CONTINUITY AND DIFFERENTIABILITY

TOTAL MARKS (15)

FOUR MARK:

Find the values of k so that function f is continuous at the indicated points in the following

1. $f(x) = \begin{cases} kx^2 \text{ if } x \leq 2 \\ 3, \text{if } x > 2 \end{cases}, \text{at } x = 2.$

   Solution: Given $f(x) = kx^2$

$f(2) = 4k$

RHL= $\lim_{x \to a^+} f(x)$

$= \lim_{x \to 2^+} 3$

$= 3$

LHL= $\lim_{x \to a^-} f(x)$

$= \lim_{x \to 2^-} kx^2$

=4k

Since f(x) is continuous at x = 2

$\therefore f(a) = RHL = LHS$

$4k = 3 = 4k$

$\therefore 4k = 3$

$k = \frac{3}{4}$

2. $f(x) = \begin{cases} kx + 1, \text{if } x \leq 5 \\ 3x - 5, \text{if } x > 5 \end{cases}$ at x = 5

Solution: Given $f(x) = kx + 1$

$f(5) = 5k + 1$

RHL= $\lim_{x \to a^+} f(x)$

$= \lim_{x \to 5^+} 3x - 5$

$= 3(5) - 5$

$= 10$

LHL= $\lim_{x \to a^-} f(x)$

$= \lim_{x \to 5^-} kx + 1$

$= 5k + 1$

Since f(x) is continuous at x = 5

$\therefore f(a) = RHL = LHS$

$5k + 1 = 10 = 5k + 1$

$5k + 1 = 10$

$\therefore k = \frac{-9}{5}$

3. $f(x) = \begin{cases} kx + 1, \text{if } x \leq \pi \\ \cos x \text{ , if } x > \pi \end{cases}$ at $x = \pi$

Solution: Given $f(x) = kx + 1$

$f(5) = \pi k + 1$

RHL= $\lim_{x \to a^+} f(x)$

$= \lim_{x \to \pi^+} cosx$

$= \cos \pi$

$= -1$

LHL= $\lim_{x \to a^-} f(x)$

$= \lim_{x \to \pi^-} kx + 1$

$= \pi k + 1$

Since f(x) is continuous at x = π

$\therefore f(a) = RHL = LHS$

$$\pi k + 1 = -1 = \pi k + 1$$
$$\pi k + 1 = -1$$
$$\therefore k = \frac{-2}{\pi}$$

4. $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ at x = $\frac{\pi}{2}$

Solution: Given $f(x) = 3$

$$\text{RHL} = \lim_{x \to a^+} f(x)$$
$$= \lim_{x \to \pi/2^+} \frac{k\cos x}{\pi - 2x}$$
$$= \lim_{h \to 0} \left[\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi - 2\left(\frac{\pi}{2}+h\right)}\right]$$
$$= \lim_{h \to 0} \left[\frac{k.(-\sin h)}{\pi - \pi - 2h}\right]$$
$$= \lim_{h \to 0} \left[\frac{-k \sin h}{-2h}\right]$$
$$= \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h}$$
$$=. \frac{k}{2}$$

$$\text{LHL} = \lim_{x \to a^-} f(x)$$
$$= \lim_{x \to \pi/2^-} \frac{k\cos x}{\pi - 2x}$$
$$= \lim_{h \to 0} \left[\frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi - 2\left(\frac{\pi}{2}-h\right)}\right]$$
$$= \lim_{h \to 0} \left[\frac{k.(\sin h)}{\pi - \pi + 2h}\right]$$
$$= \lim_{h \to 0} \left[\frac{k \sin h}{2h}\right]$$
$$= \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h}$$
$$=. \frac{k}{2}$$

Since f(x) is continuous at x = $\frac{\pi}{2}$

$$\therefore f(a) = RHL = LHS$$
$$3 = \frac{k}{2} = \frac{k}{2}$$
$$\therefore \frac{k}{2} = 3$$
$$k = 6$$

5. Find the relationship between a and b so that the function f defined by

$f(x) = \begin{cases} ax + 1, \text{if } x \leq 3 \\ bx + 3, \text{if } x > 3 \end{cases}$

Solution: Given $f(x) = ax + 1$

$$f(3) = 3a + 1$$
$$\text{RHL} = \lim_{x \to a^+} f(x)$$
$$= \lim_{x \to 3^+} bx + 3$$
$$= 3b + 1$$

LHL= $\lim_{x\to a^-} f(x)$

$= \lim_{x\to 3^-} ax+1$

$= 3a+1$

Since f(x) is continuous at x = 3

$\therefore f(a) = RHL = LHS$

$3a+1 = 3b+3 = 3a+1$

$\therefore\ 3a+1 = 3b+3$

3a= 3b+2

$a = \frac{3b+2}{3}$

a = $b + \frac{2}{3}$.

