DPP - Daily Practice Problems

Date : ____ Start Time : ____ End Time : ____

MATHEMATICS CM26

SYLLABUS : Three Dimensional Geometry

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQ's. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. The line, $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to

(a) ± 1 (b) $\pm\frac{1}{3}$

(c) $\pm\sqrt{5}$ (d) None of these

2. Two systems of rectangular axes have the same origin If a plane cuts them at the distance a, b, c and a′, b′, c′ respectively from the origin, then

$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=k\left(\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}\right)$, where k =

(a) 1 (b) 2

(c) 4 (d) $\frac{1}{2}$

3. The length intercepted by a line with direction ratios 2, 7, –5 between the lines

$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$ and $\frac{x+3}{-3}=\frac{y-3}{2}=\frac{z-6}{4}$ is

(a) $\sqrt{75}$ (b) $\sqrt{78}$

(c) $\sqrt{83}$ (d) None of these

4. From the point (1, –2, 3) lines are drawn to meet the sphere $x^2 + y^2 + z^2 = 4$ and they are divided internally in the ratio 2 : 3. The locus of the point of division is

(a) $5x^2+5y^2+5z^2-6x+12y+22=0$

(b) $5(x^2+y^2+z^2)=22$

(c) $5x^2+5y^2+5z^2-2xy-3yz-zx-6x+12y+5z+22=0$

(d) $5x^2+5y^2+5z^2-6x+12y-18z+22=0$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. If two lines L_1 and L_2 in space, are defined by

$L_1 = \{x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1),\ z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\}$ and

$L_2 = \{x = \sqrt{\mu}y + (1 - \sqrt{\mu}),\ z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$

then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :

(a) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (b) $\lambda \neq \mu$

(c) $\lambda + \mu = 0$ (d) $\lambda = \mu$

6. The locus of a point, such that the sum of the squares of its distances from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ is 9, is

(a) $x^2 + y^2 + z^2 = 3$ (b) $x^2 + y^2 + z^2 = 6$

(c) $x^2 + y^2 + z^2 = 9$ (d) $x^2 + y^2 + z^2 = 12$

7. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two mutually perpendicular lines, Then the direction cosines of the line perpendicular to both of them are

(a) $(m_1n_2 - m_2n_1), (n_1l_2 - n_2l_1), (l_1m_2 - l_2m_1)$

(b) $l_1 + l_2, m_1 + m_2, n_1 + n_2,$

(c) $l_1\, l_2, m_1 m_2, n_1 n_2$

(d) $\frac{l_1}{l_2}, \frac{m_1}{m_2}, \frac{n_1}{n_2}$

8. A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular from the origin to this plane is given by

(a) $x^2 + y^2 + z^2 - 14 = 0$

(b) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

(c) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$

(d) None of these

9. The direction cosines l, m, n, of one of the two lines connected by the relations

$l - 5m + 3n = 0,\ 7l^2 + 5m^2 - 3n^2 = 0$ are

(a) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (b) $\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

(c) $\frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (d) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$

10. The equation of a sphere is $x^2 + y^2 + z^2 - 10z = 0$. If one end point of a diameter of the sphere is $(-3, -4, 5)$, what is the other end point ?

(a) $(-3, -4, -5)$ (b) $(3, 4, 5)$

(c) $(3, 4, -5)$ (d) $(-3, 4, -5)$

11. A line makes the same angle α with each of the x and y axes. If the angle θ, which it makes with the z-axis, is such that $\sin^2\theta = 2\sin^2\alpha$, then what is the value of α ?

(a) $\pi/4$ (b) $\pi/6$

(c) $\pi/3$ (d) $\pi/2$

12. If Q is the image of the point P(2, 3, 4) under the reflection in the plane $x - 2y + 5z = 6$, then the equation of the line PQ is

(a) $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z-4}{5}$ (b) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$

(c) $\frac{x-2}{-1} = \frac{y-3}{-2} = \frac{z-4}{5}$ (d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{5}$

