# TOPIC 2 Kinematics

## Objectives

**Candidates should be able to:**

(a) state what is meant by speed and velocity
(b) calculate average speed using *distance travelled / time taken*
(c) state what is meant by uniform acceleration and calculate the value of an acceleration using *change in velocity / time taken*
(d) interpret given examples of non-uniform acceleration
(e) plot and interpret a displacement-time graph and a velocity-time graph
(f) deduce from the shape of a displacement-time graph when a body is:
  (i) at rest
  (ii) moving with uniform velocity
  (iii) moving with non-uniform velocity
(g) deduce from the shape of a velocity-time graph when a body is:
  (i) at rest
  (ii) moving with uniform velocity
  (iii) moving with uniform acceleration
  (iv) moving with non-uniform acceleration
(h) calculate the area under a velocity-time graph to determine the displacement travelled for motion with uniform velocity or uniform acceleration
(i) state that the acceleration of free fall for a body near to the Earth is constant and is approximately 10 m/s$^2$
(j) describe the motion of bodies with constant weight falling with or without air resistance, including reference to terminal velocity

## NOTES

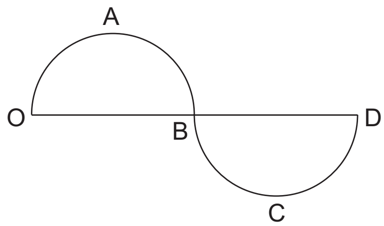
### 2.1 Distance vs Displacement and Speed vs Velocity

1.

| **Scalar** | **Vector** |
|---|---|
| Distance | Displacement |
| Speed | Velocity |

***Example 2.1***

A car travelled from point O to D along the curved path OABCD.




The distance travelled by the car is OABCD.

The displacement of the car from point O is OD (to the right of O).

2. When measuring/ calculating the displacement of an object, one has to include its starting point.

***Example 2.2***

**Wrong:** "The displacement of the bus is 500 m." (*500 m from where?*)

**Right:** "The displacement of the bus from point A is 500 m in the backward direction." or "The displacement of the bus from point A is –500 m (taking the forward direction as positive)."

3. (a) The formula for calculating speed is

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

(b) $\text{Average speed} = \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

(c) Velocity is the rate of change of displacement of an object from a fixed point (displacement per unit time).

(d) $\text{Average velocity} = \dfrac{\text{Resultant displacement from a fixed point}}{\text{Total time taken}}$

The average velocity $v_{avg}$ of an object moving through a displacement ($\Delta x$) along a straight line in a given time ($\Delta t$) is:

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

where $\Delta x = x_{final} - x_{initial}$

$x_{initial}$ : Initial displacement from starting point

$x_{final}$ : Final displacement from starting point

## 2.2 Acceleration

1. Acceleration is the rate of change of velocity.

2. $a = \frac{\Delta x}{\Delta t} = \frac{v - u}{\Delta t} =$ where $v$ is the final velocity, $u$ is the initial velocity and $\Delta t$ is the time taken.

3. Acceleration is a vector quantity. (You need to give both the magnitude and direction when writing down the answer.)

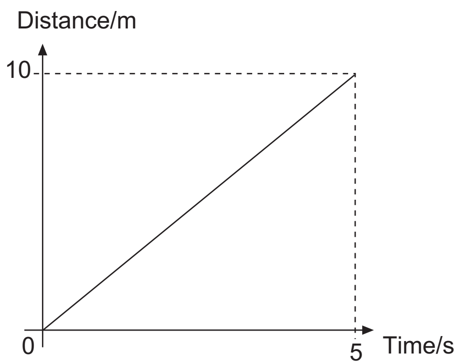
## 2.3 Graph of Distance vs Time

1. The distance-time graph of a moving object along a straight road is used to find its speed.

2. The gradient of the graph gives the speed of the object.

***Example 2.3***

Object moving at uniform speed



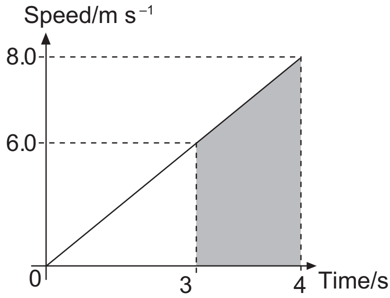

$$\text{Speed} = \frac{10 - 0}{5 - 0} = \frac{10}{5} = 2 \text{ m/s or } 2 \text{ m s}^{-1}$$

