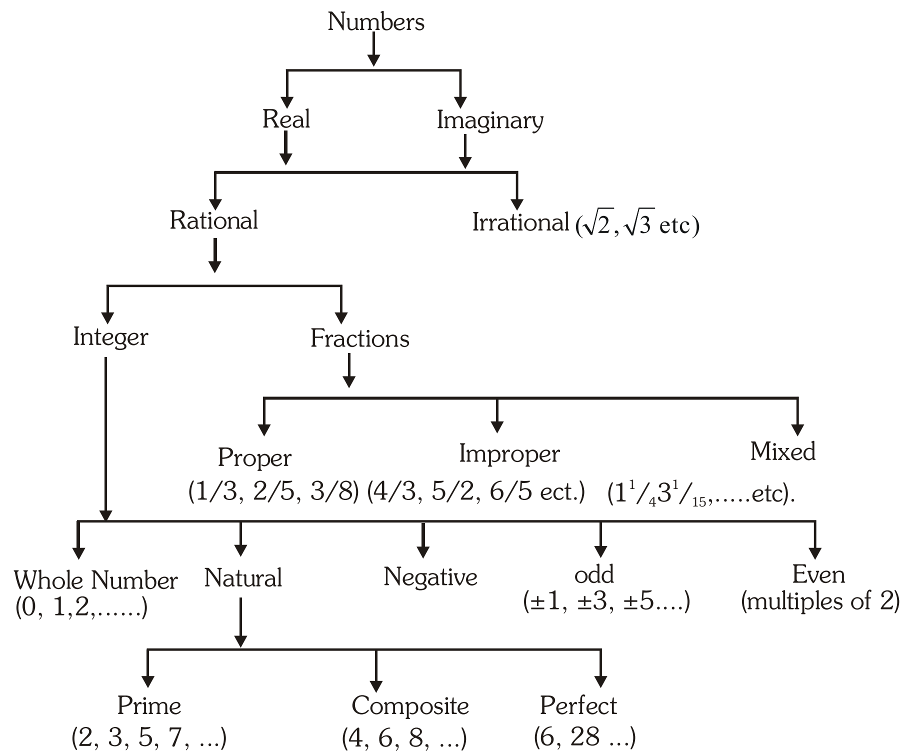
2. NUMBER SYSTEM

■ System of numbers



- **Natural numbers**

  Counting numbers 1, 2, 3,4, 5,.....are known as natural numbers. The set of all natural numbers can be represented by

  N={1, 2, 3, 4, 5....}

- **Whole numbers**

  If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5.....are called whole numbers. The set of whole numbers can be represented by

  W={0, 1, 2, 3, 4, 5,.... }

  Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

- **Integers**

  All counting numbers and their negatives including zero are known as integers. The set of integers can be represented by

  Z or {.....–4, –3, –2, –1, 0, 1, 2, 3, 4,......}

- **Positive integers**

  The set I^+ = {1, 2, 3, 4,....} is the set of all positive integers, Clearly, positive integers and natural numbers are synonyms.

- **Negative integers**

  The set I^-= {–1, –2, –3,.....} is the set of all negative integers. 0 is neither positive nor negative.

- **Non-negative integers**

  The set {0, 1, 2, 3,......} is the set of all non-negative integers.

- **Rational numbers**

The numbers of the form $\frac{p}{q}$, where p and q are integers and q≠0, are known as rational numbers, e.g. $\frac{4}{7}, \frac{3}{2}, \frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc. The set of all rational numbers is denoted by Q.

i.e. $Q = \left\{ x : x = \frac{p}{q}; p, q \in I, q \neq 0 \right\}$

Since every natural number a can be written as $\frac{a}{1}$, so is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$, so it is also a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or non-terminating repeating decimals.

For example, $\frac{1}{5} = 0.2$, $\frac{1}{3} = 0.333....$, $\frac{22}{7}$ =3.1428714287, $\frac{8}{44} = 0.18188$ ...., etc.

The recurring decimals have been given a short notation as

$0.333... = 0.\overline{3}$

$4.1555... = 4.1\overline{5}$

$0.323232... = 0.\overline{32}$.

- **Irrational numbers**

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, etc.

- Note, that the exact value of π is not $\frac{22}{7} \cdot \frac{22}{7}$ is rational while π is irrational numbers. $\frac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

- **Real numbers**

The rational and irrational numbers combined together are called real numbers, e.g. $\frac{13}{21}, \frac{2}{5}, -\frac{3}{7}$, $\sqrt{3}, 4 + \sqrt{2}$, etc. are real numbers. The set of real numbers is denoted by R.

- Note, that the sum, difference or product of a rational and irrational number is irrational, e.g. 3 + $\sqrt{2}$, $4 - \sqrt{3}$, $\frac{2}{3} - \sqrt{5}, 4\sqrt{3}, -7\sqrt{5}$ are all irrational.

- **Even numbers**

All those numbers which are exactly divisible by 2 are called even numbers, e.g. 2,6,8,10 etc. are even numbers.

- **Odd numbers**

All those numbers which are not exactly divisible by 2 are called odd numbers, e.g. 1,3,5,7, etc, are odd numbers.

- **Prime numbers**

Except 1 each natural number which is divisible by only 1 and itself is called as prime number e.g., 2, 3, 5, 7, 11, 13, 17, 19,23,29, 31,.....etc.

- There are total 25 prime numbers upto 100
- There are total 46 prime numbers upto 200
- 2 is the only even prime number and the least prime number.
- 1 is neither prime nor composite number.
- There are infinite prime numbers.
- A list of all prime numbers upto 100 is given below.

Table of prime Numbers (1-100):

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Test to find whether a given number is a prime

Step 1 Select a least positive integer n such that $n^2 >$ given number.

Step 2 Test the divisibility of given number by every prime number less than n.

Step 3 The given number is prime only if it is not divisible by any of these primes.

Ex. Investigate whether 571 is a prime number.

Sol. Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore$ n=24

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number.

- **Co-prime**

A pair of two natural numbers having no common factor, other than 1, is called a pair of co-primes.

For example: (3, 5), (4, 5), (5, 6), (7, 9), (6, 7) etc., are co-primes.

- **Twin primes**

Prime numbers differing by two are called twin primes, e.g. (3, 5), (5, 7), (11, 13) etc, are called twin primes.

- **Prime triplet**

A set of three consecutive primes differing by 2, such as (3, 5, 7) is called a prime triplet

"every prime number except 2 is odd but every odd number need not be prime."

- **Fractions**

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator < Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction : Numerator > Denominator i.e. $\frac{5}{3}$.

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

(f) Compound fraction : Fraction whose number and denominator themselves are fractions i.e. $\frac{2/3}{5/7}$

- **Composite numbers**

All natural numbers, which are not prime are composite numbers. If C is the set of composite number than C={4, 6, 8, 9, 10, 12.....}

- **Imaginary numbers**

  All the numbers whose square is negative are called imaginary numbers. e.g. 3i, –4i,....; where $i = \sqrt{-1}$.

- **Complex numbers**

  The combined form of real and imaginary numbers is known as complex number.

  It is denote by Z=A+iB, where A is real and B is imaginary part of Z and A, B $\in$ R.

Square & Square roots

The second power of number is called the square of that number. In other words the square of a number is the product of the number with the number itself.

