6 Ratio and Proportion

When going shopping, children often look at the prices of various groceries. A parent can easily explain ratios to her child using two different boxes of cereal. For example, if a 1 kg box of cereal costs Rs 100 and a 200 gm box of cereal costs Rs 25, the 1 kg box is the better value because each gm of cereal is cheaper.

When you prepare recipes, paint your house, or repair gears in a large machine or in a car transmission, you use ratios and proportions.

- Say a recipe to make brownie requires 4 cups of flour for 6 persons.
- You may want to know how much flower to put for 24 persons.

One of the most important mathematical relationships used in the day to day world is Ratio. Proportions are built from ratios. A "ratio" is just a comparison between two different things. For example: Suppose there are thirty-five people, fifteen of whom are men. Then the ratio of men to women is 15 to 20. Notice that, in the expression "the ratio of men to women", "men" came first. This order is very important, and must be respected: whichever word came first, its number must come first. If the expression had been "the ratio of women to men", then the numbers would have been "20 to 15".

Ratio: The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as a: b. In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.

Proportion: The equality of two ratios is called proportion. If a : b = c : d, we write a : b**::**c : d and we say that a, b, c, d are in proportion. Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes

Thus, $a:b::c:d \Rightarrow (b\times c)=(a\times d)$.

- **Fourth Proportional:**
  If a : b = c : d, then d is called the fourth proportional to a, b, c.
- **Third Proportional:**
  a : b = c : d, then c is called the third proportion to a and b.
- **Mean Proportional:**
  Mean proportional between a and b is ab.
- **Compounded Ratio:**
  The compounded ratio of the ratios: (a : b), (c : d), (e : f) is (ace : bdf).
- **Duplicate Ratios:**

Duplicate ratio of $(a:b)$ is $(a^2:b^2)$.

Sub-duplicate ratio of $(a:b)$ is $(a:b)$.

Triplicate ratio of $(a:b)$ is $(a^3:b^3)$.

Sub-triplicate ratio of $(a:b)$ is $(a^{1/3}:b^{1/3})$.

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (componendo and dividendo)

Example 1. Compare the ratios 2 : 3 and 4 : 7

Solution: $2:3=\frac{2}{3}$ & $4:7=\frac{4}{7}$.

L.C.M. of denominators of $\frac{2}{3}$ & $\frac{4}{7}$ is 21.

$\therefore \quad \frac{2}{3}=\frac{2\times 7}{3\times 7}=\frac{14}{21}$

And $\frac{4}{7}=\frac{4\times 3}{7\times 3}=\frac{12}{21}$.

Clearly, $\frac{14}{21}>\frac{12}{21}$ or $(2:3)>(4:7)$

Example 2. If $a:b=4:5$ & $b:c=6:8$, *find* $a:c$.

Solution: $\frac{a}{b}=\frac{4}{b}$ & $\frac{b}{c}=\frac{6}{7}$.

$\therefore \quad \frac{a}{c}=\frac{a}{b}\times\frac{b}{c}=\frac{4}{5}\times\frac{6}{7}=\frac{24}{35}.$

Hence, $a:c=24:25$.

Example 3. In a ratio which is equal to 7:8, if the antecedent is 35, what is the consequent?

Solution: If antecedent is 7, then consequent = 8.

If antecedent is 35, then consequent $=\frac{8}{7}\times 35=40$.

Example 4. The ratio between two numbers is 2 : 3. If each number is increased by 4, the ratio between them becomes 5 : 7. What are the numbers?

Solution: Let the numbers be $2x$ and $3x$.Then,

$\frac{2x+4}{3x+4}=\frac{5}{7}$ or $14x+28=15x+20$ or $x=8$.

$\therefore$ The numbers are 16 and 24.

Example 5. (i) Find the fourth proportional to 3, 5 and 21.

(ii) Find the mean proportional between 64 and 81.

(iii) Find the third proportional to 9 and 12.

Solution: (i) Let $3:5::21:x$.

Then, $\frac{3}{5}=\frac{21}{x}$ *or* $x=\frac{5\times 21}{3}=35$.

(ii) Let $64:x::x:81$.

Then, $\frac{64}{x}=\frac{x}{81}$ *or* $x=\sqrt{64\times 81}=12$.

(iii) Since third proportional to 9 and 12 is the same as fourth proportional to 9, 12, 12.

Let $9:12::12:x$.

Then, $\frac{9}{12}=\frac{12}{x}$ *or* $x=\frac{12\times 12}{9}=16$.

