Algebraic Expressions

Exercise – 3.1

Solution 1:

i. $2a + b + 7;\ 4a + 2b + 3$

$$\begin{array}{r} 2a + b + 7 \\ +\ 4a + 2b + 3 \\ \hline 6a + 3b + 10 \\ \hline \end{array}$$

ii. $3x + y - 8;\ y + 4 - 7x$

$$\begin{array}{r} 3x + y - 8 \\ -7x + y + 4 \\ \hline -4x + 2y - 4 \\ \hline \end{array}$$

iii. $3x^2 + 5x - 4;\ 8x - 2x^2 + 11$

$$\begin{array}{r} 3x^2 + 5x - 4 \\ -2x^2 + 8x + 11 \\ \hline x^2 + 13x + 7 \\ \hline \end{array}$$

iv. $5\sqrt{x} - 4\sqrt{y} + 2;\ 2\sqrt{x} + 7\sqrt{y} - 5$

$$\begin{array}{r} 5\sqrt{x} - 4\sqrt{y} + 2 \\ +\ 2\sqrt{x} + 7\sqrt{y} - 5 \\ \hline 7\sqrt{x} + 3\sqrt{y} - 3 \\ \hline \end{array}$$

Solution 2:

i. $5x^2 - 6xy + 2;\ 3x^2 + 10xy - 8$

$$\begin{array}{r} 5x^2 - 6xy + 2 \\ -3x^2 + 10xy - 8 \\ (-)\ (-)\quad (+) \\ \hline 2x^2 - 16xy + 10 \\ \hline \end{array}$$

ii. $m^2n - 8 + mn^2;\ 7 - m^2n - mn^2$

$= (m^2n - 8 + mn^2) - (7 - m^2n - mn^2)$

$= m^2n - 8 + mn^2 - 7 + m^2n + mn^2$

$= m^2n + m^2n + mn^2 + mn^2 - 8 - 7$

$= 2m^2n + 2mn^2 - 15$

iii. $5x^2 + 4y^2 - 6y + 8;\ x^2 - 5y^2 + 2xy + 3y - 10$

$= (5x^2 + 4y^2 - 6y + 8) - (x^2 - 5y^2 + 2xy + 3y - 10)$

$= 5x^2 + 4y^2 - 6y + 8 - x^2 + 5y^2 - 2xy - 3y + 10$

$= 5x^2 - x^2 + 4y^2 + 5y^2 - 6y - 3y - 2xy + 10 + 8$

$= 4x^2 + 9y^2 - 9y - 2xy + 18$

Solution 3:

Let 'a' be the expression to be added to $5x^2 + 2xy + y^2$ to get $3x^2 + 4xy$.

$\therefore 5x^2 + 2xy + y^2 + a = 3x^2 + 4xy$

$\therefore 5x^2 - 3x^2 - 4xy + 2xy + y^2 + a = 0$

$\therefore 2x^2 - 2xy + y^2 + a = 0$

$\therefore a = -2x^2 + 2xy - y^2$

Solution 4:

Let ‘x’ be the expression to be subtracted from 2a + 6b – 5 to get -3a + 2b + 3.

∴ 2a + 6b – 5 – x = -3a + 2b + 3.

∴ 2a + 6b – 5 – x + 3a – 2b – 3= 0

∴ 2a + 3a + 6b – 2b – 5 – 3 – x = 0

∴ 5a + 4b – 8 – x = 0

∴x = 5a + 4b – 8

Solution 5:

First let us add 3x – 2y + 7 and 5x – 3y – 8
(3x – 2y + 7) + (5x – 3y – 8)
= 3x – 2y + 7 + 5x – 3y – 8
= 3x + 5x – 2y – 3y + 7 – 8
= 8x – 5y – 1
Let us subtract 4x + y + 2 from 8x – 5y – 1
(8x – 5y – 1) – (4x + y + 2)
= 8x – 5y – 1 – 4x – y – 2
= 8x – 4x – 5y – y – 1 – 2
= 4x – 6y – 3.

Solution 6:

i. $5x(2x + 3y)$

$= (5x \times 2x) + (5x \times 3y)$

$= 10x^2 + 15xy$

ii. $(2x - y)(3x + 5y)$

$= (2x \times 3x) + (2x \times 5y) - (y \times 3x) - (y \times 5y)$

$= 6x^2 + 10xy - 3xy - 5y^2$

$= 6x^2 + 7xy - 5y^2$

iii. $(3xy^2 + 4x^2)(xy - 3x^2)$

$= (3xy^2 \times xy) - (3xy^2 \times 3x^2) + (4x^2 \times xy) - (4x^2 \times 3x^2)$

$= 3x^2y^3 - 9x^3y^2 + 4x^3y - 12x^4$

Solution 7:

$a^2 - b^2;\ (a-b)$

$$\begin{array}{r|l} & a+b \\ \hline (a-b) & a^2 - b^2 \\ & a^2 - ab \\ & -\quad + \\ \hline & 0 + ab - b^2 \\ & \quad ab - b^2 \\ & \quad -\quad + \\ \hline & \quad 0 \end{array}$$

Quotient = $(a+b)$;
Remainder = 0

$x^2 - \frac{1}{4x^2};\ x - \frac{1}{2x}$

$$\begin{array}{r|l} & x + \frac{1}{2x} \\ \hline x - \frac{1}{2x} & x^2 - \frac{1}{4x^2} \\ & x^2 - \frac{1}{2} \\ & -\quad + \\ \hline & 0 + \frac{1}{2} - \frac{1}{4x^2} \\ & \quad \frac{1}{2} - \frac{1}{4x^2} \\ & \quad -\quad + \\ \hline & \quad 0 \end{array}$$

Quotient = $\left(x + \frac{1}{2x}\right)$;
Remainder = 0

Exercise – 3.2

Solution 1:

$4x - 8y$
$= 4(x - 2y)$

Solution 2:

$5t + 25t^2$
$= 5t(1 + 5t)$

Solution 3:

$x^4y^5 - 3x^5y^4$
$= x^4y^4(y - 3x)$

Solution 4:

$x^2 + xy - 3x - 3y$
$= x\underline{(x + y)} - 3\underline{(x + y)}$
$= (x + y)(x - 3)$

Solution 5:

$6ax - 6by - 4ay + 9bx$
$= \underline{6ax - 4ay} + \underline{9bx - 6by}$
$= 2a\,\underline{(3x - 2y)} + 3b\,\underline{(3x - 2y)}$
$= (3x - 2y)(2a + 3b)$

Solution 6:

$7x^2 - 21x + 2xy - 6y$
$= 7x\,\underline{(x - 3)} + 2y\underline{(x - 3)}$
$= (x - 3)(7x + 2y)$

Solution 7:

$2x^2 - 3xy - 8xy^2 + 12y^3$
$= x\underline{(2x - 3y)} - 4y^2\underline{(2x - 3y)}$
$= (2x - 3y)(x - 4y^2)$

Solution 8:

$81x^2 - 64y^2$
$= (9x)^2 - (8y)^2$
$= (9x + 8y)(9x - 8y)$

Solution 9:

$27a^2 - 75b^2$
$= 3(9a^2 - 25b^2)$
$= 3[(3a)^2 - (5b)^2]$
$= 3(3a + 5b)(3a - 5b)$

Solution 10:

$3a^3 - 3a$
$= 3a(a^2 - 1)$
$= 3a(a + 1)(a - 1)$

Solution 11:

