

Chapter : 2. POLYNOMIALS

Exercise : 2A

Question: 1

Find the zeros of

Solution:

$$\text{Let } f(x) = x^2 + 7x + 12$$

$$\text{Put } f(x) = 0$$

$$x^2 + 7x + 12 = 0$$

$$x^2 + 4x + 3x + 12 = 0$$

$$3(x + 4) + x(x + 4) = 0$$

$$(3 + x)(x + 4) = 0$$

$$\therefore x = -4 \text{ or } x = -3$$

Now,

$$\text{sum of zeroes} = -3 + (-4) = -7 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (-3) \times (-4) = \frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 2

Find the zeros of

Solution:

$$\text{Let } f(x) = x^2 + 2x - 8$$

$$\text{Put } f(x) = 0$$

$$x^2 + 2x - 8 = 0$$

$$x^2 + 2x - 4x - 8 = 0$$

$$x(x + 2) - 4(x + 2) = 0$$

$$(2 + x)(x - 4) = 0$$

$$\therefore x = 4 \text{ or } x = -2$$

$$\text{Now, sum of zeroes} = -2 + 4 = 2 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (4) \times (-2) = \frac{8}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 3

Find the zeros of

Solution:

$$\text{Let } f(x) = x^2 + 3x - 10$$

$$\text{Put } f(x) = 0$$

$$x^2 + 3x - 10 = 0$$

$$x^2 + 5x - 2x - 10 = 0$$

$$x(x + 5) - 2(x + 5) = 0$$

$$(5 + x)(x - 2) = 0$$

$$\therefore x = -5 \text{ or } x = 2$$

$$\text{Now, sum of zeroes} = -5 + (2) = -3 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (-5) \times (2) = \frac{10}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 4

Find the zeros of

Solution:

$$\text{Let } f(x) = 4x^2 - 4x - 3$$

$$\text{Put } f(x) = 0$$

$$4x^2 - 4x - 3 = 0$$

$$4x^2 - 6x + 2x - 3 = 0$$

$$2x(2x - 3) + 1(2x - 3) = 0$$

$$(2x + 1)(2x - 3) = 0$$

$$\therefore x = -\frac{1}{2} \text{ or } x = \frac{3}{2}$$

$$\text{Now, sum of zeroes} = \left(-\frac{1}{2}\right) + \left(\frac{3}{2}\right) = 1 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(-\frac{1}{2}\right) \times \left(\frac{3}{2}\right) = \frac{-3}{4} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 5

Find the zeros of

Solution:

$$\text{Let } f(x) = 5x^2 - 4 - 8x$$

$$\text{Put } f(x) = 0$$

$$5x^2 - 8x - 4 = 0$$

$$5x^2 - 10x + 2x - 4 = 0$$

$$5x(x - 2) + 2(x - 2) = 0$$

$$(5x + 2)(x - 2) = 0$$

$$\therefore x = 2 \text{ or } x = -\frac{2}{5}$$

$$\text{Now, sum of zeroes} = 2 + \left(-\frac{2}{5}\right) = \frac{8}{5} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (2) \times \left(-\frac{2}{5}\right) = \frac{-4}{5} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 6

Find the zeros of

Solution:

$$\text{Let } f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$$

$$\text{Put } f(x) = 0$$

$$2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$$

$$2\sqrt{3}x^2 - 2x - 3x + \sqrt{3} = 0$$

$$2x(\sqrt{3}x - 1) - \sqrt{3}(\sqrt{3}x - 1) = 0$$

$$(\sqrt{3}x - 1)(2x - \sqrt{3}) = 0$$

$$x = \frac{1}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{2} \text{ After rationalizing the denominator, we get,}$$

$$\therefore x = \frac{\sqrt{3}}{3} \text{ or } x = \frac{\sqrt{3}}{2}$$

Now,

$$\text{sum of zeroes} = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{6} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{\sqrt{3}}{3}\right) \times \left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 7

Find the zeros of

Solution:

$$\text{Let } f(x) = 2x^2 - 11x + 15$$

$$\text{Put } f(x) = 0$$

$$2x^2 - 11x + 15 = 0$$

$$2x^2 - 6x - 5x + 15 = 0$$

$$2x(x - 3) - 5(x - 3) = 0$$

$$(2x - 5)(x - 3) = 0$$

$$\therefore x = 3 \text{ or } x = \frac{5}{2}$$

$$\text{Now, sum of zeroes} = 3 + \left(\frac{5}{2}\right) = \frac{11}{2} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (3) \times \left(\frac{5}{2}\right) = \frac{-15}{2} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 8

Find the zeros of

Solution:

$$\text{Let } f(x) = 4x^2 - 4x + 1$$

$$\text{Put } f(x) = 0$$

$$4x^2 - 4x + 1 = 0$$

$$(2x)^2 - 2(2x)(1) + (1)^2 = 0$$

$$(2x - 1)^2 = 0$$

$$\therefore x = \frac{1}{2} \text{ or } x = \frac{1}{2}$$

Now,

$$\text{sum of zeroes} = \frac{1}{2} + \left(\frac{1}{2}\right) = 1 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{1}{4} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 9

Find the zeros of

Solution:

$$\text{Let } f(x) = x^2 - 5$$

$$\text{Put } f(x) = 0$$

$$x^2 - 5 = 0$$

$$(x - \sqrt{5})(x + \sqrt{5}) = 0$$

$$\therefore x = \sqrt{5} \text{ or } x = -\sqrt{5}$$

$$\text{Now, sum of zeroes} = \sqrt{5} + (-\sqrt{5}) = 0 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (\sqrt{5}) \times (-\sqrt{5}) = \frac{5}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 10

Find the zeros of

Solution:

$$\text{Let } f(x) = 8x^2 - 4$$

$$\text{Put } f(x) = 0$$

$$8x^2 - 4 = 0$$

$$(2\sqrt{2}x - 2)(2\sqrt{2}x + 2) = 0$$

$$\therefore x = \frac{1}{\sqrt{2}} \text{ or } x = -\frac{1}{\sqrt{2}}$$

$$\text{Now, sum of zeroes} = \frac{1}{\sqrt{2}} + \left(-\frac{1}{\sqrt{2}}\right) = 0 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{1}{\sqrt{2}}\right) \times \left(-\frac{1}{\sqrt{2}}\right) = \frac{-1}{2} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 11

Find the zeros of

Solution:

$$\text{Let } f(x) = 5y^2 + 10y$$

$$\text{Put } f(x) = 0$$

$$5y^2 + 10y = 0$$

$$(5y)(y + 2) = 0$$

$$\therefore x = 0 \text{ or } x = -2$$

$$\text{Now, sum of zeroes} = 0 + (-2) = -2 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (0) \times (-2) = 0 = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 12

Find the zeros of

Solution:

$$\text{Let } f(x) = 3x^2 - x - 4$$

$$\text{Put } f(x) = 0$$

$$3x^2 - x - 4 = 0$$

$$3x^2 - 4x + 3x - 4 = 0$$

$$x(3x - 4) + 1(3x - 4) = 0$$

$$(x + 1)(3x - 4) = 0$$

$$\therefore x = -1 \text{ or } x = \frac{4}{3}$$

$$\text{Now, sum of zeroes} = -1 + \left(\frac{4}{3}\right) = \frac{1}{3} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (-1) \times \left(\frac{4}{3}\right) = \frac{-4}{3} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 13

Find the quadrati

Solution:

$$\text{Let } \alpha = 2 \text{ and } \beta = -6$$

$$\text{Now, Sum of zeros, } \alpha + \beta = 2 - 6 = -4$$

$$\text{And, product of zeroes, } \alpha\beta = 2(-6) = -12$$

We know that,

$$\text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-4)x + (-12)$$

$$= x^2 + 4x - 12$$

$$\text{Now, sum of zeroes} = 2 + (-6) = -4 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (2) \times (-6) = -12 = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 14

Find the quadrati

Solution:

$$\text{Let } \alpha = \frac{2}{3} \text{ and } \beta = -\frac{1}{4}$$

$$\text{Now, Sum of zeros, } \alpha + \beta = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

$$\text{And, product of zeroes, } \alpha\beta = \left(\frac{2}{3}\right)\left(-\frac{1}{4}\right) = -\frac{1}{6}$$

We know that,

$$\text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - \left(\frac{5}{12}\right)x + \left(-\frac{1}{6}\right)$$

$$= x^2 - \frac{5}{12}x - \frac{1}{6}$$

$$= 12x^2 - 5x - 2$$

$$\text{Now, sum of zeroes} = \frac{2}{3} + \left(-\frac{1}{4}\right) = 5/12 = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{2}{3}\right) \times \left(-\frac{1}{4}\right) = -\frac{1}{6} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence, relationship verified.