13. The foot of the perpendicular from (2, 4, –1) to the line

$x + 5 = \frac{1}{4}(y + 3) = -\frac{1}{9}(z - 6)$

(a) $(-4, 1, -3)$ (b) $(4, -1, -3)$

(c) $(-4, -1, 3)$ (d) $(-4, -1, -3)$

14. The equation of the plane which makes with co-ordinate axes, a triangle with its centroid (α, β, γ) is

(a) $\alpha x + \beta y + \gamma z = 3$ (b) $\alpha x + \beta y + \gamma z = 1$

(c) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$ (d) $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$

RESPONSE GRID					
	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. The equation of two lines through the origin, which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$ each, are

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{1};\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$

(b) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$

(c) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1};\frac{x}{1}=\frac{y}{-1}=\frac{z}{-2}$

(d) None of the above

16. A rectangular parallelopiped is formed by drawing planes through the points (–1, 2, 5) and (1, –1, –1) and parallel to the coordinate planes. The length of the diagonal of the parallelopiped is

(a) 2 (b) 3
(c) 6 (d) 7

17. The planes 3x – y + z + 1 = 0, 5x + y + 3z = 0 intersect in the line PQ. The equation of the plane through the point (2, 1, 4) and the perpendicular to PQ is

(a) $x+y-2z=5$ (b) $x+y+2z=-5$
(c) $x+y+2z=5$ (d) $x+y-2z=-5$

18. The line $\frac{x-1}{1}=\frac{y-2}{-2}=\frac{z-1}{3}$ and the plane $x+2y+z=6$ meet in

(a) no point (b) only one point
(c) infinitely many points (d) None of these

19. If from a point *P* (*a*, *b*, *c*) perpendiculars *PA* and *PB* are drawn to *yz* and *zx* planes, then the equation of the plane *OAB* is

(a) $bcx + cay + abz = 0$ (b) $bcx + cay - abz = 0$
(c) $bcx - cay + abz = 0$ (d) $-bcx + cay + abz = 0$

20. Under what condition do the planes $bx - ay = n$, $cy - bz = l$, $az - cx = m$ intersect in a line?

(a) $a+b+c=0$ (b) $a=b=c$
(c) $al+bm+cn=0$ (d) $l+m+n=0$

21. The distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ is

(a) 1 (b) 2
(c) 4 (d) $2\sqrt{3}$

22. A variable plane which remains at a constant distance 3p from the origin cut the coordinate axes at A, B and C. The locus of the centroid of triangle ABC is

(a) $x^{-1}+y^{-1}+z^{-1}=p^{-1}$ (b) $x^{-2}+y^{-2}+z^{-2}=p^{-2}$
(c) $x+y+z=p$ (d) $x^2+y^2+z^2=p^2$

23. The radius of the sphere $x^2+y^2+z^2=49$, $2x+3y-z-5\sqrt{14}=0$ is

(a) $\sqrt{6}$ (b) $2\sqrt{6}$
(c) $4\sqrt{6}$ (d) $6\sqrt{6}$

24. Two spheres of radii 3 and 4 cut orthogonally The radius of common circle is

(a) 12 (b) $\frac{12}{5}$
(c) $\frac{\sqrt{12}}{5}$ (d) $\sqrt{12}$

25. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals

(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17)

26. Equation of line in the plane $\pi \equiv 2x-y+z-4=0$ which is perpendicular to the line ℓ whose equation is $\frac{x-2}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}$ and which passes through the point of intersection of ℓ and π is –

(a) $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z-1}{-1}$ (b) $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

(c) $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z+1}{1}$ (d) $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

RESPONSE GRID					
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ			

27. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals

(a) -1 (b) 1
(c) -2 (d) 2

28. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1 ,–1) is

(a) $5x - 11y + z = 17$ (b) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(c) $x + y + z = \sqrt{3}$ (d) $x - \sqrt{2}y = 1 - \sqrt{2}$

29. A mirror and a source of light are situated at the origin O and at a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane are 1, –1, 1, then direction cosines of the reflected rays are

(a) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (b) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

(c) $-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

30. **Statement 1 :** Let θ be the angle between the line $\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and the plane $x + y - z = 5$.

Then $\theta = \sin^{-1}\frac{1}{\sqrt{51}}$

Statement 2 : Angle between a straight line and a plane is the complement of angle between the line and normal to the plane.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement -1
(b) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
(c) Statement -1 is False, Statement -2 is True
(d) Statement - 1 is True, Statement- 2 is False

RESPONSE GRID	27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 26 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	35	Qualifying Score	52
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(c)** We have, z = 0 for the point where the line intersects the curve.