$x^2 - y^2 - 6x - 6y$
$= (x)^2 - (y)^2 - 6(x + y)$
$= \underline{(x + y)}(x - y) - 6\underline{(x + y)}$
$= (x + y)\{(x - y) - 6\}$
$= (x + y)(x - y - 6)$

Solution 12:

$(a + b)(c + d) - a^2 + b^2$
$= (a + b)(c + d) - (a^2 - b^2)$
$= \underline{(a + b)}(c + d) - \underline{(a + b)}(a - b)$
$= (a + b)\{(c + d) - (a - b)\}$
$= (a + b)(c + d - a + b)$
$= (a + b)(-a + b + c + d)$

Solution 13:

$x^2 + 8x + 24y - 9y^2$
$= x^2 - 9y^2 + 8x + 24y$
$= \{(x^2) - (3y)^2\} + 8(x + 3y)$

$= (x + 3y)(x – 3y) + 8(x + 3y)$
$= (x + 3y)(x – 3y + 8)$

Solution 14:

$a^2 – 12ab + 36b^2 – 25$
$= (a – 6b)^2 – (5)^2$
$= (a – 6b + 5)(a – 6b – 5)$

Solution 15:

$= x^2 + 6xy + 9y^2 – 25m^2 + 40mn – 16n^2$
$= (x^2 + 6xy + 9y^2) – (25m^2 – 40mn + 16n^2)$
$= (x + 3y)^2 – (5m – 4n)^2$
$= [(x + 3y) + (5m – 4n)][(x + 3y) – (5m – 4n)]$
$= (x + 3y + 5m – 4n)(x + 3y – 5m + 4n)$

Exercise – 3.3

Solution 1:

$8x^3 + 125y^3$
$= (2x)^3 + (5y)^3$
$= (2x + 5y)(4x^2 – 10xy + 25y^2)$

Solution 2:

$2a^3 – 54b^3$
$= 2(a^3 – 27b^3)$
$= 2[(a)^3 – (3b)^3]$
$= 2(a – 3b)(a^2 + 3ab + 9b^2)$

Solution 3:

$(a + b)^3 – 8$
$= (a + b)^3 – (2)^3$
$= (a + b – 2)[(a + b)^2 + (a + b) \times 2 + (2)^2]$
$= (a + b – 2)(a^2 + 2ab + b^2 + 2a + 2b + 4)$

Solution 4:

$$\frac{m^3}{64} + \frac{n^3}{27}$$
$$= \left(\frac{m}{4}\right)^3 + \left(\frac{n}{3}\right)^3$$
$$= \left(\frac{m}{4} + \frac{n}{3}\right)\left(\frac{m^2}{16} - \frac{m}{4} \times \frac{n}{3} + \frac{n^2}{9}\right)$$
$$= \left(\frac{m}{4} + \frac{n}{3}\right)\left(\frac{m^2}{16} - \frac{mn}{12} + \frac{n^2}{9}\right)$$

Solution 5:

$$8y^3 - \frac{125}{y^3}$$
$$= (2y)^3 - \left(\frac{5}{y}\right)^3$$
$$= \left(2y - \frac{5}{y}\right)\left(4y^2 + 2y \times \frac{5}{y} + \frac{25}{y^2}\right)$$
$$= \left(2y - \frac{5}{y}\right)\left(4y^2 + 10 + \frac{25}{y^2}\right)$$

Solution 6:

Let $a + b = x$ and $a - b = y$,
$(a - b)^3 - (a - b)^3 = x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Substituting the values of x and y,
$[(a + b) - (a - b)][(a + b)^2 + (a + b)(a - b) + (a - b)^2]$
$= (a + b - a + b)(a^2 + 2ab + b^2 + a^2 - b^2 + a^2 - 2ab + b^2)$
$= (a - a + b + b)(a^2 + a^2 + a^2 + 2ab - 2ab + b^2 - b^2 + b^2)$
$= 2b(3a^2 + b^2)$

Solution 7:

Let $2m + 3n = a$ and $3m + 2n = b$
$(2m + 3n)^3 - (3m + 2n)^3$
$= a^3 - b^3$

$= (a – b)(a^2 + ab + b^2)$
Substituting the values of a and b,
$= [(2m + 3n) – (3m + 2n)] [(2m + 3n)^2 + (2m + 3n) (3m + 2n) + (3m + 2n)^2]$
$= (2m + 3n – 3m – 2n)(\underline{4m^2} + \underline{12mn} + 9n^2 + \underline{6m^2} + \underline{13mn} + 6n^2 + \underline{9m^2} + \underline{12mn} + 4n^2)$
$= (-m + n) (6m^2 + 9m^2 + 12mn + 13mn + 12mn + 9n^2 + 6n^2 + 4n^2)$
$=(-m + n) (19m^2 + 37mn + 19n^2)$

Solution 8:

Substituting 3x + 5y = a and 2x – y = b,
$(3x + 5y)^3 – (2x – y)^3$
$= a^3 – b^3$
$= (a – b) (a^2 + ab + b^2)$
Substituting the values of a and b,
$= [(3x + 5y) – (2x – y)][(3x + 5y)^2 + (3x + 5y) (2x -y) + (2x – y)^2]$
$= (3x + 5y – 2x + y)(\underline{9x^2} + \underline{30xy} + 25y^2 + \underline{6x^2} + \underline{7xy} – 5y^2 + \underline{4x^2} – 4xy + y^2)$
$= (x + 6y)(9x^2 + 6x^2 + 4x^2 + 30xy + 7xy – 4xy + 25y^2 – 5y^2 + y^2)$
$= (x + 6y)(19x^2 + 33xy + 21y^2)$

Solution 9:

Substituting x – 1 = a,
$27(x – 1)^3 + y^3$
$= 27a^3 + y^3$
$= (3a)^3 + y^3$
$= (3a + y) (9a^2 – 3ay + y^2)$
Substituting the value of a,
$= [3(x – 1) + y] [9(x – 1)^2 – 3(x – 1) (y) + y^2]$
$= (3x – 3 + y) [9(x^2 – 2x + 1) – 3xy + 3y + y^2]$
$= (3x + y – 3) (9x^2 – 18x + 9 – 3xy + 3y + y^2)$
$= (3x + y – 3) (9x^2 – 18x + 9 – 3xy + 3y + y^2)$

Solution 10:

$a^6 – b^6$
$= (a^3)^2 – (b^3)^2$
$= (a^3 + b^3)(a^3 – b^3)$
$= (a + b) (a^2 – ab + b^2)(a – b) (a^2 + ab + b^2)$

Exercise – 3.4

Solution 1:

$2x^2 + 3x – 5$
$= 2x^2 + 5x – 2x – 5$ …………….. ∵ $(-5) \times 2 = -10$
$= x(2x + 5) – 1(2x + 5)$ ………. $5 \times (-2) = -10$
$= (2x + 5) (x – 1)$ ………………… and $5 + (-2) = 3$

Solution 2:

$3x^2 – 14x + 8$
$= 3x^2 – 12x – 2x + 8$ ………… ∵ $3 \times 8 = 24$
$= 3x(x – 4) -2(x – 4)$ ………… $(-12) \times (- 2) = 24$
$= (x – 4) (3x – 2)$ ……………….. and $– 12 – 2 = -14$

Solution 3:

$6x^2 + 11x – 10$
$= 6x^2 + 15x – 4x – 10$ ………………… ∵ $6 \times (-10) = -60$
$= 3x(2x + 5) -2(2x + 5)$ ……………. $15 \times (- 4) = -60$
$= (2x + 5) (3x – 2)$ …………………….. and $15 – 4 = 11$

