6 Introduction to Trigonometry

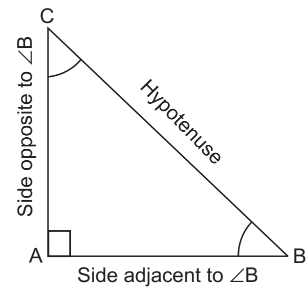
Topics Covered

1. Trigonometric Ratios
2. Trigonometric Identities

1. Trigonometric Ratios

Understanding Trigonometric Ratios

- Let ABC be a right triangle in which $\angle A = 90°$, side adjacent to $\angle B = AB$, side opposite to $\angle B = AC$ and hypotenuse $= BC$. With reference to angle B, we define the following ratios known as trigonometric ratios.



(*i*) sine of $\angle B = \sin B = \dfrac{\text{Side opposite to } \angle B}{\text{Hypotenuse}} = \dfrac{AC}{BC}$

(*ii*) cosine of $\angle B = \cos B = \dfrac{\text{Side adjacent to } \angle B}{\text{Hypotenuse}} = \dfrac{AB}{BC}$

(*iii*) tangent of $\angle B = \tan B = \dfrac{\text{Side opposite to } \angle B}{\text{Side adjacent to } \angle B} = \dfrac{AC}{AB}$

(*iv*) cosecant of $\angle B = \operatorname{cosec} B = \dfrac{1}{\sin B} = \dfrac{\text{Hypotenuse}}{\text{Side opposite to } \angle B} = \dfrac{BC}{AC}$

(*v*) secant of $\angle B = \sec B = \dfrac{1}{\cos B} = \dfrac{\text{Hypotenuse}}{\text{Side adjacent to } \angle B} = \dfrac{BC}{AB}$

(*vi*) cotangent of $\angle B = \cot B = \dfrac{1}{\tan B} = \dfrac{\text{Side adjacent to } \angle B}{\text{Side opposite to } \angle B} = \dfrac{AB}{AC}$

(*vii*) $\tan B = \dfrac{\sin B}{\cos B}$ (*viii*) $\cot B = \dfrac{\cos B}{\sin B}$

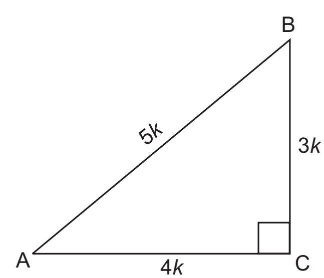
The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

- If one trigonometric ratio of an angle is given, the other trigonometric ratios of the angle can be determined.

Example 1. If $\cos A = \dfrac{4}{5}$, then sin A and sec A respectively are

(*a*) $\dfrac{3}{4}, \dfrac{4}{3}$ (*b*) $\dfrac{5}{3}, \dfrac{3}{4}$

(*c*) $\dfrac{3}{5}, \dfrac{5}{4}$ (*d*) None of these



Solution. Given: $\cos A = \dfrac{4}{5}$

Since, $\cos A = \dfrac{4}{5} = \dfrac{CA}{AB}$

$\therefore$ $CA = 4k$, $AB = 5k$, where k is a positive number.

In right-angled triangle ACB, BC = $\sqrt{25k^2 - 16k^2} = \sqrt{9k^2} = 3k$

So $\qquad \sin A = \frac{BC}{AB} = \frac{3k}{5k} = \frac{3}{5}$, $\sec A = \frac{1}{\cos A} = \frac{5}{4}$

Hence, option (*c*) is the correct answer.

Example 2. If 4 tan θ = 3, then the value of $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$ is

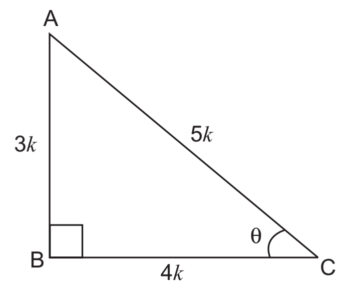
(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{5}$

Solution. ∵ $\quad 4\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{4} \Rightarrow \tan\theta = \frac{AB}{BC}$

Let $\quad AB = 3k, BC = 4k$

Then, $\quad AC = \sqrt{9k^2 + 16k^2} = 5k$

(using Pythagoras theorem)



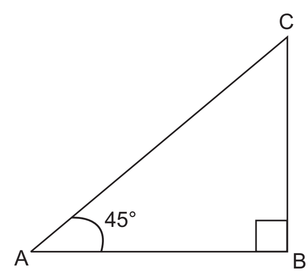
∴ $\quad \sin\theta = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$ and $\cos\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

∴ $\quad \frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\times\frac{3}{5} - \frac{4}{5}}{4\times\frac{3}{5} + \frac{4}{5}} = \frac{\frac{12-4}{5}}{\frac{12+4}{5}} = \frac{8}{5}\times\frac{5}{16} = \frac{8}{16} = \frac{1}{2}$

Hence, option (*a*) is the correct answer.