A given number is a perfect square, if it is expressed as a product of pairs of equal factors.

- **Important properties**

(i) A natural number having 2, 3, 7 or 8 in the unit's place is never a perfect square (or squared number)
17, 23, 118, 222 are not perfect squares.

(ii) The square of an even number is always an even number.
$2^2=4$, $6^2= 36$, $10^2=100$, $12^2=144$.

(iii) The square of an odd number is always an odd number.
$3^2=9$, $7^2=49$, $13^2=169$, $15^2=225$.

(iv) The number of zeroes at the end of a perfect square is never odd.
100, 400, 3600, 640000 are perfect squares and 1000, 4000, 6400000 are not perfect squares.

(v) The square of a natural number n is equal to the sum of the first n odd numbers.
$1^2=1$= sum of the first 1 odd number.
$2^2=1+3$ = sum of the first 2 odd numbers.
$3^2=1+3+5$= sum of the first 3 odd numbers.

(vi) For every natural numbers n,
$(n+1)^2 - n^2=(n+1+n)(n+1-n)=(n+1)+n$
$4^2-3^2=(3+1)+3=7$.
$16^2-15^2= (15+1)+15=31$.

(vii) A perfect square (other than 1) is either a multiple of 3 exceeds a multiple of 3 by 1.
$49 = (7)^2= 3\times16+1$, $169 = (13)^2 = 3\times56 + 1$.

(viii) A perfect square (other than 1) is either a multiple of 4 or exceeds a multiple of 4 by 4.
$441 = (21)^2 = 4\times110+1$.

- **Some other properties**

(i). If the unit digit of the number is zero then the unit digit of the square of this number will also be zero and the number of zeros will be double in the square than that of its root.
e.g., $(60)^2= 3600$, $(130)^2 = 16900$

(ii) If the unit digit of the number is 5 then the unit digit of its square is also 5 and the number formed by last two digits is 25.
e.g., $(35)^2= 1225$, $(45)^2= 2025$, $(55)^2= 3025$ etc.

(iii) If the unit digit of any number is 1 or 9 then the unit digit of the square of its number is always 1.
e.g., $(71)^2 = 5041$, $(31)^2 = 961$, $(19)^2 = 361$

(iv) If the unit digit of any number is 2 or 8 then the unit digit of the square of its number is always 4.

(v) If the unit digit of any number is 3 or 7 then the unit digit of its square is always 9.

e.g., $(23)^2 = 529$, $(27)^2 = 729$

(vi) If the unit digit of any number is 4 or 6 then the unit digit of its square is always 6.

e.g., $(26)^2 = 676$, $(24)^2 = 576$,

$(14)^2 = 196$, $(16)^2 = 256$ etc.

(vii) The square of any number is always positive irrespective of the nature of the given number.

(viii) Non square numbers between the squares of two consecutive natural numbers n & n +1 $\rightarrow (n + 1)^2 - n^2 - 1$

$= n^2 + 1 + 2n - n^2 - 1 = 2n$

(ix) If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

(x) Square root of a negative integer is an imaginary number.

(xi) Square roots of integers that are not perfect squares are always irrational numbers.

(xii) Every composite number can be uniquely factored as a product of prime numbers only.

(xiii) If a perfect square is of n-digits, then its square root will have n/2 digits if n is even or $\frac{(n+1)}{2}$ if n is odd.

Cube & cube roots

Cube of a number is obtained by multiplying the number itself thrice.

For example, 27 is the cube of 3 as $27 = 3 \times 3 \times 3$.

Cube root

Cube root of a given number is that number which when raised to the third power produces the given numbers, that is the cube root of a number x is the number whose cube is x.

- The cube root of x is written as $\sqrt[3]{x}$.

  For example, cube root of 64 is 4 as $4 \times 4 \times 4 = 64$.

Short-cut method of finding cube roots of exact cubes consisting of up to 6 digits:

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful

(i) $1^3= 1$; $2^3= 8$; $3^3 = 27$; $4^3= 64$; $5^3= 125$; $6^3=216$; $7^3= 343$; $8^3= 512$;

$9^3 = 729$; $10^3=1000$.

(ii) If the cube ends in 1, then its cube root ends in 1

If the cube ends in 2, then its cube root ends in 8

If the cube ends in 3, then its cube root ends in 7

If the cube ends in 4, then its cube root ends in 4

If the cube ends in 5, then its cube root ends in 5

If the cube ends in 6, then its cube root ends in 6

If the cube ends in 7, then its cube root ends in 3

If the cube ends in 8, then its cube root ends in 2

If the cube ends in 9, then its cube root ends in 9

If the cube ends in 0, then its cube root ends in 0

- **Clearly from above**

  1↔1, 4↔4, 5↔5, 6↔6, 9↔9, 0↔0

  2↔8, 3↔7.

- **Some other properties**

  (i) Cubes of all odd natural numbers are odd.

  (ii) Cubes of all even natural numbers are even.

  (iii) The cube of a natural number which is a multiple of 3 is a multiple of 27.

  (iv) The cube of a natural number which of the form 3n+1 (e.g., 4, 7,10.........) is also a number of the form 3n+1.

  (v) The cube of natural number which is of the form 3n+2 (e.g., 5, 8, 11,.....) is also a number of the form 3n+2.

■ Squares (short-cut methods)

- **To square any number ending with 5.**

  Method : $(A5)^2 = A(A+1)/25$

- **To square a number in which every digit is one.**

  Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

- **To square number which is nearer to 10x.**

  Method: Use the formula.

  $x^2 = (x^2 - y^2) + y^2 = (x+y)(x-y) + y^2$

Ex. Find the squares of following:

Sol. (i) $(25)^2 = 2\ (2+1)/25 = 6/25 = 625$

(ii) $(45^2) = 4\ (4+1)/25 = 20/25 = 2025$

(iii) $(85)^2 = 8\ (8+1)/25 = 72/25 = 7225$

(iv) $11^2 = 121$

(v) $11^2 = 12321$

(vi) $1111^2 = 1234321$

(vii) $222^2 = 2^2(111)^2 = 4\ (12321) = 49284$

(viii) $3333^2 = (1111)^2 = 9\ (1234321) = 11108889$

(ix) $(97)^2 = (97+3)\ (97-3)+3^2 = 9400+9=9409$

(x) $(102)^2 = (102-2)\ (102+2)+2^2 = 10400+4=10404$

(xi) $(994)^2 = (994+6)\ (994-6) + 6^2 = 988000 + 36 = 988036$

(xii) $(1005)^2 = (1005-5)\ (1005+5) + 5^2 = 1010000 + 25 = 1010025$

■ Multiplication (short-cut methods)

- **Multiplication of a given number by 9, 99, 999, *etc.*, that is by 10^{n-1}**

  Method: Put as many zeros to the right of the multiplication as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

Ex. Multiply

(i) 3893 by 99 (ii) 4327 by 999 (iii) 5863 by 9999

Sol. (i) $3893 \times 99 = 389300 - 3893 = 385407$.

(ii) $4327 \times 999 = 4327000 - 4327 = 4322673$

(iii) $5863 \times 9999 = 58630000 - 5863 = 58624137$

- **Multiplication of a given number by 11, 101, 1001, etc., that is by 10^n+1.**

  Method: Place n zeros to the right of the multipliciant and then add the multiplicant to the number so obtained.

