Straight Lines

1. EQUATION OF STRAIGHT LINE

Every linear equation in two variable x and y always represents a straight line eg. $3x + 4y = 5$

General form of straight line is given by $ax + by + c = 0$

(1) Equation of x-axis : $y = 0$

(2) Equation a line parallel to x axis at a distance 'a' from it is $y = a$

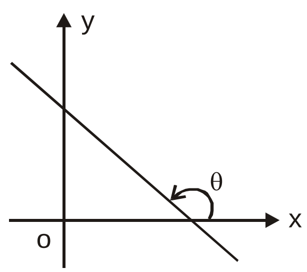
(3) Equation of y-axis : $x = 0$

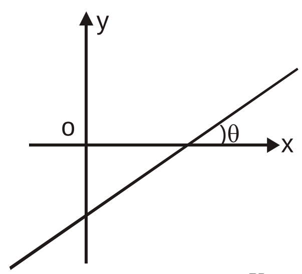
(4) Equation of a line parallel to y axis at a distance 'a' from it is $x = a$

2. SLOPE

The slope of a line is equal to the tangent of the angle which it makes with the positive side of x-axis and it is generally denoted by m.

Thus if a line makes an angle θ with x-axis then its $\boxed{\text{slope} = m = \tan\theta}$





The slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

The slope of a line $ax + by + c = 0$ is $-\dfrac{a}{b}$

Note :

(i) Slope of x axis or a line parallel to x axis is $\tan 0° = 0$

(ii) Slope of y axis or a line parallel to y axis is $\tan 90° = \infty$

3. DIFFERENT FORMS OF THE EQUATION OF STRAIGHT LINE

3.1 Slope intercept form :- $\boxed{y = mx + c}$

Where 'm' is the slope of the line and 'c' is the length of the intercept made by it on y axis

3.2 Point Slope form : The equation of a line with slope m and passing through a point (x_1, y_1) is $y - y_1 = m(x - x_1)$

3.3 Two point form : The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

3.4 Intercept form : The equation of straight line which cuts off intercepts a and b on the axes of x and y respectively is $\dfrac{x}{a} + \dfrac{b}{b} = 1$

3.5 Normal (Perpendicular) form of a line

$x \cos\alpha + y \sin\alpha = p$ is the equation of a straight line on which the length of the perpendicular from the origin is p and α is the inclination of the perpendicular with positive direction of x-axis.

3.6 Parametric form

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$

Is the equation of a straight line passing through a given point A (x_1, y_1) and makes an angle θ with x axis. The coordinates (x , y) of any point P on this line are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$. The distance of this point P from the given point A is $\sqrt{(x_1 + r\cos\theta - x_1)^2 + (y_1 + r\sin\theta - y_1)^2} = r$

4. REDUCTION OF GENERAL FORM OF EQUATION INTO STANDARD FORM

General form of equation is $ax + by + c = 0$ then its

4.1 Slope intercept form is

$$y = -\frac{ax}{b} - \frac{c}{b}, \text{ here slope } m = -\frac{a}{b}, \text{ intercept } = \frac{-c}{b}$$

4.2 Intercept form is

$$\frac{x}{-c/a} + \frac{y}{-c/b} = 1, \text{ here x intercept is } = -\frac{c}{a}, \text{ y intercept is } = -\frac{c}{b}$$

4.3 Normal form is : To change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$ like

$$-\frac{ax}{\sqrt{a^2 + b^2}} - \frac{by}{\sqrt{a^2 + b^2}} = \frac{c}{\sqrt{a^2 + b^2}} \quad (c > 0), \text{ here } \cos\alpha = -\frac{a}{\sqrt{a^2 + b^2}}$$

$$\sin\alpha = -\frac{b}{\sqrt{a^2 + b^2}} \text{ and } p = \frac{c}{\sqrt{a^2 + b^2}}$$

5. POSITION OF A POINT RELATIVE TO A LINE

(i) The point (x_1, y_1) lies on the line $ax + by + c = 0$ if, $ax_1 + by_1 + c = 0$

(ii) If P (x_1, y_1) and Q (x_2, y_2) do not lie on the line $ax + by + c = 0$ then they are on the same side of the line, if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign. They lie on the opposite side of line if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the opposite sign.