Trigonometric Ratios of Some Specific Angles

- **Trigonometric Ratios of 45°:**

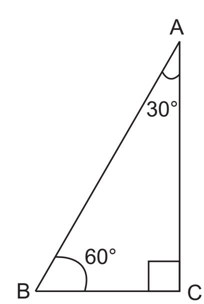


(*i*) $\sin 45° = \frac{1}{\sqrt{2}}$ (*ii*) $\cos 45° = \frac{1}{\sqrt{2}}$

(*iii*) $\tan 45° = 1$ (*iv*) $\text{cosec } 45° = \sqrt{2}$

(*v*) $\sec 45° = \sqrt{2}$ (*vi*) $\cot 45° = 1$

- **Trigonometric Ratios of 30° and 60°:**



(*i*) $\sin 30° = \frac{1}{2}$ (*ii*) $\cos 30° = \frac{\sqrt{3}}{2}$

(*iii*) $\tan 30° = \frac{1}{\sqrt{3}}$ (*iv*) $\text{cosec } 30° = 2$

(*v*) $\sec 30° = \frac{2}{\sqrt{3}}$ (*vi*) $\cot 30° = \sqrt{3}$

(*vii*) $\sin 60° = \frac{\sqrt{3}}{2}$ (*viii*) $\cos 60° = \frac{1}{2}$

(*ix*) $\tan 60° = \sqrt{3}$ (*x*) $\text{cosec } 60° = \frac{2}{\sqrt{3}}$

(*xi*) $\sec 60° = 2$ (*xii*) $\cot 60° = \frac{1}{\sqrt{3}}$

- The value of sin θ or cos θ never exceeds 1, whereas the value of sec θ or cosec θ is always greater than or equal to 1.

Example 3. The value of (sin 30° + cos 60°)

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) None of these

Solution. $\sin 30° + \cos 60° = \frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$

Hence, option (*a*) is the correct answer.

Example 4. The value of $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$ is

(*a*) $\frac{32}{35}$ (*b*) $\frac{14}{55}$ (*c*) $\frac{67}{12}$ (*d*) $\frac{19}{33}$

Solution. $\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}} = \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{\frac{67}{12}}{1} = \frac{67}{12}$$

Hence, option (*c*) is the correct answer.

Example 5. The value of $\frac{3\sin 30° + 4\cos^2 45° - \cot^2 30°}{\cos^2 30° + \sin^2 30°}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{3}{8}$

Solution. $$\frac{3\sin 30° + 4\cos^2 45° - \cot^2 30°}{\cos^2 30° + \sin^2 30°} = \frac{3\times\frac{1}{2} + 4\times\left(\frac{1}{\sqrt{2}}\right)^2 - (\sqrt{3})^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\frac{3}{2}+\frac{4}{2}-3}{\frac{3}{4}+\frac{1}{4}}$$

$$= \frac{\frac{3+4-6}{2}}{\frac{4}{4}} = \frac{1}{2}$$

Hence, option (*a*) is the correct answer.

Example 6. Given $\sin(A - B) = \frac{\sqrt{3}}{2}$ and $\cos(A + B) = \frac{\sqrt{3}}{2}$. Then A and B respectively are

(*a*) 30°, 45° (*b*) 45°, –15° (*c*) 60°, 45° (*d*) None of these

Solution. Since, $\sin(A - B) = \frac{\sqrt{3}}{2}$ and $\cos(A + B) = \frac{\sqrt{3}}{2}$

⇒ sin (A – B) = sin 60° and cos (A + B) = cos 30°

$\Rightarrow$ $A - B = 60°$ ...(*i*)

and $A + B = 30°$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$$2A = 90° \quad \Rightarrow \quad A = 45°, B = -15°$$

Hence, option (*b*) is the correct answer.