Ex. Multiply

(i) 4782×11 (ii) 9836×101 (iii) 6538×1001

Sol. (i) $4782 \times 11 = 47820 + 4782 = 52602$

(ii) $9836 \times 101 = 983600 + 9836 = 993436$

(iii) $6538 \times 1001 = 6538000 + 6538 = 6544538$

- **Multiplication of a given number by 15, 25, 35, etc.**

  Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

Ex. Multiply

(i) 7054×15 (ii) 3897×25 (iii) 4563×35

Sol. (i) $7054 \times 15 = \frac{1}{2}(7054 \times 30) = \frac{1}{2}(211620) = 105810.$

(ii) $3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850) = 97425$

(iii) $4563 \times 35 = \frac{1}{2}(4563 \times 70) = \frac{1}{2}(319410) = 159705$

- **Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.**

  Method: Place as many zeros to the right of the multplicant as in the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

Ex. Multiply

(i) 3982×5 (ii) 4739×25 (iii) 7894×125 (iv) 4863×625

Sol. (i) $3982 \times 2 = \frac{39820}{2} = 19910$

(ii) $4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = 118475$

(iii) $7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8} = 986750$

(iv) $4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16} = 3039375$

Test of divisibility

- **Divisibility by 2:** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

  For example, 4, 12, 30, 18, 102, etc, are all divisible by 2.

- **Divisibility by 3:** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

  For example, the number 3792 is divisible by 3, since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

- **Divisibility by 4:** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

  For example, the number 2616 is divisible by 4, since 16 is divisible by 4.

- **Divisibility by 5:** A number divisible by 5 if the unit's digit in the number is 0 or 5.

  For example 13520, 7805, 640, 745, ect. are all divisible by 5.

- **Divisibility by 6:** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.
  For example, the number 4518 is divisible by 6 since it is even and sum of its digits 4+5+1+8=18 is divisible by 3.
- **Divisibility by 7:** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.
  For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16.
  Then, 44–16 = 28.
  Since 28 is divisible by 7, 448 is divisible by 7.
- **Divisibility by 8:** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.
  For example, the number 41784 is divisible by 8 as the number formed by last three digits i.e. 784 is divisible by 8.
- **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.
  For example, the number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9.
- **Divisibility by 10:** A number is divisible by 10, if ends in zero.
  For example, the last digit of 580 is zero, therefore 580 is divisible by 10.
- **Divisibility by 11:** A number is divisible by 11, if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.
  For example, in the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17=0, so the number is divisible by 11.
- **Divisibility by 12:** A number is divisible by 12 if it is divisible by 3 and 4.
- **Divisbility by 25:** A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or last two digits are zero.
  For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.
- **Divisibility by 125:** A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.
  For example, the number 5250 is divisible by 125 as 250 is divisible by 125.
- **Divisibility by 18:** An even number satisfying the divisibility test of 9 is divisible by 18.
- **Divisibility by 88:** A number is divisible by 88 if it is divisible by 11 and 8.

Decimal fractions

- **Decimal fraction :** Fractions in which the denominators are the powers of 10 are called decimal fractions.
  In general, the decimal fractions are of the following types
- **Recurring decimals :** If in a decimal fraction, a figure or a set of figures is repeated continually, then such a number is called a recurring decimal.
  If a single figure is repeated, it is shown by putting a dot on it. But if a set of a figures is repeated, we express if either by putting one dot at the starting digit and one dot at the last digit of the repeating digits or by placing a bar on the repeating digit (s).

  (i) 2/3 = 0.6666....= $0.\dot{6} = 0.\overline{6}$

  (ii) 22/7=3.142857142857 = $3.\dot{1}4285\dot{7} = 3.14285\dot{7} = 3.\overline{142857}$

  (iii) 95/6 = 15.83333...= $15.8\dot{3} = 15.8\overline{3}$

- **Pure recurring decimals :** A decimal in which some figures after the decimal point repeat is called a pure recurring decimal.

Ex. $0.\dot{6}$, $3.\dot{1}4285\dot{7}$ etc.

- **Conversion of a pure recurring decimal into fraction**

Rule : Write the recurring figures only once in the numerator and take as many nines in the denominator as the number of repeating figures.

(1) $0.\overline{6} = 6/9 = 2/3$.

(2) $16.\dot{6} = 16 + 0.\overline{6} = 16+6/9 = 16 + 2/3 = 50/3$.

To convert a mixed recurring decimal into fraction

$$\left(\frac{p}{q}\right)\text{form} = \frac{(\text{Complete numbers}) - (\text{number formed by Non} - \text{repeating digit})}{\text{No. of } 9 \text{ as no. of repeating digits after that write no. of } 0 \text{ as no. of non repeating digits.}}$$

e.g. (i) $0.\overline{35} = \frac{35-0}{99} = \frac{35}{99}$ (ii) $0.4\overline{35} = \frac{435-4}{990} = \frac{431}{990}$

H.C.F & L.C.M of numbers

- **The least number which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3 respectively, such that $(d_1-r_1) = (d_2-r_2) = (d_3-r_3)$ is (L.C.M. of d_1, d_2 and d_3) – (d_1-r_1) or (d_2-r_2) or (d_3-r_3).**

Ex. Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Sol. Here, 9–4 = 10–5=15–10=5

Also, L.C.M. (9, 10, 15)= 90

$\therefore$ the required least number = 90–5=85.

- **A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is $(d_1 \times r_2 + r_1)$.**

Ex. A number on being divided by 10 and 11 successively leaves the reminders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Sol. The required remainder

$= d_1 \times r_2 + r_1 = 10 \times 7 + 5 = 75$.

- **To find the number of numbers divisible by a certain integer.**

Ex. (i) How many numbers up to 532 are divisible by 15 ?

Sol. We divide 532 by 15.

$532 = \underline{35} \times 15 + 7$

The quotient obtained is the required number of numbers. Thus there are 35 such numbers.

Ex. (ii) How many numbers up to 300 are divisible by 5 and 7 together ?

Sol. L.C.M. of 5 and 7 = 35

We divide 300 by 35

$300 = \underline{8} \times 35 + 20$

Thus there are 8 such numbers.

- **Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.**

Ex. Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Sol. The required divisor

$= 437 + 298 - 236 = 499.$

- **Product of two numbers. = L.C.M. of the numbers × H.C.F. of the numbers.**

Ex. The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Sol. The required number $= \dfrac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First number}} = \dfrac{1260 \times 63}{315} = 252$

- **To find the greatest number that will exactly divide x, y and z.**

  Required number = H.C.F. of x, y, and z.

Ex. Find the greatest number that will exactly divide 200 and 320.

Sol. The required greatest number

= H.C.F. of 200 and 320 = 40.

- **To find the greatest number that will divide x, y, and z leaving remainders a, b, and c, respectviely. Required number = H.C.F. fo (x–a), (y–b) and (z–c).**

Ex. Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Sol. The required greatest number

= H.C.F. of (148–4), (246–6) and (623–11),

i.e. H.C.F. of 144, 240 and 612 = 12.

- **To find the least number which is exactly divisible by x, y and z.**

  Required number = L.C.M. of x, y and z.

Ex. What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Sol. The required smallest number

= L.C.M. of 36, 45, 63 and 80

= 5040.