5. Find the value of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & if\ x \le 2 \\ ax+b & if\ 2 < x < 10 \\ 21 & if\ x \ge 10 \end{cases}$$ TRY

Solution: At $x = 2$

$f(x) = 5,\quad RHL = 2a+b,\ LHL = 5$

$\therefore 2a+b = 5 \dots\dots(1)$

At $x = 10$

$f(x) = 21,\quad RHL = 21,\ LHL = 10a+b$

$\therefore 10a+b = 21 \dots\dots(2)$

From (1) and (2) we get $a = 2\ and\ b = 1$

SECOND ORDER DERIVATIVES

FIVE MARK:

1. If $y = A\sin x + B\cos x$ then prove that $\frac{d^2y}{dx^2} + y = 0$

Ans : Given, $y = A\sin x + B\cos x$

Differentiate w.r.to x

$\Rightarrow \frac{dy}{dx} = A\cos x - B\sin x$

Differentiate w.r.to x

$$\Rightarrow \frac{d^2y}{dx^2} = -A\sin x - B\cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -(A\sin x + B\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -y$$

$$\therefore \quad \frac{d^2y}{dx^2} + y = 0$$

2. If $y = 5\cos x - 3\sin x$ then prove that $\frac{d^2y}{dx^2} + y = 0$

Ans : Given, $y = 5cosx - 3sinx$

Differentiate w.r.to x

$$\Rightarrow \frac{dy}{dx} = -5\sin x - 3\cos x$$

Differentiate w.r.to x

$$\Rightarrow \frac{d^2y}{dx^2} = -5\cos x + 3\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -(5\cos x - 3\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\therefore \frac{d^2y}{dx^2} + y = 0$$

3. If $y = 3e^{2x} + 2e^{3x}$ prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Ans : Given, $y = 3e^{2x} + 2e^{3x}$

Differentiate w.r.to x

$$\frac{dy}{dx} = 3e^{2x}(2) + 2e^{3x}(3)$$

$$\Rightarrow \frac{dy}{dx} = 6e^{2x} + 6e^{3x}$$

Differentiate w.r.to x

$$\frac{d^2y}{dx^2} = 6e^{2x}(2) + 6e^{3x}(3)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$$

Consider, LHS= $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y$

$$= 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x})$$

$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$

$= (12 - 30 + 18)e^{2x} + (18 - 30 + 12)e^{3x}$

$= 0$

$= RHS$

$\therefore \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

4. If $y = Ae^{mx} + Be^{nx}$ Show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Ans : Given, $y = Ae^{mx} + Be^{nx}$

Differentiate w.r.to x, we get

$\Rightarrow \frac{dy}{dx} = Ae^{mx}(m) + Be^{nx}(n)$

$\Rightarrow \frac{dy}{dx} = Ame^{mx} + Bne^{nx}$

Differentiate w.r.to x, we get

$\Rightarrow \frac{d^2y}{dx^2} = Ame^{mx}(m) + Be^{nx}(n)$

$\Rightarrow \frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx}$

Consider, $LHS = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$

$= Am^2e^{mx} + Bn^2e^{nx} - (m+n)\ (Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx} + Bmne^{nx}$

$= (Am^2 - Am^2 - Amn + Amn)e^{mx} + (Bn^2 - Bn^2 - Bmn + Bmn)e^{nx}$

$= 0$

$= RHS$

$\therefore \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

5. If $y = 500e^{7x} + 600e^{-7x}$ Show that $\frac{d^2y}{dx^2} = 49y$

Ans : Given, $y = 500e^{7x} + 600e^{-7x}$

Differentiate w.r.to x

$\frac{dy}{dx} = 7(500e^{7x}) - 7(600e^{-7x})$

Differentiate w.r.to x

$$\frac{d^2y}{dx^2} = 49(500e^{7x}) + 49(600e^{-7x})$$

$$\frac{d^2y}{dx^2} = 49(500e^{7x} + 600e^{-7x})$$

$$\therefore \frac{d^2y}{dx^2} = 49y$$

6. If $y = 3\cos(\log x) + 4\sin(\log x)$ show that $x^2y_2 + xy_1 + y = 0$

Ans : Given, $y = 3\cos(\log x) + 4\sin(\log x)$

Differentiate w.r.to x

$$\Rightarrow \frac{dy}{dx} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

Multiplying x on both sides

$$\Rightarrow x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

Differentiate w.r.to x

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

Multiplying x on both sides

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -[3\cos(\log x) + 4\sin(\log x)]$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