Exercise 6.1

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. If triangle ABC is right angled at C, then the value of sec (A + B) is [CBSE Standard SP 2019-20]

   (*a*) 0 (*b*) 1 (*c*) $\frac{2}{\sqrt{3}}$ (*d*) not defined

2. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90°)$ then the value of $\tan\theta$ is [CBSE Standard SP 2019-20]

   (*a*) $\sqrt{2}-1$ (*b*) $\sqrt{2}+1$ (*c*) $\sqrt{2}$ (*d*) $-\sqrt{2}$

3. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is [CBSE Standard SP 2019-20]

   (*a*) 0° (*b*) 90° (*c*) 60° (*d*) 30°

4. The value of $\sin 60° \cdot \cos 30° + \sin 30° \cdot \cos 60°$ is

   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 8

5. Value of $\cos 0° \cdot \cos 30° \cdot \cos 45° \cdot \cos 60° \cdot \cos 90°$ is

   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 9

6. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$

   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 5

7. The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$

   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) None of these

8. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, A > B, then the value of A is

   (*a*) A = 30° (*b*) A = 60° (*c*) A = 90° (*d*) A = 45°

9. The value of $\sin 60° \cos 30° + \sin 30° \cos 60°$ is

   (*a*) 1 (*b*) 2 (*c*) 11 (*d*) 0

10. $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ equals

    (*a*) 1 (*b*) 2 (*c*) 5 (*d*) 6

11. If $\sin\theta = x$ and $\sec\theta = y$, then the value of $\cot\theta$ is

    (*a*) xy (*b*) $2xy$ (*c*) $\frac{1}{xy}$ (*d*) $x + y$

12. If $(1 + \cos A)(1 - \cos A) = \frac{3}{4}$, the value of sec A is

    (*a*) 2 (*b*) –2 (*c*) ±2 (*d*) 0

13. If 15 cot A = 8, then the value of cosec A is

    (*a*) $\frac{15}{12}$ (*b*) $\frac{13}{15}$ (*c*) $\frac{4}{15}$ (*d*) $\frac{17}{15}$

14. If 5 tan θ = 3, then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

15. Evaluate: $4\sin^2 60° + 3\tan^2 30° - 8\sin 45° \cos 45°$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 5

16. Evaluate: $\frac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$.

(*a*) $\frac{3\sqrt{3}+2}{3\sqrt{3}-2}$ (*b*) $\frac{3\sqrt{3}-4}{3\sqrt{3}+4}$ (*c*) $\frac{3\sqrt{3}+8}{3\sqrt{3}-9}$ (*d*) None of these

17. If sin θ = cos θ, then find the value of $2\tan\theta + \cos^2\theta$.

(*a*) $\frac{2}{3}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{5}{2}$ (*d*) 0

18. If tan A = $\frac{5}{12}$, then (sin A + cos A)·sec A is equal to

(*a*) $\frac{11}{12}$ (*b*) $\frac{13}{12}$ (*c*) $\frac{12}{19}$ (*d*) $\frac{17}{12}$

19. The value of $\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°}$ is

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{2}{\sqrt{3}}$ (*c*) $\frac{1}{\sqrt{2}}$ (*d*) 0

20. ABC is a triangle right angled at C and AC = $\sqrt{3}$ BC. Then ∠ABC =

(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 0°

21. If tan θ + cot θ = 2, the value of $\sqrt{\tan^2\theta + \cot^2\theta}$ is

(*a*) 2 (*b*) $\sqrt{2}$ (*c*) 3 (*d*) $\sqrt{3}$

22. ABC is right-angled triangle, right-angled at B. If BC = 7 cm and AC – AB = 1 cm, then cos A + sin A equals

(*a*) $\frac{31}{25}$ (*b*) $\frac{51}{61}$ (*c*) $\frac{17}{39}$ (*d*) $\frac{51}{53}$

23. The value of *x* such that $2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$, is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement reason (R). Choose the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** In a right-angled triangle, if tan θ = $\frac{3}{4}$, the greatest side of the triangle is 5 units.

Reason (R): $(\text{Greatest side})^2 = (\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$.

2. Assertion (A): In a right-angled triangle, if $\cos\theta = \frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$, then $\tan\theta = \sqrt{3}$.

