METHODS OF DIFFERENTIATION

1. DERIVATIVE OF f(x) FROM THE FIRST PRINCIPLE :

Obtaining the derivative using the definition

$\lim\limits_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim\limits_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x} = f'(x) = \frac{dy}{dx}$ is called calculating derivative using first principle or ab initio or delta method.

2. FUNDAMENTAL THEOREMS :

If f and g are derivable function of x, then,

(a) $\frac{d}{dx}(f \pm g) = \frac{df}{dx} \pm \frac{dg}{dx}$, known as **SUM RULE**

(b) $\frac{d}{dx}(cf) = c\frac{df}{dx}$, where c is any constant

(c) $\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$, known as **PRODUCT RULE**

(d) $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\left(\frac{df}{dx}\right) - f\left(\frac{dg}{dx}\right)}{g^2}$,

where $g \neq 0$ known as **QUOTIENT RULE**

(e) If $y = f(u)$ & $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$, known as **CHAIN RULE**

Note : In general if $y = f(u)$, then $\frac{dy}{dx} = f'(u).\frac{du}{dx}$.

3. DERIVATIVE OF STANDARD FUNCTIONS :

	f(x)	f'(x)
(i)	x^n	nx^{n-1}
(ii)	e^x	e^x
(iii)	a^x	$a^x \ell na,\ a > 0$
(iv)	ℓnx	$1/x$
(v)	$\log_a x$	$(1/x) \log_a e,\ a > 0,\ a \neq 1$
(vi)	sinx	cosx
(vii)	cosx	– sinx
(viii)	tanx	$\sec^2 x$
(ix)	secx	secx tanx
(x)	cosecx	– cosecx . cotx
(xi)	cotx	$-\text{cosec}^2 x$
(xii)	constant	0
(xiii)	$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}},\ -1 < x < 1$
(xiv)	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}},\ -1 < x < 1$
(xv)	$\tan^{-1} x$	$\frac{1}{1+x^2},\ x \in R$
(xvi)	$\sec^{-1} x$	$\frac{1}{\lvert x \rvert \sqrt{x^2-1}},\ \lvert x \rvert > 1$
(xvii)	$\text{cosec}^{-1} x$	$\frac{-1}{\lvert x \rvert \sqrt{x^2-1}},\ \lvert x \rvert > 1$
(xviii)	$\cot^{-1} x$	$\frac{-1}{1+x^2},\ x \in R$

4. LOGARITHMIC DIFFERENTIATION :

To find the derivative of :

(a) A function which is the product or quotient of a number of function or

(b) A function of the form $[f(x)]^{g(x)}$ where f & g are both derivable, it is convenient to take the logarithm of the function first & then differentiate.

5. DIFFERENTIATION OF IMPLICIT FUNCTION :

(a) Let function is $\phi(x, y) = 0$ then to find dy /dx, in the case of implicit functions, we differentiate each term w.r.t. x regarding y as a functions of x & then collect terms in dy / dx together on one side to finally find dy / dx

OR $$\frac{dy}{dx} = \frac{-\partial\phi/\partial x}{\partial\phi/\partial y}$$ where $\frac{\partial\phi}{\partial x}$ & $\frac{\partial\phi}{\partial y}$ are partial differential coefficient of $\phi(x, y)$ w.r.to x & y respectively.

(b) In expression of dy/dx in the case of implicit functions, both x & y are present.

6. PARAMETRIC DIFFERENTIATION :

If $y = f(\theta)$ & $x = g(\theta)$ where θ is a parameter, then $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

7. DERIVATIVE OF A FUNCTION W.R.T. ANOTHER FUNCTION :

Let $y = f(x)$; $z = g(x)$, then $\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{f'(x)}{g'(x)}$

8. DERIVATIVE OF A FUNCTION AND ITS INVERSE FUNCTION :

If inverse of $y = f(x)$ is denoted as $g(x) = f^{-1}(x)$, then $g(f(x)) = x$

$\Rightarrow g'(f(x))f'(x)=1$

9. HIGHER ORDER DERIVATIVE :

Let a function y = f(x) be defined on an open interval (a, b). It's derivative, if it exists on (a, b) is a certain function $f'(x)$ [or (dy / dx) or y'] & it is called the first derivative of y w. r. t. x. If it happens that the first derivative has a derivative on (a , b) then this derivative is called second derivative of y w.r.t. x & is denoted by $f''(x)$ or (d^2y/dx^2) or y''. Similarly, the 3rd order derivative of y w.r.t x, if it exists, is defined by $\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$. It is also denoted by $f'''(x)$ or y''' & so on.

10. DIFFERENTIATION OF DETERMINANTS :

If $F(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix}$, where f, g, h. l, m, n, u, v, w are differentiable functions of x, then

$$F'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l(x) & m(x) & n(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l'(x) & m'(x) & n'(x) \\ u(x) & v(x) & w(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ l(x) & m(x) & n(x) \\ u'(x) & v'(x) & w'(x) \end{vmatrix}$$

Similarly one can also proceed columnwise.

11. L' HÔPITAL'S RULE :

(a) Applicable while calculating limits of indeterminate forms of the type $\frac{0}{0}, \frac{\infty}{\infty}$. If the function f(x) and g(x) are differentiable in certain neighbourhood of the point a, except, may be, at the point a itself, and $g'(x) \neq 0$, and if

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0 \quad \text{or} \quad \lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \infty,$$

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

provided the limit $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists (L' Hôpital's rule). The point 'a' may be either finite or improper $+\infty$ or $-\infty$.

(b) Indeterminate forms of the type $0.\infty$ or $\infty - \infty$ are reduced to forms of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$ by algebraic transformations.

(c) Indeterminate forms of the type 1^{∞}, ∞^0 or 0^0 are reduced to forms of the type $0.\infty$ by taking logarithms or by the transformation $[f(x)]^{\phi(x)} = e^{\phi(x).\ell n f(x)}$.