$$\therefore x^2y_2 + xy_1 + y = 0$$

7. If $y = \sin^{-1}x$ show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

Ans : Given, $y = \sin^{-1}x$

Differentiate w.r.to x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Multiplying $\sqrt{1 - x^2}$ on both sides

$$\Rightarrow \sqrt{1 - x^2}\,\frac{dy}{dx} = 1$$

Differentiate w.r.to x, we get

$$\sqrt{1 - x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}\,(0 - 2x) = 0$$

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{dy}{dx}\frac{x}{\sqrt{1-x^2}} = 0$$

Again, multiplying $\sqrt{1-x^2}$ on both sides

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$$

8. If $y = \left(\tan^{-1} x\right)^2$ show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2.$

Ans : Given, $y = (\tan^{-1} x)^2$

Differentiate w.r.to x

$$\Rightarrow \frac{dy}{dx} = \frac{2\tan^{-1} x}{1+x^2}$$

Multiplying $(1+x^2)$ on both sides

$$(1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$$

Differentiate w.r.to x, we get

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(0+2x) += 2\frac{1}{1+x^2}$$

Again multiplying $(1+x^2)$ on both sides

$$(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$$

$$\therefore\ (x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2.$$

9. If $e^y(x+1) = 1$ Show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Ans : Given, $e^y(x+1) = 1$

$$(x+1) = \frac{1}{e^y}$$

$$\therefore\ (x+1) = e^{-y}$$

Differentiate w.r.to x, we get

$$1 = -e^{-y}\frac{dy}{dx}$$

$$\therefore\ \frac{dy}{dx} = -e^y \quad \text{.........(1)}$$

Differentiate w.r.to x, we get

$$\frac{d^2y}{dx^2} = -e^y\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) \quad \text{using (1)}$$

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

10. If $y = e^{a\cos^{-1}x}$ Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

Ans : Given, $y = e^{a\cos^{-1}x}$

$$\Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x}\left(\frac{-a}{\sqrt{1-x^2}}\right)$$

Multiplying $\sqrt{1-x^2}$ on both sides

$$\sqrt{1-x^2}\,\frac{dy}{dx} = -ae^{a\cos^{-1}x}$$

Differentiate w.r.to x, we get

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\,\frac{1}{2\sqrt{1-x^2}}(0-2x) = -ae^{a\cos^{-1}x}\left(\frac{-a}{\sqrt{1-x^2}}\right)$$

Multiplying $\sqrt{1-x^2}$ on both sides

$$(1-x^2)\frac{d^2y}{dx^2} - \frac{dy}{dx}\frac{x}{\sqrt{1-x^2}} = a^2e^{a\cos^{-1}x}$$

$$(1-x^2)\frac{d^2y}{dx^2} - \frac{dy}{dx}\frac{x}{\sqrt{1-x^2}} = a^2y$$

$$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

THREE MARKS

Rolle's Theorem

Let $f:[a,b] \to R$ be continuous on $[a,b]$ and differentiable on (a,b) such that $f(a) = f(b)$ where a and b are some real numbers. Then there exists some c in (a,b) such that $f'(c) = 0$

Problems

1. Verify Rolle's theorem for the function $y = x^2 + 2,\ a = -2$ and $b = 2$

Solution: The given function $f(x) = x^2 + 2$ is polynomial function, is continuous in $[-2, 2]$

And $f(x) = x^2 + 2$

Diff w.r.t x

$f'(x) = 2x$

$\therefore f(x) is$ differntiable in on (-2, 2)