Solution 4:

$2x^2 – 7x – 15$
$= 2x^2 – 10x + 3x – 15$ ………………… ∵ $2 \times (-15) = -30$
$= 2x(x – 5) + 3(x – 5)$ ………………… $-10 \times 3 = -30$
$= (x – 5) (2x + 3)$ ……………………….. and $-10 + 3 = -7$

Solution 5:

$x^2 + 9xy + 18y^2$
$= x^2 + 6xy + 3xy + 18y^2$ ………………… ∵ $6 \times 3 = 18$
$= x(x + 6y) + 3y(x + 6y)$ ………………. and $6 + 3 = 9$
$= (x + 6y) (x + 3y)$

Solution 6:

$a^2 – 5ab – 36b^2$
$= a^2 + 4ab – 9ab – 36b^2$ …………….. ∵ $4 \times (-9) = -36$

= a(a + 4b) – 9b(a + 4b) …………… and 4 – 9 = -5
= (a + 4b) (a – 9b)

Solution 7:

$a^2 + 14ab - 51b^2$
$= a^2 - 3ab + 17ab - 51b^2$ ………. ∵ -3 × 17 = -51
= a (a – 3b) +17b(a – 3b) …….. and -3 + 17 = 14
= (a – 3b) (a + 17b)

Solution 8:

$2m^2 + 19mn + 30n^2$
$= 2m^2 + 4mn + 15mn + 30n^2$ …….. ∵ 2 × 30 = 60
= 2m(m + 2n) + 15n (m + 2n) ….. 4 × 15 = 60
= (m + 2n) (2m + 15n) ………………. and 4 + 15 = 19

Solution 9:

$3a^2 - 11ab + 6b^2$
$= 3a^2 - 9ab - 2ab + 6b^2$ ……………. ∵ 3 × 6 = 18
= 3a (a – 3b) – 2b(a – 3b) ………… -9 × -2 = 18
= (a – 3b) (3a – 2b) ………………….. and -9 – 2 = -11

Solution 10:

$6x^2 - 7xy - 13y^2$
$= 6x^2 + 6xy - 13xy - 13y^2$ ………… ∵ 6 × (-13) = -78
= 6x (x + y) -13y(x + y) ……………. and 6 – 13 = -7
= (x + y) (6x – 13y)

Solution 11:

$\sqrt{2x^2} + 3x + \sqrt{2}$
$= x^2\sqrt{2} + 2x + x + \sqrt{2}$ ∵ $\sqrt{2} \times \sqrt{2} = 2$
$= x\sqrt{2}(x + \sqrt{2}) + 1(x + \sqrt{2})$ $2 \times 1 = 2$
$= (x + \sqrt{2})(x\sqrt{2} + 1)$ and $2 + 1 = 3$

Exercise – 3.5

Solution 1:

$x^4 - 8x^2y^2 + 12y^4$
Let $x^2 = p$ and $y^2 = q$
Then $x^4 = p^2$, $= q^2$ and $x^2y^2 = pq$
$\therefore x^4 - 8x^2y^2 + 12y^4$
$= p^2 - 8pq + 12q^2$ ……………… $\because -6 \times -2 = 12$
$= p^2 - 6pq - 2pq + 12q^2$ …….. and $-6 - 2 = -8$
$= p(p - 6q) - 2q(p - 6q)$
$= (p - 6q)(p - 2q)$
Re-substituting the values of p and q we get,
$= (x^2 - 6y^2)(x^2 - 2y^2)$

Solution 2:

$2x^4 - 13x^2y^2 + 15y^4$
Let $x^2 = a$ and $y^2 = b$.
Then $x^4 = a^2$, $y^4 = b^2$ and $x^2y^2 = ab$
$\therefore 2x^4 - 13x^2y^2 + 15y^4$ ……………… $\because 2 \times 15 = 30$
$= 2a^2 - 13ab + 15ab^2$ ………………. $-10 \times -3 = 30$
$= 2a^2 - 10ab - 3ab + 15b^2$ ………. and $-10 - 3 = -13$
$= 2a(a - 5b) - 3b(a - 5b)$
$= (a - 5b)(2a - 3b)$
Re-substituting the values of a and b we get,
$= (x^2 - 5y^2)(2x^2 - 3y^2)$

Solution 3:

$6a^4 + 11a^2b^2 - 10b^4$
Let $a^2 = m$ and $b^2 = n$.
Then $a^4 = m^2$, $b^4 = n^2$ and $a^2b^2 = mn$
$\therefore 6a^4 + 11a^2b^2 - 10b^4$ …………….. $\because 6 \times (-10) = -60$
$= 6m^2 + 11mn - 10n^2$ ……………… $15 \times -4 = -60$
$= 6m^2 + 15mn - 4mn - 10n^2$ ….. and $15 - 4 = 11$
$= 3m(2m + 5n) - 2n(2m + 5n)$
$= (2m + 5n)(3m - 2n)$
Re-substituting the values of m and n we get,
$= (2a^2 + 5b^2)(3a^2 - 2b^2)$

Solution 4:

$3(x^2 – 5x)^2 – 2(x^2 – 5x + 5) – 6$
Substituting $(x^2 – 5x) = m$
$= 3m^2 – 2(m + 5) – 6$
$= 3m^2 – 2m – 10 – 6$
$= 3m^2 – 2m – 16$ ………………….. $\because 3 \times (-16) = -48$
$= 3m^2 – 8m + 6m – 16$ ………… $-2 \times -10 = 20$
$= m(3m – 8) + 2 (3m – 8)$ ….. and $-8 + 6 = -2$
$= m(3m – 8) + 2(3m – 8)$
$= (3m – 8) (m + 2)$
Re-substituting the value of m we get,
$= [3(x^2 – 5x) – 8] [(x^2 – 5x) + 2]$
$= (3x^2 – 15x – 8)(x^2 – 5x + 2)$

Solution 5:

$(y^2 + 5y)(y^2 + 5y – 2) – 24$
$= m(m – 2) – 24$
$= m^2 – 6m + 4m – 24$
$= m^2 – 2m – 24$ …………………….. $\because -6 \times -4 = 24$
$= m^2 – 6m + 4m – 24$ …………… and $-6 + 4 = -2$
$= m(m – 6) + 4 (m – 6)$
$= (m – 6) (m + 4)$
Re-substituting the value of m we get,
$= (y^2 + 5y – 6) (y^2 + 5y + 4))$
$= (y^2 + 6y – y – 6) (y^2 + 4y + y + 4)$
$= [y(y + 6) – 1(y + 6)][y(y + 4) + 1(y + 4)]$
$= (y + 6)(y – 1)(y + 4) (y + 1)$

Exercise – 3.6

Solution 1:

$x^3 – 27y^3 +125 + 45xy$
$= (x)^3 + (-3y)^3 + (5)^3 – 3(x)(-3y)(5)$
$= (x – 3y + 5)(x^2 + 9y^2 + 25 + 3xy + 15y – 5x)$

Solution 2:

$a^3 – b^3 + 8c^3 + 6abc$
$= (a)^3 + (-b)^3 + (2c)^3 – 3(a)(-b)(2c)$
$= (a – b + 2c)(a^2 + b^2 + 4c^2 + ab + 2bc -2ca)$

Solution 3:

$8a^3 + 27b^3 + 64c^3 - 72abc$
$= (2a)^3 + (3b)^3 + (4c)^3 - 3(2a)(3b)(4c)$
$= (2a + 3b + 4c)(4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 8ca)$

