UNIT 1: Number System

CHAPTER 1

REAL NUMBERS

Syllabus

- Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, proofs of irrationality of $\sqrt{2}, \sqrt{3}$ and $\sqrt{5}$. Decimal representation of rational numbers in terms of terminating/non-terminating recurring decimals.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Questions based on HCF & LCM	1 Q (1 M)	1 Q (3 M)	1 Q (1 M) 3 Q (2 M)	1 Q (1 M) 1 Q (2 M) 2 Q (3 M)	2 Q (1 M) 2 Q (4 M)	2 Q (1 M)
Irrational numbers, Terminating and non-terminating Recurring Decimals	1 Q (2 M)		1 Q (1 M) 2 Q (3 M)	1 Q (3 M)		1 Q (4 M)

TOPIC - 1
Euclid's Division Lemma and Fundamental Theorem of Arithmetic

Revision Notes

- **Algorithm:** An **algorithm** is a series of well defined steps which gives a procedure for solving a mathematical problem.
- **Lemma:** A lemma is a proven statement used for proving another statement.
- **Euclid's Division Lemma:** For given positive integers a and b, there exist unique integers q and r, satisfying

$$a = bq + r \text{ where } 0 \leq r < b.$$

Here, a = Dividend, b = Divisor, q = Quotient and r = Remainder *i.e.*,

Dividend = (Divisor × Quotient) + Remainder

For example,

$$\begin{array}{rl} & \quad 1 \rightarrow \text{Quotient} \\ \text{Divisor} \quad 3) & \overline{\quad 5 \rightarrow \text{Dividend}} \\ & \;-3 \\ & \overline{\quad 2 \rightarrow \text{Remainder}} \end{array}$$

As per Euclid's Division Lemma, $5 = (3 \times 1) + 2$

- **Euclid's division algorithm is applicable** for positive integers only but it can be extended for all integers except zero.
- When 'a' and 'b' are two positive integers then

$$a = bq + r, \text{ where } 0 \leq r < b,$$

- **Steps to find the HCF of two positive integers by Euclid's division algorithm:**

(i) Let two integers be a and b such that $a > b$.

(ii) Take greater number a as dividend and the smaller number b as divisor.

(iii) Now, find whole numbers 'q' and 'r' as quotient and remainder respectively.

$\therefore \quad a = bq + r$ where $0 \leq r < b$.

then $\quad$ HCF (a, b) = HCF (b, r).

(iv) If $r = 0$, b is the HCF of a and b. If $r \neq 0$, then take r as divisor and b as dividend.

(v) Repeat step (iii), till the remainder is zero, the divisor thus obtained at last stage is the required HCF.

- **The Fundamental Theorem of Arithmetic**

Every composite number can be expressed as a product of primes and this factorisation is unique, apart from the order in which the prime factors occur. Fundamental theorem of arithmetic is also called a **Unique Factorization Theorem.**

Composite number = Product of prime numbers

Or

Any integer greater than 1 can either be a prime number or can be written as a unique product of prime numbers.

e.g.,

(i) $2 \times 11 = 22$ is the same as $11 \times 2 = 22$.

(ii) 6 can be written as 2×3 or 3×2, where 2 and 3 are prime numbers.

(iii) 15 can be written as 3×5 or 5×3, where 3 and 5 are prime numbers.

The prime factorization of a natural number is unique, except to the order of its factors.

e.g., 12 detained by multiplying the prime numbers 2, 2 and 3 together,

$$12 = 2 \times 2 \times 3$$

We would probably write it as $\quad 12 = 2^2 \times 3$

- By using Fundamental Theorem of Arithmetic, we shall find the HCF and LCM of given numbers (two or more).

This method is also called **Prime Factorization Method.**

- **Prime Factorization Method to find HCF and LCM:**

(i) Find all the prime factors of given numbers.

(ii) HCF of two or more numbers = Product of the smallest power of each common prime factor, involved in the numbers.

(iii) LCM of two or more numbers = Product of the greatest power of each prime factor, involved in the numbers.

Know the Formulae

For two positive integers a and b, we have

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

and

$$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

Mnemonics

Euclid's Division Lemma ($a = bq + r$)

Alibaba's **b**est product **q**uotation is **a**ssent **r**eward.

Concept: $a = bq + r$

Interpretation:

Alibaba's A $= a$

best's B $= b$

quotation's Q $= q$

assent's A = addition of bq and r

reward's R $= r$

Then $a = b \times q + r$.

How is it done on the GREENBOARD?

Q.1. Show that 6^n can never end with digit 0 for any natural number n.

Solution

Step I: Any number which ends in zero must have at least 2 and 5 as its prime factors.

Step II: Since, $6 = 2 \times 3$

Therefore, $6^n = (2 \times 3)^n \Rightarrow 2^n \times 3^n$

Hence, prime factors of 6 are 2 and 3.

Step III: Since 6^n does not contain 5 as a prime factor, hence 6^n can never end in 0.

Step IV: Since 6^n does not contain 5 as a prime factor, hence 6^n can never end in 0.

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. If $xy = 180$ and HCF $(x, y) = 3$, then find the LCM (x, y)** A [CBSE SQP, 2020-21]

Sol. We know that,

"LCM × HCF = Product of two numbers".

(LCM) (3) = 180 ½

LCM = 60. ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Product of two numbers,

$$xy = 180$$

$$\text{HCF}(x, y) = 3$$

By using fundamental theorem,

$$\text{HCF}(x, y) \times \text{LCM}(x, y) = x \times y$$

$$\Rightarrow 3 \times \text{LCM}(x, y) = 180$$

$$\Rightarrow \text{LCM}(x, y) = 60.$$

Q. 2. Find the total number of factors of prime numbers. U [CBSE Delhi Set-I, 2020]

Sol. We have only two factors (1 and number itself) of any prime number, such as:

$2 = 2 \times 1$ (2 and 1)

$3 = 3 \times 1$ (3 and 1) ....etc. 1

Q. 3. Find the HCF and the LCM of 12, 21 and 15. U [CBSE Delhi Set-I, 2020]

Sol. Prime factors of 12 $= 2 \times 2 \times 3$

Prime factors of 21 $= 3 \times 7$

and Prime factors of 15 $= 3 \times 5$

$\therefore$ HCF of 12, 21 and 15 = 3. ½

and LCM of 12, 21 and 15 $= 2 \times 2 \times 3 \times 5 \times 7$

$= 420$. ½

AI Q. 4. **Find the sum of exponents of prime factors in the prime factorisation of 196.**

U [CBSE OD Set-I, 2020]

Sol. Prime factors of 196 $= 2^2 \times 7^2$ ½

The sum of exponents of prime factors

$= 2 + 2 = 4.$ ½

Q. 5. Complete the following statements:

Euclid's Division Lemma states that for two positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, where ....

R [CBSE OD Set-I, 2020]

Sol. For given positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$ where $0 \le r < b$. [CBSE Marking Scheme, 2020]

AI Q. 6. **Find the LCM of smallest two digit composite number and smallest composite number.**

U [CBSE SQP, 2020]

Sol. Since, the smallest composite number = 4

and smallest 2 digit composite number = 10 ½

$\therefore$ LCM of 4 and 10.

$$\begin{aligned} 4 &= 2 \times 2 \\ 10 &= 5 \times 2 \\ \text{L.C.M.} &= 2 \times 5 \times 2 \\ &= 20 \end{aligned}$$

$\therefore$ LCM of 4 and 10 = 20 ½

COMMONLY MADE ERROR

- Sometimes students assume smallest composite number as 2 as they get confused in prime number and composite number.

ANSWERING TIP

- Understand the clear difference in prime numbers and composite numbers.

