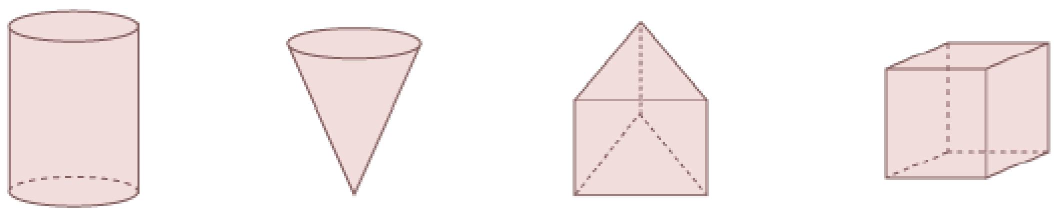
Recognition of Solids

Identification of Three-dimensional Shapes

Similar to two-dimensional shapes, we have various types of three-dimensional objects, which are classified on the basis of the nature of arrangement and orientation of various faces of the shape.

Example 1:

Identify the following shapes.



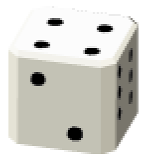
(i) **(ii)** **(iii)** **(iv)**

Solution:

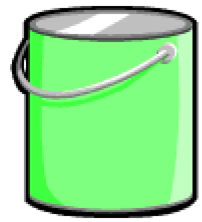
1. Cylinder
2. Cone
3. Prism
4. Cube

Example 2:

Identify the following three-dimensional shapes.



(a)



(b)

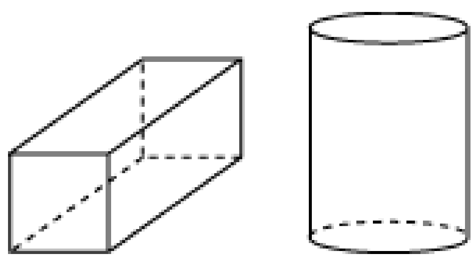
Solution:

1. This figure is cubical in shape. A dice has six sides and all of them are equal. Such types of shapes are known as cubes.

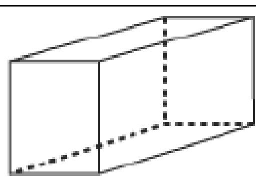
(b) This figure is cylindrical in shape.

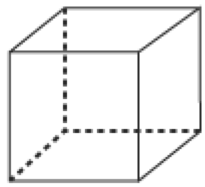
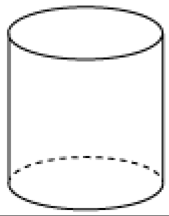
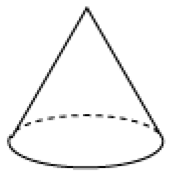
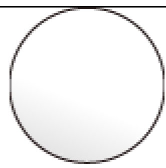
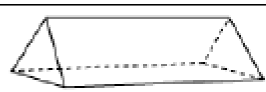
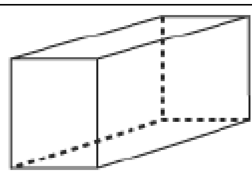
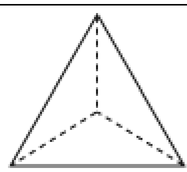
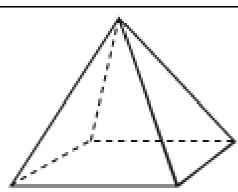
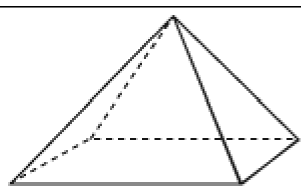
Attributes of Three-dimensional Shapes

Consider the following figures.



The following table helps us to understand the attributes of three-dimensional figures.

Name	Shape	No. of straight edges	No. of faces	No. of Vertices	Example
Cuboid		12	6	8	Pencil box, notebook

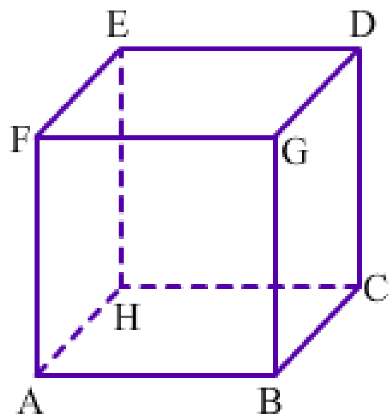
Cube		12	6	8	Dice
Cylinder		None	Two flat faces and one curved surface	None	Can, cooking cylinder
Cone		None	One flat face and one curved surface	1	Softy cone, birthday cap
Sphere		None	None	None	Ball
Triangular prism		9	5	6	Laboratory prisms
Rectangular prism		12	6	8	A rectangular glass slab
Triangular pyramid		6	4	4	
Square pyramid		8	5	5	The great pyramids of Egypt
Rectangular pyramid		8	5	5	

We know about the top and base of the solid, let us learn about its lateral face(s).