Solution 4:

$-27x^3 + y^3 - z^3 - 9xyz$
$= (-3x)^3 + (y)^3 + (-z)^3 -3(-3x)(y)(-z)$
$= (-3x + y - z)(9x^2 + y^2 + z^2 + 3xy + yz - 3zx)$

Solution 5:

$y^6 + 32y^3 - 64$
$= y^6 + 8y^3 - 64 + 24y^3$
$= (y^2)^3 + (2y)^3 + (-4)^3 - 3(y^2)(2y)(-4)$
$= (y^2 + 2y - 4)(y^4 + 4y^2 + 16 - 2y^3 + 8y + 4y^2)$
$= (y^2 + 2y - 4)(y^4 - 2y^3 + 8y^2 + 8y + 16)$

Solution 6:

$x^6 - 10x^3 - 27$
$= x^6 - x^3 - 27 - 9x^3$
$= (x^2)^3 + (-x)^3 + (-3)^3 - 3(x^2)(-x)(-3)$
$= (x^2 - x - 3)(x^4 + \underline{x}^2 + 9 + x^3 - 3x + \underline{3}x^2)$
$= (x^2 - x - 3)(x^4 + x^3 + 4x^2 - 3x + 9)$

Solution 7:

$$a^3 + 4 - \frac{1}{a^3}$$

$$a^3 + 1 - \frac{1}{a^3} + 3$$

$$= (a)^3 + (1)^3 + \left(-\frac{1}{a}\right)^3 - 3(a)(1)\left(-\frac{1}{a}\right)$$

$$= \left(a + 1 - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - a + \frac{1}{a} + 1\right)$$

Solution 8:

$(p - 3q)^3 + (3q - 7r)^3 + (7r - p)^3$
Let $(p - 3q) = a$, $(3q - 7r) = b$ and $(7r - p)=c$.
$\therefore (p - 3q)^3 + (3q - 7r)^3 + (7r - p)^3 = a^3 + b^3 + c^3$
Here, $a + b + c = p - 3q + 3q - 7r + 7r - p = 0$
$\therefore a + b + c = 0$
If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$
$\therefore (p - 3q)^3 + (3q - 7r)^3 + (7r - p)^3 = 3(p - 3q)(3q - 7r)(7r - p)$

Solution 9:

$(5x - 6y)^3 + (7z - 5x)^3 + (6y - 7z)^3$
Let $(5x - 6y) = p$, $(7z - 5x) = q$ and $(6y - 7z) = r$
$\therefore (5x - 6y)^3 + (7z - 5x)^3 + (6y - 7z)^3 = p^3 + q^3 + r^3$
Now, $p + q + r = 5x - 6y + 7z - 5x + 6y -7z = 0$
$\therefore p + q + r = 0$
If $p + q + r = 0$, $p^3 + q^3 + r^3 = 3pqr$
$\therefore (5x - 6y)^3 + (7z - 5x)^3 + (6y -7z)^3 = 3(5x - 6y)(7z - 5x)(6y - 7z)$

Solution 10:

$27(a - b)^3 + (2a - b)^3 + (4b - 5a)^3$
$= (3)^3(a - b)^3 + (2a - b)^3 + (4b - 5a)^3$
$= [3(a - b)]^3 + (2a - b)^3 + (4b - 5a)^3$
$= (3a - 3b)^3 + (2a - b)^3 + (4b - 5a)^3$
Let $3a - 3b = x$, $2a - b = y$ and $4b - 5a = z$
$= x^3 + y^3 + z^3$
Here, $x + y + z = 3a - 3b + 2a - b + 4b - 5a = 0$
$\therefore x + y + z = 0$
If $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$
$\therefore 27(a - b)^3 + (2a - b)^3 + (4b - 5a)^3 = 3(3a - 3b)(2a - b) (4b - 5a)$
$= 9 (a - b) (2a - b) (4b - 5a)$

Exercise – 3.7

Solution 1:

Expressions (i), (ii), (iv), (v) and (vii) are polynomials.

Solution 2:

i. The standard form of the expression in descending type is
$-x^4 + 5x^3 + 2x^2 + 3x - 5$
ii. The standard form of the expression in descending type is
$x^3 + 3x + 5$
iii. The standard form of the expression in descending type is
$4t^2 + \sqrt{5}t + 7$
iv. The standard form of the expression in descending type is
$\frac{11}{8}x^2 - 7x + 2$
v. The standard form of the expression in descending type is
$\sqrt{3}x^3 + 4x^2 + 7x - 14$

Solution 3:

1. The degree of the polynomial is 0.
2. The degree of the polynomial is 0.
3. The degree of the polynomial is 1.
4. The degree of the polynomial is 2.
5. The degree of the polynomial is 12.
6. The degree of the polynomial is 3.
7. The degree of the polynomial is 3.
8. The degree of the polynomial is 8.
9. The degree of the polynomial is 7.
10. The degree of the polynomial is 7.

Exercise – 3.8

Solution 1(i):

$2x^3 – 7x^2 + 3x + 4; 2x^3 – 3x^2 + 4x + 1$
$= (2x^3 – 7x^2 + 3x + 4) + (2x^3 – 3x^2 + 4x + 1)$
$= 2x^3 – 7x^2 + 3x + 4 + 2x^3 – 3x^2 + 4x + 1$
$= 2x^3 + 2x^3 – 7x^2 – 3x^2 + 3x + 4x + 4 + 1$
(Arranging the like terms together)
$= 4x^2 – 10x^2 + 7x + 5$ and degree 3.

Solution 1(ii):

$3x^2 + 5x – x^7$; $– 3x^2 + 5x + 8$
$= (3x^2 + 5x – x^7) + (- 3x^2 + 5x + 8)$
$= 3x^2 + 5x – x^7 – 3x^2 + 5x + 8$
$= – x^7 + 3x^2 – 3x^2 +5x + 5x + 8$
(Arranging the like terms together)
$= x^7 + 10x + 8$ and degree 7.

Solution 1(iii):

$x^4 + 5x^3 + 7x$; $4x^3 – 3x^2 + 5$
$= (x^4 + 5x^3 + 7x) + (4x^3 – 3x^2 + 5)$
$= x^4 + 5x^3 + 7x + 4x^3 – 3x^2 + 5$
$= x^4 + 5x^3 + 4x^3 – 3x^2 + 7x + 5$
(Arranging the like terms together)
$= x^4 + 9x^3 – 3x^2 + 7x + 5$ and degree 4.

Solution 1(iv):

$y^2 + 2y – 5$; $y^3 + 2y^2 + 3y + 4$; $y^3 + 7y – 2$
$= (y^2 + 2y – 5) + (y^3 + 2y^2 + 3y + 4) + (y^3 + 7y – 2)$
$= y^2 + 2y – 5 + y^3 + 2y^2 + 3y + 4+ y^3 + 7y – 2$
$= y^3 + y^3 + y^2 + 2y^2 + 2y + 3y + 7y – 5 + 4 – 2$
(Arranging the like terms together)
$= 2y^3 + 3y^2 + 12y – 3$ and degree 3.

Solution 1(v):

$5m^2 + 3m + 8$; $m^3 – 6m^2 + 4m$; $m^3 – m^2 – m + 5$
$= (5m^2 + 3m + 8) + (m^3 – 6m^2 + 4m) + (m^3 – m^2 – m + 5)$
$= 5m^2 + 3m + 8 + m^3 – 6m^2 + 4m + m^3 – m^2 – m + 5$
$= m^3 + m^3 + 5m^2 – 6m^2 – m^2 +3m + 4m – m + 8 + 5$
(Arranging the like terms together)
$= 2m^3 – 2m^2 + 6m + 13$ and degree 3.