Q. 7. Express 429 as a product of its prime factors. A + R [CBSE Delhi Set-I, 2019]

Topper Answer, 2019

Sol.

429 can be expressed as -

$$\begin{array}{r|l} 3 & 429 \\ 11 & 143 \\ 13 & 13 \\ & 1 \end{array}$$

$429 = 3 \times 11 \times 13.$

Q. 8. Two positive integers a and b can be written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers. Find LCM (a, b).

A [CBSE Delhi Set-III, 2019]

Sol. LCM $(x^3y^2, xy^3) = x^3y^3$. 1

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, positive integers are

$$a = x^3y^2$$

and $$b = xy^3$$

Here, x and y are prime numbers

$\therefore$ LCM (a, b) = product of greatest power of x and y.

$= x^3y^3$

Q. 9. If HCF (336, 54) = 6, find LCM (336, 54) A

[CBSE OD Set-I, II, III, 2019]

Sol. Since, HCF × LCM = Product of numbers

$$6 \times \text{LCM} = 336 \times 54$$

$$\text{LCM} = \frac{336 \times 54}{6}$$

$$\text{LCM} = 56 \times 54$$

$$\text{LCM} = 3024$$

[CBSE Marking Scheme, 2019] 1

AI Q. 10. **What is the HCF of smallest prime number and the smallest composite number ?**

A + R [CBSE Delhi, OD, 2018]

Sol. The required numbers are 2 and 4 and the HCF of 2 and 4 is 2. [CBSE Marking Scheme, 2018] 1

Detailed Solution:

Topper Answer, 2018

Smallest prime = 2

Smallest composite = 4

HCF (2, 4) = 2

$\boxed{\text{The HCF of the smallest prime and smallest composite is 2.}}$ 1

Q. 11. Explain why 13233343563715 is a composite number ? R [Board Term-I, 2016]

Sol. Since, the given number ends in 5. Hence, it is a multiple of 5. Therefore, it is a composite number.

[CBSE Marking Scheme, 2016] **1**

Short Answer Type Questions-I

2 marks each

AI **Q. 1. 3 bells ring at an interval of 4, 7 and 14 minutes. All three bells rang at 6 am, when the three bells will be ring together next ?**

C + A [CBSE SQP, 2020-21]

Sol. We know that,

The three bells again ring together on that time which is the LCM of individual time of each bell

$$4 = 2 \times 2$$
$$7 = 7 \times 1$$
$$14 = 2 \times 7$$
$$\text{LCM} = 2 \times 2 \times 7 = 28$$

The three bells will ring together again at 6: 28 am

[CBSE Marking Scheme, 2020-21] **2**

Q. 2. If HCF of 65 and 117 is expressible in the form $65n - 117$, then find the value of n. A + R [CBSE Delhi, 2019]

Topper Answer, 2019

Sol. Using Euclid's Division Lemma (states that $a = bq + r$, $0 \le r < b$) we can find the HCF of 65 and 117.

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$52 = 13 \times 4 + 0$

$\therefore$ HCF of 65, 117 = 13

But,

$65n - 117 = 13$

$\Rightarrow \quad 65n = 13 + 117$

$\Rightarrow \quad n = \dfrac{\cancel{130}\ 10}{\cancel{65}\ 5}$

$\Rightarrow \quad \boxed{n = 2}$

Q. 3. Find the HCF of 1260 and 7344 using Euclid's algorithm. A [CBSE Delhi Set-I, II, III 2019]

Sol. Using Euclid's Algorithm

$$7344 = 1260 \times 5 + 1044$$
$$1260 = 1044 \times 1 + 216 \qquad ½$$
$$1044 = 216 \times 4 + 180$$
$$216 = 180 \times 1 + 36$$
$$180 = 36 \times 5 + 0 \qquad ½$$

$\therefore$ HCF of 1260 and 7344 is 36. **1**

[CBSE Marking Scheme, 2019]

COMMONLY MADE ERROR

- Some students find the HCF directly without using Euclid's algorithm.

ANSWERING TIP

- Practice more such questions based on Euclid's algorithm as it is the most important topic from examination perspective.

Q. 4. Show that every positive odd integer is of the form $(4q + 1)$ or $(4q + 3)$, where q is some integer. A [CBSE Delhi Set-I, 2019]

Sol. Using Euclid's Algorithm

$a = 4q + r, 0 \le r < 4$

$\Rightarrow a = 4q, a = 4q + 1, a = 4q + 2$ and $a = 4q + 3$. **1**

Now $a = 4q$ and $a = 4q + 2$ are even numbers. ½

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

As per Euclid's Division Lemma

Let positive integer be $b = 4$

If a and b are two positive integers,

then $a = bq + r$, where $0 \le r < b$.

Hence, $a = 4q + r$, where $0 \le r < 4$ ...(i)

Here, r can take values 0, 1, 2 or 3

If $r = 0$ and 2, a is an even integer.

If $r = 1$, eq (i) becomes

$$a = 4q + 1,$$

this will always be an odd integer,

If $r = 3$, eq (i) becomes

$$a = 4q + 3,$$
$$= 2(2q + 1) + 1$$

this will always be an odd integer,

Hence, any odd integer is of the form $4q+1$ or $4q + 3$.

Hence Proved.

Q. 5. Write the smallest number which is divisible by both 306 and 657. A [CBSE OD Set-I, II, III, 2019]

Sol. Smallest number divisible by 306 and 657

= LCM (306, 657) **1**

LCM (306, 657) = 22338 **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers

Using Euclid's Algorithm

$$657 = 306 \times 2 + 45$$
$$306 = 45 \times 6 + 36$$
$$45 = 36 \times 1 + 9$$
$$36 = 9 \times 4 + 0$$

$\therefore$ HCF(657, 306) = 9

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}(657, 306)}$$

$$= \frac{657 \times 306}{9} = 657 \times 34$$

LCM(657, 306) = 22338

Hence, the smallest number which is divisible by 306 and 657 is 22338.

Q. 6. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, find the other number. R [CBSE SQP 2019]

Sol. Since, HCF × LCM = Product of two numbers

Then, $9 \times 360 = 45 \times 2^{nd}$ number **1**

$$2^{nd} \text{ number} = \frac{(9 \times 360)}{45}$$

Thus, 2^{nd} number = 72 **1**

[CBSE SQP Marking Scheme, 2019]

AI **Q. 7. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, where a and b are prime numbers then verify.**

LCM (p, q) × HCF (p, q) = $p.q$.

A [CBSE SQP 2017]

Sol. Since, LCM $(p, q) = a^3b^3$ ½

and HCF $(p, q) = a^2b$ ½

Hence, LCM (p, q) × HCF (p, q) $= a^3b^3 \times a^2b$

$= a^5b^4$ ½

$= a^2b^3 \times a^3b$

$= pq$ **Hence Verified.** ½

AI **Q. 8. Explain whether 3× 12 × 101 + 4 is a prime number or a composite number.**

U [Board Term-I 2017, 2015]

Sol. $3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1)$ **1**

$= 4(909 + 1)$

$= 4(910)$

$= 2 \times 2 \times 2 \times 5 \times 7 \times 13$ **1**

= a composite number

[$\because$ Product of more than two prime factors]

[CBSE Marking Scheme, 2015]

AI **Q. 9. Find the HCF and LCM of 90 and 144 by the method of prime factorization.**

U [Board Term-I, 2016]

Sol. Since, $90 = 2 \times 3^2 \times 5$

and $144 = 2^4 \times 3^2$

Hence, HCF $= 2 \times 3^2 = 18$ **1**

and LCM $= 2^4 \times 3^2 \times 5 = 720$ **1**

Q. 10. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

A [Board Term-I, 2016]

Sol. Given, Length = 8 m 50 cm = 850 cm

breadth = 6 m 25 cm = 625 cm

height = 4 m 75 cm = 475 cm

Since, the length of the longest rod is equal to HCF of 850, 625 and 475

625) 850 (1
625
225) 625 (2
450
175) 225 (1
175
50) 175 (3
150
25) 50 (2 1½
50
0

HCF (625, 850) = 25

$\because$ 25 divides 475

i.e., 475 = 25 × 19

Hence, HCF (625, 850, 475) = 25 ½

Thus, the longest rod that can measure the dimensions of the room exactly = 25 cm.