- **To find the least number which when divided by x,y, and z leaves the remainders a,b, and c, respectively. It is a always observed that (x–a) = (y–b)=(z–c) = k(say)**

  $\therefore$ **Required number = (L.C.M. of x, y, and z)–k.**

Ex. Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Sol. Since (36–25) = (48–37) = (64–53) = 11, therefore the required smallest number

= (L.C.M. of 36, 48 and 64) –11

= 576–11 = 565.

- **To find the least number which when divided by x, y and z leaves the same remainder r in each case. Required number = (L.C.M. of x, y, and z) +r.**

Ex. Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Sol. The required smallest number

= (L.C.M.of 12, 16 and 18) + 15

= 144 + 5 = 149.

- **To find the greatest number that will divide x, y, and z leaving the same remainder in each case.**

  (a) When the value of remainder r is given :

  Required number = H.C.F. of (x–r), (y–r) and (z–r).

  (b) When the value of reminder is not given:

  Required number = H.C.F. of $|(x-y)|$, $|(y-z)|$ **and** $|(z-x)|$

Ex. (a) Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Sol. The required greatest number

= H.C.F. of (772–5) and (2778–5)

= H.C.F. pf 767 and 2773

= 59.

Ex. (b) Find the greatest number which on dividing 152, 277 and 427 leaves remainder.

Sol. The required greatest number.

= H.C.F. of $|(x-y)|$, $|(y-z)$ and $(z-x)|$

= H.C.F. of $|(152-27)|$, $|(277-427)|$

and $|(427-152)|$

= H.C.F. of 125, 275 and 150

= 25.

- **To find the n-digit greatest number which, when divided by x, y and z.**

(a) leaves no remainder (i.e., exactly divisible)

Step 1 – L.C.M. of x, y and z = L

Step 2 – $\dfrac{L\overline{)\text{n - digit greatest number}}}{\text{Remainder} = R}$

Step 3 – Required number = n-digit greatest number – R

(b) leaves reminder K in each case

Required number = (n-digit greatest number–R) + K.

Ex. Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28, leaves 3 as a remainder in each case.

Sol. L.C.M. of 12, 18, 21 and 28 = 252.

$$\begin{array}{r} 252\overline{)9999}(39 \\ \underline{9828} \\ 171 \end{array}$$

∴ The required number = (9999–171)+3= 9931.

- **To find the n-digit smallest number which when divided by x, y and z**

(a) leaves no remainder (i.e. exactly divisible)

Step 1 – L.C.M. of x, y and z= L

Step 2 – $\dfrac{L\overline{)\text{n-digit smallest number}}(}{\text{Remainder} = R}$

Step 3 – Required number = n-digit smallest number + (L–R).

(b) leaves remainder K in each case.

Required number = n-dight smallest number + (L–R) + k.

Ex. (a) Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Sol. L.C.M. of 4, 6, 8, and 10 = 120.

$$120\overline{)1000}(8$$
$$\underline{960}$$
$$40$$

$\therefore$ The required number = 1000 + (120–40)= 1080.

Ex. (b) Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Sol. L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)1000}(3$$
$$\underline{756}$$
$$244$$

$\therefore$ The required number

= 1000+(252–244)+3

= 1011.

■ Number of factors of a given number

- **Number of factors (or divisors) of a given number (composite number)**

Lets us assume a composite number say 24 then find the number of factors.

24 = 1×24

2×12

3×8

4×6

We see that there are total 8 factors namely,

1, 2, 3, 4, 6, 8, 12 and 24.

Let there be a composite number N and its prime factors be a, b, c, d,....... etc and p,q, r,s...etc. be the indices (or powers) of the a, b, c, d..... etc. respectively i.e., if N can be expressed as

$\mathbf{N = a^p. b^q.c^r. d^s...}$

then, the number of total divisors or factors of N is

(p+1) (q+1) (r+1) (s+1)......

Ex. Find the total number of factors of 540:

(1) 24 (2) 20 (3) 30 (4) None of these

Sol. $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$540 = 2^2 \times 3^3 \times 5^1$

Therefore total number of factors of 540 is

(2+1) (3+1) (1+1) = 24

■ Sum of factors of given number

Once again if you want to find the sum of smaller composite numbers, then you can do it manually, but for larger numbers its a problem.

e.g. Sum of factors of 24

= 1 + 2 + 3 + 4 + 6+ 8 + 12 + 24 = 60

Let N be the composite number and a, b, c, d.. be its prime factors and p,q, r,s be the indices (or powers) of a,b,c,d i.e., if N can be expressed as

$$\mathbf{N= a^p. b^p. c^r. d^s...}$$

then the sum of all the divisors (or factors) of N $= \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$

Ex. Find the sum of factors of 270.

Sol. $270 = 2 \times 3^3 \times 5$

$\therefore$ Sum of factors of 270 $= \dfrac{(2^{1+1}-1)(3^{3+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$

$= \dfrac{3 \times 80 \times 24}{1 \times 2 \times 4} = 720$

■ Product of factors

Let us assume a very small number 24 and see the factors

$24 = 1 \times 24$

$= 2 \times 12$

$= 3 \times 8$

$= 4 \times 6$

Note, it is obvious from the above explanation that the product of factors of 24

$= (1 \times 24) \times (2 \times 12) \times (3 \times 8) \times (4 \times 6)$

$= 24 \times 24 \times 24 \times 24 = (24)^4$

Thus, **the product of factors of composite number $N = N^{n/2}$, where n is the total number of factors of N.**

Ex. Product of divisors of 7056 is :

(1) $(84)^{48}$ (2) $(84)^{44}$ (3) $(84)^{45}$ (4) None of these

Sol. $\therefore 7056 = 2^4 \times 3^2 \times 7^2$

$\therefore$ Number of factors /divisors of 7056

$= (4+1)(2+1)(2+1) = 45$

$\therefore$ product of factors $= (7056)^{45/2} = (84)^{45}$

Hence (c) is the correct option.

■ Number of odd factors of given number

Let us assume a smaller number e.g.,, 24

$24 = 1 \times 24$ also $24 = 2^3 \times 3^1$

$= 2 \times 12$

$= 3 \times 8$

We can see that there are total 2 odd factors namely 1 and 3.

Further assume another number say 36

$36 = 1 \times 36$ also $36 = 2^2 \times 3^2$

$= 2 \times 18$

$= 3 \times 12$

$= 4 \times 9$

$= 6 \times 6$

So, we can see that there are only 3 odd factors viz., 1, 3 and 9.

Once again we assume another number say 90

then $90 = 1 \times 90$ also $90 = 2 \times 3^2 \times 5$

$= 2 \times 45$

$= 3 \times 30$

$= 5 \times 18$

$= 6 \times 15$

$= 9 \times 10$

Thus there are only 6 odd factors namely 1, 3, 5, 9, 15, 45.

To get the number of odd factors of a numbers N first of all express the number N as

$$\mathbf{N = (p_1^a \times p_2^b \times p_3^c \times ....) \times (e^x)}$$

(where, p_1, p_2, p_3, ... are the odd prime factors and e is the even prime factor)

Then the total number of odd factors

= (a+1) (b+1) (c+1)...

Ex. The number of odd factors of 36 is ....