Solution 2(i):

$x^4 + x^2 + x – 1$; $x^4 – x^3 – x^2 + 1$
$= (x^4 + x^2 + x – 1) – (x^4 – x^3 – x^2 + 1)$
$= x^4 + x^2 + x – 1 – x^4 + x^3 + x^2 – 1$
$= x^4 – x^4 + x^3 + x^2 + x- 1 – 1$

(Arranging the like terms together)
$= x^3 + 2x^2 + x – 2$ and degree 3.

Solution 2(ii):

$n^3 – 5n^2 + 6; n^3 – 3n + 8$
$= (n^3 – 5n^2 + 6) – (n^3 – 3n + 8)$
$= n^3 – 5n^2 + 6 – n^3 + 3n – 8$
$= n^3 – n^3 – 5n^2 + 3n + 6 – 8$
(Arranging the like terms together)
$= – 5n^2 + 3n – 2$ and degree 2.

Solution 2(iii):

$2a + 3a^2 – 7; 3a^2 – 12 + 2a$
$= (2a + 3a^2 – 7) – (3a^2 – 12 + 2a)$
$= 2a + 3a^2 – 7 – 3a^2 + 12 – 2a$
$= 3a^2 – 3a^2 + 2a – 2a – 7 + 12$
(Arranging the like terms together)
= 5 and degree 0.

Solution 3(i):

$(3x^2 – 2x + 1) + (x^2 + 5x -3) + (4x^2 + 8)$
$= 3x^2 – 2x + 1 + x^2 + 5x -3 + 4x^2 + 8$
$= \underline{3x^2 + x^2 + 4x^2} – \underline{2x + 5x} + \underline{1 – 3 + 8}$
(Arranging the like terms together)
$= 8x^2 + 3x + 6.$

Solution 3(ii):

$(2y^3 + 3y -7) – (8y – 6) + (4y^3 – 2y + 1)$
$= 2y^3 + 3y – 7 – 8y + 6 + 4y^3 – 2y + 1$
$= \underline{2y^3 + 4y^3} + \underline{3y – 8y – 2y} – \underline{7 + 6 + 1}$
(Arranging the like terms together)
$= 6y^3 -7y.$

Solution 3(iii):

$5m^3 – m + 6m^2 – (3m^2 – 2 + m)$
$(5m^3 – m + 6m^2) – (3m^2 – 2 + m)$
$= 5m^3 – m + 6m^2 -3m^2 + 2 – m$

$= 5m^3 + 6m^2 – 3m^2 – m – m + 2$
$= 5m^3 + 3m^2 – 2m + 2$.

Solution 4:

Let the polynomial to be added be p(x)
$(2x^4 – 3x^2 + 5x + 8) + p(x) = (2x^2 – 5x + 4)$
$p(x) = (2x^2 – 5x + 4) – (2x^4 – 3x^2 + 5x + 8)$
$= 2x^2 -5x + 4 – 2x^4 + 3x^2 – 5x – 8$
$= -2x^4 + 2x^2 + 3x^2 -5x – 5x + 4 – 8$
$= -2x^4 + 5x^2 – 10x – 4$
Therefore, $– 2x^4 + 5x^2 – 10x – 4$ should be added to $2x^4 – 3x^2 + 5x + 8$ to get $2x^2 – 5x + 4$.

Solution 5:

Let the polynomial to be subtracted be p(y).
$(y^3 + 2y^2 + 5y -1) – p(y) = 2y^2 + 12$
$\therefore (y^3 + 2y^2 + 5y – 1) – (2y^2 + 12) = p(y)$
$\therefore p (y) = y^3 + 2y^2 + 5y – 1 – 2y^2 – 12$
$= y^3 + \underline{2y^2 – 2y^2} + 5y – \underline{1 – 12}$
$= y^3 + 5y – 13$
Therefore, $y^3 + 5y – 13$ should be subtracted from $y^3 + 2y^2 + 5y – 1$ to get $2y^2 + 12$.

Solution 6:

$= (z^3 + 3z^2 + 5z + 8) + (4z^3 + 2z^2 – 7z – 2) – (2z^3 – 3z^2 + z – 4)$
$= z^3 + 3z^2 + 5z + 8 + 4z^3 + 2z^2 – 7z – 2 – 2z^3 + 3z^2 – z + 4$
$= \underline{z^3 + 4z^3 – 2z^3} + \underline{3z^2 + 2z^2 + 3z^2} + \underline{5z – 7z – z} + \underline{8 – 2 + 4}$
$= 3z^3 + 8z^2 – 3z + 10$

Exercise – 3.9

Solution 1(i):
$(x^2 + 3x + 1)(2x – 3)$
$= x^2(2x – 3) + 3x(2x – 3) + 1(2x – 3)$
$= 2x^3 – 3x^2 + 6x^2 – 9x + 2x – 3$
$= 2x^3 + 3x^2 – 7x – 3$ and degree 3.

Solution 1(ii):

$(3x^2 + 5x)(x^2 + 2x + 1)$
$= 3x^2(x^2 + 2x + 1) + 5x(x^2 + 2x + 1)$
$= 3x^4 + 6x^3 + 3x^2 + 5x^3 + 10x^2 + 5x$
$= 3x^4 + 6x^3 + 5x^3 + 3x^2 + 10x^2 + 5x$
(Arranging the like terms together)
$= 3x^4 + 11x^3 + 13x^2 + 5x$ and degree 4.

Solution 1(iii):

$(x^3 + 4x + 2)(x^2 + x + 5)$
$= x^3(x^2 + x + 5) + 4x(x^2 + x + 5) + 2(x^2 + x + 5)$
$= x^5 + x^4 + 5x^3 + 4x^3 + 4x^2 + 20x + 2x^2 + 2x + 10$
$= x^5 + x^4 + 9x^3 + 4x^2 + 2x^2 + 20x + 2x + 2x + 10$ (Arranging the like terms together)
$= x^5 + x^4 + 9x^3 + 6x^2 + 22x + 10$ and degree 5.

Solution 1(iv):

$(x^3 – 1)(x^2 – x + 4)$
$= x^3(x^2 – x + 4) – 1(x^2 – x + 4)$
$= x^5 – x^4 + 4x^3 – x^2 + x – 4$ and degree 5.

Solution 1(v):

$(2y^2 + 3)(3y^3 + 1)$
$= 2y^2(3y^3 + 1) + 3(3y^3 + 1)$
$= 6y^5 + 2y^2 + 9y^3 + 3$
$= 6y^5 + 9y^3 + 2y^2 + 3$ and degree 5.

