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Volume and Surface Area

The amount of space occupied by the three dimensional object is called its volume.

Cube

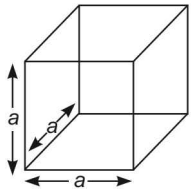
If the length, height and width of a room are equal, then the room is said to be cubical room.

In our daily life, there are so many things having the shape of cube.

e.g. Dice, cubical tank of water etc.

Volume of Cube

If the side of the cube is a unit long, then the volume of that cube must be $(\text{Side})^3$ $\text{unit}^3 = a^3\ \text{unit}^3$



Example 1 Find the volume of a cube whose one edge is 4 cm.

(a) 65 cm^3 (b) 64 cm^3
(c) 16 m^3 (d) 4 m^3

Sol. *(b)* Volume of the cube

$= (\text{Edge} \times \text{Edge} \times \text{Edge})\ \text{unit}^3$

$= (4 \times 4 \times 4)$ cu cm $= 64\ \text{cm}^3$

Whole Surface Area of Cube

Whole surface area is the sum of all surfaces of cube. Since, cube has six surfaces and all are in the form of square, hence the whole surface area of the cube is given by $= 6 \times$ (Area of one square) $= 6 \times a^2 = 6a^2$

Example 2 The surface area of a cube is 486 sq cm. Find its volume.

(a) 729 cm^3 (b) 297 cm^3
(c) 927 cm^3 (d) 1000 cm^3

Sol. *(a)* Let the edge of cube $= a$

According to the question,

$$6a^2 = 486 \Rightarrow a^2 = \frac{486}{6} = 81$$

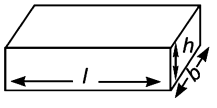
$$\therefore a = \sqrt{81} = 9 \text{ cm}$$

$$\therefore \text{Volume} = a^3 = 9^3 = 9 \times 9 \times 9 = 729\ \text{cm}^3$$

Cuboid

If the length, height and width of a room are of different measurement, then the room is said to be cuboidal room.

In our daily life, we are very used to of such things having shape of cuboid.



e.g. Room, bricks, matchbox etc.

Volume of Cuboid

Let l, b, h be the length, breadth and height of cuboid respectively, then

Volume of cuboid = Product of all three edges

$$= l \times b \times h \text{ unit}^3 = lbh \text{ unit}^3$$

Example 3 Find the volume of a cuboid whose length is 5 cm, breadth is 4 cm and height is 2 cm.

(a) 40 cm^3 (b) 50 cm^3 (c) 60 cm^3 (d) 70 cm^3

Sol. *(a)* Volume of a cuboid

$$= (\text{Length} \times \text{Breadth} \times \text{Height}) \text{ unit}^3$$

$$= (5 \times 4 \times 2) \text{ cu cm} = 40 \text{ cm}^3$$

Whole Surface Area of Cuboid

Whole surface area of cuboid = 2 (lb + bh + hl)

Example 4 Find the surface area of a cuboid 18m long, 14 m broad and 7m hight.

(a) 592 sq m (b) 952 sq m
(c) 295 sq m (d) 400 sq m

Sol. *(b)* Surface area = 2 (lb + bh + lh)

$$= 2(18 \times 14 + 14 \times 7 + 18 \times 7)$$

$$= 2(252 + 98 + 126)$$

$$= (2 \times 476) = 952 \text{ sq m}$$

Example 5 The length, breadth and height of soapcake are 7 cm, 5 cm and 2.5 cm, respectively. How many soapcakes can be put in the card board box of length, breadth and height 56 cm, 40 cm and 25 cm, respectively?

(a) 650 (b) 630 (c) 640 (d) 660

Sol. *(c)* Length of card board box $L = 56$ cm

Breadth of card board box $B = 40$ cm

Height of card board box $H = 25$ cm

Volume of card board box

$$V = (\text{Length} \times \text{Breadth} \times \text{Height})$$

$$= (l \times b \times h) \text{ unit}^3$$

$$= (56 \times 40 \times 25) \text{ cm}^3 = 56000 \text{ cm}^3$$

Length of soapcake $l = 7$ cm

Breadth of the soapcake $b = 5$ cm

Height of the soapcake $h = 2.5$ cm

Volume of the soapcake $= (l \times b \times h) \text{ unit}^3$

$$= (7 \times 5 \times 2.5) \text{ cm}^3 = 87.5 \text{ cm}^3$$

Number of soapcakes can be put in the card board box

$$= \frac{\text{Volume of card board box}}{\text{Volume of the soapcake}} = \frac{56000}{87.5} = 640$$