[CBSE Marking Scheme, 2016]

COMMONLY MADE ERRORS

- Mostly candidates are unable to determine about what they have to find. Actually, most of the candidates don't get to know that the question is about HCF or LCM.
- Sometimes students calculate the longest length of rod that lies in the room by finding its diagonal.

ANSWERING TIPS

- Adequate practice is necessary for such type of questions and basic concept of HCF and LCM should be clear.
- Students should read the question properly.

Q. 11. Show that any positive even integer can be written in the form $6q$, $6q + 2$ or $6q + 4$, where q is an integer. A [Board Term-I, 2016]

Sol. Let a be any positive integer ½
By division algorithm
$a = 6q + r$, where $0 \le r < 6$
$\therefore \quad a = 6q, 6q + 1, 6q + 2, 6q + 3, 6q + 4$ or $6q+5$ ½
Here, $6q$, $6q + 2$, and $6q + 4$ are divisible by 2 and so $6q$, $6q + 2$, or $6q + 4$ are even positive integers. ½
Hence, a is an even integer and can be written as
$a = 6q, 6q + 2$, or $6q + 4$ ½
[CBSE Marking Scheme, 2016]

AI **Q. 12. Find the smallest natural number by which 1,200 should be multiplied so that the square root of the product is a rational number.** U [Board Term-I, 2015]

Sol. Since, $1200 = 4 \times 3 \times (2 \times 5)^2$ 1
$= 2^4 \times 3 \times 5^2$ 1
Hence, the required smallest natural number is 3.
[CBSE Marking Scheme, 2015]

Q. 13. Can two numbers have 15 as their HCF and 175 as their LCM ? Give reasons. R [Board Term-I, 2015]

Sol. Since, 15 does not divide 175 and
LCM of two numbers should be exactly divisible by their HCF. 1
$\therefore$ Two numbers cannot have their HCF as 15 and LCM as 175. 1

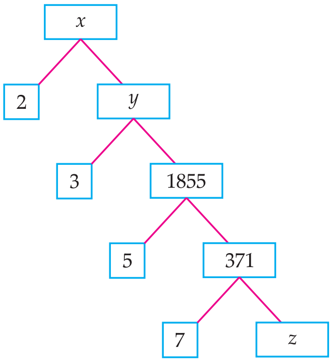
Q. 14. Check whether 4^n can end with the digit 0 for any natural number n. A [Board Term-I, 2015]

Sol. If the number 4^n, for any natural number n, were to end with the digit zero, then it would be divisible by 5 and 2.
That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because $4^n = (2)^{2n}$; so, the only prime in the factorization of 4^n is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. 2
[CBSE Marking Scheme, 2015]

Q. 15. Find HCF of the numbers given below:
k, $2k$, $3k$, $4k$ and $5k$, where k is a positive integer.
U [Board Term-I, 2015]

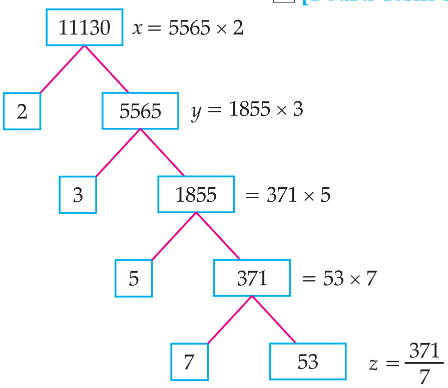
Sol. HCF of $\left.\begin{array}{l} k \\ k.2 \\ k.3 \\ k.2^2 \\ \text{and } k.5 \end{array}\right\}$ is k.
2
[CBSE Marking Scheme, 2015]

AI **Q. 16. Complete the following factor tree and find the composite number x:**



U [Board Term-I, 2015]

Sol.



$\therefore \quad x = 11{,}130$ 2

Short Answer Type Questions-II — 3 marks each

Q. 1. If HCF of 144 and 180 is expressed in the form $13m - 16$. Find the value of m. A [CBSE SQP, 2020]

Sol.
$180 = 144 \times 1 + 36$ 2
$144 = 36 \times 4 + 0$
$\therefore \quad \text{HCF}(180, 144) = 36$ 1
$36 = 13m - 16$
Solving, we get $m = 4$.
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given, HCF of 144 and 180 is in form of $13m - 16$,

Now, HCF of 144 and 180.

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^4 \times 3^2$$
$$180 = 2 \times 2 \times 3 \times 3 \times 5$$
$$= 2^2 \times 3^2 \times 5$$
$$\therefore \quad \text{HCF }(144, 180) = 2^2 \times 3^2$$
$$= 36$$

$\because$ HCF of 144 and 180 is given in the form of $13m - 16$,

$$\therefore \quad 13m - 16 = 36$$
$$13m = 36 + 16$$
$$13m = 52$$
$$m = \frac{52}{13}$$
$$m = 4$$

COMMONLY MADE ERROR

- Students often commit errors in finding HCF and LCM. Some students find HCF but used rule LCM.

ANSWERING TIP

- Understand the rule of the difference between LCM and HCF.

Q. 2. Using Euclid's Algorithm, find the HCF of 2048 and 960. U [CBSE OD Set-I, 2019]

Sol.

$$2048 = 960 \times 2 + 128 \qquad 1$$
$$960 = 128 \times 7 + 64 \qquad 1$$
$$128 = 64 \times 2 + 0$$

Hence, HCF (2048, 960) = 64. 1

[CBSE Marking Scheme, 2019]

AI **Q. 3. Using Euclid's Division Algorithm find the HCF of 726 and 275.** U [CBSE SQP, 2019]
[CBSE comptt. Set I, II, III, 2018]
[CBSE OD Set I, II, III, 2019]

Sol. Euclid's division lemma

$$726 = 275 \times 2 + 176 \qquad ½$$
$$275 = 176 \times 1 + 99 \qquad ½$$
$$176 = 99 \times 1 + 77 \qquad ½$$
$$99 = 77 \times 1 + 22 \qquad ½$$
$$77 = 22 \times 3 + 11 \qquad ½$$
$$22 = 11 \times 2 + 0$$
$$\text{Thus, HCF} = 11 \qquad ½$$

[CBSE SQP Marking Scheme, 2018]

Q. 4. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.
R + U [CBSE Delhi, OD, 2018]

Sol. Since, $404 = 2 \times 2 \times 101 = 2^2 \times 101$

and $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$\therefore$ HCF of 404 and 96 $= 2^2 = 4$ 1

LCM of 404 and 96 $= 101 \times 2^5 \times 3 = 9696$

HCF × LCM $= 4 \times 9696 = 38784$ 1

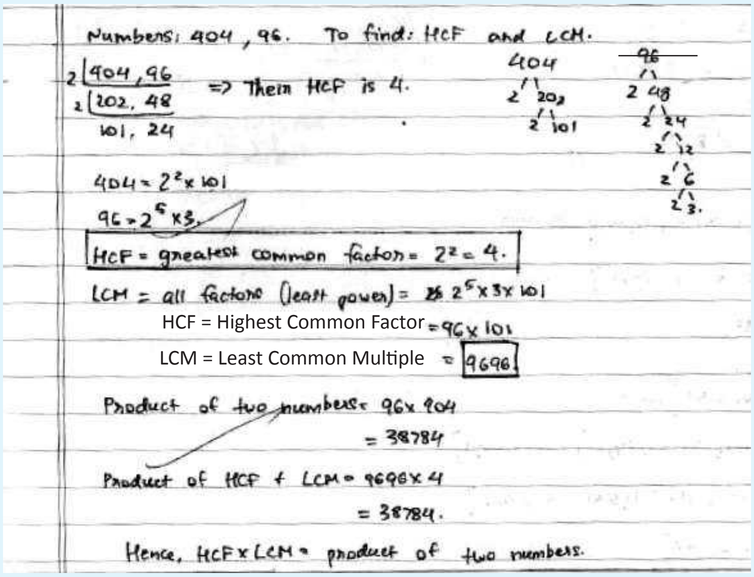
Also, $404 \times 96 = 38784$

Hence, HCF × LCM = Product of 404 and 96. **Hence Verified. 1**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018


Numbers: 404, 96. To find: HCF and LCM.