Multiple Choice Questions

1. If A : B = 5 : 7, B : C = 6 : 11 then A : B : C is

a. 55 : 77 : 66 **b.** 30 : 42 : 77

c. 35 : 49 : 42 **d.** 55 : 42 : 96

2. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then $\frac{a+b+c}{c}$ is equal to

a. 7 **b.** 2 **c.** $\frac{1}{2}$ **d.** $7\frac{1}{7}$

3. If 15% of $x=20\%$ of Y, then X : Y is

a. 3 : 4 **b.** 4 : 3 **c.** 17 : 16 **d.** 16 : 17

4. The third proportional to 0.36 and 0.48 is–

a. 0.64 **b.** 0.1728 **c.** 0.42 **d.** 0.99

5. The sum of the two numbers is 30 and their different is 12. Find the ratio of these two numbers.

a. 3 : 7 **b.** 4 : 7 **c.** 5 : 6 **d.** 7 : 3

6. The ratio between two numbers is 3:4 and their L.C.M. is 180. The first number is:

a. 60 **b.** 45 **c.** 20 **d.** 15

7. An alloy is to contain copper and zinc in the ratio 9 : 4. If quantity of zinc is 24 kg., the quantity of copper is–

a. $10\frac{2}{3}$ kg **b.** $10\frac{1}{3}$ kg **c.** $9\frac{2}{3}$ kg **d.** 9 kg

8. Gold is 19 times as heavy as water and copper is 9 times as heavy as water. In what ratio should these be mixed to get an alloy 15 times as heavy as water?

a. 1 : 1 **b.** 2 : 3 **c.** 1 : 2 **d.** 3 : 2

9. 15 liters of mixture contains 20% alcohol and the rest of water. If 3 liters of water be mixed with it, the percentage of alcohol in the new mixture would be:

a. 15% **b.** $16\frac{2}{3}\%$ **c.** 17% **d.** $18\frac{2}{3}\%$

10. The average age of three boys is 25 years and their ages are in the ratio 3 : 5 : 7. The age of the youngest boy is:

a. 21 years **b.** 18 years **c.** 15 years **d.** 9 years

11. The speeds of three cars are in the ratio 5 : 4 : 6. The ratio between the time taken by them to travel the same distance is:

a. 5 : 4 : 6 **b.** 6 : 4 : 5

c. 10 : 12 :15 **d.** 12 : 15 : 10

12. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ and its perimeter is 104 cm. The length of the longest side is:

a. 52 cm **b.** 48 cm **c.** 32 cm **d.** 26 cm

13. Three containers have their volumes in the ratio 3 : 4 : 5. They are full of mixtures of milk and water. The mixtures contain milk and water in the ratio (4:1), (3:1), and (5:2) respectively. The contents of all these three containers are poured into fourth container. The ratio of milk and water in the fourth container is:

a. 4 : 1 **b.** 151 : 48 **c.** 157 : 53 **d.** 5 : 2

14. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper n C will be:

a. 5 : 7 **b.** 5 : 9 **c.** 7 : 6 **d.** 7 : 5

15. A sum of Rs. 53 is divided among A, B, C in such a way that A gets Rs. 7 more than what B gets and B gets Rs. 8 more than what C gets. The ratio of their shares is:

a. 16 : 9 : 18 **b.** 27 : 18 : 10

c. 18 : 25 : 10 **d.** 25:18:10

16. Find the ratio compounded of the four ratios: 4: 3, 9 : 13, 26 : 5 and 2 : 15?

a. 16 : 25 **b.**16 : 26 **c.** 16 : 27 **d.** 16 : 28

17. Divide 1458 into two parts such that one may be to the other as 2 : 7.

a. 1134 **b.** 1136 **c.** 1138 **d.** 1140

18. The sum of three numbers is 98. If the ratio between the first and second be 2 : 3 and that between the second and third be 5 : 8, then find the second number.

a. 10 **b.** 20 **c.** 30 **d.** 40

19. The ratio of the money with Ratio and Sita is 7: 15 that with Sita and Kavita is 7 : 16. If Rita has Rs. 490, how much money does Kavita have?

a. Rs. 2000 **b.** Rs. 2200

c.Rs.2300 **d.**Rs. 2400

20. A hound pursues a hare and takes 5 leaps for every 6 leaps of the hare, but 4 leaps of the hound are equal to 5 leaps of the hare. Compare the rates of the hound and the hare.

a. 25 : 24 **b.**25 : 26

c.25 : 27 **d.**25 : 28

ANSWERS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	b	b	a	d	b	a	d	b	c
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
d	b	c	d	d	a	a	c	d	a

SOLUTIONS

1. **(b)** A : B 5 : 7

B : C 6 : 11

A : B : C = 30 : 42 : 77

2. **(b)** $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=k$ then

$a=3k,\ b=4k,\ c=7k$

$\Rightarrow \frac{a+b+c}{c}=\frac{3k+4k+7k}{7k}=\frac{14k}{7k}=2$

3. **(b)** 15% of $x=20\%$ of y

$\frac{5x}{100}=\frac{20y}{100}=\frac{x}{y}=\left(\frac{20}{100}\times\frac{100}{15}\right)=\frac{4}{3}$

4. **(a)** Let the third proportional to 0.36 and 0.48 be × 0.36 : 0.48 : : 0.48 : x

$x=\frac{0.48\times0.48}{0.36}=0.64$

5. **(d)** Let two numbers are x and y

ratio of two numbers

x : y = (30 + 12) : (30 – 12) = 42 : 18 = 7 : 3

6. **(b)** Let the required numbers be 3x and 4x.

Then, their L.C.M. is12x. $12x=180\Leftrightarrow x=15.$

Hence, the first number is 45.