Reason (R): $\frac{\sin\theta}{\cos\theta}$

Case Study Based Questions

I. Three children were playing with sticks. As they had one stick each of them, they put all the three sticks together. Finding all the three sticks equal, they pick up the sticks and put them in a triangular form in such a way that the ends of each stick touch the other. They were surprised. Now they thought of a plane. They took another stick and put it as in the adjacent figure. The stick AD is just touching the stick BC. Somehow, they measured each angle. Finding that each angle. $\angle A = \angle B = \angle C = 60°$ (equal) and $\angle BAD = \angle CAD = 30°$. Likewise, they measured BD = CD, and $\angle ADB = \angle ADC = 90°$. Taking AB = BC = CA = $2a$, you are required to answer the following questions:

1. The length of AD is

(*a*) a (*b*) $2a$ (*c*) $\sqrt{2}a$ (*d*) $\sqrt{3}a$

2. Using the above figure, the value of sin 30° is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{\sqrt{2}}$ (*c*) $\frac{\sqrt{3}}{2}$ (*d*) 1

3. Using the above figure, the value of cos 60° is

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) $\frac{\sqrt{3}}{2}$

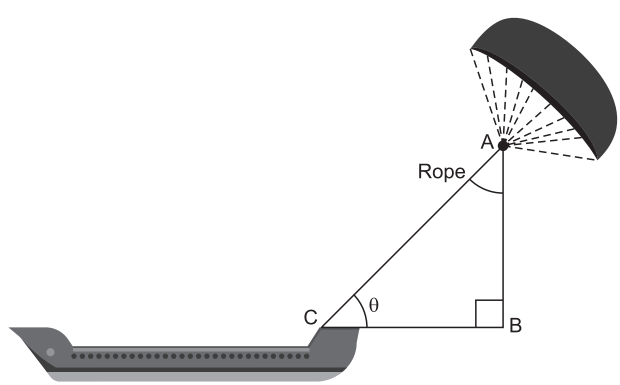
4. Using the above figure, the value of tan 30° is

(*a*) $\sqrt{3}$ (*b*) 1 (*c*) 0 (*d*) $\frac{1}{\sqrt{3}}$

5. Using the above figure, the value of cosec 60° is

(*a*) $\frac{2}{\sqrt{3}}$ (*b*) $\frac{\sqrt{3}}{2}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{2}}$

II. Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres to 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.



1. In the given figures, if $\sin\theta = \cos(3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then the value of θ is

(*a*) 30° (*b*) 60° (*c*) 45° (*d*) None of these.

2. What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated in part (*a*) and be at a vertical height of 200 m?

(*a*) 300 m (*b*) 400 m (*c*) 500 m (*d*) 600 m

3. If BC = 15 m, θ = 30°, then AB is

(*a*) $2\sqrt{3}$ m (*b*) 15 m (*c*) 24 m (*d*) $5\sqrt{3}$ m

4. Suppose AB = BC = 12 m, then θ =

(*a*) 0° (*b*) 30° (*c*) 45° (*d*) 60°

5. Given that BC = 6 m and θ = 45°. The values of AB and AC are respectively

(*a*) AB = 4 m, AC = $4\sqrt{2}$ m (*b*) AB = 7 m, AC = $7\sqrt{5}$ m

(*c*) AB = 9 m, AC = $9\sqrt{3}$ m (*d*) AB = 6 m, AC = $6\sqrt{2}$ m

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*d*) not defined
2. (*a*) $\sqrt{2}-1$
3. (*d*) 30°
4. (*b*) 1
5. (*a*) 0
6. (*b*) 1
7. (*b*) 1
8. (*d*) A = 45°
9. (*a*) 1
10. (*b*) 2
11. (*c*) $\frac{1}{xy}$

Given $\sin\theta = x$ and $\sec\theta = y$

$\Rightarrow \cos\theta = \frac{1}{y}$

Now, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{xy}$

Hence, $\cot\theta = \frac{1}{xy}$

12. (*c*) ±2

$(1+\cos A)(1-\cos A) = \frac{3}{4}$

$\Rightarrow 1-\cos^2 A = \frac{3}{4} \Rightarrow 1-\frac{3}{4} = \cos^2 A$

$\Rightarrow \frac{1}{4} = \cos^2 A \Rightarrow \sec^2 A = 4$

$\Rightarrow \sec A = \pm 2$

13. (*d*) $\text{cosec } A = \frac{17}{15}$

14. (*a*) 0

Given $5\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{5}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3}{5}$

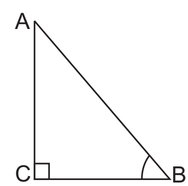
$\Rightarrow \sin\theta = 3x$ and $\cos\theta = 5x$

$\therefore \frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\times 3x - 3\times 5x}{4\times 3x + 3\times 5x} = \frac{15x-15x}{12x+15x} = \frac{0}{27x} = 0$

15. (*a*) 0
16. (*b*) $\frac{3\sqrt{3}-4}{3\sqrt{3}+4}$
17. (*c*) $\frac{5}{2}$
18. (*d*) $\frac{17}{12}$
19. (*a*) $\frac{\sqrt{3}}{2}$
20. (*b*) 60°