Sol. $36 = 2^2 \times 3^2$

$\therefore$ Number of odd factors = (2 + 1) = 3

■ Number of even factors a composite number

Number of even factors of a number

= (Total number of factors of the given number) – (Total number of odd factors)

Ex. (a) Find the number of even factors (or divisors) of 24

Sol. number of even factors = 8 – 2 = 6

Ex. (b) Find the number of even factors of 36

Sol. number of even factors = 9 – 3 = 6

Ex. (c) Find the number of even factors of 90

Sol. number of even factors = 12 – 6 =6

■ Number of ways of expressing c composite number as a product of two factors

Let us consider an example of small composite number say, 24

then 24 = 1×24

2×12

3×8

4×6

So it is clear that

the number of ways of expressing a composite no. as a product of two factors $= \frac{1}{2} \times$ **the no. of total factors**

Ex. Find the number of ways of expressing 180 as a product of two factors.

Sol. $180 = 2^2 \times 3^3 \times 5^1$

Number of factors = (2+1) (2+1) (1+1) = 18

Hence, there are total $\frac{18}{2}$ =9 ways in which 180 can be expressed as a product of two factors.

Note - As you know when you express any perfect square number 'N' as a product of two factors namely $\sqrt{N}$ and $\sqrt{N}$, and also know that since in this case $\sqrt{N}$ appears two times but it is considered only once while calculating the no. of factors so we get always an odd number as number of factors so we can not divide the odd number exactly by 2 as in the above formula. So if we have to consider these two same factors then we find the number of ways of expressing N as a **product of two factors** $= \mathbf{\frac{(Number\ of\ factors + 1)}{2}}$.

Again if it is asked that find no. of ways of expressing N as a product of two distinct factors then we do not consider 1 way (i.e. N = $\sqrt{N} + \sqrt{N}$) then **no. of ways** $= \mathbf{\frac{(Number\ of\ factors - 1)}{2}}$

Ex. (a) Find the number of ways expressing 36 as a product of two factors.

Sol. $36 = 2^2 \times 3^2$

Number of factors = (2+1) (2+1) = 9

Hence the no. of ways of expressing 36 as a product of two factors = $\frac{(9+1)}{2} = 5$

as 36 = 1×36, 2×18, 3×12, 4×9 and 6×6

Ex. (b) In how many ways can 576 be expressed as the product of two distinct factors?

Sol. $576 = 2^6 \times 3^2$

$\therefore$ Total number of factors = (6 + 1) (2 + 1) = 21

So the number of ways of expressing 576 as a product of two distinct factors = $= \frac{21-1}{2} = 10$

Note – Since the word 'distinct' has been used therefore we do not include 576 = 26 × 26.

Cyclicity

We are having 10 digits in our number systems and some of them shows special characteristics like they, repeat their unit digit after a cycle, for example 1 repeat its unit digit after every consecutive power. So its cyclicity is 1 on the other hand digit 2 repeat its unit digit after every four power, hence the cyclicity of 2 is four. The cyclicity of digits are as follows

Digit	**Cyclicity**
0, 1, 5 and 6	1
4 and 9	2
2, 3, 7 and 8	4

So, if we want to find last digit of 2^{45}, divide 45 by 4. The remainder is 1 so the last digit of 2^{45} would be same as the last digit of 2^1 which is 2.

- **To Find the Unit Digit in Exponential Expression:**

(i) Where there is 2 in unit's place of any number.

Since, in 2^1 unit digit is 2, in 2^2 unit digit is 4, in 2^3 unit digit is 8, in 2^4 unit digit is 6, after that the unit's digit repeats e.g. unit digit $(12)^{12}$ is equal to the unit digit of 2^4 i.e. 6

Ex. In $(32)^{33}$ unit digit is equal to the unit digit of 21 i.e. 2.

(ii) When there is 3 in unit's place of any number.

Since, in 3^1 unit digit is 3, in 3^2 unit digit is 9, in 3^3 unit digit is 7, in 3^4 unit digit is 1, after that the unit's digit repeats.

Ex. In $(43)^{46}$ unit digit be 9

(iii) When there is 4 in units place of any number.

Since, in 4^1 unit digit is 4, in 4^2 unit digit is 6, after that the unit's digit repeats.

Ex. In $(34)^{14}$ unit digit is 6

Ex. In $(34)^{33}$ unit digit is 4

(iv) When there is 5 in unit's place of any number.

Since , in 5^1 unit digit is 5, in 5^2 unit digit is 5 and so on.

Ex. In $(25)^{15}$ unit digit is 5

(v) When there is 6 in unit's place of any number.

Since, in 6^1 unit digit is 6, in 6^2 unit digit is 6 & so on.

Ex. In $(46)^{13}$ unit digit is 6

(vi) When there is 7 in unit's place of any number.

Since, in 7^1 unit digit is 7, in 7^2 unit digit is 9, in 7^3 unit digit is 3, in 7^4 unit digit is 1, after that the unit's digit repeats.

Ex. In $(57)^9$ unit digit is 7

Ex. In $(97)^9$ unit digit is 3

(vii) When there is 8 is unit's place of any number.

Since in 8^1 unit digit is 8, in 8^2 unit digit is 4, in 8^3 unit digit is 2, in 8^4 unit is 6, after that unit's digit repeats after a group of 4.

(viii) When there is 9 in unit's place of any number.

Since, in 9^1 unit's digit is 9, in 9^2 unit's digit is 1, after that unit's digit repeats after a group of 2.

(ix) When there is zero in unit's place of any number.

There will always be zero in unit's place.

Ex. (a) Find the last digit of

(i) 3^{57} (ii) 13^{59}

Sol. (i) The cyclicity of 3 is 4. Hence, $\frac{57}{4}$ gives the remainder 1. So the last digit of 3^{57} is same as the last digit of 3^1, i.e. 3.

(ii) The number of digits in the base will not make a difference to the last digit. It is last digit of the base which decides the last digit of the number itself. For 13^{59}, we find $\frac{59}{4}$ which gives a remainder 3. So the last digit of 13^{59} is same as the last digit of 3^3, i.e 7.

(b) Find the last digit of the product $7^{23} \times 8^{13}$.

Sol. Both 7 and 8 exhibit a cyclicity of 4. the last digit are

$7^1 = 7$	$8^1 = 8$
$7^2 = 9$	$8^2 = 4$
$7^3 = 3$	$8^3 = 2$
$7^4 = 1$	$8^4 = 6$
$7^5 = 7$	$8^5 = 8$

The cycle would repeat itself for higher powers. 7^{23} ends with the same last digit as 7^3, i.e. 3.

8^{13} ends with the same last digit as 8^1, i.e. 8. Hence, the product of the two numbers would end with the same last digit as that of 3×8, i.e. 4.

Ex. (c) Find unit's digit in $y = 7^{17} + 7^{34}$

Sol. $7^{17} + 7^{34} = 7^1 + 7^2 = 56$, Hence the unit digit is 6

Ex. (d) What will be the last digit of $(73)^{75^{64^{76}}}$

Sol. Let $(73)^{75^{64^{76}}} = (73)^x$ where $x = 75^{64^{76}} = (75)^{\text{even power}}$

$\because$ Cyclicity of 3 is 4

$\therefore$ To find the last digit we have to find the remainder when x is divided by 4.