Therefore, $\frac{x-2}{3}=\frac{y+1}{2}=\frac{0-1}{-1}$

$\Rightarrow \frac{x-2}{3}=1$ and $\frac{y+1}{2}=1$

$\Rightarrow x=5$ and $y=1$

Put these value in $xy=c^2$, we get, $5=c^2 \Rightarrow c=\pm\sqrt{5}$

2. **(a)** Let a, b, c be the intercepts when Ox, Oy, Oz are taken as axes; then the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Also let a′, b′, c′ be the intercepts when OX, OY, OZ are taken as axes ; then in this case equation of the same plane is $\frac{X}{a'}+\frac{Y}{b'}+\frac{Z}{c'}=1$

Now (1) and (2) are equations of the same plane and in both the cases the origin is same. Hence length of the perpendicular drawn from the origin to the plane in both the case must be the same.

i.e $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$

or $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2} \quad \therefore k=1$

3. **(b)** The general points on the given lines are respectively $P(5+3t,\ 7-t,\ -2+t)$ and $Q(-3-3s,\ 3+2s,\ 6+4s)$.

Direction ratios of PQ are

$<-3-3s-5-3t,\ 3+2s-7+t,\ 6+4s+2-t>$

i.e., $<-8-3s-3t,\ -4+2s+t,\ 8+4s-t>$

If PQ is the desired line then direction ratios of PQ should be proportional to <2, 7, –5>, therefore,

$\frac{-8-3s-3t}{2}=\frac{-4+2s+t}{7}=\frac{8+4s-t}{-5}$

Taking first and second numbers, we get

$-56-21s-21t=-8+4s+2t$

$\Rightarrow 25s+23t=-48 \qquad \ldots(i)$

Taking second and third members, we get

$20-10s-5t=56+28s-7t$

$\Rightarrow 38s-2t=-36 \qquad \ldots(ii)$

Solving (i) and (ii) for t and s, we get $s=-1$ and $t=-1$.

The coordinates of P and Q are respectively

$(5+3(-1),\ 7-(-1),\ -2-1)=(2,\ 8,\ -3)$

and $(-3-3(-1),\ 3+2(-1),\ 6+4(-1))=(0,\ 1,\ 2)$

$\therefore$ The required line intersects the given lines in the points (2, 8, –3) and (0, 1, 2) respectively.

Length of the line intercepted between the given lines

$=|PQ|=\sqrt{(0-2)^2+(1-8)^2+(2+3)^2}=\sqrt{78}$.

4. **(d)** Suppose any line through the given point (1, –2, 3) meets the sphere $x^2+y^2+z^2=4$ in the point

(x_1, y_1, z_1). Then $x_1^2+y_1^2+z_1^2=4 \qquad \ldots(1)$

Now let the co-ordinates of the points which divides the join of (1, –2, 3) and (x_1, y_1, z_1) in the ratio 2 : 3 be (x_2, y_2, z_2). Then we have

$$\left.\begin{array}{l} x_2 = \dfrac{2.x_1+3.1}{2+3} \text{ or } x_1 = \dfrac{5x_2-3}{2} \\ y_2 = \dfrac{2.y_1+3.(-2)}{2+3} \text{ or } y_1 = \dfrac{5y_2+6}{2} \\ z_2 = \dfrac{2.z_1+3.3}{2+3} \text{ or } z_1 = \dfrac{5z_2-9}{2} \end{array}\right] \quad ...(2)$$

Putting the values of x_1, y_1, z_1, from (2) in (1), we have

$$(5x_2-3)^2+(5y_2+6)^2+(5z_2-9)^2 = 4\times4$$

or $25(x_2^2+y_2^2+z_2^2)-30x_2+60y_2-90z_2+110=0$

or $5(x_2^2+y_2^2+z_2^2)-6(x_2-2y_2+3z_2)+22=0$

$\therefore$ The locus of (x_2, y_2, z_2) is

$5(x^2+y^2+z^2)-6(x-2y+3z)+22=0$.