Question: 15

Find the quadrati

Solution:

Let the zero of the polynomial be α and β

According to the question,

$$\alpha + \beta = 8$$

$$\alpha\beta = 12$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$$

$$= x^2 - 8x + 12$$

$$\therefore \text{required polynomial } f(x) = x^2 - 8x + 12$$

$$\text{Put } f(x) = 0$$

$$x^2 - 8x + 12 = 0$$

$$x^2 - 6x - 2x + 12 = 0$$

$$x(x - 6) - 2(x - 6) = 0$$

$$(x - 6)(x - 2) = 0$$

$$x = 6 \text{ or } x = 2$$

Question: 16

Find the quadrati

Solution:

Let the zero of the polynomial be α and β

According to the question,

$$\alpha + \beta = 0$$

$$\alpha\beta = -1$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$$

$$= x^2 - 0x - 1$$

$$\therefore \text{required polynomial } f(x) = x^2 - 1$$

$$\text{Put } f(x) = 0$$

$$x^2 - 1 = 0$$

$$(x - 1)(x + 1) = 0$$

$$x = 1 \text{ or } x = -1$$

Question: 17

Find the quadratic

Solution:

Let the zero of the polynomial be α and β

According to the question:

Sum of zeroes:

$$\alpha + \beta = \frac{5}{2}$$

Product of zeroes:

$$\alpha\beta = 1$$

we know, A quadratic equation can be formed using its sum of roots and product of roots with the form

$$\therefore f(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$$

$$= x^2 - \frac{5}{2}x + 1$$

$$\therefore \text{required polynomial } f(x) = x^2 - \frac{5}{2}x + 1$$

$$\text{Put } f(x) = 0$$

$$x^2 - (5/2)x + 1 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x - 2) - 1(x - 2) = 0$$

$$(x - 2)(2x - 1) = 0$$

$$x = 2 \text{ or } x = 1/2$$

Question: 18

Find the quadratic

Solution:

Let the zero of the polynomial be α and β

According to the question,

$$\alpha + \beta = \sqrt{2}$$

$$\alpha\beta = \frac{1}{3}$$

$$\therefore f(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$$

$$= x^2 - \sqrt{2}x + \frac{1}{3}$$

$$\therefore \text{required polynomial } f(x) = x^2 - \sqrt{2}x + 1/3$$

$$\text{Put } f(x) = 0$$

$$x^2 - \sqrt{2}x + 1/3 = 0$$

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

Question: 19

If Now, Sum of zeros = $\frac{2}{3} + (-3) = \frac{2-9}{3} = -\frac{7}{3} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$$\Rightarrow -\frac{7}{3} = -\frac{7}{a}$$

$$\therefore a = 3 \text{ (i)}$$

Product of zeroes = $\left(\frac{2}{3}\right) \times (-3) = -\frac{2}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2} = -\frac{2}{1} = \frac{b}{3}$ (From i)

$$\therefore b = -6$$

Question: 20

If $(x + a)$ is a f

Solution:

Since, $x + a$ is a factor of $2x^2 + 2ax + 5x + 10$

$$\therefore x + a = 0$$

$$x = -a$$

Put $x = -a$ in $2x^2 + 2ax + 5x + 10 = 0$

$$2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$2a^2 - 2a^2 - 5a + 10 = 0$$

$$-5a = -10$$

$$a = 2$$

Question: 21

One zero of the p

Solution:

It is given in the question that,

$x = 2/3$ is one of the zeros of the given polynomial $3x^3 + 16x^2 + 15x - 18$

We have, $x = 2/3$

$$x - 2/3 = 0$$

To find the quotient we have to divide the given polynomial by $x - 2/3$

$$\begin{array}{r} \phantom{x - \frac{2}{3}} \overline{3x^2 + 18x + 27} \\ x - \frac{2}{3} \overline{) 3x^3 + 16x^2 + 15x - 18} \\ \underline{3x^3 - 2x^2} \\ 18x^2 + 15x \\ \underline{18x^2 - 12x} \\ 27x - 18 \\ \underline{27x - 18} \\ 0 \end{array}$$

$$\text{Quotient} = 3x^2 + 18x + 27$$

$$\therefore 3x^2 + 18x + 27 = 0$$

$$3x^2 + 9x + 9x + 27 = 0$$

$$3x(x + 3) + 9(x + 3) = 0$$

$$(x + 3)(3x + 9) = 0$$

$$(x + 3) = 0 \text{ or } (3x + 9) = 0$$

$$\text{Hence, } x = -3 \text{ or } x = -3$$

Exercise : 2B

Question: 1

Verify that 3, -2

Solution:

It is given in the question that,

$$p(x) = x^3 - 2x^2 - 5x + 6$$

Also, 3, -2 and 1 are the zeros of the given polynomial

$$\therefore p(3) = (3)^3 - 2(3)^2 - 5(3) + 6$$

$$= 27 - 18 - 15 + 6$$

$$= 33 - 33$$

$$= 0$$

$$p(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$$

$$= -8 - 8 + 10 + 6$$

$$= -16 + 16$$

$$= 0$$

$$\text{And, } p(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$

$$= 1 - 2 - 5 + 6$$

$$= 7 - 7$$

$$= 0$$

Verification of the relation is as follows:

Let us assume $\alpha = 3$, $\beta = -2$ and $\gamma = 1$

$$\alpha + \beta + \gamma = 3 - 2 + 1$$

$$= 2$$

$$\therefore -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{2}{1} = 2 = \alpha + \beta + \gamma$$

$$\text{Also, } \alpha\beta + \beta\gamma + \alpha\gamma = 3(-2) + (-2)(1) + 1(3)$$

$$= -6 - 2 + 3$$

$$= -5$$

$$\therefore \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = -\frac{5}{1} = -5 = \alpha\beta + \beta\gamma + \alpha\gamma$$

$$\text{And, } \alpha\beta\gamma = 3 \times (-2) \times 1$$

$$= -6$$

$$\therefore \frac{\text{constant term}}{\text{Coefficient of } x^3} = -\frac{6}{1} = -6 = \alpha\beta\gamma$$

Question: 2

Verify that 5, -2

Solution:

It is given in the question that,

$$p(x) = 3x^3 - 10x^2 - 27x + 10$$

Also, 5, -2 and $\frac{1}{3}$ are the zeros of the given polynomial

$$\therefore p(5) = 3(5)^3 - 10(5)^2 - 27(5) + 10$$

$$= 3 \times 125 - 250 - 135 + 10$$

$$= 385 - 385$$

$$= 0$$

$$p(-2) = 3(-2)^3 - 10(-2)^2 - 27(-2) + 10$$

$$= -24 - 40 + 54 + 10$$

$$= -64 + 64$$

$$= 0$$

$$\text{And, } p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - 10\left(\frac{1}{3}\right)^2 - 27\left(\frac{1}{3}\right) + 10$$

$$= \frac{1}{8} - \frac{10}{9} - 9 + 10$$

$$= \frac{1}{9} - \frac{10}{9} - 9 + 10$$

$$= \frac{1}{9} - \frac{10}{9} + 1$$

$$= \frac{1}{9} - \frac{1}{9}$$

$$= 0$$

Verification of the relation is as follows:

Let us assume $\alpha = 5$, $\beta = -2$ and $\gamma = 1/3$

$$\alpha + \beta + \gamma = 5 - 2 + 1/3 = \frac{10}{3}$$

$$\therefore -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\frac{(-10)}{3} = \frac{10}{3} = \alpha + \beta + \gamma$$

$$\text{Also, } \alpha\beta + \beta\gamma + \alpha\gamma = 5(-2) + (-2)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)(5)$$

$$= -27/3$$

$$= -9$$

$$\therefore \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = -\frac{27}{3} = -9 = \alpha\beta + \beta\gamma + \alpha\gamma$$

$$\text{And, } \alpha\beta\gamma = 5 \times (-2) \times \frac{1}{3} = -\frac{10}{3}$$

$$\therefore \frac{\text{constant term}}{\text{Coefficient of } x^3} = -\frac{10}{3} = \alpha\beta\gamma$$