Also, $f(a) = f(-2) = 4 + 2 = 6$

$f(b) = f(2) = 4 + 2 = 6$

$f(a) = f(b)$

Then there exists atleast one real number c ϵ (-2, 2) such that $f'(c) = 0$

$\therefore 2c = 0$

$c = \frac{0}{2} = 0 \in (-2, 2)$

Hence Rolle's Theorem is varified

2. Verify Rolle's theorem for the function $y = x^2 + 2x - 8$, $a = -4$ and $b = 2$

Solution: The given function $f(x) = x^2 + 2x - 8$ is polynomial function, is continuous in $[-4, 2]$

And, $f(x) = x^2 + 2x - 8$

Diff w,r.t to x

$f'(x) = 2x + 2$

$\therefore f(x) is$ differntiable in on (-4, 2)

Also, $f(a) = (-4)^2 + 2(-4) - 8 = 16 - 8 - 8 = 0$

$f(b) = (2)^2 + 2(2) - 8 = 4 + 4 - 8 = 0$

$f(a) = f(b)$

Then there exists atleast one real number c ϵ (-2, 2) such that $f'(c) = 0$

$\therefore 2c + 2 = 0$

$2c = -2$

$c = -2/2$

$c = -1 \in (-4, 2)$

Hence Rolle's Theorem is varified

Mean Value Theorem

Let $f: [a, b] \to R$ be continuous on $[a, b]$ and differentiable on (a, b). Then there exists some c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

3. Verify mean theorem if $f(x) = x^2$ in the interval [a, b] where a = 2 b = 4

Solution: The given function $f(x) = x^2$ is polynomial function, is continuous in $[2, 4]$

And, $f(x) = x^2$

Diff w,r.t to x

$f'(x) = 2x$

$\therefore f(x) is$ differntiable in on (2, 4)

Also, $f(a) = (2)^2 = 4$

$f(b) = (4)^2 = 16$

$f(a) \neq f(b)$

Then there exists atleast one real number c ϵ (2, 4) such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$2c = \frac{16-4}{4-2}$

$2c = \frac{12}{2}$

$2c = 6$

$c = 3 \in (2, 4)$

Hence Mean value Theorem is varified

4. Verify mean theorem if $f(x) = x^2 - 4x - 3$ in the interval [a, b] where a = 1 b = 4

Solution: The given function $f(x) = x^2 - 4x - 3$ is polynomial function, is continuous in $[1, 4]$

And, $f(x) = x^2 - 4x - 3$

Diff w.r.t to x

$f'(x) = 2x - 4$

$\therefore f(x) is$ differntiable in on $(1, 4)$

Also, $f(a) = 1 - 4 - 3 = -6$

$f(b) = 16 - 16 - 3 = -3$

Then there exists atleast one real number c ϵ (1, 4) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$2c - 4 = \frac{-3-(-6)}{4-1}$$

$$2c - 4 = \frac{-3+6}{3}$$

$$2c - 4 = 1$$

$$2c = 1 + 4$$

$$c = \frac{5}{2} \in (1, 4)$$

Hence Mean value Theorem is verified

5. Verify mean value theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[1, 3]$

Solution: The given function $f(x) = x^3 - 5x^2 - 3x$ is polynomial function, is continuous

in $[1, 3]$

And, $f(x) = x^3 - 5x^2 - 3x$

Diff w.r.t x

$f'(x) = 3x^2 - 10x - 3$

$\therefore f(x) is$ differntiable in on $(1, 3)$

Also, $f(a) = (1)^3 - 5(1)^2 - 3(1) = 1 - 5 - 3 = -7$

$f(b) = (3)^3 - 5(3)^2 - 3(3) = 27 - 45 - 9 = -27$

Then there exists atleast one real number c ϵ (1, 3) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$3c^2 - 10c - 3 = \frac{(-27)-(-7)}{3-1}$$

$$3c^2 - 10c - 3 = \frac{(-27)-(-7)}{3-1}$$

$$3c^2 - 10c - 3 = \frac{-27+7}{2}$$

$$3c^2 - 10c - 3 = \frac{-20}{2}$$

$$3c^2 - 10c - 3 = -10$$

$$3c^2 - 10c = -10 + 3$$

$$3c^2 - 10c = -7$$

$$3c^2 - 10c + 7 = 0$$

$$3c^2 - 3c - 7c + 7 = 0$$

$$3c(c - 1) - 7(c - 1) = 0$$

$$(3c - 7)(c - 1) = 0$$

$$(3c - 7) = 0 \; and \; (c - 1) = 0$$

$c = \frac{7}{3} \epsilon$ c $(1, 3)$ and $c = 1 \in (1, 3)$

Hence Mean value Theorem is verified

THREE MARKS

IMPLICIT FUNCTIONS

Find $\frac{dy}{dx}$ in the following :