$x = (75)^{\text{even power}} = (76-1)^{\text{even power}}$, where n is dvided by 4 so remainder will be1

Therefore, the last digit of $(73)^{75^{64^{76}}}$ will be $3^1 = 3$

■ Euclid's division lemma

Euclid's division lemma states that "For any two positive integers a and b, there exist integers q and r such that $a = bq + r$, $0 \le r < b$."

e.g.(i) Consider number 23 and 5, then :

$23 = 5 \times 4 + 3$

Comparing with $a = bq + r$

we get, $a = 23$, $b = 5$, $q = 4$, $r = 3$ and $0 \le r < b$ (as $0 < 3 < 5$)

(ii) Consider positive integers 18 and 4

$18 = 4 \times 4 + 2$

For 18 (= a) and 4 (= b) we have $q = 4$, $r = 2$ and $0 \le r < b$

In the relation $a = bq + r$, where $0 \le r < b$ is nothing but a statement of the long division of number a by b in which q is the quotient obtained and r is the remainder.

■ Euclid's divison algorithm

In mathematics, the Euclid's Algorithm is an efficient method for computing the greatest common divisor (GCD) or highest common factor (HCF).

So, let us state Euclid's division algorithm clearly.

To obtain the HCF of two positive integers, say c and d, with $c > d$ follow the steps below :

Step-1 : Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.

Step-2 : If $r = 0$, d is the HCF of c and d. If $r \ne o$, apply the division lemma to d and r.

Step-3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because HCF (c,d) = HCF (d,r) where the symbol HCF (c,d) denotes the HCF of c and d.

Ex. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

Solution

In order to find the HCF of 441, 567 & 693, we first find the HCF of 441 & 567 by Euclid's division algorithm.

Using division algorithm, we get

$567 = 441 \times 1 + 126$

$441 = 126 \times 3 + 63$

$126 = 63 \times 2 + 0$

So, HCF (567, 441) = 63

Now, we find the HCF of 63 and 693

$693 = 63 \times 11 + 0$

$\therefore$ HCF (63, 693) = 63

Hence HCF (441, 567, 693) = 63

Fundamental theorem of arithmetic

The fundamental theorem of arithmetic (FTA) tells us something important about the relationship between composite numbers and prime numbers. It is usually stated as follows :

"Every composite number can be expressed as a product of primes, and their decomposition is unique, a part from the order in which the prime factors occur."

e.g. $12600 = 2^3.3^2.5^2.7$

Thus we have expressed the composite number 12600 as product of powers of primes in ascending order and this decomposition is unique.

Points to remember

- For any three positive integers p,q,r, HCF (p,q,r) × LCM (p,q,r) ≠ p × q × r. However, the following results hold good. LCM (p,q,r) = $\frac{p.q.r\ HCF(p,q,r)}{HCF(p,q) \cdot HCF(q,r) \cdot HCF(p,r)}$
- All odd numbers can be obtained by the formula, f(n) = 2n + 1, n = 1,2,3,....

  while even numbers can be obtained by the formula, g(n) = 2n, n = 1,2,3,....
- L.C.M. of pair of coprimes = Product of co-primes
- The denominator of the rational number must be in the form $2^m.5^n$ (where m & n are non-negative integers) so as to have the decimal expansion of that rational number as terminating.
- Any positive odd integer is of the form 6q + 1, or 6q + 3 or 6q + 5, where q is some integer.
- The square of any positive integer is either of the form 3m or 3m + 1 for some integer m.
- The cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.
- The square of any positive odd integer is of the form 8m + 1 for some integer m.

1. If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by
(1) 5 (2) 13
(3) Both (1) and (2) (4) Neither (1) nor (2)

2. N is a natural number such that when N^3 is divided by 9, it leaves remainder a. It can be concluded that
(1) a is a perfect square (2) a is a perfect cube
(3) Both (1) and (2) (4) Neither (1) nor (2)

3. The remainder of any perfect square divided by 3 is
(1) 0 (2) 1
(3) Either (1) or (2) (4) Neither (1) nor (2)

4. Find the HCF of 432 and 504 using prime factorization method.
(1) 36 (2) 72 (3) 96 (4) 108

5. If n is any natural number, then $6^n - 5^n$ always ends with
(1) 1 (2) 3 (3) 5 (4) 7

6. The LCM of two numbers is 1200. Which of the following cannot be their HCF ?
(1) 600 (2) 500 (3) 200 (4) 400

7. Which of the following is always true ?
(1) The rationalising factor of a number is unique
(2) The sum of two distinct irrational numbers is rational
(3) The product of two distinct irrational numbers is irrational
(4) None of these

8. Find the remainder when the square of any number is divided by 4.
(1) 0 (2) 1
(3) Either (1) or (2) (4) Neither (1) nor (2)

9. Ashok has two vessels which contain 720 ml and 405 ml of milk respectively. Milk in each vessel is poured into glasses of equal capacity of their brim. find the minimum number of glasses which can be filled with milk.
(1) 45 (2) 35 (3) 25 (4) 30

10. If n is an odd natural number, $3^{2n} + 2^{2n}$ is always divisible by
(1) 13 (2) 5 (3) 17 (4) 19

11. If the product of two irrational numbers is rational, then which of the following can be concluded ?
(1) The ratio of the greater and the smaller numbers is an integer
(2) The sum of the numbers must be rational
(3) The excess of the greater irrational number over the smaller irrational number must be rational
(4) None of these

12. The LCM and HCF of two numbers are equal, then the numbers must be
(1) Prime (2) Co-prime
(3) Composite (4) Equal

13. The sum of LCM and HCF of two numbers is 1260. If their LCM is 900 more than their HCF, find the product of two numbers.
(1) 203400 (2) 194400
(3) 198400 (4) 205400

14. Find the remainder when the square of any prime number greater than 3 is divided by 6.
(1) 1 (2) 3 (3) 2 (4) 4

15. If HCF (72, q) = 12 then how many values can q take ? (Assume q be a product of a power of 2 and a power of 3 only)
(1) 1 (2) 2 (3) 3 (4) 4

16. Find the HCF of 120 and 156 using Euclid's division algorithm.
(1) 18 (2) 12 (3) 6 (4) 24

17. What is the digit in the tens place in the product of the first 35 even natural numbers ?
(1) 6 (2) 2 (3) 0 (4) 5

18. The LCM of $\frac{1}{4}$ and $\frac{2}{5}$ is
(1) 1 (2) $\frac{1}{10}$ (3) 2 (4) $\frac{1}{20}$

19. The multiplicative inverse of $(x + 1) + \frac{1}{(x-1)}$ is
(1) $\frac{1}{(x+1)} + (x-1)$ (2) $(x-1) - \frac{1}{(x+1)}$
(3) $\frac{x-1}{x^2}$ (4) $\frac{x+1}{x^2}$

20. Find the unit's digit in the product of the first 50 odd natural numbers.
(1) 0 (2) 5 (3) 7 (4) None

21. There are 20 balls. The balls are numbered consecutively starting from anyone of the numbers from 1 to 20. For any case, the sum of the numbers on all the balls will be a/an
(1) odd number (2) even number
(3) prime number (4) Cannot say