Hence, verified

Question: 3

Find a cubic poly

Solution:

Let the zeros of the polynomial be a, b and c

Where a = 2, b = - 3 and c = 4

The cubic polynomial can be calculated as follows:

$$x^3 - (a + b + c) x^2 + (ab + bc + ca)x - abc$$

Putting the values of a, b and c in the above equation we get:

$$= x^3 - (2 - 3 + 4) x^2 + (-6 - 12 + 8) x - (- 24)$$

$$= x^3 - 3x^2 - 10x + 24$$

Question: 4

Find a cubic poly

Solution:

Let the zeros of the polynomial be a, b and c

Where a = 1/2, b = 1 and c = - 3

The cubic polynomial can be calculated as follows:

$$x^3 - (a + b + c) x^2 + (ab + bc + ca) x - abc$$

Putting the values of a, b and c in the above equation we get:

$$= x^3 - (1/2 + 1 - 3) x^2 + (1/2 - 3 - 3/2)x - (- 3/2)$$

$$= x^3 - (-3/2) x^2 - 4x + 3/2$$

$$= 2x^3 + 3x^2 - 8x + 3$$

Question: 5

Find a cubic poly

Solution:

The required cubic polynomial can be calculated as:

$$x^3 - (\text{Sum of the zeros}) x^2 + (\text{Sum of the product of the zeros taking two at a time}) x - \text{Product of Zeros}$$

It is given that, sum of the product of its zeros taken two at a time, and the product of its zeros as 5, -2 and -24 respectively

Putting these values in the equation, we get:

$$x^3 - 5x^2 - 2x + 24$$

Question: 6

Find the quotient

Solution:

It is given in the question that,

$$f(x) = x^3 - 3x^2 + 5x - 3$$

$$\text{And, } g(x) = x^2 - 2$$

$$\begin{array}{r} \overline{) \begin{array}{r} x^3 - 3x^2 + 5x - 3 \\ x^3 - 2x \\ \hline - 3x^2 + 7x - 3 \\ - 3x^2 + 6 \\ \hline + 7x - 9 \end{array}} \end{array}$$

Hence,

Quotient $q(x) = x - 3$

Remainder $r(x) = 7x - 9$

Question: 7

Find the quotient

Solution:

It is given in the question that,

$$f(x) = x^4 - 3x^2 + 4x + 5$$

And, $g(x) = x^2 + 1 - x$

$$\begin{array}{r}
 x^2 + x - 3 \\
 x^2 - x + 1 \overline{)x^4 + 0x^3 - 3x^2 + 4x + 5} \\
 \underline{x^4 - x^3 + x^2} \\
 x^3 + 4x^2 + 4x + 5 \\
 \underline{x^3 - x^2 + x} \\
 -3x^2 + 3x + 5 \\
 \underline{-3x^2 + 3x - 3} \\
 + \quad - \quad + \\
 \hline
 8
 \end{array}$$

Hence,

Quotient $q(x) = x^2 + x - 3$

Remainder $r(x) = 8$

Question: 8

Find the quotient

Solution:

It is given in the question that,

$$f(x) = x^4 - 5x + 6 = x^4 + 0x^3 + 0x^2 - 5x + 6$$

And, $g(x) = x^2 + 1 - x$

$$\begin{array}{r}
 -x^2 + 2 \overline{\left) x^4 + 0x^3 + 0x^2 - 5x + 6 \right.} \\
 \underline{x^4 \quad - 2x^2} \\
 2x^2 - 5x + 6 \\
 \underline{2x^2 \quad - 4} \\
 -5x + 10
 \end{array}$$

Hence,

Quotient $q(x) = x^2 - 2$

Remainder $r(x) = 5x + 10$

Question: 9

By actual divisio

Solution:

It is given in the question that,

$$f(x) = 2x^4 + 3x^3 - 2x^2 - 9x - 12$$

And, $g(x) = x^3 - 3$

$$\begin{array}{r}
 2x^2 + 3x + 4 \\
 x^2 - 3 \overline{) 2x^4 + 3x^3 - 2x^2 - 9x - 12} \\
 \underline{2x^4 - 6x^2} \\
 - + \\
 \underline{3x^3 + 4x^2 - 9x - 12} \\
 3x^3 - 9x \\
 \underline{- + } \\
 4x^2 - 12 \\
 4x^2 - 12 \\
 \underline{- + } \\
 0
 \end{array}$$

Hence,

Quotient $q(x) = 2x^2 + 3x + 4$

Remainder $r(x) = 0$

As the remainder is 0

$$\therefore x^2 - 3 \text{ is a factor of } 2x^4 + 3x^3 - 2x^2 - 9x - 12$$

Question: 10

On dividing $3x$

Solution:

We know that,

According to the division rule, we have

$$g(x) = -3x^2 + 5x + 2$$

$$\begin{array}{r}
 2x + 3 \\
 -3x^2 + 5x + 2 \overline{) -6x^3 + x^2 + 20x + 8} \\
 \underline{-6x^3 + 10x^2 + 4x} \\
 + \quad - \quad - \\
 \underline{ -9x^2 + 16x + 8} \\
 -9x^2 + 15x + 6 \\
 + \quad - \quad - \\
 \underline{ x + 2}
 \end{array}$$

\therefore Quotient = $2x + 3$

Remainder = $x + 2$

We know that, according to division rule:

Dividend = Quotient \times Divisor + Remainder

Putting the values in the above formula, we get:

$$-6x^3 + x^2 + 20x + 8 = (-3x^2 + 5x + 2)(2x + 3) + (x + 2)$$

$$-6x^3 + x^2 + 20x + 8 = -6x^3 + 10x^2 + 4x - 9x^2 + 15x + 6 + x + 2$$

$$-6x^3 + x^2 + 20x + 8 = -6x^3 + x^2 + 20x + 8$$

Question: 12

It is given that

Solution:

Let us assume $f(x) = x^3 + 2x^2 - 11x - 12$

It is given in the question that, -1 is a zero of the polynomial

$\therefore (x + 1)$ is a factor of $f(x)$

Now on dividing $f(x)$ by $(x + 1)$, we get

$$\begin{array}{r}
 x^2 + x + 12 \\
 x + 1 \overline{) x^3 + 2x^2 - 11x - 12} \\
 \underline{x^3 + x^2} \\
 - \quad - \\
 x^2 - 11x - 12 \\
 x^2 + x \\
 \underline{ -} \\
 -12x - 12 \\
 -12x - 12 \\
 \underline{ + } \\
 0
 \end{array}$$

$$f(x) = x^3 + 2x^2 - 11x - 12$$

$$= (x + 1)(x^2 + x - 12)$$

$$\begin{aligned}
 &= (x + 1) \{x^2 + 4x - 3x - 12\} \\
 &= (x + 1) \{x(x + 4) - 3(x + 4)\} \\
 &= (x + 1)(x - 3)(x + 4)
 \end{aligned}$$

$$\therefore f(x) = 0$$

$$(x + 1)(x - 3)(x + 4) = 0$$

$$(x + 1) = 0 \text{ or } (x - 3) = 0 \text{ or } (x + 4) = 0$$

$$x = -1 \text{ or } x = 3 \text{ or } x = -4$$

Hence, zeros of the polynomial are -1, 3 and -4

Question: 13

If 1 and -2 are t

Solution:

Let us assume $f(x) = x^3 - 4x^2 - 7x + 10$

As 1 and -2 are the zeros of the given polynomial therefore each one of $(x - 1)$ and $(x + 2)$ is a factor of $f(x)$

Consequently, $(x - 1)(x + 2) = (x^2 + x - 2)$ is a factor of $f(x)$

Now, on dividing $f(x)$ by $(x^2 + x - 2)$ we get:

$$\begin{array}{r}
 \overline{) x^3 - 4x^2 - 7x + 10} \\
 \underline{x^3 + x^2 - 2x} \\
 -5x^2 - 5x + 10 \\
 \underline{-5x^2 - 5x + 10} \\
 0
 \end{array}$$

$$f(x) = 0$$

$$(x^2 + x - 2)(x - 5) = 0$$

$$(x - 1)(x + 2)(x - 5) = 0$$

$$\therefore x = 1 \text{ or } x = -2 \text{ or } x = 5$$

Hence, the third zero is 5

Question: 14

If 3 and -3 are t

Solution:

Let us assume $f(x) = x^4 + x^3 - 11x^2 - 9x + 18$

As 3 and -3 are the zeros of the given polynomial therefore each one of $(x + 3)$ and $(x - 3)$ is a factor of $f(x)$

Consequently, $(x - 3)(x + 3) = (x^2 - 9)$ is a factor of $f(x)$

Now, on dividing $f(x)$ by $(x^2 - 9)$ we get:

$$\begin{array}{r}
 x^2 + x - 2 \\
 x^2 - 9 \overline{) x^4 + x^3 - 11x^2 - 9x + 18} \\
 \underline{x^4 \quad - 9x^2} \\
 x^3 - 2x^2 - 9x + 18 \\
 \underline{x^3 \quad - 9x} \\
 -2x^2 + 18 \\
 \underline{-2x^2 + 18} \\
 0
 \end{array}$$

$$f(x) = 0$$

$$(x^2 + x - 2)(x^2 - 9) = 0$$

$$(x - 1)(x + 2)(x - 3)(x + 3) = 0$$

$$\therefore x = 1 \text{ or } x = -2 \text{ or } x = 3 \text{ or } x = -3$$

Hence, all the zeros of the given polynomial are 1, -2, 3 and -3

Question: 15

If 2 and -2 are t

Solution:

$$\text{Let us assume } f(x) = x^4 + x^3 - 34x^2 - 4x + 120$$

As 2 and -2 are the zeros of the given polynomial therefore each one of $(x - 2)$ and $(x + 2)$ is a factor of $f(x)$

Consequently, $(x - 2)(x + 2) = (x^2 - 4)$ is a factor of $f(x)$

Now, on dividing $f(x)$ by $(x^2 - 4)$ we get:

$$\begin{array}{r}
 x^2 + x - 30 \\
 x^2 - 4 \overline{) x^4 + x^3 - 34x^2 - 4x + 120} \\
 \underline{x^4 \quad - 4x^2} \\
 x^3 - 30x^2 - 4x + 120 \\
 \underline{x^3 \quad - 4x} \\
 -30x^2 + 120 \\
 \underline{-30x^2 + 120} \\
 0
 \end{array}$$

$$f(x) = 0$$

$$(x^2 + x - 30)(x^2 - 4) = 0$$

$$(x^2 + 6x - 5x - 30)(x - 2)(x + 2)$$

$$[x(x+6) - 5(x+6)](x-2)(x+2)$$

$$\begin{array}{r}
 \overline{2x^2-3x+1} \\
 x^2-3 \Big) 2x^4-3x^3-5x^2+9x-3 \\
 \underline{2x^4 -6x^2} \\
 -3x^3+9x-3 \\
 \underline{-3x^3+9x} \\
 + - \\
 x^2-3 \\
 \underline{x^2-3} \\
 - + \\
 0
 \end{array}$$

$$f(x) = 0$$

$$2x^2 - 3x^2 - 5x^2 + 9x - 3 = 0$$

$$(x^2 - 3)(2x^2 - 3x + 1) = 0$$

$$(x^2 - 3)(2x^2 - 2x - x + 1)(2x - 1)(x - 1) = 0$$

$$(x - \sqrt{3})(x + \sqrt{3})(2x - 1)(x - 1) = 0$$

$$\therefore x = \sqrt{3} \text{ or } x = -\sqrt{3} \text{ or } x = \frac{1}{2} \text{ or } x = 1$$

Hence, all the zeros of the given polynomial are $\sqrt{3}, -\sqrt{3}, \frac{1}{2}$ and 1

Question: 18

Obtain all other

Solution:

$$\text{Let us assume } f(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$$

As $(x-\sqrt{5})$ and $(x+\sqrt{5})$ are the zeros of the given polynomial therefore each one of $(x-\sqrt{5})$ and $(x+\sqrt{5})$ is a factor of $f(x)$

Consequently, $(x-\sqrt{5})(x+\sqrt{5}) = (x^2 - 5)$ is a factor of $f(x)$

Now, on dividing $f(x)$ by $(x^2 - 5)$ we get:

$$\begin{array}{r}
 \overline{2x^2-3x+1} \\
 x^2-5 \Big) x^4+4x^3-2x^2-20x-15 \\
 \underline{x^4 -5x^2} \\
 4x^3+3x^2-20x-15 \\
 \underline{4x^3 -20x} \\
 3x^2-15 \\
 \underline{3x^2-15} \\
 - + \\
 0
 \end{array}$$

$$f(x) = 0$$

$$x^4 + 4x^3 - 7x^2 - 20x - 15 = 0$$

$$(x^2 - 5)(x^2 + 4x + 3) = 0$$

$$(x - \sqrt{5})(x + \sqrt{5})(x + 1)(x + 3) = 0$$

$$\therefore x = \sqrt{5} \text{ or } x = -\sqrt{5} \text{ or } x = -1 \text{ or } x = -3$$

Hence, all the zeros of the given polynomial are $\sqrt{5}$, $-\sqrt{5}$, -1 and -3

Question: 19

Find all the zero

Solution:

Let us assume $f(x) = 2x^4 - 11x^3 + 7x^2 + 13x - 7$

As $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$ are the zeros of the given polynomial therefore each one of $(x + 3 + \sqrt{2})$ and $(x + 3 - \sqrt{2})$ is a factor of $f(x)$

Consequently, $[(x - (3 + \sqrt{2}))][(x - (3 - \sqrt{2}))]$

$$= [(x - 3) - \sqrt{2}][(x - 3) + \sqrt{2}]$$

$$= [(x - 3)^2 - 2] = x^2 - 6x + 7 \text{ is a factor of } f(x)$$

Now, on dividing $f(x)$ by $(x^2 - 6x + 7)$ we get:

$$\begin{array}{r}
 \overline{2x^4 - 11x^3 + 7x^2 + 13x - 7} \\
x^2 - 6x + 7 \overline{) 2x^4 - 11x^3 + 7x^2 + 13x - 7} \\
\underline{2x^4 - 12x^3 + 14x^2} \\
- + - \\
 x^3 - 7x^2 + 13x - 7 \\
 \underline{x^3 - 6x^2 + 7x} \\
 - + - - 7 \\
 \underline{- x^2 + 6x - 7} \\
 - 7 \\
 \underline{- x^2 + 6x - 7} \\
 \underline{+ - 7} \\
 0
\end{array}$$

$$f(x) = 0$$

$$2x^4 - 11x^3 + 7x^2 + 13x - 7 = 0$$

$$(x^2 - 6x + 7)(2x^2 + x - 1) = 0$$

$$(x + 3 + \sqrt{2})(x + 3 - \sqrt{2})(2x - 1)(x + 1) = 0$$

$$\therefore x = -3 - \sqrt{2} \text{ or } x = -3 + \sqrt{2} \text{ or } x = 1/2 \text{ or } x = -1$$

Hence, all the zeros of the given polynomial are $(-3 - \sqrt{2})$, $(-3 + \sqrt{2})$, $1/2$ and -1

Exercise : 2C

Question: 1

If one zero of the

Solution:

It is given in the question that, zero of the polynomial $x^2 - 4x + 1$ is $(2 + \sqrt{3})$

Let the other zero of the polynomial be a

We know that,

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\therefore 2 + \sqrt{3} + a = \frac{-(-4)}{1}$$

$$a = 2 - \sqrt{3}$$

Hence, the other zero of the given polynomial is $(2 - \sqrt{3})$

Question: 2

Find the zeros of

Solution:

We have,

$$f(x) = x^2 + x - p(p + 1)$$

$$f(x) = x^2 + (p + 1 - p)x - p(p + 1)$$

$$= x^2 + (p + 1)x - px - p(p + 1)$$

$$= x[x + (p + 1)] - p[x + (p + 1)]$$

$$= [x + (p + 1)](x - p)$$

To find the zeroes of $f(x)$, let $f(x) = 0$

$$[x + (p + 1)](x - p) = 0$$

$$[x + (p + 1)] = 0 \text{ or } (x - p) = 0$$

$$x = p \text{ or } x = -(p + 1)$$

Hence, the zeros of the given polynomial are p and $-(p + 1)$

Question: 3

Find the zeros of

Solution:

We have,

$$f(x) = x^2 - 3x - m(m + 3)$$

Now, by adding and subtracting mx , we get

$$f(x) = x^2 - mx - 3x + mx - m(m + 3)$$

$$= x[x - (m + 3)] + m[x - (m + 3)]$$

$$= [x - (m + 3)](x + m)$$

$$\therefore f(x) = 0$$

$$[x - (m + 3)](x + m) = 0$$

$$[x - (m + 3)] = 0 \text{ or } (x + m) = 0$$

$$x = m + 3 \text{ or } x = -m$$

Hence, the zeros of the given polynomial are $-m$ and $m + 3$

Question: 4

Find α, β are the

Solution:

It is given in the question that,

$$\alpha + \beta = 6$$

And, $\alpha\beta = 4$

We know that,

If the zeros of the polynomial are α and β then the quadratic polynomial can be found as $x^2 - (\alpha + \beta)x + \alpha\beta$ (i)

Now substituting the values in (i), we get

$$x^2 - 6x + 4$$

Question: 5

If one zeros of t

Solution:

It is given in the question that,

One of the zero of the polynomial $kx^2 + 3x + k$ is 2

\therefore It will satisfy the above polynomial

Now, we have

$$k(2)^2 + 3(2) + k = 0$$

$$4k + 6 + k = 0$$

$$5k + 6 = 0$$

$$5k = -6$$

$$\therefore k = -6/5$$

Question: 6

If 3 is a zero of

Solution:

It is given in the question that,

One of the zero of the polynomial $2x^2 + x + k$ is 3

\therefore It will satisfy the above polynomial

Now, we have

$$2(3)^2 + 3 + k = 0$$

$$21 + k = 0$$

$$k = -21$$

Question: 7

If -4 is a zero o

Solution:

It is given in the question that,

One of the zero of the polynomial $x^2 - x - (2k + 2)$ is -4

\therefore It will satisfy the above polynomial

Now, we have

$$(-4)^2 - (-4) - (2k + 2) = 0$$

$$16 + 4 - 2k - 2 = 0$$

$$-2k = -18$$

$$k = 18/2$$

$$\therefore k = 9$$

Question: 8

If 1 is a zero of

Solution:

It is given in the question that,

One of the zero of the polynomial $ax^2 - 3(a - 1)x - 1$ is 1

\therefore It will satisfy the above polynomial

Now, we have

$$a(1)^2 - 3(a - 1)1 - 1 = 0$$

$$a - 3a + 3 - 1 = 0$$

$$-2a = -2$$

$$\therefore a = 1$$

Question: 9

If -2 is a zero o

Solution:

It is given in the question that,

One of the zero of the polynomial $3x^2 + 4x + 2k$ is -2

\therefore It will satisfy the above polynomial

Now, we have

$$3(-2)^2 + 4(-2) + 2k = 0$$

$$12 - 8 + 2k = 0$$

$$4 + 2k = 0$$

$$2k = -4$$

$$\therefore k = -2$$

Question: 10

Write the zeros o

Solution:

We have,

$$f(x) = x^2 - x - 6$$

$$= x^2 - 3x + 2x - 6$$

$$= x(x - 3) + 2(x - 3)$$

$$= (x - 3)(x + 2)$$

$$f(x) = 0$$

$$(x - 3)(x + 2) = 0$$

$$(x - 3) = 0 \text{ or } (x + 2) = 0$$

$$x = 3 \text{ or } x = -2$$

\therefore The zeros of the given polynomial are 3 and -2

Question: 11

If the sum of the

Solution:

It is given in the question that, zero of the polynomial $kx^2 - 3x + 5$ is 1

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\Rightarrow 1 = \frac{-(-3)}{k}$$

$$\Rightarrow k = 3$$

Question: 12

If the product of

Solution:

It is given in the question that, zero of the polynomial $x^2 - 4x + k$ is 3

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$3 = k/1$$

$$k = 3$$

Question: 13

If $(x + a)$ is a f

Solution:

It is given in the question that,

$(x + 4)$ is a factor of $2x^2 + 2ax + 5x + 10$

Now, we have

$$x + a = 0$$

$$x = -a$$

As $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$

Thus, it will satisfy the given polynomial

$$\therefore 2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$$

$$-5a + 10 = 0$$

$$a = 2$$

Question: 14

If $(a - b)$, a and

Solution:

It is given in the question that, zero of the polynomial $2x^3 - 6x^2 + 5x - 7$ is $(a - b)$, a and $(a + b)$

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$a - b + a + a + b = (-(-6))/2$$

$$3a = 3$$

$$a = 1$$

Question: 15

If x^3

Solution:

Firstly, equating $x^2 - x$ to 0 to find the zeros we get:

$$x(x - 1) = 0$$

$$x = 0 \text{ or } x - 1 = 0$$

$$x = 0 \text{ or } x = 1$$

As $x^3 + x^2 - ax + b$ is divisible by $x^2 - x$

\therefore The zeros of $x^2 - x$ will satisfy $x^3 + x^2 - ax + b$

$$\text{Hence, } (0)^3 + 0^2 - a(0) + b = 0$$

$$b = 0$$

Also,

$$(1)^3 + 1^2 - a(1) + 0 = 0$$

$$\therefore a = 2$$

Question: 16

If α and β are the

Solution:

It is given in the question that, zeros of the polynomial $2x^2 + 7x + 5$ are α and β

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \text{ and product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-7}{2} \text{ and } \alpha\beta = \frac{5}{2}$$

$$\therefore \alpha + \beta + \alpha\beta = -\frac{7}{2} + \frac{5}{2} = -1$$

Question: 17

State division al

Solution:

The Division algorithm for polynomials is as follows:

If we have two polynomials $f(x)$ and $g(x)$ and the degree of $f(x)$ is greater than the degree of $g(x)$, where $g(x) \neq 0$ then there exist two unique polynomials $q(x)$ and $r(x)$ such that:

$$f(x) = g(x) \times q(x) + r(x)$$

where $r(x) = 0$ or degree of $r(x) < \text{degree of } g(x)$

Question: 18

The sum of the ze

Solution:

We know that,

We can find the quadratic polynomial if we know the sum of the roots and product of the roots by using the formula:

$$x^2 - (\text{Sum of the zeroes})x + \text{Product of zeros}$$

$$\therefore x^2 - (-1/2)x + (-3)$$

$$x^2 + \frac{1}{2}x - 3$$

∴ The required polynomial is $x^2 + \frac{1}{2}x - 3$

Question: 19

Write the zeros o

Solution:

We know that,

To find the zeros of the quadratic polynomial we have to equate the $f(x)$ to 0

$$\therefore f(x) = 0$$

$$6x^2 - 3 = 0$$

$$3(2x^2 - 1) = 0$$

$$2x^2 - 1 = 0$$

$$2x^2 = 1$$

$$x^2 = 1/2$$

$$x = \pm 1/\sqrt{2}$$

Hence, the zeros of the quadratic polynomial $f(x)$ are $1/\sqrt{2}$ and $-1/\sqrt{2}$

Question: 20

Write the zeros o

Solution:

We know that,

To find the zeros of the quadratic polynomial we have to equate the $f(x)$ to 0

$$\therefore f(x) = 0$$

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$(\sqrt{3}x + 2) = 0 \text{ or } (4x - \sqrt{3}) = 0$$

$$x = -\frac{2}{\sqrt{3}} \text{ or } x = \frac{\sqrt{3}}{4}$$

Hence, the zeros of the quadratic polynomial $f(x)$ are $-\frac{2}{\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$

Question: 21

If α and β are th

Solution:

It is given in the question that,

Zeros of the polynomial $x^2 - 5x + k$ are α and β

Also,

$$\alpha - \beta = 1$$

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = -\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \text{ and product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-(-5)}{1} \text{ and } \alpha\beta = \frac{k}{1}$$

$$\alpha + \beta = 5 \text{ and } \alpha\beta = k/1$$

Now solving $\alpha - \beta = 1$ and $\alpha + \beta = 5$, we get:

$$\alpha = 3 \text{ and } \beta = 2$$

Now, substituting these values in $\alpha\beta$ we get:

$$k = 6$$

Question: 22

If α and β are the

Solution:

It is given in the question that,

Zeros of the polynomial $6x^2 + x - 2$ are α and β

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \text{ and product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-1}{6} \text{ and } \alpha\beta = -\frac{2}{3}$$

Now we have:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{-1}{6}\right)^2 - 2\left(-\frac{2}{3}\right)}{\frac{-1}{3}}$$

$$= \frac{\frac{1}{36} + \frac{4}{3}}{\frac{-1}{3}}$$

$$= -\frac{25}{12}$$

Question: 23

If α and β are the

Solution:

It is given in the question that,

Zeros of the polynomial $5x^2 - 7x + 1$ are α and β

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \text{ and product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-(-7)}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

$$\alpha + \beta = \frac{7}{5} \text{ and } \alpha\beta = \frac{1}{5}$$

Now, we have:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{7}{1}$$

$$= 7$$

Question: 24

If α and β are the

Solution:

It is given in the question that,

Zeros of the polynomial $x^2 + x - 2$ are α and β

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} \text{ and product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-1}{1} \text{ and } \alpha\beta = \frac{-2}{1}$$

$$\alpha + \beta = -1 \text{ and } \alpha\beta = -2$$

Now, we have:

$$\frac{1}{\alpha} - \frac{1}{\beta} = \left(\frac{\beta - \alpha}{\alpha\beta}\right)^2$$

$$= \frac{(\alpha + \beta)^2 - 4\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(-1)^2 - 4(-2)}{(-2)^2}$$

$$= \frac{1 + 8}{4}$$

$$= \frac{9}{4}$$

$$\therefore \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \frac{9}{4}$$

$$\left(\frac{1}{\alpha} - \frac{1}{\beta}\right) = \pm \frac{3}{2}$$

Question: 25

If the zeros of the

Solution:

It is given in the question that,

Zeros of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$

Now by using the relationship between the zeros of the quadratic polynomial we have:

$$\text{Sum of zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$a - b + a + a + b = \frac{-3}{1}$$

$$3a = 3$$

$$a = \frac{3}{3} = 1$$

$$\text{Now, we have product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$(a - b)(a)(a + b) = \frac{-1}{1}$$

$$(1 - b)(1)(1 + b) = -1$$

$$1 - b^2 = -1$$

$$b^2 = 2$$

$$\therefore b = \pm\sqrt{2}$$

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

Which of the foll

Solution:

We know that, Polynomial is an expression consist of constants, variables and exponents which are connected by addition, subtraction, multiplication and division but the exponent of variables cannot be in fraction and has to be a positive integer.

Hence, option D is correct.

Question: 2

Which of the foll

Solution:

We can observe that in the second expression the power of x is -1 but we know that the exponent of the variable in a polynomial has to be a positive term.

Hence, option D is correct.

Question: 3

The zeros of the

Solution:

We have, $f(x) = x^2 - 2x - 3$

Now, put $f(x) = 0$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(x + 1) = 0$$

Thus, $x = 3, -1$

Hence, option C is correct.

Question: 4

The zeros of the

Solution:

We have, $f(x) = x^2 - \sqrt{2}x - 12$

Now, put $f(x) = 0$

$$x^2 - \sqrt{2}x - 12 = 0$$

$$x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 12 = 0$$

$$x(x - 3\sqrt{2}) + 2\sqrt{2}(x - 3\sqrt{2}) = 0$$

$$(x - 3\sqrt{2}) + (x + 2\sqrt{2}) = 0$$

Thus, $x = 3\sqrt{2}, -2\sqrt{2}$

Hence option B is correct.

Question: 5

The zeros of the

Solution:

We have, $f(x) = 4x^2 + 5\sqrt{2}x - 3$

Now, put $f(x) = 0$

$$4x^2 + 5\sqrt{2}x - 3 = 0$$

$$4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3 = 0$$

$$2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3) = 0$$

$$(2\sqrt{2}x - 1)(\sqrt{2}x + 3) = 0$$

$$x = -\frac{3}{\sqrt{2}}, \frac{\sqrt{2}}{4}$$

Hence, option C is correct.

Question: 6

The zeros of the

Solution:

We have, $f(x) = x^2 + \frac{1}{6}x - 2$

$$f(x) = 6x^2 + x - 12 = 0$$

Now, put $f(x) = 0$

$$6x^2 + x - 12 = 0$$

$$6x^2 + 9x - 8x - 12 = 0$$

$$3x(2x + 3) - 4(2x + 3) = 0$$

$$\text{Thus, } -\frac{3}{2}, \frac{4}{3}$$

Hence, option B is correct.

Question: 7

The zeros of the

Solution:

We have, $f(x) = 7x^2 - \frac{11x}{3} - \frac{2}{3}$

$$f(x) = 21x^2 - 11x - 2 = 0$$

Now, put $f(x) = 0$

$$21x^2 - 11x - 2 = 0$$

$$21x^2 - 14x + 3x - 2 = 0$$

$$7x(3x - 2) + 1(3x - 2) = 0$$

$$(3x - 2)(7x + 1) = 0$$

$$\text{Thus, } x = \frac{2}{3}, -\frac{1}{7}$$

Hence, option A is correct.

Question: 8

The sum and the p

Solution:

We have, sum of zeroes($\alpha + \beta$) = 3

Product of zeroes($\alpha\beta$) = -10

We know that,

Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - 3x - 10$$

Hence, option C is correct.

Question: 9

A quadratic polyn

Solution:

We have, $\alpha = 5, \beta = -3$

Thus, sum of zeros($\alpha + \beta$) = $5 - 3 = 2$

Product of zeros($\alpha\beta$) = $5(-3) = -15$

We know that,

Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - 2x - 15$$

Hence, option C is correct.

Question: 10

A quadratic polyn

Solution:

We have, $\alpha = \frac{3}{5}, \beta = -\frac{1}{2}$

Thus, sum of zeros($\alpha + \beta$)

$$= \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$$

Product of zeros($\alpha\beta$) = $\frac{3}{5} \left(-\frac{1}{2}\right) = -\frac{3}{10}$

We know that,

Required polynomial = $x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - \frac{1}{10}x - \frac{3}{10}$$

Hence, option D is correct.

Question: 11

The zeros of the

Solution:

Let the zeros be α and β

We know that, $\alpha + \beta = -88$ (sum of zeros)

$\alpha\beta = 125$ (product of zeros)

This is only possible when both the zeroes are negative.

Question: 12

If α and β are th

Solution:

Since, α and β are the zeroes of polynomial $x^2 + 5x + 8$

And we know that,

$x^2 - (\alpha + \beta)x + \alpha\beta$ is the polynomial with α and β as its zeros

Thus, $(\alpha + \beta) = -5$

Hence, option B is correct.

Question: 13

If α and β are th

Solution:

Since, α and β are the zeroes of polynomial $2x^2 + 5x - 9$

And we know that,

$x^2 - (\alpha + \beta)x + \alpha\beta$ is the polynomial with α and β as its zeros

Thus, $(\alpha\beta) = -\frac{9}{2}$

Hence, option C is correct.

Question: 14

If one zero of th

Solution:

Given: 2 is the zero of the polynomial $f(x) = kx^2 + 3x + k$

Put $f(2) = 0$

$k(2)^2 + 3(2) + k = 0$

$4k + 6 + k = 0$

$k = -\frac{6}{5}$

Hence, option D is correct.

Question: 15

If one zero of th

Solution:

Given: -4 is the zero of the polynomial $f(x) = (k - 1)x^2 + kx + 1$

Put $f(-4) = 0$

$(k - 1)(-4)^2 + k(-4) + 1 = 0$

$16k - 16 - 4k + 1 = 0$

$k = \frac{5}{4}$

Hence, option B is correct.