Solution 2(i):

$$
\begin{array}{r|l}
 & x^2 - 7x + 18 \\
\hline
x + 2 & x^3 - 5x^2 + 4x + 8 \\
 & x^3 + 2x^2 \\
 & - \quad - \\
\hline
 & -7x^2 + 4x + 8 \\
 & -7x^2 - 14x \\
 & + \quad + \\
\hline
 & 18x + 8 \\
 & 18x + 36 \\
 & - \quad - \\
\hline
 & -28
\end{array}
$$

$$x^3 - 5x^2 + 4x + 8 = (x + 2)(x^2 - 7x + 18) - 28$$

Solution 2(ii):

$$
\begin{array}{r|l}
 & y^2 - 5y + 1 \\
\hline
y - 1 & y^3 - 6x^2 + 6y + 1 \\
 & y^3 - y^2 \\
 & - \quad + \\
\hline
 & -5y^2 + 6y + 1 \\
 & -5y^2 + 5y \\
 & + \quad - \\
\hline
 & y + 1 \\
 & y - 1 \\
 & - \quad + \\
\hline
 & 2
\end{array}
$$

$$y^3 - 6x^2 + 6y + 1 = (y - 1)(y^2 - 5y + 1) + 2$$

Solution 2(iii):

$$
\begin{array}{r|l}
 & y^2 + 4y + 16 \\
\hline
y - 4 & y^3 + 0y^2 + 0y - 64 \\
 & y^3 - 4y^2 \\
 & - \quad + \\
\hline
 & 4y^2 - 0y - 64 \\
 & 4y^2 - 16y \\
 & - \quad + \\
\hline
 & 16y - 64 \\
 & 16y - 64 \\
 & - \quad + \\
\hline
 & 0
\end{array}
$$

$y^3 - 64 = (y - 4)(y^2 + 4y + 16) + 0.$

Solution 2(iv):

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 5 \\
\hline
3x - 2 & 6x^3 + 5x^2 - 21x + 10 \\
 & 6x^3 - 4x^2 \\
 & - \quad + \\
\hline
 & 9x^2 - 21x + 10 \\
 & 9x^2 - 6x \\
 & - \quad + \\
\hline
 & -15x + 10 \\
 & -15x + 10 \\
 & + \quad - \\
\hline
 & 0
\end{array}
$$

$6x^3 + 5x^2 - 21x + 10 = (3x - 2)(2x^2 + 3x - 5) + 0.$

Solution 2(v):

$$
\begin{array}{r|l}
 & 3x^3 - 4x^2 + 12x - 12 \\
\hline
x^2 - 3 & 3x^5 - 4x^4 + 3x^3 + 0x^2 + 2x + 0 \\
 & 3x^5 \qquad - 9x^3 \\
 & - \qquad\quad + \\
\hline
 & - 4x^4 + 12x^3 + 0x^2 + 2x + 0 \\
 & - 4x^4 \qquad + 12x^2 \\
 & + \qquad\quad - \\
\hline
 & 12x^3 - 12x^2 + 2x + 0 \\
 & 12x^3 \qquad - 36x \\
 & - \qquad\quad + \\
\hline
 & - 12x^2 + 38x + 0 \\
 & - 12x^2 \qquad + 36 \\
 & + \qquad\quad - \\
\hline
 & 38x - 36
\end{array}
$$

$$3x^5 - 4x^4 + 3x^3 + 2x = (x^2 - 3)(3x^3 - 4x^2 + 12x - 12) + 36x - 36$$

Exercise – 3.10

Solution 1:

1. The coefficient form of the polynomial $2x^2 + 5x + 12$ is (2, 5, 12).
2. The index form of the polynomial $y^4 – 3y^2 + 2y – 7 = y^4 + 0y^3 – 3y^2 + 2y – 7$.∴The coefficient form = (1, 0, -3, 2, -7).
3. The index form of the polynomial $x^5 + 3x^2 = x^5 + 0x^4 + 0x^3 + 3x^2 + 0x + 0$.∴The coefficient form = (1, 0, 0, 3, 0, 0).
4. The index form of the polynomial $y^4 – 3 = y^4 + 0y^3 + 0y^2 + 0y – 3$.∴The coefficient form = (1, 0, 0, 0, -3).
5. The index form of the polynomial $9x = 9x + 0$. ∴ The coefficient form = (9, 0).

Solution 2:

1. Number of coefficients = 3.∴The degree of the polynomial = 3 – 1 = 2
   ∴The index form of the given polynomial is $3x^2 + 2x + 7$.
2. The number of coefficients = 4.∴The degree of the polynomial = 4-1=3
   ∴The index form of the given polynomial is $2x^3 – 4$.

3. Number of coefficients = 5.∴The degree of the polynomial = 5 – 1= 4
   ∴The index form of the given polynomial is $x^4 - 3x^2 + x + 5$.
4. Number of coefficients = 4.∴The degree of the polynomial = 4 – 1= 3,
   ∴The index form of the given polynomial is $-x^3 + 3x^2 - 5x + 6$.
5. Number of coefficients = 7.∴The degree of the polynomial = 7 – 1 = 6
   ∴The index form of the given polynomial is $x^6 + 64$.

Solution 3(i):

Dividend is $x^3 - 4x^2 - 2x + 1$

Index form is $x^3 - 4x^2 - 2x + 1$

Coefficient form is $(1, -4, -2, 1)$

Comparing the divisor $x - 3$ with $x - a$,

$\therefore a = 3$

$$\begin{array}{r|rrrr} 3 & 1 & -4 & -2 & 1 \\ & & 3 & -3 & -15 \\ \hline & 1 & -1 & -5 & \boxed{-14} \end{array}$$

∴ Quotient in cofficient form $= (1, -1, -5)$

∴ Quotient $= x^2 - x - 5$ in the variable x

Remainder $= -14$

$\therefore (x^3 - 4x^2 - 2x + 1) = (x - 3)(x^2 - x - 5) + (-14)$.

Solution 3(ii):

Dividend is $2x^3 - 3x^2 + 4x + 2$

Index form is $2x^3 - 3x^2 + 4x + 2$

Coefficient form is $(2, -3, 4, 2)$

is .

Comparing the divisor $x - 1$ with $x - a$,

$\therefore a = 1$.

$$\begin{array}{r|rrrr} 1 & 2 & -3 & 4 & 2 \\ & & 2 & -1 & 3 \\ \hline & 2 & -1 & 3 & \boxed{5} \end{array}$$

∴ Quotient in cofficient form $= (2, -1, 3)$.

∴ Quotient $= 2x^2 - x + 3$ in the variable x

Remainder $= 5$

$2x^3 - 3x^2 + 4x + 2 = (x - 1)(2x^2 - x + 3) + 5$.

Solution 3(iii):

Dividend is $y^3 + 343$

Index form is $y^3 + 0y^2 + 0y + 343$

Coefficient form is $(1, 0, 0, 343)$

Comparing the divisor $y + 7$ with $y - a$,

$\therefore a = -7.$

-7	1	0	0	343
		-7	49	-343
	1	-7	49	$\boxed{0}$

$\therefore$ Quotient in cofficient form = $(1, -7, 49)$.

$\therefore$ Quotient = $y^2 - 7y + 49$ in the variable y

Remainder = 0

$y^3 + 343 = (y + 7)(y^2 - 7y + 49) + 0.$

Solution 3(iv):

Dividend is $(x^5 + x^3 + x^2 - 2x + 4)$

Index form is $(x^5 + 0x^4 + x^3 + x^2 - 2x + 4)$

Coefficient form is $(1, 0, 1, 1, -2, 4)$

Comparing the divisor $x + 3$ with $x - a$,

$\therefore a = -3$

-3	1	0	1	1	-2	4
		-3	9	-30	87	-255
	1	-3	10	-29	85	$\boxed{-251}$

$\therefore$ Quotient in cofficient form = $(1, -3, 10, -29, 85)$

$\therefore$ Quotient = $x^4 - 3x^3 + 10x^2 - 29x + 85$ in the variable x

Remainder = -251

$\therefore (x^5 + x^3 + x^2 - 2x + 4) = (x + 3)(x^4 - 3x^3 + 10x^2 - 29x + 85) + (-251).$

Solution 3(v):

Dividend is $x^3 + 2x^2 + x + 2$

Index form is $x^3 + 2x^2 + x + 2$

Coefficient form is $(1, 2, 1, 2)$

Comparing the divisor $x - 1$ with $x - a$,

$\therefore a = 1.$

1	1	2	1	2
		1	3	4
	1	3	4	$\boxed{6}$

$\therefore$ Quotient in cofficient form $= (1, 3, 4)$.