2 | 404, 96
2 | 202, 48
101, 24

=> Their HCF is 4.

404 = $2^2 \times 101$

96 = $2^5 \times 3$

HCF = greatest common factor = $2^2 = 4$.

LCM = all factors (least power) = $2^5 \times 3 \times 101$

HCF = Highest Common Factor = 96×101

LCM = Least Common Multiple = $\boxed{9696}$

Product of two numbers = 96×404

= 38784

Product of HCF + LCM = 9696×4

= 38784.

Hence, HCF × LCM = product of two numbers.

AI **Q. 5. Show that exactly one of the number n, $n + 2$ or $n + 4$ is divisible by 3.** A [CBSE SQP 2017]

Sol. Let n be any positive integer and $b = 3$

Then, $n = 3q + r$

where, q is the quotient and r is the remainder and $0 \le r < 3$

So, the remainders may be 0, 1 or 2 and n may be in the form of $3q$, $3q + 1$, $3q + 2$

Let $n = 3q$, $3q + 1$ or $3q + 2$.

(i) When $n = 3q$

$\Rightarrow n$ is divisible by 3.

$n + 2 = 3q + 2$

$\Rightarrow n + 2$ is not divisible by 3.

$n + 4 = 3q + 4 = 3(q + 1) + 1$

$\Rightarrow n + 4$ is not divisible by 3. **1**

(ii) When $n = 3q + 1$

$\Rightarrow n$ is not divisible by 3.

$n + 2 = (3q + 1) + 2 = 3q + 3 = 3(q + 1)$

$\Rightarrow n + 2$ is divisible by 3.

$n + 4 = (3q + 1) + 4 = 3q + 5 = 3(q + 1) + 2$

$\Rightarrow n + 4$ is not divisible by 3. **1**

(iii) When $n = 3q + 2$

$\Rightarrow n$ is not divisible by 3.

$n + 2 = (3q + 2) + 2 = 3q + 4 = 3(q + 1) + 1$

$\Rightarrow$ $n + 2$ is not divisible by 3.

$n + 4 = (3q + 2) + 4 = 3q + 6 = 3(q + 2)$

$\Rightarrow$ $n + 4$ is divisible by 3.

Hence, exactly one of the numbers n, $n + 2$ or $n + 4$ is divisible by 3. **1**

Q. 6. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm. U [Board Term-I, 2016]

Sol. Since,

$$324 = 252 \times 1 + 72$$

$$252 = 72 \times 3 + 36 \quad \textbf{1}$$

$$72 = 36 \times 2 + 0$$

$\therefore$ $\text{HCF}(324, 252) = 36$ **1**

$$180 = 36 \times 5 + 0$$

$\therefore$ $\text{HCF}(36, 180) = 36$ **1**

$\therefore$ HCF of 180, 252 and 324 is 36.

[CBSE Marking Scheme, 2016]

AI **Q. 7. Find the greatest number of six digits exactly divisible by 18, 24 and 36.** A [Board Term-I, 2016]

Sol. LCM of 18, 24 and 36

$$18 = 2 \times 3^2$$

$$24 = 2^3 \times 3$$

$$36 = 2^2 \times 3^2$$

$$\text{LCM}(18, 24, 36) = 2^3 \times 3^2$$

$$= 72 \quad \textbf{1}$$

The largest 6 digit number is 999999 **1**

$$\begin{array}{r|l} & 13888 \quad \text{Quotient} \\ 72 & 999999 \\ & -72 \\ & 279 \\ & -216 \\ & 639 \\ & -576 \\ & 639 \\ & -576 \\ & 639 \\ & -576 \\ & 63 \rightarrow \text{Remainder} \end{array}$$ **½**

$\therefore$ The required number $= 999999 - 63 = 999936$. **½**

[CBSE Marking Scheme, 2016]

Q. 8. Use Euclid division lemma to show that the square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$ for some integer m. A [Board Term-I, 2015]

Sol. Let n be any positive integer.

By Eucild's division lemma, $n = 5q + r$, where, $0 \le r < 5$

Then, $n = 5q, 5q + 1, 5q + 2, 5q + 3$ or $5q + 4$, where $q \in W$ **1**

q is a whole number.

now $n^2 = (5q)^2 = 25q^2 = 5(5q^2) \Rightarrow 5m$

where $m = 5q^2$

$n^2 = (5q + 1)^2 = 25q^2 + 10q + 1$

$= 5(5q^2 + 2q) + 1 = 5m + 1$

where $m = 5q^2 + 2q$

$n^2 = (5q + 2)^2 = 25q^2 + 20q + 4$

$= 5(5q^2 + 4q) + 4 = 5m + 4$

where $m = 5q^2 + 4q$

$n^2 = (5q + 3)^2 = 25q^2 + 30q + 9$

$= 5(5q^2 + 6q + 1) + 4 = 5m + 4$

where $m = 5q^2 + 6q + 1$

$n^2 = (5q + 4)^2 = 25q^2 + 40q + 16$

$= 5(5q^2 + 8q + 3) + 1 = 5m + 1$

where $m = 5q^2 + 8q + 3$

Thus, square of any positive integer cannot be of the form $5m + 2$ or $5m + 3$. **2**

Long Answer Type Questions — 5 marks each

AI **Q. 1. Show that the square of any positive integer can not be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q.** A [CBSE Delhi Set-I, 2020]

Sol. See Q.8. from SATQ-II Page 10.

Q. 2. Prove that one of every three consecutive positive integers is divisible by 3. A + R [CBSE Delhi Set-I, 2020]

Sol. Let n be any positive integer.

$\therefore$ $n = 3q + r$, where $r = 0, 1, 2$ **1**

Putting $r = 0$,

$n = 3q + 0 = 3q$,

which is divisible by 3. **1**

Putting $r = 1$,

$n = 3q + 1$,

which is not divisible by 3. **1**

Putting $r = 2$,

$n = 3q + 2$,

which is not divisible by 3. **1**

Hence, one of every three consecutive positive integers is divisible by 3. **1**

Hence Proved.

[CBSE Marking Scheme, 2020]

Q. 3. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization method. R [Board Term-I, 2016]

Sol. Fundamental theorem of arithmetic: Every composite number can be expressed as the product of powers of primes and this factorization is unique.