22. Which pair of numbers below are twin primes ?

(1) 8 and 9 (2) 2 and 3

(3) 3 and 7 (4) 41 and 43

23. Which of the following values are even ?

(a) $21 + 18 + 9 + 2 + 19$

(b) $34 \times 28 \times 37 \times 94 \times 12712$

(c) $33 \times 35 \times 37 \times 39 \times 41 \times 43$

(d) $11 \times 11 \times 11 \times 11 \times 11 \times ....$

(e) 1^{10}

(f) $39 - 24$

(1) a,b,c (2) d,e,f

(3) b (4) a,b,d,e

24. What is the number in the units place of $(763)^{84}$?

(1) 1 (2) 3 (3) 7 (4) 9

25. If the numbers a – b and a + b are twin primes, then a and b are necessarily

(1) Twin primes (2) Co-primes

(3) Cannot say (4) None

26. The HCF of all the natural numbers from 200 to 478 is

(1) 2 (2) 1 (3) 478 (4) 3

27. Find the greatest number that divides 59 and 54 leaving remainders 3 and 5 respectively.

(1) 3 (2) 7 (3) 8 (4) 5

28. Find the unit digit in the expansion of $(44)^{44} + (55)^{55} + (88)^{88}$.

(1) 7 (2) 5 (3) 4 (4) 3

29. Find the digit in the units place of $(676)^{99}$.

(1) 9 (2) 2 (3) 4 (4) 6

30. The LCM of $\frac{5}{12}, \frac{6}{5}, \frac{3}{2}$ and $\frac{4}{17}$ is

(1) 60 (2) $\frac{1}{60}$

(3) 180 (4) None

31. Find the number of factors of 1080.

(1) 32 (2) 28 (3) 24 (4) 36

32. If p,q and r are prime numbers such that r = q + 2 and q = p + 2, then the number of triplets of the form (p,q,r) is

(1) 0 (2) 1 (3) 2 (4) 3

33. The absolute value of $25 - (25 + 10) + 25 \div 125 \times 25$ is

(1) – 5 (2) 3 (3) 15 (4) 5

34. The greatest five digit number exactly divisible by 9 and 13 is

(1) 99945 (2) 99918

(3) 99964 (4) 99972

35. If the number 2345p60q is exactly divisible by 3 and 5, then the maximum value of p + q is

(1) 12 (2) 13

(3) 14 (4) 15

36. If $1 \le k \le 25$, how many prime numbers are there which are of the form 6k + 1 ?

(1) 15 (2) 16

(3) 17 (4) 18

37. If a,b,c and d are four positive real numbers such that sum of a,b, and c is even and the sum of b,c and d is odd, then $a^2 - d^2$ is necessarily

(1) odd (2) even

(3) prime (4) Either (1) or (2)

38. Mukesh bought 3 apples, 5 bananas and 7 custard apples for certain amount (which is even). The cost of apples, bananas and custard apples could be (in Rs.)

(1) 5,7,9 (2) 9,8,6

(3) 2,4,5 (4) 9,10,11

39. In a class there are 72 boys and 64 girls. If the class is to be divided into least number of groups such that each group contains either only boys or only girls, then how many groups will be formed ?

(1) 17 (2) 34 (3) 24 (4) None

40. The HCF of two numbers, obtained in three steps of division, is 7 and the first 3 quotients are 2,4 and 6 respectively. Find the numbers.

(1) 175, 392 (2) 189, 392

(3) 168, 385 (4) None

41. Find the greatest four digit number which when divided by 18 and 12 leaves a remainder of 4 in each case

(1) 9976 (2) 9940 (3) 9904 (4) 9868

42. Rahul wanted to type of first 180 natural numbers. Find the number of times he had to press the numbered keys.

(1) 384 (2) 432 (3) 416 (4) 448

43. If the seven digit number 4567 X 75 is divisible by 15 then find the least possible value of X.

(1) 2 (2) 1 (3) 0 (4) 3

44. A rational number can be expressed as a terminating decimal if the denominator has factors

(1) 2 or 5 (2) 2 ,3 or 5

(3) 3 or 5 (4) None of these

45. The only prime number which is even is

(1) 2 (2) 4

(3) 6 (4) none of these

46. The value of $23.\overline{43}+5.\overline{2}$ is

(1) $\frac{2395}{990}$ (2) $\frac{2527}{99}$

(3) $\frac{5169}{990}$ (4) $\frac{2837}{99}$

47. If $2 = x + \cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$, then value of x is

(1) $\frac{12}{17}$ (2) $\frac{13}{17}$

(3) $\frac{18}{17}$ (4) $\frac{21}{17}$

48. If R "Every fraction is a rational number" and T "Every rational number is a fraction", then which of the following is correct?

(1) R is True and T is False.
(2) R is False and T is True.
(3) Both R and T are True.
(4) Both R and T are False.

49. $5.\overline{2}$ is equal to

(1) $\frac{45}{9}$ (2) $\frac{46}{9}$

(3) $\frac{47}{9}$ (4) None of these

50. For any two rational numbers A and B, which of the following properties are correct?

(i) A < B (ii) A = B
(iii) A > B

(1) Only (i) and (ii) are correct.
(2) Only (ii) and (iii) are correct.
(3) Only (ii) is correct.
(4) All (i), (ii), (iii) are correct.

51. If $x = \frac{3+\sqrt{2}}{3-\sqrt{2}}$ and $y = \frac{3-\sqrt{2}}{3+\sqrt{2}}$, the value of (x + y) is

(1) $\frac{306}{49}$ (2) $\frac{484}{49}$

(3) $\frac{22}{7}$ (4) $\frac{73}{7}$

52. If x: Every whole number is a natural number and y: 0 is not a natural number, Then which of the following statement is true?

(1) x is false and y is the correct explanation of x.
(2) x is true and y is the correct explanation of x.
(3) x is true and y is false.
(4) Both x and y are true.

53. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is

(1) 34 (2) 35
(3) 36 (4) 37

54. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^3 + b^3$ is

(1) 194 (2) 196
(3) 198 (4) 200

55. If $x = \left(7+4\sqrt{3}\right)$, then the value of $x^2 + \frac{1}{x^2}$ is

(1) 193 (2) 194
(3) 195 (4) 196

56. If $x = 7 + 4\sqrt{3}$, then the value of $\sqrt{x}+\frac{1}{\sqrt{x}}$ is

(1) 8 (2) 6
(3) 5 (4) 4

57. $8 - 8 \times \dfrac{2\frac{1}{5}-1\frac{2}{7}}{2-\cfrac{1}{6-\cfrac{1}{6}}}$ is equal to

(1) 6 (2) 2
(3) 4 (4) 8

58. The rational form of $2.74\overline{35}$ is

(1) $\frac{27161}{9999}$ (2) $\frac{27}{99}$

(3) $\frac{27161}{9900}$ (4) $\frac{27161}{9000}$

59. If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, then the value of 'a' and 'b' is

(1) a = 2, b = – 1
(2) a = 2, b = 1
(3) a = – 2, b = 1
(4) a = – 2, b = – 1

60. The value of $\frac{1}{\sqrt{3}+\sqrt{2}-1}$ on simplifying upto 3 decimal places, given that $\sqrt{2}$ = 1.4142 and $\sqrt{6}$ = 2.4495 is

(1) 0.166 (2) 0.366
(3) 0.466 (4) 0.566

61. π is

(1) Rational (2) Irrational
(3) Imaginary (4) An integer

62. An irrational number is

(1) A terminating and nonrepeating decimal
(2) A nonterminating and nonrepeating decimal
(3) A terminating and repeating decimal
(4) A nonterminating and repeating decimal

63. Which of the following statement is true ?

(1) Every point on the number line represents a rational number
(2) Irrational numbers cannot be represented by points on the number line
(3) $\frac{22}{7}$ is a rational number
(4) None of these

64. The sum of rational and irrational number is always

(1) Rational (2) Irrational
(3) Both (4) Can't say

65. The product of rational and irrational number is always

(1) Rational (2) Irrational
(3) Both (4) Can't say

66. Rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(1) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (2) $\frac{\sqrt{2}\times\sqrt{3}}{2}$
(3) 1.5 (4) 1.8

67. The number $\frac{5-\sqrt{5}}{5+\sqrt{5}}$ is

(1) Rational (2) Irrational
(3) Both (4) Can't say

68. $0.\overline{23}+0.\overline{22}$ = ?