5. **(d)** For L_1,

$$x=\sqrt{\lambda}y+(\sqrt{\lambda}-1) \Rightarrow y=\frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} \quad ...(i)$$

$$z=(\sqrt{\lambda}-1)y+\sqrt{\lambda} \Rightarrow y=\frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad ...(ii)$$

From (i) and (ii)

$$\frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}}=\frac{y-0}{1}=\frac{z-\sqrt{\lambda}}{\sqrt{\lambda}-1} \quad ...(A)$$

The equation (A) is the equation of line L_1.
Similarly equation of line L_2 is

$$\frac{x-(1-\sqrt{\mu})}{\sqrt{\mu}}=\frac{y-0}{1}=\frac{z-\sqrt{\mu}}{1-\sqrt{\mu}} \quad ...(B)$$

Since $L_1 \perp L_2$, therefore

$\sqrt{\lambda}\sqrt{\mu}+1\times1+(\sqrt{\lambda}-1)(1-\sqrt{\mu})=0$

$\Rightarrow \sqrt{\lambda}+\sqrt{\mu}=0 \Rightarrow \sqrt{\lambda}=-\sqrt{\mu} \Rightarrow \lambda=\mu$

6. **(c)** Let the variable point be (α, β, γ) then according to question

$$\left(\frac{|\alpha+\beta+\gamma|}{\sqrt{3}}\right)^2+\left(\frac{|\alpha-\gamma|}{\sqrt{2}}\right)^2+\left(\frac{|\alpha-2\beta+\gamma|}{\sqrt{6}}\right)^2=9$$

$\Rightarrow \alpha^2+\beta^2+\gamma^2=9$.

So, the locus of the point is $x^2+y^2+z^2=9$

7. **(a)** Let l, m, n be the direction cosines of the line perpendicular to each one of the given lines.

Then, $ll_1+mm_1+nn_1=0$ ...(1)

$ll_2+mm_2+nn_2=0$ ...(2)

Solving (1) and (2) by cross-multiplicatin, we get :

$$\frac{l}{(m_1n_2-m_2n_1)}=\frac{m}{(n_1l_2-n_2l_1)}=\frac{n}{(l_1m_2-l_1m_1)}$$

$$=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{\Sigma(m_1n_2-m_2n_1)^2}} \text{ or } \frac{l}{(m_1n_2-m_2n_1)}$$

$$=\frac{m}{(n_1l_2-n_2l_1)}=\frac{n}{(l_1m_2-l_2m_1)}=\frac{1}{\sin\theta},$$

where θ is the angle between the given lines.

But, $\theta=\frac{\pi}{2}$ and therefore, $\sin\theta=1$

$\therefore l=(m_1n_2-m_2n_1)$; $m=(n_1l_2-n_2l_1)$ and $n=(l_1m_2-l_2m_1)$

Hence, the direction cosines of the required line are $(m_1n_2-m_2n_1)$ $(n_1l_2-n_2l_1)$, $(l_1m_2-l_2m_1)$

8. **(c)** Let $P(\alpha, \beta, \gamma)$ be the foot of the perpendicular from the origin O(0, 0, 0) to the plane So, the plane passes through $P(\alpha,\beta,\gamma)$ and is perpendicular to OP. Clearly direction ratios of OP i.e., normal to the plane are α, β, γ. Therefore, equation of the plane is $\alpha(x-\alpha)+\beta(y-\beta)+\gamma(z-\gamma)=0$

This plane passes through the fixed point (1, 2, 3), so

$\alpha(1-\alpha)+\beta(2-\beta)+\gamma(3-\gamma)=0$

or $\alpha^2+\beta^2+\gamma^2-\alpha-2\beta-3\gamma=0$

Generalizing α, β and γ, locus of P (α, β, γ) is

$x^2+y^2+z^2-x-2y-3z=0$

9. **(a)** From the first relation, $l=5m-3n$. Putting this value of l in second relation

$7(5m-3n)^2+5m^2-3n^2=0$

$\Rightarrow 180m^2-210mn+60n^2=0$

or $6m^2-7mn+2n^2=0$

Note that it, being quadratic in m, n, gives two sets of values of m, n, and hence gives the d.r.s. of two lines. Now, factorising it, we get

$6m^2-3mn+4mn+2n^2=0$

or $(2m-n)(3m-2n)=0$

$\Rightarrow$ either $2m-n=0$, or $3m-2n=0$

Taking $2m-n=0$ we get $2m=n$.