Here, in ΔABC, ∠C = 90° and AC = $\sqrt{3}$ BC [given]

$\Rightarrow \frac{AC}{BC} = \sqrt{3}$ ...(*i*)



Also, $\tan B = \frac{AC}{BC}$

$\Rightarrow \tan B = \sqrt{3}$ [using (*i*)]

$\Rightarrow B = 60°$

21. (*b*) $\sqrt{2}$

Given: $\tan\theta + \cot\theta = 2$

Squaring both sides, we get

$\tan^2\theta + \cot^2\theta + 2 = 4$

$\Rightarrow \tan^2\theta + \cot^2\theta = 2$

Taking square root on both sides, we get

$\sqrt{\tan^2\theta + \cot^2\theta} = \sqrt{2}$

22. (*a*) $\frac{31}{25}$

23. (*c*) 3

$2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$

$\Rightarrow 2\times(2)^2 + x\times\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\times\left(\frac{1}{\sqrt{3}}\right)^2 = 10$

$\Rightarrow 8 + x\times\frac{3}{4} - \frac{1}{4} = 10$

$$\Rightarrow \quad \frac{3x}{4} = 10 - 8 + \frac{1}{4}$$

$$\Rightarrow \quad \frac{3x}{4} = \frac{9}{4} \quad \Rightarrow \quad x = \frac{9}{4} \times \frac{4}{3}$$

$$\Rightarrow \quad x = 3$$

B. Assertion-Reason Type Questions

1. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
2. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Case Study Based Questions

I. 1. (*d*) $\sqrt{3}a$ **2.** (*a*) $\frac{1}{2}$

3. (*c*) $\frac{1}{2}$ **4.** (*d*) $\frac{1}{\sqrt{3}}$

5. (*a*) $\frac{2}{\sqrt{3}}$

II. 1. (*a*) 30° **2.** (*b*) 400 m

3. (*d*) $5\sqrt{3}$ m **4.** (*c*) 45°

5. (*d*) AB = 6m, AC = $6\sqrt{2}$ m

2. Trigonometric Identities

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

Some trigonometric ratios are listed below:

(*i*) $\sin^2 A + \cos^2 A = 1, 0° \le A \le 90°$ (*ii*) $1 + \tan^2 A = \sec^2 A, 0° \le A < 90°$

(*iii*) $\cot^2 A + 1 = \operatorname{cosec}^2 A, 0° < A \le 90°$

Example 1. The value of $\sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta \cos^2 \theta$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) $\frac{1}{4}$

Solution. $\sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta \cos^2 \theta$

$$= (\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^4 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^4 \theta + 3 \sin^2 \theta \cos^2 \theta$$
$$= 1 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) + 3 \sin^2 \theta \cos^2 \theta$$
$$= 1 - 3 \sin^2 \theta \cos^2 \theta + 3 \sin^2 \theta \cos^2 \theta = 1$$

Hence, option (*b*) is the correct answer.

Example 2. The value of $(\sin^4\theta - \cos^4\theta + 1) \operatorname{cosec}^2\theta$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 5

Solution. $(\sin^4 \theta - \cos^4 \theta + 1) \operatorname{cosec}^2 \theta = \{(\sin^2 \theta + \cos^2 \theta) (\sin^2 \theta - \cos^2 \theta) + 1\} \operatorname{cosec}^2 \theta$

$$= (\sin^2 \theta - \cos^2 \theta + 1) \operatorname{cosec}^2 \theta = (\sin^2 \theta + \sin^2 \theta) \operatorname{cosec}^2 \theta$$
$$= (2 \sin^2 \theta) \operatorname{cosec}^2 \theta = 2 \sin^2 \theta \times \frac{1}{\sin^2 \theta} = 2$$

Hence, option (*c*) is the correct answer.

Example 3. If $\tan A = n \tan B$ and $\sin A = m \sin B$, then $\cos^2 A =$

(*a*) $\frac{m^2 - 1}{n^2 - 1}$ (*b*) $\frac{m^2 + 1}{n^2 + 1}$ (*c*) $\frac{1 - m^2}{1 + m^2}$ (*d*) $\frac{1 + m^2}{1 - m^2}$

Solution. $\tan A = n \tan B \Rightarrow \tan B = \frac{1}{n} \tan A \Rightarrow \cot B = \frac{n}{\tan A}$

and $\sin A = m \sin B \Rightarrow \sin B = \frac{1}{m} \sin A \Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$

Now, $\operatorname{cosec}^2 B - \cot^2 B = 1$

$$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1 \quad \Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1 \quad \Rightarrow \quad \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \quad m^2 - n^2 \cos^2 A = \sin^2 A \quad \Rightarrow \quad m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow \quad m^2 - 1 = (n^2 - 1) \cos^2 A \quad \Rightarrow \quad \cos^2 A = \frac{m^2 - 1}{n^2 - 1}$$

Hence, option (*a*) is the correct answer.