Since, $2520 = 2^3 \times 3^2 \times 5 \times 7$ **1½**

and $10530 = 2 \times 3^4 \times 5 \times 13$ **1**

$\therefore$ LCM $= 2^3 \times 3^4 \times 5 \times 7 \times 13$

$= 294840$ **2½**

[CBSE Marking Scheme, 2016]

Q. 4. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets.

A [Board Term-I, 2016]

Sol. HCF of 990 and 945

$$945\,)\,990\,(\,1$$
$$-945$$
$$45\,)\,945\,(\,21$$
$$-90$$
$$45$$
$$-45$$
$$0$$

2

$990 = 945 \times 1 + 45$

$945 = 45 \times 21 + 0$

Since, HCF of 990 and 945 is 45. **1½**

Thus, the fruit vendor should put 45 fruits in each basket to have minimum number of baskets. **1½**

[CBSE Marking Scheme, 2016]

Q. 5. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

A [Board Term-I, 2015]

Sol. If 6^n ends with 0 or 5, then it must have 5 as a factor.

But only prime factors of 6^n are 2 and 3.

$\therefore 6^n = (2 \times 3)^n = 2^n \times 3^n$ **2**

From the fundamental theorem of arithmetic, the prime factorization of every composite number is unique. **1½**

$\therefore 6^n$ can never end with 0 or 5. **1½**

[CBSE Marking Scheme, 2015]

Q. 6. For any positive integer n, prove that $n^3 - n$ is divisible by 6. A [Board Term-I, 2015]

Sol. $n^3 - n = n(n^2 - 1)$

$= n(n-1)(n+1)$

Let $a = 3q + r$

where $r = 0, 1, 2$

Case I: When $r = 0$, $a = 3q$

$n^3 - n = n(n-1)(n+1)$

$= 3q(3q-1)(3q+1)$

$= 3\,m$

where $m = q(3q-1)(3q+1)$

$\therefore (n^3 - n)$ is divisible by 3

Case II: When $r = 1$, $a = 3q + 1$

$n^3 - n = n(n-1)(n+1)$

$= (3q+1)(3q)(3q+2)$

$= 3q(3q+1)(3q+2)$

$= 3\,m$

where $m = q(3q+1)(3q+2)$

$\therefore n^3 - n$ is divisible by 3

Case III: When $r = 2$, $a = 3q + 2$

$n^3 - n = n(n-1)(n+1)$

$= (3q+2)(3q+1)(3q+3)$

$= 3(q+1)(3q+2)(3q+1)$

$= 3\,m$

where $m = (q+1)(3q+2)(3q+1)$

$\therefore n^3 - n$ is divisible by 3

Let $a = 2q + r$

where $r = 0, 1$

Case I: When $r = 0$

$n^3 - n = n(n-1)(n+1)$

$= 2q(2q-1)(2q+1)$

$= 2\,m$

where $m = q(2q-1)(2q+1)$

$\therefore n^3 - n$ is divisible by 2

Case II: When $r = 1$ $a = 2q + 1$

$n^3 - n = n(n-1)(n+1)$

$= 2q(2q+1)(2q+2)$

$= 2\,m$

where $m = q\,(2q+1)\,(2q+2)$

So we can say that one of the numbers among $n,(n-1)$ and $(n-1)$ is always divisible by 2 and 3

$\therefore$ As per the divisibility rule of 6

The given number is divisible by 6

$\therefore n^3 - n$ is divisible by 6 **Hence Proved**

TOPIC - 2
Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

Revision Notes

- **Rational Numbers:** A number in the form $\frac{p}{q}$, where p and q are co-prime numbers and $q \neq 0$, is known as rational number.

  For example: $2, -3, \frac{3}{7}, -\frac{2}{5}$, etc. are rational numbers.

- **Irrational Numbers:** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ are irrational numbers.
- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- **Terminating Decimals:** If decimal expansion of rational number $\frac{p}{q}$ comes to an end, then the decimal obtained from $\frac{p}{q}$ is called terminating decimal.
- **Non-terminating Repeating (or Recurring) Decimals:** The decimal expansion obtained from $\frac{p}{q}$ repeats periodically, then it is called non-terminating repeating (or recurring) decimal.
- Just divide the numerator by the denominator of a fraction. If you end up with a remainder of 0, you have a terminating decimal otherwise repeating or recurring decimal.
- The sum or difference of a rational and an irrational number is irrational.
- The product and quotient of a non-zero rational and an irrational number is irrational.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which terminates.
- Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in the form $\frac{p}{q}$, where p and q are co-primes and the prime factorization of q is of the form 2^m5^n, where m and n are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^m5^n, where n and m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating.

How is it done on the GREENBOARD?

Q.1. Show that $2\sqrt{3} + 5$ is an irrational number.

Solution

Step I: Let $2\sqrt{3} + 5$ be a rational number.

A rational number can be expressed as $\frac{a}{b}$, where $b \neq 0$ and a and b are integers.

Step II: Then, $2\sqrt{3} + 5 = \frac{a}{b}$

or $2\sqrt{3} = \frac{a}{b} - 5$

or $\sqrt{3} = \frac{1}{2}\left(\frac{a}{b} - 5\right)$

Here, R.H.S $= \frac{1}{2}\left(\frac{a}{b} - 5\right)$ is rational

while, L.H.S, $\sqrt{3}$ is irrational which is not possible.

Step III: Hence, our assumption that $2\sqrt{3} + 5$ is a rational number is wrong.

Hence, $2\sqrt{3} + 5$ is an irrational number.

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. The decimal representation of $\frac{14587}{2^1 \times 5^4}$ will terminate after how many decimal places ?**

R [CBSE SQP, 2020-21]

Sol. Four decimal places. 1

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

$$\frac{14587}{2\times5^4} = \frac{14587}{2\times5^4}\times\frac{2^3}{2^3}$$

$$= \frac{14587\times(2)^3}{10^4} = \frac{116696}{10000}$$

$$= 11.6696$$

Thus the given rational number terminates after four decimal places. **1**

AI **Q. 2. The decimal representation of $\frac{11}{2^3\times5}$ will terminate after how many decimal places ?**

R **[CBSE SQP, 2020-21]**

Sol. Since $$\frac{11}{2^3\times5} = \frac{11}{8\times5}$$

$$= \frac{11}{40} = \frac{11}{40}\times\frac{25}{25} = \frac{275}{1000}$$

$$= 0.275$$

Thus, $\frac{11}{2^3\times5}$ will terminate after 3 decimal places.**1**

COMMONLY MADE ERROR

- Students commit errors in converting the fraction into decimal.

ANSWERING TIP

- First convert the fraction into decimal, then give the answer.

Q. 3. Write one rational and one irrational number lying between 0.25 and 0.32. A **[CBSE SQP, 2020]**

Sol. Rational number $= 0.30$ ½

Irrational number $= 0.3010203040......$ ½

Or any other correct rational and irrational number.

[CBSE Marking Scheme, 2020]

Q. 4. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

A **[CBSE Delhi Set- I, II, III, 2019]**

Sol. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.)

e.g., 1.5, 1.6, 1.63 etc.

[CBSE Marking Scheme, 2019] 1

Detailed Solution:

Since, $\sqrt{2} = 1.414 ....$

and $\sqrt{3} = 1.732 ....$

Now, we can write '*n*' rational numbers between them *e.g.*, just greater than 1.414 and less than 1.732 and it should be terminating or not terminating recurring.

e.g., 1.415659, 1.416893, 1.715644, ...

Therefore, one rational number between $\sqrt{2}$ and $\sqrt{3}$ is 1.416893.

Q. 5. After how many decimal places will the decimal expansion of $\frac{23}{2^4\times5^3}$ terminate ?

R **[CBSE SQP, 2018]**

Sol. 4 places. **1**

[CBSE SQP Marking Scheme, 2018]

Detailed Solution:

$$\frac{23}{2^4\times5^3} = \frac{23\times5}{2^4\times5^3\times5} = \frac{23\times5}{2^4\times5^4} \quad ½$$

$$= \frac{115}{(10)^4} = \frac{115}{10000} = 0.0115$$

Hence, $\frac{23}{2^4\times5^3}$ will terminate after 4 decimal places. ½

COMMONLY MADE ERROR

- Sometimes students make errors in converting the denominator in the form of $2^m \times 5^n$.