6. ANGLE BETWEEN TWO LINES

The angle θ between two lines whose slopes are m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

(i) If the lines are parallel, then $m_1 = m_2$

(ii) If the lines are perpendicular, then $m_1 m_2 = -1$

7. EQUATION OF PARALLEL & PERPENDICULAR LINES

(i) Equation of a line which is parallel to $ax + by + c = 0$ is $ax + by + k = 0$

(ii) Equation of a line which is perpendicular to $ax + by + c = 0$ is $bx - ay + k = 0$

8. EQUATION OF A LINE MAKING A GIVEN ANGLE WITH ANOTHER LINE

The equation of a line passing through (x_1, y_1) and making an angle a with the line $y = mx + c$ is given by

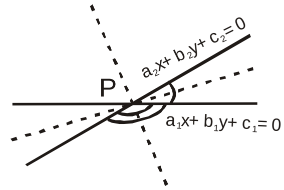
9. DISTANCE BETWEEN TWO PARALLEL LINES

The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

10. BISECTORS OF THE ANGLES BETWEEN TWO LINES

Bisectors of the two angles between two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$



First write the equation of the lines so that the constant terms are positive. Then

(a) If $a_1a_2 + b_1b_2 > 0$ then on taking positive sign in the above bisectors equation we shall get the obtuse angle bisector and on taking negative sign we shall get the acute angle bisector.

(b) If $a_1a_2 + b_1b_2 < 0$ then positive sign give the acute angle bisector and negative sign gives the obtuse angle bisector.

(c) On taking positive sign we shall get equation of the bisector of the angle which contains the origin and negative sign gives the equation of the bisector which does not contain origin.

Note : This is also the bisector of the angle in which origin lies (since c_1 , c_2 are positive and it has been obtained by taking positive sign)

11. LINES PASSING THROUGH THE POINT OF INTERSECTION OF TWO LINES

If equation of two lines $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$ then the equation of the lines passing through the point of intersection of these lines is $L_1 + \lambda L_2 = 0$, value of l is obtained with the help of the additional infomation given in the problem.

12. HOMOGENEOUS EQUATION OF SECOND DEGREE

The equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines through the origin if $h^2 \geq ab$

(i) If m_1 , m_2 be slopes of these lines, then

$$m_1 + m_2 = -\frac{2h}{b} \quad ; \qquad m_1 m_2 = \frac{a}{b}$$

(ii) If θ be the angle between these lines then $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

(iii) These lines are identical if $h^2 = ab$

(iv) These lines are perpendicular if $a + b = 0$

(v) Equations of the bisectors between these lines $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

(vi) Product of $\perp^r$ Ist drawn from (x_1, y_1) to the above pair of line is $\left|\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$

13. GENERAL EQUATION OF SECOND DEGREE

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is called the general equation of second degree in x and y it represent a pair of two straight lines if

(i) $D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ or $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

(ii) $h^2 \geq ab$, $g^2 \geq ac$, $f^2 \geq bc$

14. EQUATION OF LINES JOINING THE INTERSECTION POINTS OF A LINE & A CURVE TO THE ORIGIN

(1) The point of inter section of lines represented by $ax^2 + c = 0$ is $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

(2) If the pair of line given by $ax^2 + c = 0$ are parallel then $h^2 = ab$, $bg^2 = af^2$ and the distance between them is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2 - bc}{b(a+b)}}$

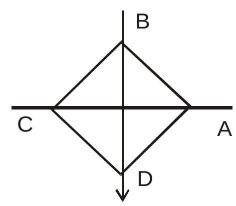
(3) The equationof the pair of the straight line joining the origin to the points of intersection of the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and the line $lx + my + n = 0$ is a homogenous equation of second degree given by

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-n}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0$$

15. BASIC IMPORTANT POINT'S

15.1 Area of quadrilateral

When its sides are $|ax| + |by| + c = 0$, then it will be a rhombus



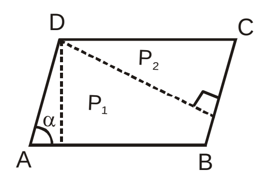
and area of this rhombus = $4 \times$ (area of Δ OAB) = $4 \times \frac{c^2}{2ab} = \frac{2c^2}{ab}$

Note : If a = b then it becomes a square whose area = $\frac{2c^2}{a^2}$

15.2 Area of Parallelogram

If p_1, p_2 be distances between the opposite sides of a parallelogram and α be one of its angle, then its area = p_1p_2 cosec α .If $y = m_1x + c_1$, $y = m_1x + c_2$, $y = m_2x + d_1$ and $y = m_2x + d_2$ are the sides of a prallelogram then

$$\text{Area} = \left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$$



16. FOOT OF THE PERPENDICULAR AND REFLECTION OF A POINT WITH RESPECT TO A LINE

(i) The foot (h, k) of the perpendicular drawn from the point (x_1, y_1) on the line ax + by + c = 0 is given by $\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\left(\frac{ax_1 - by_1 + c}{a^2 + b^2}\right)$

(ii) The image (h, k) of the point (x_1, y_1) in the line ax + by + c = 0 is given by $\frac{h - x_1}{a} = \frac{k - y_1}{b} = -2\left(\frac{ax_1 - by_1 + c}{a^2 + b^2}\right)$