Exercise 6.2

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. If $4 \tan \theta = 3$, then $\left[\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta}\right]$ is equal to

   (*a*) $\frac{2}{3}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{4}$

2. If $\sin \theta - \cos \theta = 0$, then the value of $(\sin^4 \theta + \cos^4 \theta)$ is

   (*a*) 1 (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{4}$

3. If $2 \sin \theta = \sqrt{3}$, then $\theta =$

   (*a*) 30° (*b*) 60° (*c*) 45° (*d*) 90°

4. If $x = 2 \sin^2 \theta$, $y = 2 \cos^2 \theta + 1$, then the value of $x + y$ is

   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 12

5. Simplest form of $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ is

   (*a*) $\sin^2 A$ (*b*) $\cos^2 A$ (*c*) $\sec^2 A$ (*d*) $\tan^2 A$

6. The value of $\left(\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}\right)$ equals

   (*a*) 1 (*b*) 8 (*c*) 11 (*d*) 24

7. The value of $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$ is

   (*a*) 0 (*b*) 1 (*c*) 8 (*d*) 17

8. If $3x = \operatorname{cosec} \theta$ and $\frac{3}{x} = \cot \theta$, the value of $3\left(x^2 - \frac{1}{x^2}\right)$ is

   (*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{5}$

9. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then $\cos \theta - \sin \theta$ equals

   (*a*) $\sin \theta$ (*b*) $2 \sin \theta$ (*c*) $\sqrt{2} \sin \theta$ (*d*) $\frac{\sin \theta}{2}$

10. The value of $(\operatorname{cosec}^2\theta - 1) \tan^2\theta$ is

    (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 7

11. The value of $\cot^2 \theta - \frac{1}{\sin^2 \theta}$ is

    (*a*) 0 (*b*) –1 (*c*) 2 (*d*) –8

12. If cosec θ + cot θ = x, the value of cosec θ – cot θ is

(a) x (b) $2x$ (c) $\frac{x}{2}$ (d) $\frac{1}{x}$

13. If sin A + $\sin^2$ A = 1, then the value of the expression ($\cos^2$ A + $\cos^4$ A) is

(a) 0 (b) 1 (c) 2 (d) 9

14. The magnitude of θ in the equation $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ is

(a) 0° (b) 30° (c) 60° (d) 90°

15. The value of $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$ is equal to

(a) 1 (b) 2 (c) 3 (d) None of these

16. The value of (1 + cot θ – cosec θ) (1 + tan θ + sec θ) is equal to

(a) 1 (b) 2 (c) 3 (d) 4

17. If 7 $\sin^2$A + 3 $\cos^2$A = 4, then tan A =

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

18. If tan (A + B) = 1 and tan (A – B) = $\frac{1}{\sqrt{3}}$. 0° < A + B < 90°. A > B, then the values of A and B respectively are

(a) A = 30°, B = 4.5° (b) A = 37.5°, B = 7.5° (c) A = 15°, B = 30° (d) None of these

19. If sec θ + tan θ = p, then the value of cosec θ is

(a) $\frac{p^2-1}{p^2+1}, -1$ (b) $\frac{p^2-1}{p^2+1}, 1$ (c) $\frac{p^2+1}{p^2+2}, -1$ (d) None of these

20. If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$, then $x^2 + y^2 + z^2 =$

(a) r (b) r^2 (c) $2r$ (d) $\frac{r}{2}$

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement reason (R). Choose the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** $\sin^2 67° + \cos^2 67° = 1$.

   Reason (R): For any value of θ, $\sin^2\theta + \cos^2\theta = 1$.

2. **Assertion (A):** The value of $\sec^2 10° - \cot^2 80°$ is 1.

   Reason (R): The value of sin 30° = $\frac{1}{2}$.

