\ (p \geq 0)$

For $p = 0,\ k = 0$

$$f(x) = \frac{(2x-1)2x^2}{(x+1)(x^2)}, \Rightarrow D_f : R - \{-1, 0\}, R_f : R - \{4, -2\}$$

$\therefore \alpha + \beta = 2$

For $p = 2, k = 2$

$$f(x) = \frac{(2x-1)2(x-2)^2}{(x+1)(x-2)^2} \Rightarrow D_f : R - \{-1, 2\}, R_f : R - \{4, 2\}$$

$\alpha + \beta = 6$. **Ans.**

Q.16 (ABC)

$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x^2-1} + \tan^{-1}\tan y = k$

$0 \leq x \leq 1$ and $0 \leq x^2 - 1 \leq 1$

Hence, $x = 1$

$$\frac{\pi}{2} + \frac{\pi}{2} + \tan^{-1}\tan y = k$$

$$\frac{-\pi}{2} < \tan^{-1} \tan y < \frac{\pi}{2}$$

$$\therefore \ k \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

Hence, $(p + q) = 4$.

Q.17 5

$\because \quad |x + y| \leq |x| + |y|$

But $\quad |f(x) + (x^2 + 1)| \geq |f(x)| + |x^2 + 1|$

$\therefore \quad |f(x) + (x^2 + 1)| = |f(x)| + |x^2 + 1|$ will be true iff $f(x) \cdot (x^2 + 1) \geq 0$

$\Rightarrow \quad f(x) \geq 0$

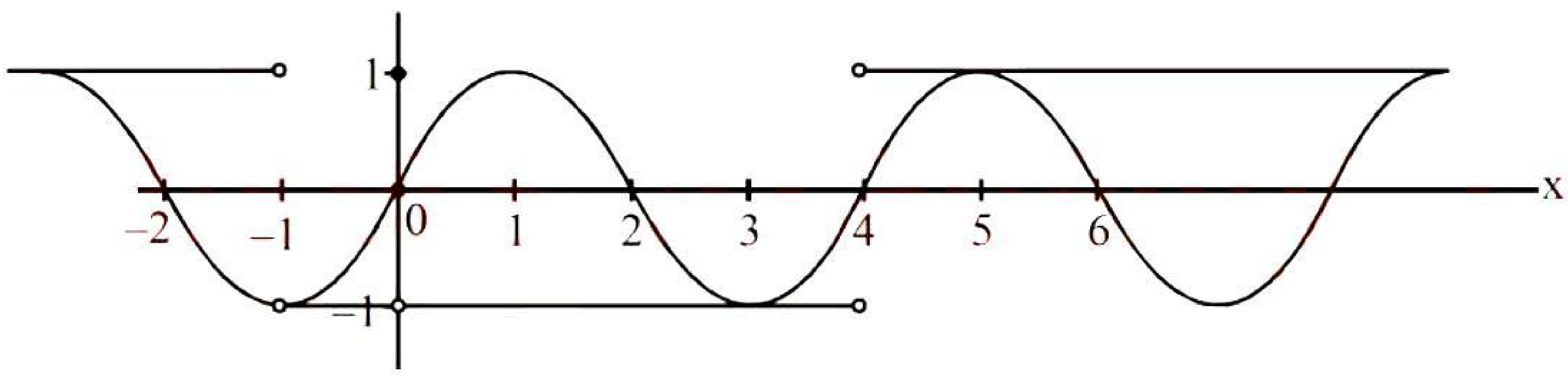
But $f(x) \leq 0$, $\therefore f(x) = 0$

$$\therefore \sum_{r=1}^{5}(1 + f(r)) = \sum_{r=1}^{5} 1 = 5 \ \textbf{Ans.}$$

Q.18 0041

$f(x) = (x^2 - 3x - 4) = (x - 4)(x + 1)$

$g(f(x)) = \text{sgn}\big((x - 4)(x + 1)\big)$, $y = \sin\dfrac{\pi x}{2}$



solutions are 3, 5, 9, 13, 17, –7, –3, 4

sum is 41.

Q.19 0055

$10 - x \geq 0 \Rightarrow x \leq 10 \quad ; \quad x \geq 0$

$x \in [0, 10]$

and $\dfrac{1}{3} \leq \dfrac{1}{2 - \sin 3x} \leq 1$

i.e., always positive

$\dfrac{1}{2 - \sin 3x} \neq 1$, when $\sin 3x \neq 1$

$3x \neq n\pi + (-)^n \dfrac{\pi}{2}$

$x \neq \dfrac{n\pi}{3} + (-)^n \dfrac{\pi}{6}$

We get $x \neq \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{9\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \dfrac{21\pi}{6}, \ldots$

but $\frac{21\pi}{6} > 10$

$$\Rightarrow L = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} + \frac{13\pi}{6} + \frac{17\pi}{6} =$$

$$L = \frac{45\pi}{6} \Rightarrow \frac{6L}{\pi} = 45$$

Q.20 2

$$\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{\tan^{-1}\left(\frac{(n+2)-(n+1)}{1+(n+1)(n+2)}\right)}{1+\tan^{-1}\left(\frac{1}{n+1}\right)\cdot\tan^{-1}\left(\frac{1}{n+2}\right)}\right)$$

$$= \sum_{n=0}^{\infty} \tan^{-1}\left(\frac{\tan^{-1}\left(\frac{1}{n+1}\right)-\tan^{-1}\left(\frac{1}{n+2}\right)}{1+\tan^{-1}\left(\frac{1}{n+1}\right)\cdot\tan^{-1}\left(\frac{1}{n+2}\right)}\right)$$

$$= \sum_{n=0}^{\infty}\left(\tan^{-1}\left(\tan^{-1}\left(\frac{1}{n+1}\right)\right)-\tan^{-1}\left(\tan^{-1}\left(\frac{1}{n+2}\right)\right)\right)$$

$$= \tan^{-1}\left(\tan^{-1}(1)\right) = \tan^{-1}\left(\frac{\pi}{4}\right)$$

$\therefore$ p = π

$\left[\cos^{-1}(\cos(\pi-1))\right] = [\pi - 1] = 2$ **Ans.**