The faces that join the bases of a solid are called **lateral faces**.

We know that a cube has six square faces. Any face of the cube can be taken as its base.

Consider the cube shown below.



Here, ABCH is the base of the cube and EFGD is the top of the cube.

Rest four faces of the cube, namely ABGF, BGCD, CDEH and AHEF are the lateral faces of the cube as these faces meet

the base as well as the top of the cube.

Let us now look at some examples.

Example 1:

Find the number of faces and vertices of the following three-dimensional shapes.

(i) Cuboid (ii) Cube (iii) Cylinder (iv) Cone

(v) Sphere

Solution:

1. A cuboid has six faces and eight vertices.
2. A cube has six faces and eight vertices.
3. A cylinder has two flat faces and one curved surface. It has no vertices.
4. A cone has one flat face and one curved surface. It has one vertex.
5. A sphere has no flat face. Also, it has no vertex.

Example 2:

Find the number of faces and edges of the following three-dimensional shapes.

1. **Triangular prism**
2. **Rectangular prism**
3. **Triangular pyramid**
4. **Rectangular pyramid**

Solution:

1. A triangular prism has five faces and nine edges.
2. A rectangular prism has six faces and twelve edges.
3. A triangular pyramid has four faces and six edges.
4. A rectangular pyramid has five faces and eight edges.

Euler's Formula

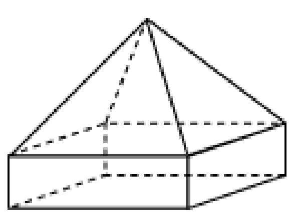
Every polyhedron has a specific number of faces, edges, and vertices (depending upon the type of polyhedron it is). However, is there any relation that can be applied to the number of faces, edges, and vertices of any polyhedron irrespective of the type of polyhedron?

Let us find that out with the help of the following video.

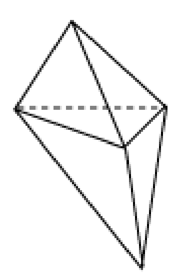
Let us discuss some examples based on Euler’s formula.

Example 1:

Verify Euler’s formula for the following solids.



(a)



(b)

Solution:

(a) The number of faces (F), edges (E), and vertices (V) for the given solid are 9, 16, and 9 respectively.

Now, $F + V - E = 9 + 9 - 16 = 18 - 16 = 2$

Thus, Euler's formula is verified for the given solid.

(b) The number of faces (F), edges (E), and vertices (V) for the given solid are 6, 9, and 5 respectively.

Now, $F + V - E = 6 + 5 - 9$

$= 11 - 9$

$= 2$

Thus, Euler's formula is verified for the given solid.

Example 2:

Is a polyhedron with 12 faces, 21 edges, and 13 vertices possible?

Solution:

The number of faces (F), edges (E), and vertices (V) is given as 12, 21, and 13 respectively.

Now, $F + V - E = 12 + 13 - 21 = 25 - 21 = 4$

However, according to Euler's formula, the relation between the number of faces (F), the number of edges (E), and the number of vertices (V) for any polyhedron is $F + V - E = 2$

Thus, a polyhedron with 12 faces, 21 edges, and 13 vertices is not possible.

Example 3:

Find the unknown values for polyhedrons in the following table.

Face	**Edge**	**Vertex**
8	**12**	**?**

12	**?**	**20**
?	**18**	**12**

Solution:

Let the number of vertices for the first polyhedron be V.

It is given that the number of faces (F) and edges (E) for this polyhedron are 8 and 12 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$8 + V - 12 = 2$

$V = 2 + 12 - 8$

$V = 6$

Thus, the number of vertices for the first polyhedron is 6.

Let the number of edges for the second polyhedron be E.

It is given that the number of faces (F) and vertices (V) for this polyhedron are 12 and 20 respectively.

Using Euler's formula, we obtain

$F + V - E = 2$

$12 + 20 - E = 2$

$E = 12 + 20 - 2$

$E = 30$

Thus, the number of edges for the second polyhedron is 30.