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (c) $\frac{1}{2}$
2. (c) $\frac{1}{2}$
3. (b) 60°
4. (c) 3
5. (d) $\tan^2$ A
6. (a) 1
7. (b) 1
8. (c) $\frac{1}{3}$

9. (*c*) $\sqrt{2}\sin\theta$

Here, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \quad \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \quad \sin\theta = (\sqrt{2}-1)\cos\theta$

$(\sqrt{2}+1)\sin\theta = (\sqrt{2}-1)(\sqrt{2}+1)\cos\theta$

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow \quad \sqrt{2}\sin\theta = \cos\theta - \sin\theta$

10. (*b*) 1

$$(\text{cosec}^2\theta - 1)\cdot\tan^2\theta = \cot^2\theta\cdot\frac{1}{\cot^2\theta} = 1$$

11. (*b*) – 1

$$\cot^2\theta - \frac{1}{\sin^2\theta} = \cot^2\theta - \text{cosec}^2\theta = -1$$

12. (*d*) $\frac{1}{x}$

$\text{cosec}\,\theta + \cot\theta = x$

As we know that $\quad \text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \quad (\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta) = 1$

$\Rightarrow \quad (\text{cosec}\,\theta - \cot\theta)x = 1$

$\Rightarrow \quad \text{cosec}\,\theta - \cot\theta = \frac{1}{x}$

13. (*b*) 1

$\sin A + \sin^2 A = 1$

$\Rightarrow \quad \sin A + 1 - \cos^2 A = 1$

$\Rightarrow \quad \sin A - \cos^2 A = 0$

$\Rightarrow \quad \sin A = \cos^2 A$

$\Rightarrow \quad \sin^2 A = \cos^4 A$

$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \quad \cos^2 A + \cos^4 A = 1$

14. (*c*) 60°

$$\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$$

$$\Rightarrow \frac{\cos^2\theta}{\cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)} = 3$$

$$\Rightarrow \frac{1}{\text{cosec}^2\theta - 1} = 3$$

$$\Rightarrow \frac{1}{\cot^2\theta} = 3$$

$\Rightarrow \quad \tan^2\theta = 3$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \theta = 60°$

15. (*a*) 1

$$\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\text{cosec}^2\theta}$$

[$\because \sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta$]

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1} + \frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1}$$

$= \sin^2\theta + \cos^2\theta = 1$

16. (*b*) 2

17. (*d*) $\frac{1}{\sqrt{3}}$

Given, $7\sin^2 A + 3\cos^2 A = 4$

Dividing both sides by $\cos^2 A$, we get

$7\tan^2 A + 3 = 4\sec^2 A$ [$\because \sec^2\theta = 1 + \tan^2\theta$]

$\Rightarrow \quad 7\tan^2 A + 3 = 4(1 + \tan^2 A)$

$\Rightarrow \quad 7\tan^2 A + 3 = 4 + 4\tan^2 A$

$\Rightarrow \quad 3\tan^2 A = 1$

$\Rightarrow \quad \tan^2 A = \frac{1}{3}$

$\Rightarrow \quad \tan A = \frac{1}{\sqrt{3}}$

18. (*b*) A = 37.5°, B = 7.5°

Given: tan (A + B) = 1 = tan 45°

[$\because$ tan 45° = 1]

$\Rightarrow \quad \tan(A + B) = \tan 45°$

$\Rightarrow \quad A + B = 45° \quad$ ...(*i*)

Also, $\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30°$

[$\because \tan 30° = \frac{1}{\sqrt{3}}$]

$\Rightarrow \quad \tan(A - B) = \tan 30°$

$\Rightarrow \quad A - B = 30° \quad$ ...(*ii*)

Adding (*i*) and (*ii*)

$2A = 75°$

$\Rightarrow \quad A = 37.5°$

From equation (*i*), $B = 45° - 37.5°$

$= 7.5°$

Hence, A = 37.5° and B = 7.5°

19. (*a*) $\frac{p^2+1}{p^2-1}$, –1

$$\sec\theta + \tan\theta = p$$

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = p$$

$$1 + \sin\theta = p\cos\theta = p\sqrt{1-\sin^2\theta}$$

$$(1+\sin\theta)^2 = p^2(1-\sin^2\theta)$$

$$1 + \sin^2\theta + 2\sin\theta = p^2 - p^2\sin^2\theta$$

$$(1+p^2)\sin^2\theta + 2\sin\theta + (1-p^2) = 0$$

$$D = 4 - 4(1+p^2)(1-p^2)$$

$$= 4 - 4(1-p^4) = 4p^4$$

$$\sin\theta = \frac{-2\pm\sqrt{4p^4}}{2(1+p^2)} = \frac{-1\pm p^2}{(1+p^2)} = \frac{p^2-1}{p^2+1}, -1$$

$$\therefore \operatorname{cosec}\theta = \frac{p^2+1}{p^2-1}, -1$$

20. (*b*) r^2

Hint: Find the sum of squares of

$$x = r\sin\theta\cos\phi \qquad \text{...(i)}$$

$$y = r\sin\theta\sin\phi \qquad \text{...(ii)}$$

and $$z = r\cos\theta \qquad \text{...(iii)}$$

B. Assertion-Reason Type Questions

1. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Experts' Opinion

Questions based on following types are very important for Exams. So, students are advised to revise them thoroughly.