Also putting $m=n/2$ in $l=5m-3n$, we get

$l=(5n/2)-3n \Rightarrow l=-n/2 \Rightarrow n=-2l$

Thus, we get, $-2l=2m=n$ or $\frac{l}{-1}=\frac{m}{1}=\frac{n}{2}$

$\Rightarrow$ d.r.s. of one line are –1, 1, 2.

Hence, the d,c,s. of one line are

$$\left[\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}\right] \text{ or } \left[\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{-2}{\sqrt{6}}\right]$$

Taking $3m-2n=0$, we get

$3m=2n$ or $m=\frac{2n}{3}$.

Putting this value in $l=5m-3n$, we obtain

$l=5\times\frac{2n}{3}-3n=\frac{n}{3}$ or $n=3l$

Thus $3l=\frac{3m}{2}=n \Rightarrow \frac{l}{1}=\frac{m}{2}=\frac{n}{3}$

$\Rightarrow$ the d.r'.ss of the second line are 1, 2, 3; and hence

d.c.s. of second line are $\left[\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right]$

or $\left[\frac{-1}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-3}{\sqrt{14}}\right]$

10. (b) The equation of the given sphere is
$x^2+y^2+z^2-10z=0$.
$\therefore$ Its centre is (0, 0, 5).
Coordinates of one end point of a diameter of the sphere is given as $(-3,-4,5)$.
Let Coordinates of another end point of this diameter (x_1, y_1, z_1)

$\therefore \dfrac{-3+x_1}{2}=0 \quad \Rightarrow x_1=3$

$\dfrac{-4+y_1}{2}=0 \quad \Rightarrow y_1=4$

and $\dfrac{5+z_1}{2}=5 \Rightarrow z_1=5$

$\therefore$ Required coordinates are (3, 4, 5).

11. (a) Since $l^2+m^2+n^2=1$
$\therefore \cos^2\alpha+\cos^2\alpha+\cos^2\theta=1$ .... (i)
($\because$ *A* line makes the same angle α with *x* and *y*-axes and θ with z-axis)
Also, $\sin^2\theta=2\sin^2\alpha$
$\Rightarrow 1-\cos^2\theta=2(1-\cos^2\alpha)$ ($\because \sin^2 A+\cos^2 A=1$)
$\Rightarrow \cos^2\theta=2\cos^2\alpha-1$ .... (ii)
$\therefore$ From Eq. (i) and (ii)
$2\cos^2\alpha+2\cos^2\alpha-1=1$

$\Rightarrow 4\cos^2\alpha=2 \Rightarrow \cos^2\alpha=\dfrac{1}{2}$

$\Rightarrow \cos\alpha=\pm\dfrac{1}{\sqrt{2}} \quad \Rightarrow \alpha=\dfrac{\pi}{4},\dfrac{3\pi}{4}$

12. (b) Let Q be the image of the point P(2, 3, 4) in the plane $x-2y+5z=6$, then PQ is normal to the plane
$\therefore$ direction ratios of PQ are <1, –2, 5 >
Since PQ passes through P(2, 3, 4) and has direction ratios 1, –2, 5

$\therefore$ Equation of PQ is $\dfrac{x-2}{1}=\dfrac{y-3}{-2}=\dfrac{z-4}{5}$

13. (a) Given equation of line is

$x+5=\dfrac{1}{4}(y+3)=-\dfrac{1}{9}(z-6)$

or $\dfrac{x+5}{1}=\dfrac{y+3}{4}=\dfrac{z-6}{-9}=\lambda(say)$
$x=\lambda-5, y=4\lambda-3, z=-9\lambda+6$
$(x,y,z)\equiv(\lambda-5, 4\lambda-3, -9\lambda+6)$ ...(i)
Let it is foot of perpendicualr
So, d.r.'s of $\perp$ line is
$(\lambda-5-2, 4\lambda-3-4, -9\lambda+6+1)$
$\equiv(\lambda-7, 4\lambda-7, -9\lambda+7)$
D.r.'s of given line is $(1, 4, -9)$ and both lines are $\perp$
$\therefore (\lambda-7).1+(4\lambda-7).4+(-9\lambda+7)(-9)=0$
$\Rightarrow 98\lambda=98 \Rightarrow \lambda=1$
$\therefore$ Point is $(-4, 1, -3)$. [Substituting $\lambda=1$ in (i)]