Let the number of faces for the third polyhedron be F.

It is given that the number of edges (E) and vertices (V) for this polyhedron are 18 and 12 respectively.

Using Euler’s formula, we obtain

$F + V - E = 2$

$F + 12 - 18 = 2$

$F = 2 + 18 - 12$

$F = 8$

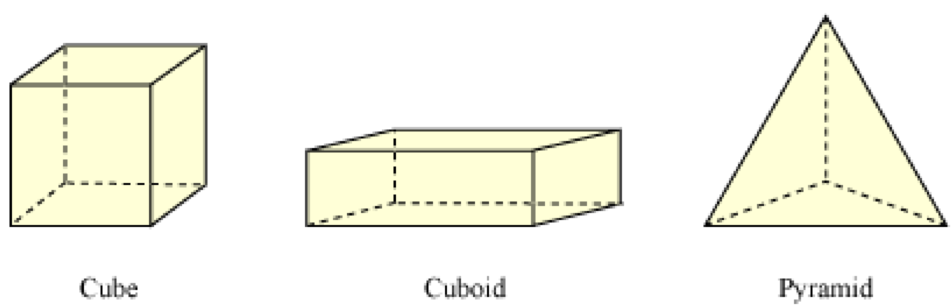
Thus, the number of faces for the third polyhedron is 8.

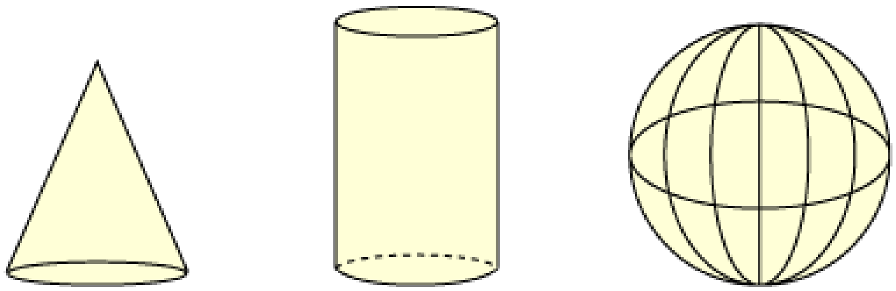
Nets of Three-Dimensional Figures

Look at the following figures.

What do you think about the shapes of the objects shown in the figure above?

These figures are three-dimensional in shape. Let us see some more three-dimensional figures.





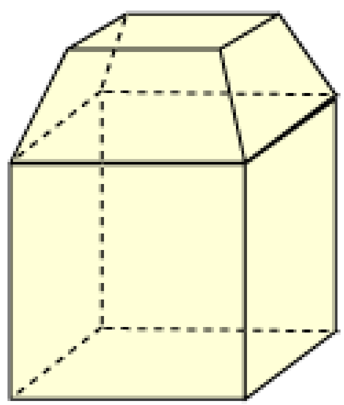
Cone Cylinder Sphere

As the paper is two-dimensional, we cannot draw these figures on paper very easily. If we try to draw them on paper, then the back edges will not be shown. But we can draw the net of the three-dimensional solids.

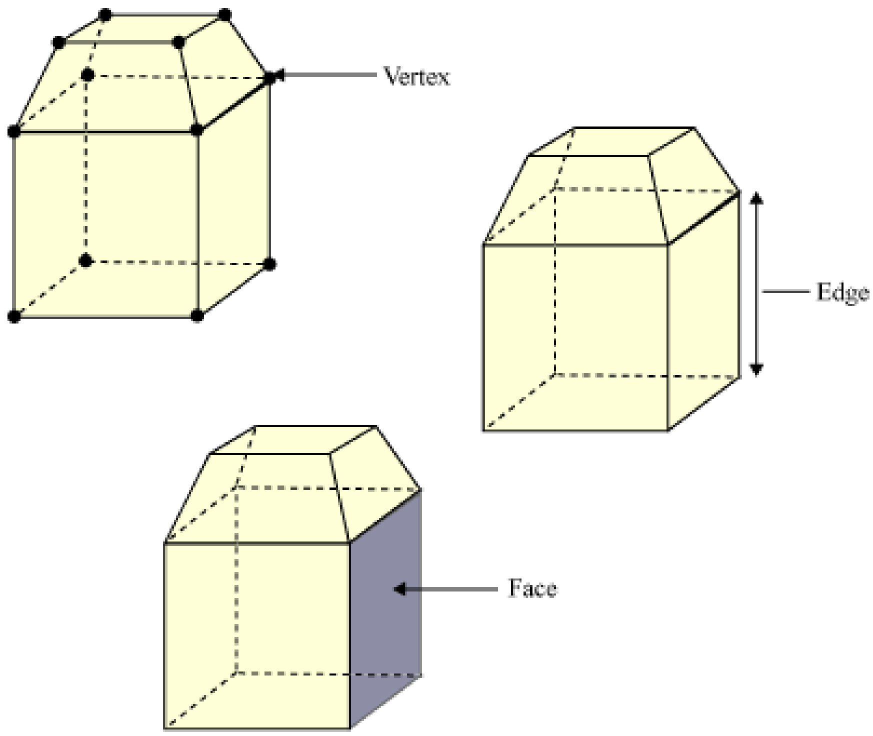
Let us learn more about three-dimensional shapes through various illustrative examples.