Important Formulae

- Possible longest length kept within the room is given by (diagonal of cuboid) = $\sqrt{l^2 + b^2 + h^2}$
  Where, l, b, h are the length, breadth and height of the room, respectively.
- Diagonal of cube = $a\sqrt{3}$ unit

Practice Exercise

1. Find the volume of cubes whose edge is 6 cm.
   (a) 215 cm^3 (b) 216 cm^3
   (c) 250 cm^3 (d) 264 cm^3
2. Find the volume of cubes whose edge is 2.5 m.
   (a) 13.625 m^3
   (b) 15.225 m^3
   (c) 15.625 m^3
   (d) 12.225 m^3
3. Find the volume of cuboid whose dimensions are length = 4 cm, breadth = 5 cm, height = 6 cm.
   (a) 120 cm^3 (b) 150 cm^3
   (c) 130 cm^3 (d) 125 cm^3
4. The length, breadth and height of a cuboid are 40 cm, 10 cm and 20 cm respectively and the length of edge of a cube is 12 cm. What is the difference in their volumes?
   (a) 6171 cm^3 (b) 6472 cm^3
   (c) 6200 cm^3 (d) 6272 cm^3

5. How many cubes of 2 m edge will fit in a cuboid box 5 m long, 4 m broad and 4 m height?
(a) 16 (b) 10
(c) 8 (d) 12

6. What can be possible volume of any box to carry 100 books 20 cm long, 14 cm broad and 2 cm high?
(a) 56500 cm^3 (b) 65000 cm^3
(c) 56000 cm^3 (d) 5600 cm^3

7. Find the volume of the cuboid whose length, breadth and height are respectively 12 cm, 10 cm and 8 cm.
(a) 960 cm^3 (b) 860 cm^3
(c) 980 cm^3 (d) 880 cm^3

8. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold?
(a) 140×10^3 (b) 145×10^3
(c) 125×10^2 (d) 135×10^3

9. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5m × 1.25m × 0.5m that can be stored in the godown.
(a) 16625 crates (b) 16000 crates
(c) 15000 crates (d) 14425 crates

10. Capacity of a tank is 60 kL. If the length and breadth of the tank are respectively 5 m and 4 m. Find the depth of the tank.
(a) 6 m (b) 5 m (c) 3 m (d) 10 m

11. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 480 m^3 of a liquid?
(a) 6 m (b) 5 m (c) 3 m (d) 10 m

12. Find the volume of the cuboid whose length, breadth and height are respectively 1.5 m, 25 cm and 15 cm.
(a) 62552 cm^3 (b) 25625 cm^3
(c) 56252 cm^3 (d) 56250 cm^3

13. A block of wood is in form a cube its edge is 4 m. How many rectangular piece of dimension 10 cm × 20 cm × 5 cm be cut from the block?
(a) 640 (b) 64 (c) 6400 (d) 64000

14. Capacity of a tank is 90 kL. If the length and breadth of the tank are respectively 6 m and 3 m, find its depth.
(a) 6 m (b) 12 m (c) 5 m (d) 10 m

15. Find the length of a wooden plank of width 2.5 m, thickness 0.025 m and volume 0.25 m^3.
(a) 4 m (b) 10 m (c) 12 m (d) 5 m

Answers

1	(b)	2	(c)	3	(a)	4	(d)	5	(b)	6	(c)	7	(a)	8	(d)	9	(b)	10	(c)
11	(a)	12	(d)	13	(d)	14	(c)	15	(a)										

Hints & Solutions

1. Volume of cube = (Edge × Edge × Edge) $unit^3$
= (6 × 6 × 6) cu cm = 216 cm^3

2. Volume of cube = (2.5 × 2.5 × 2.5) m^3
= 15.625 m^3

3. Volume of cuboid
= (Length × Breadth × Height) $unit^3$
= (4 × 5 × 6) cu cm = 120 cm^3

4. Volume of cuboid
= (Length × Breadth × Height) $unit^3$
= (40 × 10 × 20) cu cm = 8000 cm^3
Volume of cube = (Edge × Edge × Edge) $unit^3$
= (12 × 12 × 12) cu cm = 1728 cm^3
Difference in volumes
= 8000 − 1728 = 6272 cm^3

5. Volume of cuboid box
= (Length × Breadth × Height) $unit^3$
= (5 × 4 × 4) cu m = 80 m^3
Volume of cube = (Edge × Edge × Edge) $unit^3$
= (2 × 2 × 2) cu m = 8 m^3
Number of cubes will fit in the box
$= \frac{\text{Volume of cuboid box}}{\text{Volume of cube}} = \frac{80}{8} = 10$ cubes