14. (c) Let us take a triangle ABC and their vertices A (a, 0, 0), B (0, b, 0) and C (0, 0, c)
Therefore the equation of plane is

$\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ .... (i)

Now, given centroid of ΔABC is (α, β, γ)
As we know, centroid of ΔABC with vertices (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$\therefore$ By using this formula, we have

$\dfrac{a+0+0}{3}=\alpha \Rightarrow a=3\alpha$,

$\dfrac{0+b+0}{3}=\beta \Rightarrow b=3\beta$

and $\dfrac{0+0+c}{3}=\gamma \Rightarrow c=3\gamma$

Now, put the values of a, b, c in equation (i), which gives

$\dfrac{x}{3\alpha}+\dfrac{y}{3\beta}+\dfrac{z}{3\gamma}=1$

$\therefore \dfrac{x}{\alpha}+\dfrac{y}{\beta}+\dfrac{z}{\gamma}=3$

15. (b) Given equation of line is

$\dfrac{x-3}{2}=\dfrac{y-3}{1}=\dfrac{z}{1}$

$\Rightarrow$ DR's of the given line are 2, 1, 1

$\Rightarrow$ DC's of the given line are $\dfrac{2}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}$

Since, required lines make an angle $\dfrac{\pi}{3}$ with the given line
The DC's of the required lines are

$\dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}, \dfrac{-1}{\sqrt{6}}$ and $\dfrac{-1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}}$ respectively.

Also, both the required lines pass through the origin.
$\therefore$ Equation of required lines are

$\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{-1}$ and $\dfrac{x}{-1}=\dfrac{y}{1}=\dfrac{z}{-2}$

16. (d) The planes forming the parallelopiped are
$x=-1, x=1;\ y=2, y=-1$ and $z=5, z=-1$
Hence, the lengths of the edges of the parallelopiped are $1-(-1)=2$, $|-1-2|=3$ and $|-1-5|=6$
(Length of an edge of a rectangular parallelopiped is the distance between the parallel planes perpendicular to the edge)
$\therefore$ Length of diagonal of the parallelopiped

$=\sqrt{2^2+3^2+6^2}=\sqrt{49}=7$.

17. (d) Let $\{l, m, n\}$ be the direction -cosines of PQ, then
$3l-m+n=0$ and $5l+m+3n=0$

$\therefore \dfrac{l}{-3-1}=\dfrac{m}{5-9}=\dfrac{n}{3+5}$ i.e $\dfrac{l}{1}=\dfrac{m}{1}=\dfrac{n}{-2}$

Now a plane $\perp$ to PQ will have l, m , n as the coefficients of x, y and z
Hence the plane $\perp$ to PQ is $x+y-2z=\lambda$
It passes through (2, 1, 4); $\therefore 2+1-2.4=\lambda$ i.e $\lambda=-5$
Hence the required plane is $x+y-2z=-5$

18. (c) D.R. of given line are 1, –2, 3 and the d.r. of normal to the given plane are 1, 2, 1.
Since $1 \times 1 + (-2) \times 2 + 3 \times 1 = 0$, therefore, the line is parallel to the plane, Also, the base point of the line (1, 2, 1) lies in the given plane.
$(1 + 2 \times 2 + 1 = 6$ is true)
Hence, the given line lies in the given plane.
Alternatively, any point on the given line is $(t+1, -2t+2, 3t+1)$.
It lies in the given plane
$x + 2y + z = 6$ if $t + 1 + 2\,(-2t + 2) + 3t + 1 = 6$
i.e. if $0t = 0$, which is true for all real t. Hence every point on the given line lies in the given plane i.e. the line lies in the plane.