Example 1:

Find the number of vertices, edges, and faces in the following figure.



Solution:

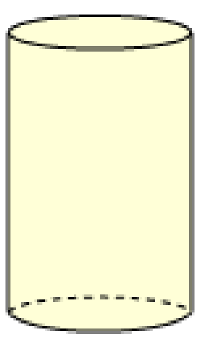


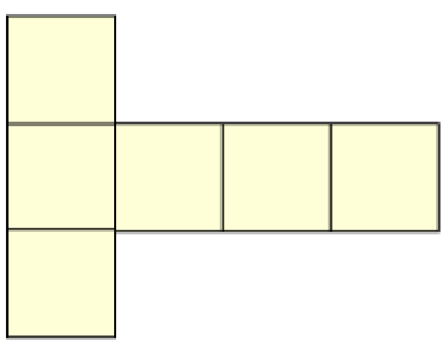
The given figure has 12 vertices, 20 edges, and 10 faces.

Example 2:

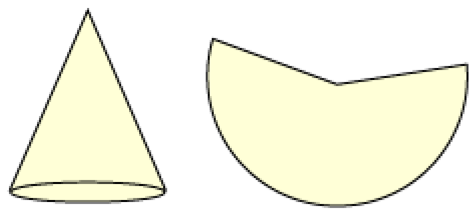
Match the following shapes with their appropriate nets.

(i) (a)

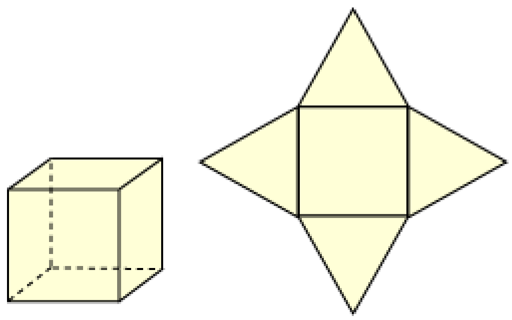




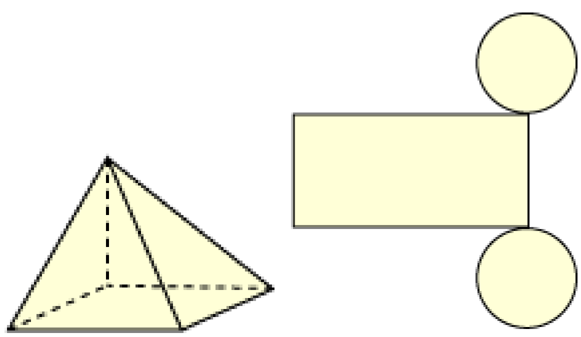
(ii) (b)



(iii) (c)

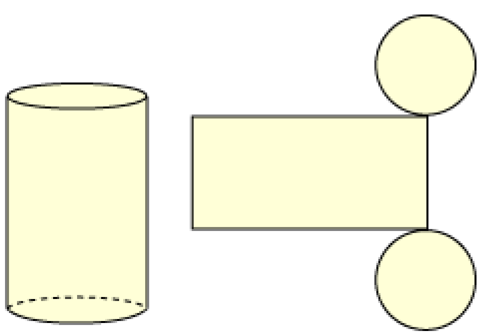


4. **(d)**

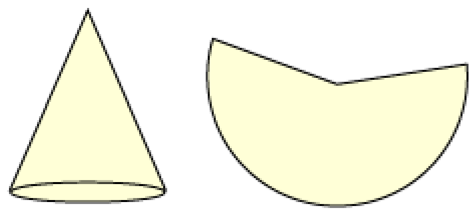


Solution:

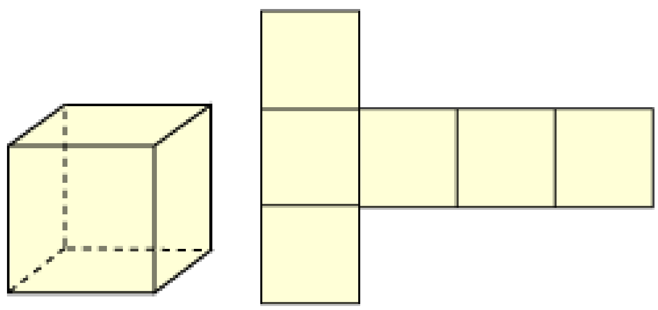
(i) (d)



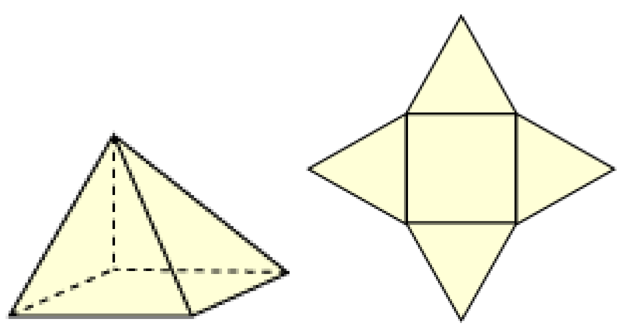
(ii) (b)



(iii) (a)



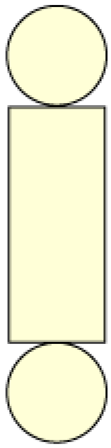
(iv) (c)



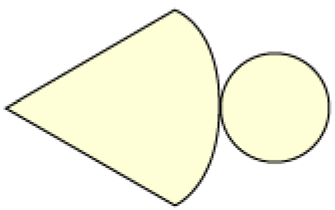
Example 3:

Draw the 3-D shapes that can be obtained from the following 2-D nets.

(i)

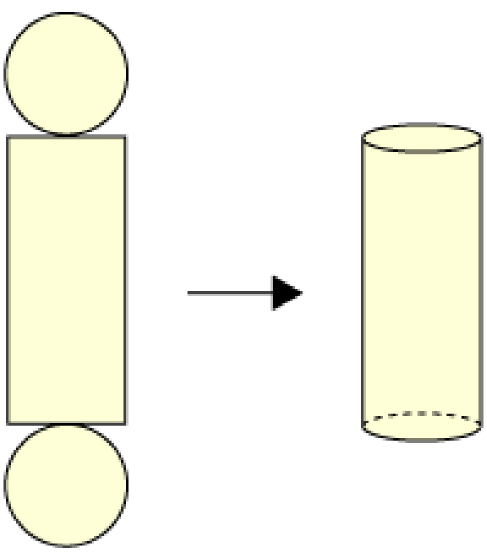


(ii)

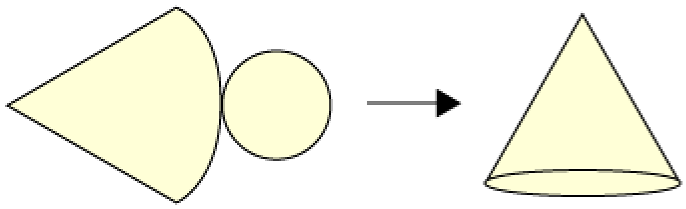


Solution:

(i) From the given net, a cylinder will be obtained.



(ii) From the given net, a cone will be obtained.



Identification And Marking Of Symbols, Scale, And Reference Point On A Map

In earlier classes, we have seen maps of India and its states. We have located mountains, rivers, etc. and marked different places on maps. Maps are not only related with history and geography. We can also draw a map of any city, town, school, etc.

To understand the concept of maps and scale of a map, look at the following video.

Sometimes, the scale of a map is written in statement form as 1 cm = 10 km. It means that if the distance between two points on the map is 1 cm, then the distance between those two points on the ground is 10 km.

If the distance between two points on the map is 5 cm, then the distance between those two points on the ground is $5 \times 10 = 50$ km.

This scale can vary from map to map but not within a map. Let us discuss some examples based on scale and symbols of a map.