$\therefore$ Quotient $= x^2 + 3x + 4$ in the variable x

Remainder = 6

$x^3 + 2x^2 + x + 2 = (x - 1)(x^2 + 3x + 4) + 6.$

Solution 3(vi):

Dividend is $y^2 - 11y + 30$

Index form is $y^2 - 11y + 30$

Coefficient form is $(1, -11, 30)$

Comparing the divisor $y - 5$ with $y - a$,

$\therefore a = 5.$

5	1	−11	30
		5	−30
	1	−6	$\boxed{0}$

$\therefore$ Quotient in cofficient form $= (1, -6)$.

$\therefore$ Quotient $= y - 6$ in the variable x

Remainder = 0

$y^2 - 11y + 30 = (y - 5)(y - 6) + 0.$

Solution 3(vii):

Dividend is $x - 3x^2 - 12x + 4$

Index form is $x - 3x^2 - 12x + 4$

Coefficient form is $(1, -3, 12, 4)$

Comparing the divisor $x - 2$ with $x - a$,

$\therefore a = 2.$

2	1	− 3	− 12	4
		2	− 2	− 28
	1	− 1	− 14	−24

$\therefore$ Quotient in cofficient form $= (1, -1, -14)$.

$\therefore$ Quotient $= x^2 - x - 14$ in the variable x

Remainder $= -24$

$$x - 3x^2 - 12x + 4 = (x - 2)(x^2 - x - 14) - 24.$$

Solution 3(viii):

Dividend is $2x^4 + 3x^2 + 5$

Index form is $2x^4 + 0x^3 + 3x^2 + 0x + 5$

Coefficient form is $(2, 3, 5)$

Comparing the divisor $x + 2$ with $x - a$,

$\therefore a = -2.$

−2	2	0	3	0	5
		−4	8	−22	44
	2	−4	11	−22	49

$\therefore$ Quotient in coefficient form $= (2, -4, 11, -22)$.

$\therefore$ Quotient $= 2x^3 - 4x^2 + 11x - 22$ in the variable x

Remainder $= 49$

$$2x^4 + 3x^2 + 5 = (x + 2)(2x^3 - 4x^2 + 11x - 22) + 49.$$

Exercise – 3.11

Solution 1:

1. $p(x) = x^2 + 2x + 5= 0 + 0 + 5$
2. $= 5$
3. $\therefore p(0) = (0)^2 + 2(0) + 5$
4. $p(x) = x^2 + 2x + 5= 9 + 6 + 5$
5. $= 20$
6. $\therefore p(3) = (3)^2 + 2(3) + 5$
7. $p(x) = x^2 + 2x + 5= 1 – 2 + 5$
8. $= 4$
9. $\therefore p(-1) = (-1)^2 + 2(-1) + 5$
10. $p(x) = x^2 + 2x + 5= 9 – 6 + 5$
11. $= 8$
12. $\therefore p(-3) = (-3)^2 + 2(-3) + 5$
13. $p(x) = x^2 + 2x + 5= a^2 + 2a + 5$
14. $\therefore p(a) = (a)^2 + 2(a) + 5$

Solution 2:

$P(y) = y^3 – 5y – 2y^2 + 3$

$= y^3 – 2y^2 – 5y + 3$ …(Standard form)

1. $p(y) = y^3 – 2y^2 – 5y + 3= 1 – 2 – 5 + 3$
2. $= -3$
3. $\therefore p(1) = (1)^3 – 2(1)^2 – 5(1) + 3$
4. $p(y) = y^3 – 2y^2 – 5y + 3= 8 – 2(4) – 10 + 3= -7$
5. $= 8 – 8 – 10 + 3$
6. $\therefore p(2) = (2)^3 – 2(2)^2 – 5(2) + 3$
7. $p(y) = y^3 – 2y^2 – 5y + 3= -8 – 2(4) + 10 + 3= -3$
8. $= -8 – 8 + 10 + 3$
9. $\therefore p(-2) = (-2)^3 – 2(-2)^2 – 5(-2) + 3$
10. $p(y) = y^3 – 2y^2 – 5y + 3= 64 – 2(16) – 20 + 3= 15$
11. $= 64 – 32 – 20 + 3$
12. $\therefore p(4) = (4)^3 – 2(4)^2 – 5(4) + 3$
13. $p(y) = y^3 – 2y^2 – 5y + 3= -b^3 – 2b^2 + 5b + 3$
14. $\therefore p(-b) = (-b)^3 – 2(-b)^2 – 5(-b) + 3$

Solution 3:

$p(x) = x^2 – mx + 7$
$\therefore p(2) = (2)^2 – m(2) + 7$
$= 4 – 2m + 7$
$= 11 – 2m$
But $p(2) = 35$ given
$\therefore 11 – 2 m = 35$
$\therefore 2m = 11 – 35$
$\therefore 2m = -24$
$\therefore m = -12$

Solution 4:

$p(y) = ay^2 + 2y – 6$
$\therefore p(-3) = a(-3)^2 + 2(-3) – 6$
$= 9a – 6 – 6$
$= 9a – 12$
But $p(y) = 15$
$\therefore 9a – 12 = 15$
$\therefore 9a = 15 + 12$
$\therefore 9a = 27$
$\therefore a = 3$

Exercise – 3.12

Solution 1:

To find zero of the polynomial we solve by equating it to zero.

i. $p(x) = x + 2$
$p(x) = 0$
$\therefore x + 2 = 0$
$\therefore x = -2$
Thus, -2 is the zero of the polynomial $x + 2$.

ii. $q(x) = 4x - 12$
$p(x) = 0$
$\therefore 4x - 12 = 0$
$\therefore 4x = 12$
$\therefore x = 3$
Thus, 3 is the zero of the polynomial $4x - 12$.

iii. $r(x) = 5 - 6x$
$r(x) = 0$
$\therefore 5 - 6x = 0$
$\therefore 6x = 5$
$\therefore x = \frac{5}{6}$
Thus, $\frac{5}{6}$ is the zero of the polynomial $5 - 6x$.

iv. $p(y) = y + 1$
$p(y) = 0$
$\therefore y + 1 = 0$
$\therefore y = -1$
Thus, 1 is the zero of the polynomial $y + 1$.

v. $p(m) = m$
$p(m) = 0$
$\therefore m = 0$
Thus, 0 is the zero of the polynomial m.

vi. $q(y) = 4y$
$q(y) = 0$
$\therefore 4y = 0$
$\therefore y = 0$
Thus, 0 is the zero of the polynomial 4y.

Solution 2(i):

$p(x) = x – 2$
$p(x) = 0$
$\therefore x – 2 = 0$
$\therefore x = 2$
$\therefore$ 2 is the zero of the given polynomial.

Solution 2(ii):

$p(x) = (x – 2)$
$p(x) = 0$
$\therefore x – 2 = 0$
$\therefore x =2$
$p(x) = (x – 9)$
$p(x) = 0$
$\therefore x – 9 = 0$
$\therefore x = 9$
$\therefore$ 2 and 9 are the zero of the given polynomial.