ANSWERING TIP

- Students should carefully do the simplification so as to avoid wastage of time.

AI **Q. 6. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives an irrational or a rational number.**

U **[CBSE Comptt. Set I, II, III 2018]**

Sol. For writing $\frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$ ½

$= 6$ which is rational. ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times5}+3\sqrt{4\times5}}{2\sqrt{5}}$$

$$\Rightarrow \quad = \frac{2\times3\sqrt{5}+3\times2\sqrt{5}}{2\sqrt{5}}$$

$$\Rightarrow \quad = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$$

$$\Rightarrow \quad = \frac{(6+6)\sqrt{5}}{2\sqrt{5}}$$

$$\Rightarrow \quad = \frac{12\sqrt{5}}{2\sqrt{5}} = 6$$

which is a rational number.

Q. 7. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal. U [CBSE SQP, 2017]

Sol.
$$\frac{7}{75} = \frac{7}{3 \times 5^2}$$

Since, denominator of given rational number is not of form $2^m \times 5^n$.

Hence, it is non-terminating recurring decimal expansion. **1**

Q. 8. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal. A [Board Term-I, 2016]

Sol. Since, $\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01$.

Thus, smallest rational number is $\frac{7}{100}$

[CBSE Marking Scheme, 2016] **1**

Q. 9. What type of decimal expansion does a rational number has ? How can you distinguish it from decimal expansion of irrational numbers ? R [Board Term-I, 2016]

Sol. A rational number may has its decimal expansion either terminating or non-terminating repeating. An irrational number has its decimal expansion non-repeating and non-terminating.

[CBSE Marking Scheme, 2016] **1**

Short Answer Type Questions-I — 2 marks each

Q. 1. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number. A [CBSE Delhi OD 2018] [CBSE Comptt. Set I, II, III, 2018]

Sol. Let us assume $\left(5+3\sqrt{2}\right)$ is a rational number.

$$\therefore \quad 5+3\sqrt{2} = \frac{p}{q} \qquad \tfrac{1}{2}$$

(where, $q \neq 0$ and p and q are integers)

$$\Rightarrow \quad \sqrt{2} = \frac{p-5q}{3q} \qquad \tfrac{1}{2}$$

This contradicts the given fact that $\sqrt{2}$ is irrational.

Hence, $(5+3\sqrt{2})$ is an irrational number. [CBSE Marking Scheme, 2018] **1**

Detailed Solution:

Topper Answer, 2018

Given, $\sqrt{2}$ is irrational

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. So it is in form $\frac{a}{b}$. ~~for a, b ∈ R, b≠0~~ [$a, b \in Z$, $b \neq 0$, HCF$(a, b) = 1$]

$\Rightarrow 5+3\sqrt{2} = \frac{a}{b}$

$3\sqrt{2} = \frac{a}{b} - 5$

$3\sqrt{2} = \frac{a-5b}{b}$

$\sqrt{2} = \frac{a-5b}{3b}$

This shows that $\sqrt{2}$ is ~~irrational~~ rational ($a-5b$ and $3b$ are integers).

But we know that $\sqrt{2}$ is irrational

This contradicts our assumption that $5+\sqrt{2}\times 3$ is rational.

$\Rightarrow 5+3\sqrt{2}$ is irrational, hence proved.

Q. 2. Show that $7-\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational. U [SQP 2018-19]

Sol. Let us assume, to the contrary that $7-\sqrt{5}$ is rational.

$$7-\sqrt{5} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are co-prime and } q \neq 0$$ **1**

$$\Rightarrow \quad \sqrt{5} = \frac{7q-p}{q}$$

$\frac{7q-p}{q}$ is rational $=$ $\sqrt{5}$ is rational. which is contradictory.

Hence, $7-\sqrt{5}$ is irrational. **1**

[CBSE Marking Scheme, 2018]

Q. 3. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$. U

Sol. Let, $x = 0.3\overline{178}$

or $x = 0.3178178178$ ½

Now, $10{,}000\,x = 3178.178178$ ...(i)

and $10\,x = 3.178178$ ...(ii) ½

Subtracting, equation (ii) from eq. (i), we get

$9990\,x = 3175$ ½

or $x = \frac{3175}{9990}$

Hence, $x = \frac{635}{1998}$. ½

Short Answer Type Questions-II

3 marks each

AI **Q. 1. Prove that $2-\sqrt{3}$ is irrational, given that $\sqrt{3}$ is irrational.** A [CBSE SQP, 2020-21]

Sol. Let $2-\sqrt{3}$ be a rational number ½

We can find co-prime numbers a and b ($b \neq 0$) such that

$$2-\sqrt{3} = \frac{a}{b}$$ ½

$$2-\frac{a}{b} = \sqrt{3}$$ ½

So, we get $\frac{2b-a}{b} = \sqrt{3}$

Since a and b are integers, we get $\frac{2b-a}{b} = \sqrt{3}$ is irrational and so $\sqrt{3}$ is rational. But $\sqrt{3}$ is an irrational number. ½

But rational number cannot be equal to an irrational number.

Which contradicts our statement ½

Therefore $2-\sqrt{3}$ is irrational. ½

[CBSE Marking Scheme, 2020-21]

AI **Q. 2. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5}-3$ is an irrational number.** A [CBSE SQP- 2020]

Sol. Let us assume, to the contrary, that $2\sqrt{5}-3$ is a rational number

$\therefore$ $2\sqrt{5}-3 = \frac{p}{q}$, where p and q are integers and $q \neq 0$

$$\Rightarrow \quad \sqrt{5} = \frac{p+3q}{2q} \quad .....(1)$$

Since p and q are integers

$\therefore$ $\frac{p+3q}{2q}$ is a rational number **1**

$\therefore$ $\sqrt{5}$ is a rational number which is contradiction as $\sqrt{5}$ is an irrational number **1**

Hence our assumption is wrong and hence $2\sqrt{5}-3$ is an irrational number.

[CBSE SQP Marking Scheme, 2020] **1**

Q. 3. Prove that $\sqrt{3}$ is an irrational number. A [CBSE Board, 2019]

Topper Answer, 2019

Sol. Let us assume, if possible, that $\sqrt{3}$ is rational

Then, $\sqrt{3}$ can be expressed as $\frac{p}{q}$ where ($q \neq 0$) and p, q are coprimes [HCF$(p,q)=1$]

$\therefore \sqrt{3} = \frac{p}{q}$ [$p, q \in \mathbb{Z}$; HCF$(p,q) = 1$]

On squaring both sides,

$$3 = \frac{p^2}{q^2}$$

$\Rightarrow p^2 = 3q^2.$ —— ①

3 divides p^2

$\therefore$ 3 divides p.

Then, p can be written as;

$p = 3a$ for some integer 'a'.

On squaring,

$p^2 = 9a^2$

Put $p = 3q^2$ from ①

$\Rightarrow \cancel{3}q^2 = \overset{3}{\cancel{9}}a^2$

$\Rightarrow q^2 = 3a^2.$

3 divides q^2

$\therefore$ 3 divides q.

$\therefore$ 3 divides both p and q, 3 is a common factor of p and q.

But, p and q are co-primes.

Therefore, our assumption is wrong

$\therefore$ $\sqrt{3}$ is irrational. **3**

AI **Q. 4. Prove that $2+5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.**

A **[CBSE OD, Set-I, 2019]**

Sol. Let $2+5\sqrt{3} = a$, where 'a' is a rational number. **1**

Then, $\sqrt{3} = \frac{a-2}{5}$ **1**

Which is a contradiction as LHS is irrational and RHS is rational

$\therefore$ $2+5\sqrt{3}$ can not be rational.