(1) $0.\overline{45}$ (2) $0.\overline{43}$
(3) $0.4\overline{5}$ (4) 0.45

69. $0.\overline{018}$ can be expressed in the rational form as

(1) $\frac{18}{1000}$ (2) $\frac{18}{990}$
(3) $\frac{18}{9900}$ (4) $\frac{18}{999}$

70. The equivalent rational form of $17.\overline{6}$ is

(1) $\frac{53}{3}$ (2) $\frac{88}{5}$
(3) $\frac{44}{25}$ (4) None of these

71. $\frac{961}{625}$ is a

(1) Terminating decimal
(2) Nonterminating decimal
(3) Cannot be determined
(4) None of these

72. When simplified, the product

$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{n}\right)$ equals ;

(1) $\frac{1}{n}$ (2) $\frac{2}{n}$
(3) $\frac{2(n-1)}{n}$ (4) $\frac{2}{n(n+1)}$

73. Which of the following has most number of divisors ?

(1) 99 (2) 101
(3) 176 (4) 182

74. A number n is said to be perfect if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is

(1) 6 (2) 9
(3) 15 (4) 21

75. The H.C.F. of $2^2\times3^3\times5^5$, $2^3\times3^2\times5^2\times7$ and $2^4\times3^4\times5\times7^2\times11$ is

(1) $2^2\times3^2\times5$
(2) $2^2\times3^2\times5\times7\times11$
(3) $2^4\times3^4\times5^5$
(4) $2^4\times3^4\times5^5\times7\times11$

76. Which of the following is a pair of co-primes ?

(1) (16, 62) (2) (18, 25)
(3) (21, 35) (4) (23, 92)

77. The L.C.M. of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$ is

(1) $2^3 \times 3^2 \times 5$

(2) $2^5 \times 3^4 \times 5^3$

(3) $2^3 \times 3^2 \times 5 \times 7 \times 11$

(4) $2^5 \times 3^4 \times 5^3 \times 7^2$ x 11

78. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is

(1) 3 (2) 13
(3) 23 (4) 33

79. The least number which is a perfect square and is divisible by each of the numbers 16, 20 and 24, is

(1) 1600 (2) 3600
(3) 6400 (4) 14400

80. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is

(1) 1008 (2) 1015
(3) 1022 (4) 1032

81. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54, is

(1) 427 (2) 859
(3) 869 (4) 4320

82. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is

(1) 504 (2) 536
(3) 544 (4) 548

83. The largest four-digit number which when divided by 4, 7 or 13 leaves a remainder of 3 in each case, is

(1) 8739 (2) 9831
(3) 9834 (4) 9893

84. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is

(1) 74 (2) 94
(3) 184 (4) 364

85. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is

(1) 11115 (2) 15110
(3) 15120 (4) 15210

86. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.

(1) 3002 (2) 3013
(3) 3024 (4) 3036

87. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is

(1) 1677 (2) 1683
(3) 2523 (4) 3363

88. The product of two numbers is 960. If H.C.F. is 8, then the numbers are

(1) 24,40 (2) 8, 120 or 24, 40
(3) 8, 140 (4) none of these

89. The least number divisible by 12, 15, 20, and is perfect square is

(1) 900 (2) 400
(3) 36 (4) 256

90. The largest number which divides 133 and 245 leaving a remainder 5 is

(1) 17 (2) 15
(3) 8 (4) 16

91. The H.C.F,. of two numbers is $\frac{1}{5}$ of their L.C.M. If the product of two number is 720, then the H.C.F. of the numbers is

(1) 13 (2) 12
(3) 14 (4) 18

92. The L.C.M. of two numbers is 39780 and their ratio is 13 : 15 then the numbers are

(1) 273,315 (2) 2652,3060
(3) 516, 685 (4) none

93. The L.C.M. of two numbers is 14 times of their H.C.F. The sum of L.C.M. and H.C.F. is 600. If one of the number is 80, then other is

(1) 280 (2) 218
(3) 25 (4) 45

94. Four bells begin to toll together and toll respectively at intervals of 5, 6, 8 and 12 seconds. How many times will they toll together in an hour excluding the one at the start

(1) 30 (2) 19
(3) 13 (4) 5

95. Two ropes of length 28 m and 36 m are to be cut into bits of same length. The greatest possible length of each is

(1) 7 (2) 3
(3) 4 (4) 5

96. 28 mango trees, 42 apple trees and 21 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. The minimum number of rows in which the above trees may be planted

(1) 13 (2) 12
(3) 11 (4) 10

97. A heap is to be formed with lots of 8, 10 and 15 pebbles of different colours. The smallest number of pebbles in the heap is

(1) 121 (2) 120
(3) 110 (4) 8

98. Three measuring tapes are 64 cm, 72 cm, 96 cm, the shortest length that can measured by any one of the tapes exactly is

(1) 576 (2) 120
(3) 8 (4) 10.

99. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is

(1) 28 (2) 32
(3) 40 (4) 64

100. $\frac{(x^{a+b})^2(x^{b+c})^2(x^{c+a})^2}{(x^a.x^b.x^c)^4} =$

(1) −1 (2) 0
(3) 1 (4) None

101. $\frac{2^{n+4}-2(2^n)}{2(2^{n+3})}+2^{-3}$ is equal to

(1) 2^{n+1} (2) $-2^{n+1}+\frac{1}{8}$

(3) $\frac{9}{8}-2^n$ (4) 1

102. The value of $\left[(x^{a-a^{-1}})^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}} =$

(1) x (2) 1/x
(3) x^a (4) $1/x^a$

103. If $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$, $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$, then the value of a + b is

(1) 14 (2) −14
(3) $8\sqrt{3}$ (4) $-\sqrt{3}$

104. The smallest number by which 3600 can be divided to make it a perfect cube is

(1) 9 (2) 50
(3) 300 (4) 450

105. The remainder when 7^{84} is divided by 342 is

(1) 0 (2) 1
(3) 49 (4) 341

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	2	3	2	1	2	4	3	3	1	4	4	2	1	2	2	3	3	3	
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	4	3	1	2	2	2	1	4	1	2	2	4	2	2	2	1	4	1	
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	2	1	1	1	4	4	1	3	4	3	1	1	3	2	4	3	3	1	
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	2	3	2	2	3	2	1	4	1	1	2	3	1	1	2	4	3	2	
Que.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	2	4	2	4	2	2	2	2	1	4	2	2	1	1	3	1	2	1	3	
Que.	101	102	103	104	105															
Ans.	4	1	1	4	2															