Question: 16

If -2 and 3 are t

Solution:

It is given in the question that, -2 and 3 are the zeros of $x^2 + (a + 1)x + b$

$$\therefore (-2)^2 + (a + 1) \times (-2) + b = 0$$

$$4 - 2a - 2 + b = 0$$

$$b - 2a = -2 \text{ (i)}$$

Also, we have

$$3^2 + (a + 1) \times 3 + b = 0$$

$$9 + 3a + 3 + b = 0$$

$$b + 3a = -12 \text{ (ii)}$$

Now, by subtracting the equation (i) from (ii) we get:

$$a = -2$$

$$\text{Also, } b = -2 - 4 = -6$$

Question: 17

If one zero of $3x^2 + 8x + k$

Solution:

Let α and $\frac{1}{\alpha}$ be the zeros of polynomial $3x^2 + 8x + k$

Now, product of the polynomial = $\frac{\text{constant}}{\text{coefficient of } x^2}$

$$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$$

$$\text{Thus, } k = 3$$

Hence, option A is correct.

Question: 18

If the sum of the

Solution:

Let the zeros of the polynomial be α and β

$$\text{Now, } \alpha + \beta = -\frac{2}{k} \text{ and } \alpha\beta = 3$$

According to the question,

$$\alpha + \beta = \alpha\beta$$

$$-\frac{2}{k} = 3$$

$$k = -\frac{2}{3}$$

Hence option D is correct.

Question: 19

If α, β are the z

Solution:

Given: α and β are zeros of the given polynomial.

$$\therefore \alpha = \beta = -6 \text{ and } \alpha\beta = 2$$

$$\text{Thus, } \left(\frac{1}{\alpha} + \frac{1}{\beta} \right) = \left(\frac{\alpha + \beta}{\alpha\beta} \right)$$

$$= -\frac{6}{2} = -3$$

Hence, option B is correct.

Question: 20

If α, β, γ are the

Solution:

Given: α, β, γ are the zeros of the polynomial $x^3 - 6x^2 - x + 30$

We know that,

$$(\alpha\beta + \gamma\alpha + \beta\gamma) = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

$$= -\frac{1}{1} = -1$$

Hence, option A is correct.

Question: 21

If α, β, γ are the

Solution:

Given: α, β, γ are the zeros of the polynomial $2x^3 + x^2 - 13x + 6$

We know that,

$$(\alpha\beta\gamma) = -\frac{\text{constant term}}{\text{coefficient of } x^3}$$

$$= -\frac{6}{2} = -3$$

Hence, option A is correct.

Question: 22

If α, β, γ be the

Solution:

According to the question,

α, β, γ be the zeros of the polynomial $p(x)$ such that $(\alpha + \beta + \gamma) = 3$, $(\alpha\beta + \beta\gamma + \gamma\alpha) = -10$ and $\alpha\beta\gamma = -24$

Thus, $p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$$x^3 - 3x^2 - 10x + 24$$

Hence, option C is correct.

Question: 23

If two of the zeros

Solution:

Let us assume $\alpha, -0$ and 0 be the zeros of the given polynomial

$$\therefore \text{Sum of zeros} = -\frac{b}{a}$$

$$\alpha + 0 + 0 = -\frac{b}{a}$$

$$\alpha = -\frac{b}{a}$$

\therefore The third zero is $-\frac{b}{a}$

Hence, option A is correct

Question: 24

If one of the zer

Solution:

Let us assume α, β and 0 be the zeros of the given polynomial

\therefore Sum of the products of zeros taking two at a time is given by:

$$(\alpha\beta + \beta \times 0 + \alpha \times 0) = \frac{c}{a}$$

$$\alpha\beta = \frac{c}{a}$$

\therefore The product of other two zeros will be $\frac{c}{a}$

Hence, option B is correct

Question: 25

If one of the zer

Solution:

It is given in the question that,

- 1 is the zero of the given polynomial $x^3 + ax^2 + bx + c$

$$\therefore (-1)^3 + a \times (-1)^2 + b \times (-1) + c = 0$$

$$a - b + c + 1 = 0$$

$$c = 1 - a + b$$

Also, the product of all zeros is:

$$\alpha\beta \times (-1) = -c$$

$$\alpha\beta = c$$

$$\alpha\beta = 1 - a + b$$

Hence, option C is correct

Question: 26

If α, β be the ze

Solution:

It is given in the question that,

α and β are the zeros of the given polynomial $2x^2 + 5x + k$

$$\therefore \alpha + \beta = -\frac{5}{2} \text{ and } \alpha\beta = \frac{k}{2}$$

Also, it is given in the question that.

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\therefore (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\left(-\frac{5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\frac{25}{4} - \frac{k}{2} = \frac{21}{4}$$

$$\frac{k}{2} = \frac{25}{4} - \frac{21}{4} = \frac{4}{4} = 1$$

$$k = 2$$

Hence, option D is correct

Question: 27

One dividing a po

Solution:

As, we know that

According to the division algorithm on polynomials, we have

either $r(x) = 0$ or $\deg r(x) < \deg g(x)$

Hence, option C is correct

Question: 28

Which of the foll

Solution:

We know that,

Monomial is that which consist only one term and in the given option we have only one term i.e. $5x^2$

\therefore Option D is correct

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

Zeros of $p(x) = x$

Solution:

It is given in the question that,

$$p(x) = x^2 - 2x - 3$$

$$\text{Let us assume } x^2 - 2x - 3 = 0$$

$$x^2 - (3 - 1)x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x - 3) + 1(x - 3) = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3, -1$$

Question: 2

If α, β, γ are th

Solution:

We have,

$$p(x) = x^3 - 6x^2 - x + 3$$

Now we will compare the given polynomial with: $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

By comparing we get:

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = -1$$

Question: 3

If α, β are the z

Solution:

We have,

$$p(x) = x^2 - 2x + 3k$$

Now by comparing the given polynomial with $ax^2 + bx + c$, we get:

$$a = 1, b = -2 \text{ and } c = 3k$$

In the question it is given that, α and β are the roots of the given polynomial

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = -\left(\frac{-2}{1}\right)$$

$$\alpha + \beta = 2 \text{ (i)}$$

Also we have:

$$\alpha\beta = \frac{c}{a}$$

$$\alpha\beta = \frac{3k}{1}$$

$$\alpha\beta = 3k \text{ (ii)}$$

Hence, by using (i) and (ii), we have

$$\alpha + \beta = \alpha\beta$$

$$2 = 3k$$

$$k = \frac{2}{3}$$

Question: 4

If is given that

Solution:

Let us assume the zeros of the polynomial be α and $\alpha + 4$

We have,

$$p(x) = 4x^2 - 8kx + 9$$

Now comparing the given polynomial with $ax^2 + bx + c$, we get:

$$a = 4, b = -8k \text{ and } c = 9$$

$$\text{Sum of roots} = -\frac{b}{a}$$

$$\alpha + \alpha + 4 = -\frac{-8k}{4}$$

$$2\alpha + 4 = 2k$$

$$\alpha + 2 = k$$

$$\alpha = (k - 2) \text{ (i)}$$

Also, we have product of roots, $\alpha\beta = \frac{c}{a}$

$$\alpha(\alpha + 4) = \frac{9}{4}$$

$$(k - 2) (k - 2 + 4) = \frac{9}{4}$$

$$(k - 2) (k + 2) = \frac{9}{4}$$

$$k^2 - 4 = \frac{9}{4}$$

$$4k^2 - 16 = 9$$

$$4k^2 = 25$$

$$k^2 = \frac{25}{4}$$

$$k = \sqrt{\frac{25}{4}}$$

$$k = \frac{5}{2}$$

Question: 5

Find the zeros of

Solution:

We have,

$$p(x) = x^2 + 2x - 195$$

Let us assume $p(x) = 0$

$$x^2 + (15 - 13)x - 195 = 0$$

$$x^2 + 15x - 13x - 195 = 0$$

$$x(x + 15) - 13(x + 15) = 0$$

$$(x + 15)(x - 13) = 0$$

$$x = -15, 13$$

Hence, the zeros of the polynomial are -15 and 13

Question: 6

If one zero of th

Solution:

We have,

$$(a + 9)x^2 - 13x + 6a = 0$$

Comparing with standard form of quadratic equation $Ax^2 + Bx + C$

$$A = (a^2 + 9), B = 13 \text{ and } C = 6a$$

Let us assume α and $\frac{1}{\alpha}$ be the zeros of the given polynomial

$$\therefore \text{Product} = \frac{C}{A}$$

$$\alpha \times \frac{1}{\alpha} = \frac{6a}{a^2 + 9}$$

$$1 = \frac{6a}{a^2 + 9}$$

$$a^2 + 9 = 6a$$

$$a^2 - 6a + 9 = 0$$

$$a^2 - 2 \times a \times 3 + 3^2 = 0$$

$$(a - 3)^2 = 0$$

$$a - 3 = 0$$

$$a = 3$$

Question: 7

Find a quadratic

Solution:

It is given in the question that the two roots of the given polynomial are 2 and - 5

Let us assume $\alpha = 2$ and $\beta = - 5$

We have:

$$\text{Sum of Zeros} = \alpha + \beta = 2 + (-5) = - 3$$

$$\text{Product of Zeros} = \alpha\beta = 2 \times (-5) = - 10$$

Hence,

$$\text{Required polynomial} = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (- 3)x + 10$$

$$= x^2 + 3x - 10$$

Question: 8

If the zeros of t

Solution:

It is given in the question that the roots of the given polynomial are (a - b), a and (a + b)

Now by comparing the given polynomial with $Ax^3 + Bx^2 + Cx + D$, we get:

$$A = 1, B = - 3, C = 1 \text{ and } D = 1$$

Now,

$$(a - b) + a + (a + b) = -\frac{B}{A}$$

$$3a = -\frac{-3}{1}$$

$$a = 1$$

Also, we have:

$$(a - b) \times a \times (a + b) = -\frac{D}{A}$$

$$a(a^2 - b^2) = -\frac{1}{1}$$

$$1(1^2 - b^2) = - 1$$

$$1 - b^2 = - 1$$

$$b^2 = 2$$

$$b = \pm\sqrt{2}$$

$$\text{Hence, } a = 1 \text{ and } b = \pm\sqrt{2}$$

Question: 9

Verify that 2 is

Solution:

We have,

$$p(x) = x^3 + 4x^2 - 3x - 18$$

Now,

$$p(2) = (2)^3 + 4 \times (2)^2 - 3(2) - 18$$

$$= 8 + 16 - 6 - 18$$

$$= 24 - 24$$

$$= 0$$

$\therefore 2$ is the zero of the given polynomial

Question: 10

Find the quadrati

Solution:

It is given in the question that,

Sum of the zeros = - 5

And, product of the zeros = 6

\therefore Required polynomial = $x^2 - (\text{Sum of the zeros})x + \text{Product of the zeros}$

$$= x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6$$

Question: 11

Find a cubic poly

Solution:

Let us assume α, β and γ be the zeros of the required polynomial

We have,

$$\alpha + \beta + \gamma = 3 + 5 + (-2) = 6$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times 5 + 5 \times (-2) + (-2) \times 3 = -1$$

$$\text{And, } \alpha\beta\gamma = 3 \times 5 \times -2 = -30$$

Now, we have:

$$p(x) = x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$$

$$= x^3 - x^2 \times 6 + x \times (-1) - (-30)$$

$$= x^3 - 6x^2 - x + 30$$

Hence, the required polynomial is $p(x) = x^3 - 6x^2 - x + 30$

Question: 12

Using remainder t

Solution:

Remainder theorem: Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$. It is given in the question that,

$$p(x) = x^3 + 3x^2 - 5x + 4$$

$$\text{Now, } p(2) = (2)^3 + 3(2)^2 - 5(2) + 4$$

$$= 8 + 12 - 10 + 4$$

$$= 14$$

Question: 13

Show that $(x + 2)$

Solution:

It is given in the question that,

$$f(x) = x^3 + 4x^2 + x - 6$$

Now, we have

$$f(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$$

$$= -8 + 16 - 2 - 6$$

$$= 0$$

Hence, $(x + 2)$ is a factor of $f(x)$

Question: 14

If α, β, γ are th

Solution:

It is given in the question that,

$$p(x) = 6x^3 + 3x^2 - 5x + 1$$

$$= 6x^3 - (-3)x^2 + (-5)x - 1$$

Now by comparing the polynomial with $x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$, we get:

$$\alpha\beta + \beta\gamma + \gamma\alpha = -5 \text{ and,}$$

$$\alpha\beta\gamma = -1$$

$$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= \left(\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}\right)$$

$$= \left(\frac{-5}{-1}\right)$$

$$= 5$$

Question: 15

If α, β are the z

Solution:

It is given in the question that,

$f(x) = x^2 - 5x + k$ such that its coefficients are $a = 1, b = -5$ and $c = k$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha + \beta = \frac{(-5)}{1}$$

$$\alpha + \beta = 5 \text{ (i)}$$

$$\text{Also, } \alpha - \beta = 1 \text{ (ii)}$$

So from (i) and (ii), we get:

$$2\alpha = 6$$

$$\alpha = \frac{6}{2} = 3$$

Now putting the value of α in (i), we get

$$3 + \beta = 5$$

$$\beta = 5 - 3 = 2$$

we know, Product of zeroes

$$\therefore \alpha\beta = \frac{c}{a}$$

$$3 \times 2 = \frac{k}{1}$$

Hence, $k = 6$

Question: 16

Show that the pol

Solution:

Let us assume $t = x^2$

$$\therefore f(t) = t^2 + 4t + 6$$

Now first of all we have to equate $f(t) = 0$ in order to find the zeros

$$\therefore t = \frac{-4 \pm \sqrt{16 - 24}}{2}$$

$$= \frac{-4 \pm \sqrt{-8}}{2}$$

$$= -2 \pm \sqrt{-2}$$

As, $t = x^2$

$$\text{So, } x^2 = -2 \pm \sqrt{-2}$$

$$x = \sqrt{-2 \pm \sqrt{-2}}$$

We know that, the zeros of the polynomial should be a real number and this is not a real number

$\therefore f(x)$ has no zeros

Question: 17

If one zero of th

Solution:

It is given in the question that,

$$p(x) = x^3 - 6x^2 + 11x - 6 \text{ having factor } (x + 3)$$

Now, we have to divide $p(x)$ by $(x - 3)$

$$x^3 - 6x^2 + 11x - 6 = (x - 3)(x^3 - 3x + 2)$$

$$= (x - 3)[(x^2 - (2 + 1)x + 2)]$$

$$= (x - 3)(x^2 - 2x - x + 2)$$

$$= (x - 3)[x(x - 2) - 1(x - 2)]$$

$$= (x - 3)(x - 1)(x - 2)$$

Hence, the two zeros of the polynomial are 1 and 2

Question: 18

If two zeros of t

Solution:

It is given in the question that,

$p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$ having zeros $\sqrt{2}$ and $-\sqrt{2}$

\therefore The polynomial is $(x + \sqrt{2})(x - \sqrt{2}) = x^2 - 2$

Let us now divide $p(x)$ by $(x^2 - 2)$

$$2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$$

$$= (x^2 - 2)[(2x^2 - (2 + 1)x + 1)]$$

$$= (x^2 - 2)(2x^2 - 2x - x + 1)$$

$$= (x^2 - 2)[(2x(x - 1) - 1(x - 1))]$$

$$= (x^2 - 2)(2x - 1)(x - 1)$$

Hence, the other two zeros are $\frac{1}{2}$ and 1

Question: 19

Find the quotient

Solution:

It is given in the question that,

$$p(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 2$$

Now we have to divide $p(x)$ by $(x^2 + 3x + 1)$, we get:

$$\begin{array}{r} \overline{3x^2 - 4x + 2} \\ x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\ \underline{3x^4 + 9x^3 + 3x^2} \\ -4x^3 - 10x^2 + 2x + 2 \\ \underline{-4x^3 - 12x^2 - 4x} \\ +2x^2 + 6x + 2 \\ \underline{2x^2 + 6x + 2} \\ - \\ \times \end{array}$$

Hence, the quotient is $3x^2 - 4x + 2$

Question: 20

Use remainder the

Solution:

Given: when $x^3 + 2x^2 + kx + 3$ is divided by $(x - 3)$, then the remainder is 21. **To find:** The value of k . **Solution:** If $x - 3$ divides the equation and leaves 21 as remainder, it means substituting $x = 3$ in the equation and putting it equal to 21 will give the value of k . Let us assume,

$$p(x) = x^3 + 2x^2 + kx + 3$$

$$\text{Now, } p(3) = (3)^3 + 2(3)^2 + 3k + 3$$

$$= 27 + 18 + 3k + 3$$

$$= 48 + 3k$$

It is given in the question that the remainder is 21

Hence, $3k + 48 = 21$

$$3k = -27$$

$$k = -9$$