1. $x - y = \pi$

   Diff. w .r .t .x

   $$1 - \frac{dy}{dx} = 0$$

   $$\frac{dy}{dx} = 1.$$

2. $x + siny = cosx.$

   Diff. w. r. t. x

   $$1 + cosy.\frac{dy}{dx} = -sinx.$$

   $$cosy.\frac{dy}{dx} = -sinx - 1$$

   $$\frac{dy}{dx} = \frac{-[1+\sin x]}{\cos y}.$$

3. $\sqrt{x} + \sqrt{y} = x$

   Diff. w. r. t. x

   $$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 1$$

   $$\frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x}}$$

   $$\frac{dy}{dx} = \left[1 - \frac{1}{2\sqrt{x}}\right] 2\sqrt{y}.$$

4. $y + \sin y = \cos x$

   Diff. w. r. t. x.

   $$\frac{dy}{dx} + cosy.\frac{dy}{dx} = -sinx$$

   $$\frac{dy}{dx}[1 + cosy] = -sinx$$

   $$\frac{dy}{dx} = \frac{-\sin x}{1 + \cos y}.$$

5. $2x + 3y = sinx$

   Diff. w. r. t. x.

   $$2 + 3\frac{dy}{dx} = cosx$$

   $$3\frac{dy}{dx} = cosx - 2$$

   $$\frac{dy}{dx} = \frac{cosx - 2}{3}$$

6. $ax + by^2 = cosy$

   Diff. w. r. t. x.

   $$a + 2by.\frac{dy}{dx} = -siny.\frac{dy}{dx}$$

   $$a + 2by.\frac{dy}{dx} + siny.\frac{dy}{dx} = 0$$

   $$a + \frac{dy}{dx}[2by + siny] = 0$$

$$\frac{dy}{dx}[2by + siny] = -a$$
$$\frac{dy}{dx} = \frac{-a}{2by+siny}.$$

7. $xy + y^2 = tanx + y$

Diff. w. r.t.x.

$$x.\frac{dy}{dx} + y + 2y.\frac{dy}{dx} = sec^2x + \frac{dy}{dx}$$
$$x.\frac{dy}{dx} + 2y.\frac{dy}{dx} - \frac{dy}{dx} = sec^2x - \mathrm{y}$$
$$x.\frac{dy}{dx} + 2y.\frac{dy}{dx} - \frac{dy}{dx} = sec^2x - y.$$
$$\frac{dy}{dx}[x + 2y - 1] = sec^2x - y.$$
$$\frac{dy}{dx} = \frac{sec^2x - y}{[x+2y-1]}$$

8. $x^2 + xy + y^2 = 100$

Diff. w. r. t. x

$$2x + x.\frac{dy}{dx} + y + 2y.\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}[x + 2y] = -2x - y$$
$$\frac{dy}{dx} = -\frac{[2x+y]}{(x+2y)}.$$

9. $x^3 + x^2y + xy^2 + y^3 = 81$

Diff. w. r. t. x

$$3x^2 + x^2.\frac{dy}{dx} + y.2x + x.2y.\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0.$$
$$\frac{dy}{dx}[x^2 + 2xy + 3y^2] = -3x^2 - 2xy - y^2$$
$$\frac{dy}{dx} = \frac{-(3x^2+y^2+2xy)}{x^2+2xy+3y^2}.$$

10. $sin^2x + cos^2y = 1$

Diff. w. r. t. x.

$$2sinx.cosx + 2cosy.(-siny).\frac{dy}{dx} = 0$$
$$2sinx.cosx = 2cosy.siny.\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{2\sin x\ cosx}{2\ cosy\ siny} = \frac{\sin 2x}{\sin 2y}.$$

11. $sin^2y + \cos(xy) = k$

$$2\ siny.cosy.\frac{dy}{dx} - \sin(xy)\left(x.\frac{dy}{dx} + y\right) = 0$$
$$\sin 2y.\frac{dy}{dx} - xsin(xy)\frac{dy}{dx} - ysin(xy) = 0$$
$$\frac{dy}{dx}[sin2y - xsin(xy)] = y\sin(xy)$$
$$\frac{dy}{dx} = \frac{ysin(xy)}{[sin2y - xsin(xy)]}.$$