Example 1:

Scale of a map is 1 cm = 15 km. If the distance between two cities on the ground is 75 km, then what is the distance between these two cities on the map?

Solution:

The scale of the map is 1 cm = 15 km

i.e., 15 km distance between the two points on the ground = 1 cm distance between those points on the map

1 km distance between the two points on the ground = $\frac{1}{15}$ cm distance between those points on the map

Therefore, 75 km distance between the two points on the ground

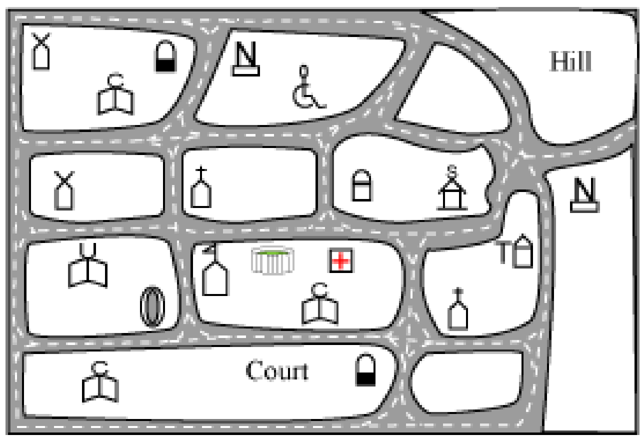
$= \left(75 \times \frac{1}{15}\right)$ cm distance between those points on the map

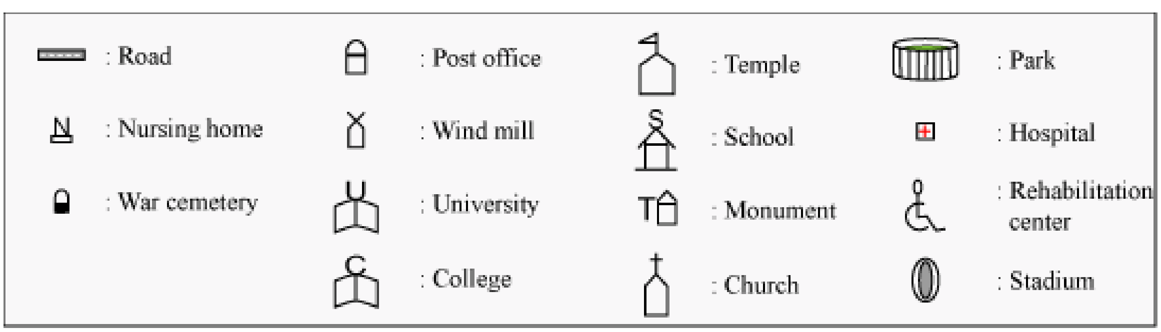
$= 5$ cm distance between those points on the map

Thus, the distance between the two cities on the map is 5 cm.

Example 2:

The given figure shows the map of a city.





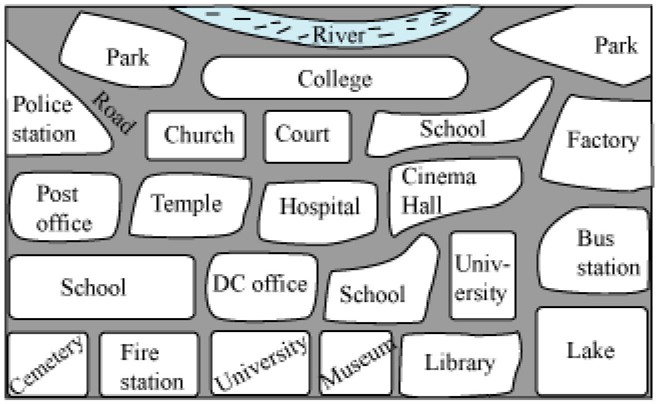
How many colleges are there in the city?

Solution:

It can be seen that in the given map, there are three colleges in the city, which are represented by the sign .

Example 3:

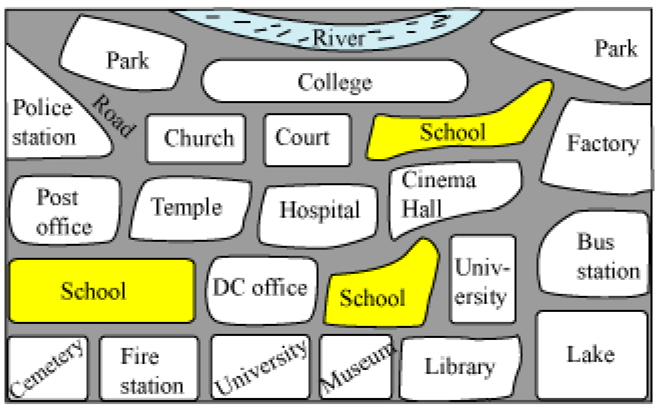
The given figure shows the map of a city.