## 2.4 Graph of Speed vs Time

1. The speed-time graph of a moving object along a straight road is used to find:
   (a) Acceleration (Using the gradient of graph)
   (b) Distance travelled (Using the area under the graph)

***Example 2.4***

Object moving with uniform acceleration:




$$\text{Acceleration} = \frac{8.0 - 0.0}{4 - 0} = 2 \text{ m/s}^2 \text{ or } 2 \text{ m s}^{-2}$$

Distance travelled from $t = 0$ to $t = 4$ s

$$= \frac{1}{2}(4 - 0)(8.0 - 0.0) = 16 \text{ m}$$

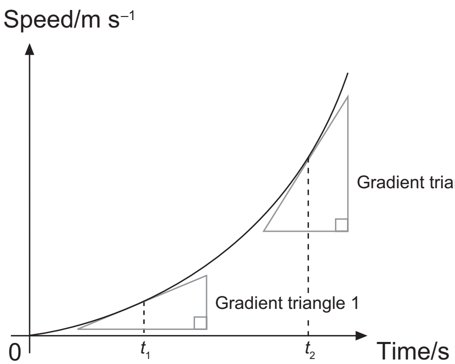
Distance travelled from $t = 3$ to $t = 4$ s

$$= \frac{1}{2}(4 - 3)(8.0 + 6.0) = 7 \text{ m}$$

2. For an object moving with constant acceleration, the speed-time graph is a sloping straight line. A constant acceleration means that speed is increasing at a constant rate.

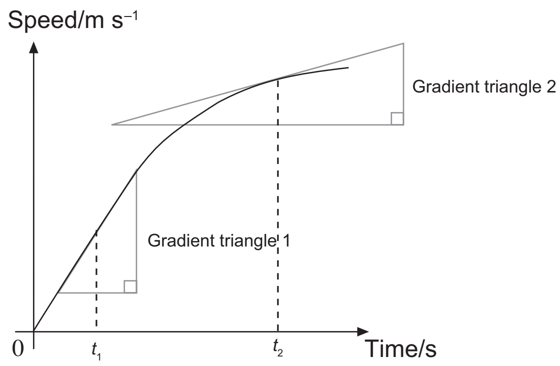
## 2.5 Interpret Other Speed-Time Graphs (Non-Uniform Acceleration)

1. Increasing acceleration:




Notice that the gradient of the graph becomes steeper.
The gradient of triangle 2 is larger than the gradient of triangle 1.
(Gradient gets more and more positive).
The **speed is increasing** with **increasing acceleration** (increasing rate).
At time = $t_1$, acceleration = $a_1$.
At time = $t_2$, acceleration = $a_2$.
$a_2 > a_1$

2. Decreasing acceleration:




Notice that the gradient of the graph becomes less steep.
The gradient of triangle 2 is smaller than the gradient of triangle 1.
(Gradient gets less and less positive).
The **speed is increasing** with **decreasing acceleration** (decreasing rate).
At time = $t_1$, acceleration = $a_1$.
At time = $t_2$, acceleration = $a_2$.
$a_2 < a_1$

## 2.6 Acceleration Due to Free-Fall

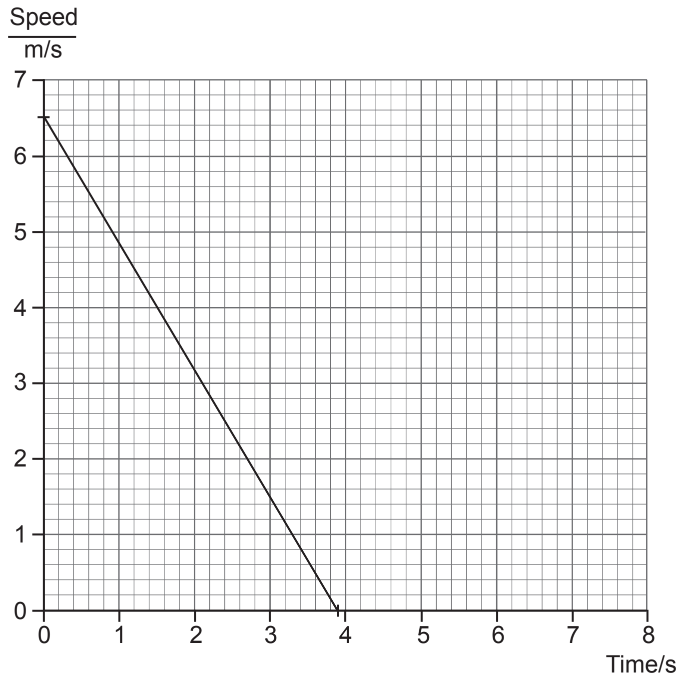
Near the surface of the earth, the acceleration of free fall for an object is constant and is approximately 10 m/s$^2$. When an object drops from the top of a building, its speed will increase from 0 m/s uniformly at a rate of 10 m/s per second.