Hence, $2+5\sqrt{3}$ is irrational. **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let us assume $2+5\sqrt{3}$ be a rational number.

Let $2+5\sqrt{3} = \frac{a}{b}$

[$b \neq 0$; a and b are integers]

$\Rightarrow \sqrt{3} = \frac{1}{5}\left(\frac{a}{b}-2\right)$ **1**

$\therefore$ $\frac{1}{5}\left(\frac{a}{b}-2\right)$ is a rational number.

But this contradicts the fact that $\sqrt{3}$ is an irrational number.

So, our assumption is wrong.

Therefore, $2+5\sqrt{3}$ is an irrational number. **2**

Q. 5. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

A [CBSE Delhi Set-III, 2019]

Sol. Let us assume $\frac{2+\sqrt{3}}{5}$ be a rational number.

Let $\frac{2+\sqrt{3}}{5} = \frac{a}{b}$ ($b \neq 0$, a and b are integers)

$\Rightarrow \quad \sqrt{3} = \frac{5a-2b}{b}$ **1**

$\Rightarrow a, b$ are integers

$\therefore \quad \frac{5a-2b}{b}$ is a rational number **1**

i.e., $\sqrt{3}$ is a rational number

which contradicts the fact that $\sqrt{3}$ is irrational

Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number. **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let $\frac{2+\sqrt{3}}{5}$ is a rational number

Therefore, we can write it in the form of $\frac{p}{q}$.

$$\therefore \quad \frac{2+\sqrt{3}}{5} = \frac{p}{q}$$

$$\Rightarrow \quad 2+\sqrt{3} = \frac{5p}{q}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5p}{q} - 2$$

$$\Rightarrow \quad \sqrt{3} = \frac{5p-2q}{q}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number.

But this contradicts the fact that $\sqrt{3}$ is an irrational number.

So, our assumption is wrong.

Therefore, $\frac{2+\sqrt{3}}{5}$ is an irrational number,

Hence Proved.

AI **Q. 6. Prove that $\sqrt{2}$ is an irrational number.**

U [CBSE Delhi Set-I, II, III, 2019]

[Borad Term-I, 2015]

Sol. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

So $\quad \sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$ **1**

Thus a^2 is a multiple of 2

$\Rightarrow a$ is a multiple of 2. ½

Let $a = 2m$ for some integer m

$\therefore \quad b^2 = 2m^2$ ½

Thus b^2 is a multiple of 2

$\Rightarrow b$ is a multiple of 2 ½

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence 2 is an irrational number. ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Let $\sqrt{2}$ be a rational number, Then, $\sqrt{2} = \frac{p}{q}$, where p, q are integers, $q \neq 0$.

If HCF $(p, q) \neq 1$, then by dividing p and q by HCF (p, q), $\sqrt{2}$ can be reduced as

$$\sqrt{2} = \frac{a}{b}, \text{ where HCF } (a, b) = 1 \quad \text{....(i)}$$

$\Rightarrow \quad \sqrt{2}\,b = a$

$\Rightarrow \quad 2b^2 = a^2$

$\Rightarrow \quad a^2$ is divisible by 2

$\Rightarrow \quad a$ is divisible by 2 .....(ii)

$\Rightarrow \quad a = 2c$, where c is an integer

$\Rightarrow \quad \sqrt{2}b = 2c$

$\Rightarrow \quad 2b^2 = 4c^2$

$\Rightarrow \quad b^2 = 2c^2$

$\Rightarrow \quad b^2$ is divisible by 2

$\Rightarrow \quad b$ is divisible by 2 .....(iii)

From (ii) and (iii), 2 is a common factor of a and b, which contradicts (i).

So, $\sqrt{2}$ is an irrational number. **Hence Proved.**

Long Answer Type Questions — 5 marks each

AI **Q. 1. Prove that $\sqrt{5}$ is an irrational number.**

A [CBSE OD Set-I, 2020]

Sol. Let $\sqrt{5}$ be a rational number.

$$\therefore \quad \sqrt{5} = \frac{p}{q}, \qquad 1$$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get

$$5 = \frac{p^2}{q^2}$$

or $\quad p^2 = 5q^2 \qquad$ 1

$\because$ p^2 is divisible by 5

$\therefore$ p is divisible by 5

Let $p = 5r$ for some positive integer r,

$$p^2 = 25r^2 \qquad 1$$

$$\therefore \quad 5q^2 = 25r^2$$

or $\quad q^2 = 5r^2$

$\because$ q^2 is divisible by 5

$\therefore$ q is divisible by 5.

Here p and q are divisible by 5, which contradicts the fact that p and q are co-primes.

Hence, our assumption is false

$\therefore$ $\sqrt{5}$ is an irrational number. **Hence Proved. 2**

[CBSE Marking Scheme, 2020]

Q. 2. Prove that $\sqrt{p}+\sqrt{q}$ is an irrational, where p and q are primes. A

Sol. We prove this by using the method of contradiction.

Assume that $\sqrt{p}+\sqrt{q}$ is a rational number.

Then, $\sqrt{p}+\sqrt{q} = \frac{a}{b}$, where HCF $(a, b) = 1 \qquad$ ½

Squaring both sides, we get

$$\left(\sqrt{p}+\sqrt{q}\right)^2 = \frac{a^2}{b^2} \qquad 1$$

$$p+q+2\sqrt{pq} = \frac{a^2}{b^2} \qquad ½$$

$$2\sqrt{pq} = \frac{a^2}{b^2}-p-q$$

$$\sqrt{pq} = \frac{1}{2}\left(\frac{a^2}{b^2}-p-q\right) \qquad 1$$

Since $\sqrt{pq}$ is an irrational number whereas $\frac{1}{2}\left(\frac{a^2}{b^2}-p-q\right)$ is a rational number. 1

So, $\sqrt{pq} = \frac{1}{2}\left(\frac{a^2}{b^2}-p-q\right)$ contradicts our assumption.

Hence, $\sqrt{p}+\sqrt{q}$ is an irrational number. 1

Q. 3. Prove that $\left(\sqrt{2}+\sqrt{3}\right)$ is an irrational number. A

Sol. Refer Q.2. from LAT Q.

Visual Case Based Questions — 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

Q. 1. To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

[CBSE QB, 2021]

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

(a) 144 **(b) 128**

(c) 288 **(d) 272**

Sol. Correct option: **(c).**

Explanation: We have to find the LCM of 32 and 36.

$$\text{LCM}(32, 36) = 25 \times 32 = 288$$

Hence, the minimum number of books required to distribute equally among students of section A and section B are 288.

(ii) **If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32 , 36) is**

(a) 2 (b) 4
(c) 6 (d) 8

Sol. Correct option: **(b).**

(iii) **36 can be expressed as a product of its primes as**

(a) $2^2 \times 3^2$ (b) $2^1 \times 3^3$
(c) $2^3 \times 3^1$ (d) $2^0 \times 3^0$

Sol. Correct option: **(a).**

(iv) **$7 \times 11 \times 13 \times 15 + 15$ is a**

(a) Prime number
(b) Composite number
(c) Neither prime nor composite
(d) None of the above

Sol. Correct option: **(b).**

(v) **If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is**

(a) ab (b) a^2b^2
(c) a^3b^2 (d) a^3b^3

Sol. Correct option: **(b).**

Explanation: Given, $p = ab^2 = a \times b \times b$

$$q = a^2b = a \times a \times b$$

$$\text{LCM of } (p, q) = a^2b^2$$

Q. 2. A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. **[CBSE QB, 2021]**

(i) **In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number participants that can accommodated in each room are**

(a) 14 (b) 12
(c) 16 (d) 18

Sol. Correct option: **(b).**

Explanation: No. of participants seated in each room would be HCF of all the three values above.

$$60 = 2 \times 2 \times 3 \times 5$$
$$84 = 2 \times 2 \times 3 \times 7$$
$$108 = 2 \times 2 \times 3 \times 3 \times 3$$

Hence, HCF = 12.