Solution 2(iii):

We know $x^2 - x - 12 = x^2 - 4x + 3x - 12$

$= x(x - 4) + 3(x - 4)$

$= (x + 3)(x - 4)$

The polynomial have zeroes when $p(x) = 0$

$\therefore\ x + 3 = 0$ or $x - 4 = 0$

$\therefore\ x = -3$ or $x = 4$

$\therefore$ The zeroes of the polynomial $x^2 - x - 12$ are -3 and 4.

Solution 3(i):

We know $x^2 + 10x + 16 = x^2 + 8x + 2x + 16$

$= x(x + 8) + 2(x + 8)$

$= (x + 2)(x + 8)$

The polynomials have zeroes when $p(x) = 0$.

$\therefore\ x + 2 = 0$ or $x + 8 = 0$

$\therefore\ x = -2$ or $x = -8$

$\therefore$ The zeroes of the polynomial $x^2 + 10x + 16$ are -2 and -8.

Now, sum of zeroes $= -2 - 8 = -10 = -\frac{10}{1} = -\frac{b}{a}$

and product of zeroes $= (-2) \times (-8) = 16 = \frac{16}{1} = \frac{c}{a}$

Solution 3(ii):

We know $x^2 - 4x - 5 = x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

$= (x + 1)(x - 5)$

The polynomials have zeroes when $p(x) = 0$.

$\therefore\ x + 1 = 0$ or $x - 5 = 0$

$\therefore\ x = -1$ or $x = 5$

$\therefore$ The zeroes of the polynomial $x^2 - 4x - 5$ are -1 and 5.

Now, sum of zeroes $= -1 + 5 = 4 = -\frac{(-4)}{1} = -\frac{b}{a}$

and product of zeroes $= (-1) \times (5) = -5 = \frac{-5}{1} = \frac{c}{a}$

Solution 4(i):

Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

We know $\alpha + \beta = 5 = -(-5) = \frac{-b}{1}$

and $\alpha\beta = -50 = \frac{c}{a}$

If a = 1, then b = −5 and c = −50.

Therefore, the quadratic polynomial which satisfies the above condition is $x^2 - 5x - 50$.

Solution 4(ii):

Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

We know $\alpha + \beta = -11 = \frac{-b}{1}$

and $\alpha\beta = 10 = \frac{c}{a}$

If a = 1, then b = 11 and c = 10.

Therefore, the quadratic polynomial which satisfies the above condition is $x^2 + 11x + 10$.

Exercise – 3.13

Solution 1(i):

$p(x) = 3x^2 + x + 7$
Divisor is x + 2
∴ Put x = -2 in p(x)
∴ By the Remainder Theorem
Remainder = $p(-2) = 3(-2)^2 + (-2) + 7$
= 3(4) – 2 + 7
= 12 – 2 + 7
= 17
∴ Remainder = 17

Solution 1(ii):

$p(x) = 4x^3 + 5x - 10$
Divisor is x – 3
∴ Put x = 3 in p(x)
∴ By the Remainder Theorem
Remainder = $p(3) = 4(3)^3 + 5(3) - 10$
= 4(27) + 15 – 10

=108 + 15 – 10
= 113
∴ Remainder = 113

Solution 1(iii):

$p(x) = x^3 – ax^2 + 2x – a$
Divisor is x – a.
∴ Put x = a in p(x)
∴ By the Remainder Theorem
Remainder = $p(a) = (a)^3 – a(a)^2 + 2 (a) – a$
= $a^3 – a^3 + 2a – a$
= a
∴ Remainder = a

Solution 2(i):

$p(x) = 2x^3 – 3x^2 + 4x – 5$
Divisor is x – 2
∴ Put x = 2 in p(x)
∴ By the Remainder Theorem
Remainder = $p(2) = 2 (2)^3 – 3(2)^2 + 4(2) – 5$
=16 – 3(4) + 8 – 5
= 16 – 12 + 8 – 5
= 7
∴ Remainder = 7

Solution 2(ii):

$p(x) = 2x^3 – 3x^2 + 4x – 5$
Divisor is x + 3
∴ Put x = -3 in p(x)
∴ By the Remainder Theorem
Remainder = $p(-3) = 2 (-3)^3 – 3(-3)^2 + 4(-3) – 5$
= 2 (- 27) – 3(9) – 12 – 5
= – 54 – 27 – 12 – 5
= -98
∴ Remainder = -98

Solution 2(iii):

$p(x) = 2x^3 – 3x^2 + 4x – 5$
Divisor is x – 1

∴ Put x = 1 in p(x)
∴ By the Remainder Theorem
Remainder = p(1) = 2 $(1)^3$ – $3(1)^2$ + 4(1) – 5
= 2 (1) – 3(1) + 4 – 5
= 2 – 3 + 4 – 5
= -2
∴ Remainder = -2.

Solution 3:

p(x) = x^3 + ax^2 + 4x – 5
Divisor is x + 1
∴ Put x = 1 in p(x)
∴ By the Remainder Theorem
Remainder = p(-1) = $(-1)^3$ + $a(-1)^2$ + 4(-1) – 5
= -1 +a – 4 – 5
= a – 10
But, remainder = 14
∴ a – 10 = 14
∴ a = 14 + 10
∴ a = 24

Exercise – 3.14

Solution 1(i):

p(x) = x^2 – 4
Put x = -2 in p(x), we get
p(-2) = $(-2)^2$ – 4
= 4 – 4
= 0
As p(-2) = 0,
∴ By the Factor Theorem (x + 2) is a factor of x^2 – 4.

Solution 1(ii):

p(x) = x^3 – 27
Put x = 3 in p(x), we get
p(3) = $(3)^3$ – 27
= 27 – 27
= 0
As p(3) = 0,
∴ By the Factor Theorem (x – 3) is a factor of x^3 – 27.

Solution 1(iii):

$p(x) = 2x^4 + 9x^3 + 6x^2 – 11x – 6$
Put x = 1 in p(x), we get
$p(1) = 2(1)^4 + 9(1)^3 + 6(1)^2 – 11(1) – 6$
$= 2 + 9 + 6 – 11 – 6$
$= 0$
As p(1) = 0,
∴ By the Factor Theorem (x – 1) is a factor of $2x^4 + 9x^3 + 6x^2 – 11x – 6$.

Solution 1(iv):

$p(x) = x^2 + 10x + 24$
Put x = -4 in p(x), we get
$p(1) = (-4)^2 + 10(-4) + 24$
$= 16 – 40 + 24$
$= 0$
As p(-4) = 0,
∴ By the Factor Theorem (x + 4) is a factor of $x^2 + 10x + 24$.

Solution 2:

$p(x) = x^3 – 3x^2 + 4x + 4$
Put x = 2 in p(x), we get
$p(1) = (2)^3 – 3(2)^2 + 4(2) + 4$
$= 8 – 3(4) + 8 + 4$
$= 8 – 12 + 8 + 4$
$= 8 \neq 0$
As $p(2) \neq 0$,
∴ By the Factor Theorem (x – 2) is not a factor of $x^3 – 3x^2 + 4x + 4$.

Solution 3:

Let $p(x) = 2x^3 – 6x^2 + 5x + a$
Put x = 2 in p(x), we get
$p(2) = 2(2)^3 – 6(2)^2 + 5(2) + a$
$= 2(8) – 6(4) + 10 + a$
$= 16 – 24 + 10 + a$
$= 2 + a$
But p(2) must be 0, because (x – 2) is a factor
∴ 2 + a = 0
∴ a = -2