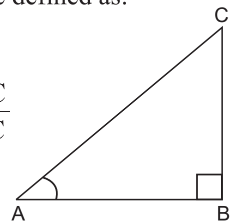
1. To use trigonometric ratios of some specific angles.
2. To use trigonometric identities.

COMMON ERRORS

Errors	Corrections
(*i*) Incorrectly interpreting that sin A means the product of 'sin' and 'A'.	(*i*) sin A is an abbreviation for 'the sine of angle A'. It is not the product of 'sin' and A. Similar interpretations follow for all the other trigonometric ratios.
(*ii*) Interpreting incorrectly that trigonometric ratios represent lengths.	(*ii*) Trigonometric ratios are numerical quantities. Each one of them represents the ratio of one length to another. They must themselves never be considered as lengths.
(*iii*) Interpreting incorrectly that trigonometric ratios depend on the lengths of the sides of the triangle.	(*iii*) The trigonometric ratios depend on the magnitude of the angle and not upon the lengths of the sides of the triangle.
(*iv*) Drawing other types of triangles instead of right-angled triangles for calculating trigonometric ratios.	(*iv*) Any right-angled triangles are to be used and hypotenuse is related to right triangles only.
(*v*) Writing trigonometric ratios without angle.	(*v*) It is not correct. Must write ratios with correct angles and do sufficient practice.
(*vi*) Considering $\sin^2\theta + \cos^2\theta = 1$ and taking its square root as $\sin\theta + \cos\theta = \pm 1$.	(*vi*) Remember that square root is taken only for whole term *i.e.*, not with + and – sign.

QUICK REVISION NOTES

- Trigonometry deals with the relationships between sides and angles of a triangle.
- Trigonometric ratios of the angle A in a right triangle ABC, right-angled at B are defined as:



(*i*) $\sin A = \frac{BC}{AC}$ (*ii*) $\cos A = \frac{AB}{AC}$

(*iii*) $\tan A = \frac{BC}{AB}$ (*iv*) $\text{cosec } A = \frac{1}{\sin A} = \frac{AC}{BC}$

(*v*) $\sec A = \frac{1}{\cos A} = \frac{AC}{AB}$ (*vi*) $\cot A = \frac{1}{\tan A} = \frac{AB}{BC}$

- **Trigonometric Ratios of some specific angles:**

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND*
$\text{cosec }\theta$	ND*	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND*
$\cot\theta$	ND*	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

* ND stands for 'Not defined'.

- **Trigonometric Relations/Identities:**

(*i*) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (*ii*) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(*iii*) $\sin^2\theta + \cos^2\theta = 1, 0° \le \theta \le 90°$ (*iv*) $\sec^2\theta = \tan^2\theta + 1, 0° \le \theta < 90°$

(*v*) $\text{cosec}^2\theta = 1 + \cot^2\theta, 0° < \theta \le 90°$

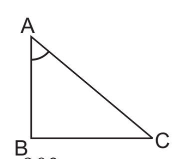
IMPORTANT FORMULAE

- In a right triangle ABC, right-angled at B,

(*i*) $\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}}$ (*ii*) $\cos A = \frac{\text{side adjacent to angle A}}{\text{hypotenuse}}$

(*iii*) $\tan A = \frac{\text{side opposite to angle A}}{\text{side adjacent to angle A}} = \frac{\sin A}{\cos A}$ (*iv*) $\text{cosec } A = \frac{1}{\sin A}$

(*v*) $\sec A = \frac{1}{\cos A}$ (*vi*) $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$



- (*i*) $\sin^2 A + \cos^2 A = 1$, for $0° \le A \le 90°$. (*ii*) $\sec^2 A - \tan^2 A = 1$, for $0° \le A < 90°$.

(*iii*) $\text{cosec}^2 A - \cot^2 A = 1$, for $0° < A \le 90°$.