(ii) **What is the minimum number of rooms required during the event?**

(a) 11 (b) 31
(c) 41 (d) 21

Sol. Correct option: **(d).**

Explanation: Minimum no. of rooms required are total number of students divided by number of students in each room.

$$\text{No. of rooms} = \frac{60+84+108}{12}$$
$$= 21$$

(iii) **The LCM of 60, 84 and 108 is**

(a) 3780 (b) 3680
(c) 4780 (d) 4680

Sol. Correct option: **(a).**

(iv) **The product of HCF and LCM of 60,84 and 108 is**

(a) 55360 (b) 35360
(c) 45500 (d) 45360

Sol. Correct option: **(d).**

(v) **108 can be expressed as a product of its primes as**

(a) $2^3 \times 3^2$ (b) $2^3 \times 3^3$
(c) $2^2 \times 3^2$ (d) $2^2 \times 3^3$

Sol. Correct option: **(d).**

Q. 3. A garden consists of 135 rose plants planted in certain number of columns. There are another set of 225 marigold plants, which is to be planted in the same number of columns. U + C

Read carefully the above paragraph and answer the following questions:

(i) **What is the maximum number of columns in which they can be planted ?**

(a) 45 (b) 40
(c) 15 (d) 35

Sol. Correct option: **(a).**

Explanation: No. of rose plants = 135

No. of marigold plants = 225

The maximum number of columns in which they can be planted

$= \text{HCF of 135 and 225}$

$\therefore$ Prime factors of 135 $= 3 \times 3 \times 3 \times 5$

and 225 $= 3 \times 3 \times 5 \times 5$

$\therefore$ Prime factors of 135 $= 3 \times 3 \times 5 = 45.$

(ii) Find the total number of plants

(a) 135 **(b) 225**

(c) 360 **(d) 45**

Sol. Correct option: **(c)**.

Explanation: Total number of plants 135 + 225

$= 360$ plants

(iii) Find the sum of exponents of the prime factors of the maximum number of columns in which they can be planted.

(a) 5 **(b) 3**

(c) 4 **(d) 6**

Sol. Correct option: **(b)**.

Explanation: We have proved that the maximum number of columns = 45

So, prime factors of 45 $= 3 \times 3 \times 5$

$= 3^2 \times 5^1$

$\therefore$ Sum of exponents $= 2 + 1 = 3.$

(iv) What is total numbers of row in which they can be planted

(a) 3 **(b) 5**

(c) 8 **(d) 15**

Sol. Correct option: **(c)**.

Explanation: Number of rows of Rose plants

$$= \frac{135}{45} = 3$$

$$\text{Number of rows of marigold plants} = \frac{225}{45} = 5$$

Total number of rows $= 3 + 5 = 8$

(v) Find the sum of exponents of the prime factors of total number of plants.

(a) 2 **(b) 3**

(c) 5 **(d) 6**

Sol. Correct option: **(d)**.

Explanation:

Total number of plants $= 135 + 225$

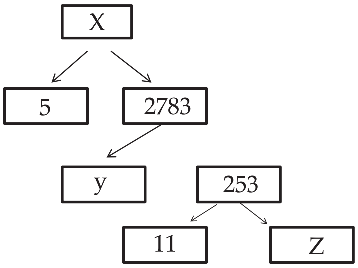
$= 360$

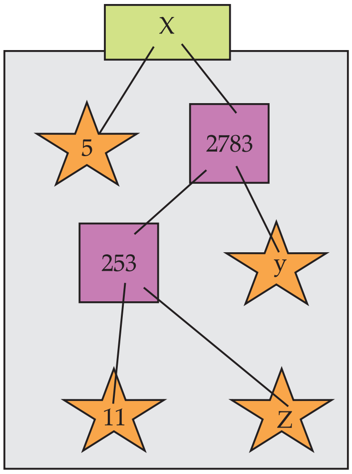
The prime factors of $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^3 \times 3^2 \times 5^1$

$\therefore$ Sum of exponents $= 3 + 2 + 1 = 6.$

Q. 4. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.





Observe the following factor tree and answer the following:

(i) What will be the value of x?

(a) 15005 **(b) 13915**

(c) 56920 **(d) 17429**

Sol. Correct option: **(b).**

Explanation: $x = 2783 \times 5$

$x = 13915$

(ii) What will be the value of y?

(a) 23 **(b) 22**

(c) 11 **(d) 19**

Sol. Correct option: **(c).**

Explanation: $2783 = y \times 253$

$$y = \frac{2783}{253}$$

$y = 11$

(iii) What will be the value of z?

(a) 22 **(b) 23**

(c) 17 **(d) 19**

Sol. Correct option: **(b).**

Explanation: $253 = 11 \times z$

$$z = \frac{253}{11}$$

$z = 23$

(iv) According to Fundamental Theorem of Arithmetic 13915 is a

(a) **Composite number**

(b) **Prime number**

(c) **Neither prime nor composite**

(d) **Even number**

Sol. Correct option: **(a).**

(v) The prime factorisation of 13915 is

(a) $5 \times 11^3 \times 13^2$ (b) $5 \times 11^3 \times 23^2$

(c) $5 \times 11^2 \times 23$ (d) $5 \times 11^2 \times 23^2$

Sol. Correct option: **(c).**

SELF ASSESSMENT TEST - 1

Maximum Time: 1 hour **MM: 25**

Q. 1. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a + b)$.

Q. 2. If HCF of two numbers is 18 and their product is 12960, then find the LCM.

Q. 3. If p is a prime number, then find LCM of p, p^2 and p^3.

Q. 4. Write whether rational number $\frac{7}{150}$ will have terminating decimal expansion or a non-terminating decimal.

Q. 5. Write whether $\frac{2\sqrt{125}+\sqrt{20}}{3\sqrt{5}}$ on simplification gives an irrational or a rational number.

Q. 6. A sweet seller has 420 Bundi Laddoo and 130 badam barfis. He wants to stack them is such a way that each stack has the same number, and they take up the least area of the tray.

Read carefully the above paragraph and answer the following questions:

(i) The prime factors of 420 are: C

(a) $2^2 \times 3^2 \times 5 \times 7$ (b) $2 \times 3 \times 5 \times 7$

(c) $2 \times 3 \times 5^2 \times 7$ (d) $2^2 \times 3 \times 5 \times 7$

(ii) Using Euclid's algorithm, the HCF of 420 and 130 is: C

(a) 10 (b) 30

(c) 20 (d) 15

(iii) The LCM of 420 and 130 is: C

(a) 4560 (b) 5460

(c) 6540 (d) 5640

(iv) The sum of exponents of prime factors in the prime factorization of 420 is: C

(a) 2 (b) 3

(c) 5 (d) 4

(v) The sum of exponents of prime factors in the prime factorization of 130 is: C

(a) 1 (b) 3

(c) 2 (d) 4

Q. 7. On a morning walk three persons steps off together and their steps measure 40 cm, 42 cm and 45 cm respectively.

What is the minimum distance each should walk so that each can cover the same distance in complete steps ? A E

Q. 8. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division. U

Q. 9. If p is a prime number, then prove that $\sqrt{p}$ is an irrational.

Q. 10. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together ? A

Q. 11. Show that there is no positive integer n, for which $\sqrt{n-1}+\sqrt{n+1}$ is rational. A