## 2.7 Effect of Air Resistance

1. In real life, a falling object will encounter air resistance on Earth, unless it is moving in a vacuum.
2. Air resistance acts against the motion of the object increasingly to reduce its downward acceleration (NOT SPEED) to zero.
3. When the air resistance increases till it is equal to the weight of a falling object, the acceleration of the object is zero.
4. With zero acceleration, the object will continue falling downward at a constant velocity.
5. The constant velocity of the object is known as “terminal velocity”.

### *Example 2.5*

An astronaut standing on the Moon's surface throws a rock vertically upwards. The figure shows the speed-time graph of the rock where at $t$ = 0 s, the rock just leaves the astronaut's hand. Air resistance on the Moon can be neglected.




(a) (i) What is the time taken for the rock to reach its maximum height?

(ii) What is the total distance travelled by the rock when it returns to its initial position?

(iii) Find the acceleration of the rock.

(b) The rock is then brought back to the Earth's surface and the astronaut repeats the same action as on the Moon. Determine whether the speed-time graph of the rock, when it is thrown on Earth, will be different. Explain your answer.

***Solution***

(a) (i) From the graph, the time taken for the rock to reach its maximum height is 3.90 seconds.

(ii) Total distance travelled = 2 × area under the graph

$$= \left(\frac{1}{2} \times (6.50 - 0.00) \times (3.90 - 0.00)\right) \times 2$$

= 25.4 m (to 3 s.f.)

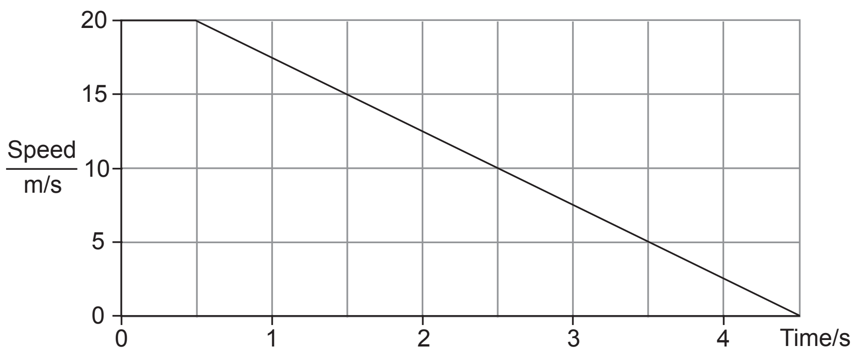
(iii) Acceleration of rock $= \dfrac{6.50 - 0.00}{0.00 - 3.90}$

$= -1.67\ \text{m s}^2$ or $-1.67\ \text{m s}^{-2}$ (to 3 s.f.)

(b) The speed-time graph of the rock on Earth is different because the speed of the rock decreases as it falls from a height. This is due to air resistance. The speed of the rock is decreasing at an increasing rate. The deceleration of the rock increases as the speed decreases. Hence, the gradient of the speed-time graph is steeper initially and becomes gentler after some time. The sketch of the speed-time graph is a curve and not a straight line.

***Example 2.6***

The graph shows the speed of a car from the time the driver saw an obstacle on the road and applied the brakes till the car came to a stop.




(a) How long did it take the driver to begin applying the brakes after seeing the obstacle?

(b) Calculate the distance travelled

(i) before the brakes were applied,

(ii) while the brakes were being applied.

(c) Calculate the average speed of the car.

***Solution***

(a) The speed remains at 20 m/s for the first 0.5 seconds, so the driver took 0.5 seconds to begin applying the brakes after seeing the obstacle.

(b) (i) Distance travelled before braking

$= 20 \times 0.5$

$= 10$ m

(ii) Distance travelled while the brakes were being applied

$= \frac{1}{2} \times 20 \times (4.5 - 0.5)$

$= 40$ m

(c) Average speed of car $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{10 + 40}{4.5}$

$= \dfrac{50}{4.5}$

$= 11.1$ m/s or $11.1$ m $s^{-1}$ (to 3 s.f.)