Colour all the schools in the map with yellow colour.

Solution:

There are three schools in the map. After colouring, we obtain the following map.



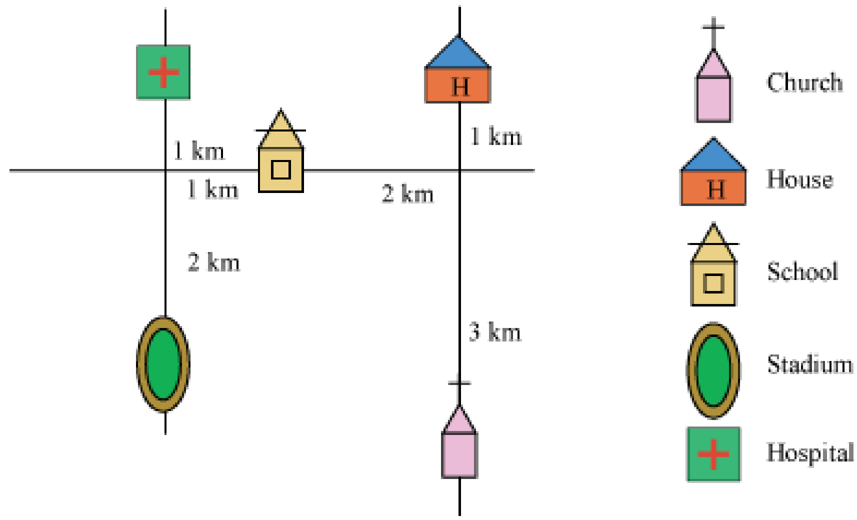
Interpretation Of A Map

A map is an important tool and it needs to be interpreted to derive any useful information out of it. The given video will help you understand the interpretation of a map.

Let us look at some more examples to understand the concept better.

Example 1:

A map is given showing some landmarks of a city. The landmarks are replaced by symbols, whose meanings are given on right side of the map.



Answer the following questions.

(1) Ankit lives in the house H. How far is Ankit's house from the school?

(2) Which is nearby from Ankit's house, stadium or hospital?

Solution:

(1) School is at a distance of 2 + 1 = 3 km from Ankit's house.

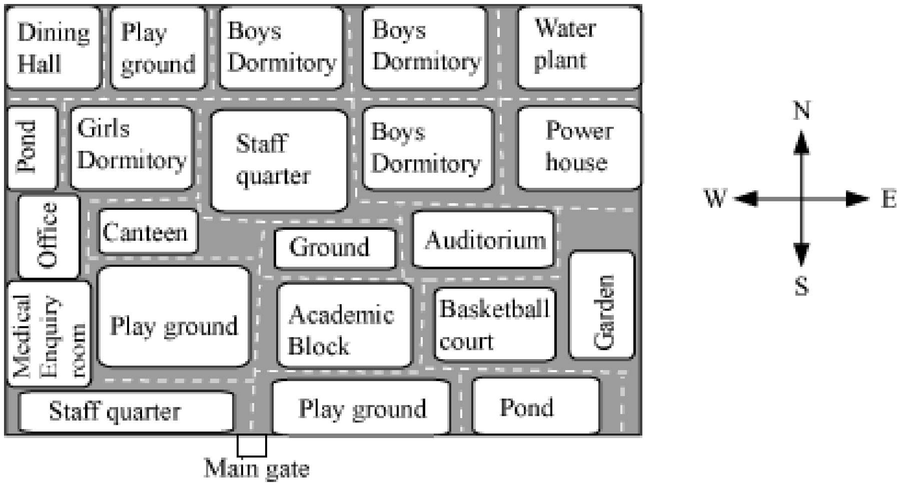
(2) Distance of stadium from Ankit's house = 1 + 2 + 1 + 2 = 6 km

Distance of hospital from Ankit's house = 1 + 2 + 1 + 1 = 5 km

Therefore, hospital is nearer than stadium from Ankit's house.

Example 2:

The given figure shows the map of a compound of a boarding school.



Answer the following questions.

(i) Which of the following landmarks is nearest to the girls dormitory?