7. **(a)** Let the required numbers be 3x kg

Then, $9:4::24:\times\Leftrightarrow 9x=4\times24$

$x=\frac{4\times24}{9}=10\frac{2}{3}$

Hence, the required quantity of copper is $10\frac{2}{3}$ kg

8. **(d)** G = 19 w and C = 9 W (W = water)

Let 1 gm of gold be mixed × gm of copper to get (1+ x) gm of the alloy.

(1 gm gold) + (x gm copper) = (x + 1) gm of alloy 19W + 9W× = (x +1) × 15W

$19+9x=15(x+1)\Leftrightarrow 6x=4\Leftrightarrow x=\frac{2}{3}$

$\therefore$ Ratio of gold with copper $=1:\frac{2}{3}=3:2$

9. **(b)** Alcohol in 15 litres of mix = 20% of 15

Litres $=\left(\frac{20}{100}\times15\right)$ litres = 3 liters

Water in it = (15–3) litres= 12 litres

New quantity of mix = (15+3) =18 litres

Quantity of alcohol in it =3 litres

Percentage of alcohol in new mix

$=\left(\frac{3}{18}\times100\right)\%=16\frac{2}{3}\%$

10. **(c)** Total age of 3 boys = (25×3) = 75 years

Ratio of their ages = 3:5:7

Age of the younges

$t=\left(75\times\frac{3}{15}\right)$ years

= 15 years

11. **(d)** Ratio of time taken $=\frac{1}{5}:\frac{1}{4}:\frac{1}{6}=12:15:10$

12. **(b)** Ratio of sides $=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$

Largest side $=\left(104\times\frac{6}{13}\right)$ cm = 48cm.

13. **(c)** Let the three containers contain $3x$, $4x$ and $5x$ liters of mixtures respectively.

Milk in 1st mix. $=\left(3x\times\frac{4}{5}\right)$ liters $=\frac{12x}{5}$ liters

Water in 1st mix. $=\left(3x-\frac{12x}{5}\right)$ liters $=\frac{3x}{5}$ liters

Milk in 2nd mix $=\left(4x\times\frac{3}{4}\right)$ liters = $3x$ liters

Water in 2nd mix. $=(4x-3x)$ liters = x liters

Milk in 3rd mix. $=\left(5x\times\frac{5}{7}\right)$ liters $=\frac{10x}{7}$ liters

Water in 3rd mix. $=\left(5x-\frac{25}{7}\right)$ liters $=\frac{10x}{7}$ liters

Total milk in final mix. $=\left(\frac{12x}{5}+3x+\frac{25x}{7}\right)$ liters

$=\frac{314x}{35}$ liters

Total water in final mix. $= \left(\frac{3x}{5} + x + \frac{10x}{7}\right)$ liters

$= \frac{106x}{35}$ liters

Required ratio of milk and water

$= \frac{314x}{35} : \frac{106x}{35} = 157 : 53$

14. **(d)** Gold in $C = \left(\frac{7}{9} + \frac{7}{18}\right)$ units.

Copper in $C = \left(\frac{2}{9} + \frac{11}{18}\right)$ units $= \frac{5}{6}$

Gold: Copper $= \frac{7}{6} : \frac{5}{6} = 7 : 5$

15. **(d)** Suppose C gets Rs. X

Then, B gets

Rs. $(x+8)$ and A gets

Rs. $(x+15)$

Then, $X + (x+8) + (x+15) = 53 \Leftrightarrow x = 10$

A:B:C = (10+15) : (10+8) :10

= 25:18:10

16. **(a)** The required ratio

$= \frac{4 \times 9 \times 26 \times 2}{3 \times 13 \times 5 \times 15} = \frac{16}{25}$

Note: When the ratio 4 : 3 is compounded with itself, the resulting ratio is $4^2 : 3^2$. It is called the duplicate ratio of 4 : 3.

Similarly, $4^3 : 3^3$. is the triplicate ratio of 4 : 3.

$\sqrt{4} : \sqrt{3}$ is called the sub-duplicate ratio of 4 : 3.

$a^{1/3} : b^{1/3}$ is sub-duplicate ratio of a and b.

17. **(a)** 1st part

$= 2 \times \frac{1428}{2+7} = 2 \times \frac{1428}{9} = 324$;

2nd part

$= 7 \times \frac{1428}{9} = 1134$

18. **(c)** The theorem does not give the direct value of the second number, but we can find the combined ratio of all the three numbers by using the above theorem. The ratio among the three numbers is:

2 : 3

5 : 8

10 : 15 : 24

$\therefore$ The second number

$= \frac{98}{10+15+24} \times 15 = 30$

19. **(d)** Rita : Sita : Kavita

7 : 15

7 : 16

49 : 105 : 240

The ratio of money with Rita, Sita and Kavita is 49 : 105 : 240, we see that

$49 \equiv$ Rs.490

$\therefore$ $240 \equiv$ Rs.2400

20. **(a)** 4 leaps of hound = 5 leaps of hare

$\therefore$ 5 leaps of hound $= \frac{25}{4}$ leaps of hare

$\therefore$ the rate of hound : rate of hare

$= \frac{25}{4} : 6 = 25 : 24$

□□□