6. Volume of 1 book
$= (\text{Length} \times \text{Breadth} \times \text{Height})\ \text{unit}^3$
$= (20 \times 14 \times 2)\ \text{cm}^3 = 560\ \text{cm}^3$
Volume of box to carry such 100 books
$= (100 \times 560)\ \text{cm}^3 = 56000\ \text{cm}^3$

7. Volume $= 12 \times 10 \times 8 = 960\ \text{cm}^3$

8. Volume of tank $= 6 \times 5 \times 4.5 = 135\ \text{m}^3$
$\because \quad 1\ \text{m}^3 = 10^3\ \text{L}$
$\therefore \quad 135\ \text{m}^3 = 10^3 \times 135 = 135 \times 10^3\ \text{L}$

9. Number of wooden crates
$$= \frac{\text{Volume of godown}}{\text{Volume of a crate}}$$
$$= \frac{40 \times 25 \times 15}{1.5 \times 1.25 \times 0.5} = \frac{15000}{0.9375} = 16000\ \text{crates}$$

10. Volume of the tank $= 60\ \text{kL} = 60000\ \text{L} = 60\ \text{m}^3$
$$\therefore \quad \text{Depth} = \frac{60}{5 \times 4} = 3\ \text{m}$$

11. $$\text{Height} = \frac{\text{Volume}}{\text{l} \times \text{b}} = \frac{480}{10 \times 8} = \frac{480}{80} = 6\ \text{m}$$

12. $1.5\ \text{m} = 150\ \text{cm}$
$\therefore$ Volume $= 150 \times 25 \times 15 = 56250\ \text{cm}^3$

13. Required rectangular piece cut from the block
$$= \frac{4 \times 4 \times 4\ \text{m}^3}{10 \times 20 \times 5\ \text{cm}^3}$$
$$= \frac{64 \times 100 \times 100 \times 100\ \text{cm}^3}{1000\ \text{cm}^3}$$
$= 64000$ piece

14. Volume of tank $= 90\ \text{kL} = 90000\ \text{L}$
$= 90\ \text{m}^3 \quad [\because 1000\ \text{L} = 1\ \text{m}^3]$
$$\therefore \quad \text{Depth} = \frac{90}{6 \times 3} = 5\ \text{m}$$

15. $$\text{Length} = \frac{\text{Volume}}{\text{Breadth} \times \text{Height}} = \frac{0.25}{2.5 \times 0.025} = 4\ \text{m}$$

Try Yourself

1) Find the volume of a cube whose edge is 3.5 m.
(a) 42.875 m^3 (b) 40.850 m^3
(c) 41.875 m^3 (d) 45.850 m^3

2) Find the volume of a cube whose edge is 13 m.
(a) 2095 m^3 (b) 2250 m^3
(c) 2197 m^3 (d) 2097 m^3

3) Find the volume of a cuboid, whose dimensions are length = 8 m, breadth = 1.6 m, height = 1.2 m.
(a) 19.36 m^3 (b) 14.50 m^3
(c) 18.05 m^3 (d) 15.36 m^3

4) How many cubes of 4 m edge will fit in a cuboid box 10 m long, 8 m broad and 8 m high?
(a) 12 (b) 10 (c) 11 (d) 15

5) A pencil box measures 17.5 cm in length, 9 cm in breadth and 2 cm in height. How many such pencil boxes can be packed in a cartoon of 45 cm long, 25 cm broad and 21 cm high?
(a) 70 (b) 78 (c) 75 (d) 76

6) Find the volume of cuboid whose length, breadth and height are respectively 8 m, 70 cm and 90 cm.
(a) 5.04 m^3 (b) 3.04 m^3
(c) 6.04 m^3 (d) 4.25 m^3

7) Find the cost of digging a cuboidal pit 6 m long, 5 m broad and 3 m deep at the rate of ₹ 30 per m^3.
(a) ₹ 2656 (b) ₹ 2720
(c) ₹ 2560 (d) ₹ 2700

8) Two cubes, each of side 8 cm are joined end to end. Find the volume of the resulting cuboid.
(a) 1020 cm^3
(b) 1024 cm^3
(c) 1030 cm^3
(d) 1026 cm^3

9) A matchbox measures 4 cm by 2.5 cm by 1.5 cm. What will be the volume of a packet containing a maximum of 12 such matchboxes?
(a) 175 cm^3 (b) 185 cm^3
(c) 179 cm^3 (d) 180 cm^3

10) How many wooden cubical blocks of edge 20 cm can be cut from a log of wood of size 8 m by 5 m by 80 cm, assuming there is no wastage?
(a) 5100 (b) 4050
(c) 4000 (d) 3535

Answers

1	(a)	2	(c)	3	(d)	4	(b)	5	(c)
6	(a)	7	(d)	8	(b)	9	(d)	10	(c)