A. Boys dormitory

B. Academic block

C. Auditorium

D. Power house

(ii) Which landmark is situated at the North-West corner of the school compound?

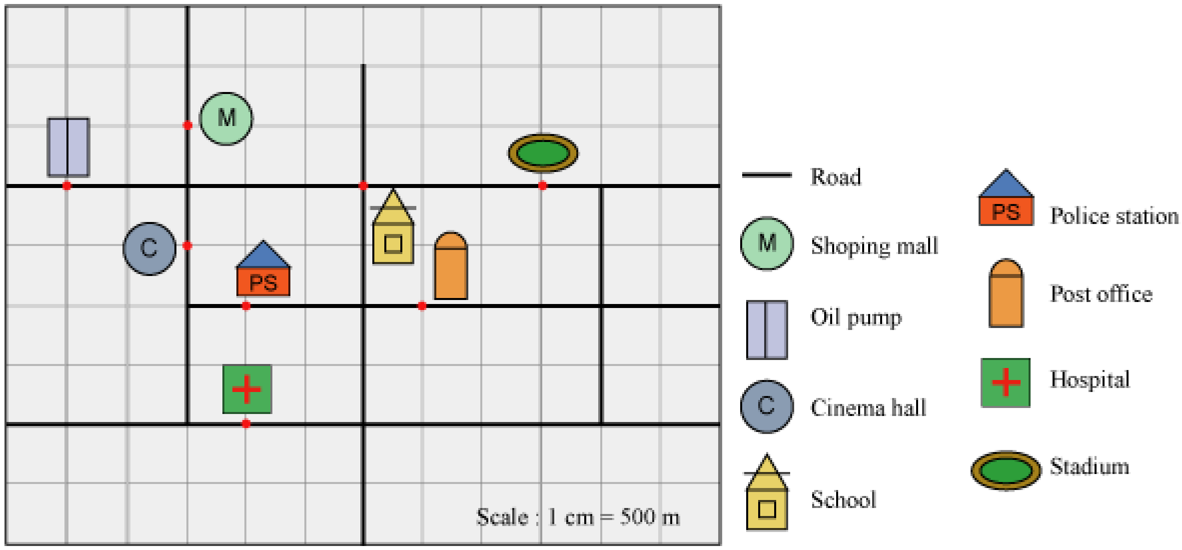
Solution:

(i) It can be clearly seen in the map that boys dormitory is nearer to girls dormitory as compared to academic block, auditorium, and power house.

(ii) The North West corner of the school compound in the given map is the dining hall.

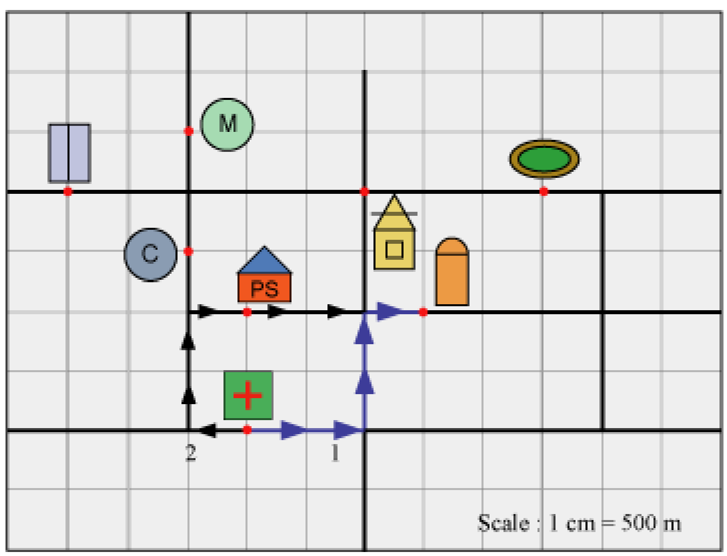
Example 3:

The given figure shows the map of a locality which is drawn on a centimetre grid paper.



What is the shortest distance between the hospital and the post office?

Solution:



It can be seen in the map that there are two ways of reaching the post office from the hospital and the shortest is the one which is shown by blue arrows.

The scale used in the map is 1 cm = 500 m

Therefore, shortest distance between hospital and post office

= 5 × 500 m = 2500 m

We know that, 1000 m = 1 km

$\therefore$ 2500 m = $\frac{2500}{1000}$ km = $2\frac{1}{2}$ km

Thus, the shortest distance between the hospital and the post office is $2\frac{1}{2}$ km.