19. (b) $A\,(0, b, c)$ in yz-plane and $B(a, 0, c)$ in zx-plane.
Plane through O is $px + qy + rz = 0$. It passes through A and B.
$\therefore\ 0p + qb + rc = 0$ and $pa + 0q + rc = 0$
$\Rightarrow\ \dfrac{p}{bc} = \dfrac{q}{ca} = \dfrac{r}{-ab} = k$
$\Rightarrow\ p = bck,\ q = cak$ and $r = -abk$.
Hence required plane is $bcx + cay - abz = 0$.

20. (c) The planes $bx - ay = n$, $cy - bz = l$ and $az - cx = m$ intersect in a line, if $al + bm + cn = 0$.

21. (a) Equation of the line through (1, –2, 3) parallel to the line
$\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z-1}{-6}$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = r \text{ (say)} \qquad ...(1)$$

Then any point on (1) is $(2r + 1, 3r - 2, -6r + 3)$
If this point lies on the plane $x - y + z = 5$ then

$$(2r+1) - (3r-2) + (-6r+3) = 5 \Rightarrow r = \frac{1}{7}$$

Hence the point is $\left(\dfrac{9}{7}, -\dfrac{11}{7}, \dfrac{15}{7}\right)$

Distance between (1, –2, 3) and $\left(\dfrac{9}{7}, -\dfrac{11}{7}, \dfrac{15}{7}\right)$

$$= \sqrt{\left(\frac{4}{49} + \frac{9}{49} + \frac{36}{49}\right)} = \sqrt{\left(\frac{49}{49}\right)} = 1$$

22. (b) Let equation of the variable plane be $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$
This meets the coordinate axes at A(a, 0, 0), B (0, b, 0) and C(0, 0, c).
Let $P(\alpha, \beta, \gamma)$ be the centroid of the ΔABC. Then

$$\alpha = \frac{a+0+0}{3}, \beta = \frac{0+b+0}{3}, \gamma = \frac{0+0+c}{3}$$

$\therefore\ a = 3\alpha, b = 3\beta, c = 3\gamma$ ...(2)
Plane (1) is at constant distance 3p from the origin, so

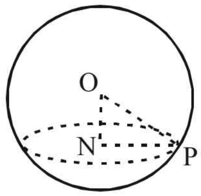
$$3p = \frac{\left|\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1\right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \qquad ...(3)$$

From (2) and (3), we get

$$\frac{1}{9\alpha^2} + \frac{1}{9\beta^2} + \frac{1}{9\gamma^2} = \frac{1}{9p^2} \quad \Rightarrow \alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$$

Generalizing α, β, γ, locus of centroid $P(\alpha, \beta, \gamma)$ is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

23. (b) The sphere $x^2 + y^2 + z^2 = 49$ has centre at the origin (0, 0, 0) and radius 7.
Disance of the plane



$2x + 3y - z - 5\sqrt{14} = 0$
from the origin

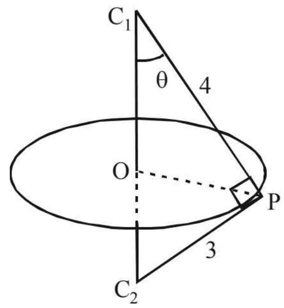
$$= \frac{\left|2(0) + 3(0) - (0) - 5\sqrt{14}\right|}{\sqrt{2^2 + 3^2 + (-1)^2}}$$

$$= \frac{\left|-5\sqrt{14}\right|}{\sqrt{14}} = \frac{5\sqrt{14}}{\sqrt{14}} = 5$$

Thus in Figure
OP = 7, ON = 5
$NP^2 = OP^2 - ON^2 = (7)^2 - (5)^2 = 49 - 25 = 24$
$\therefore\ NP = 2\sqrt{6}$

Hence the radius of the circle = NP = $2\sqrt{6}$

24. (b)



For the orthogonal section C_1P and C_2P are pendicular where C_1 and C_2 are centres of sphere of radii 4 and 3 respectively

Now $C_1P = 4$ and $C_2P = 3$, so $\tan\theta = \dfrac{3}{4}$
$\therefore$ Radius of circle of intersection

$$OP = C_1P \sin\theta = 4 \times \frac{3}{5} = \frac{12}{5}$$

25. (a) $\because$ The line $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$
$\therefore$ Point (2, 1, – 2) lies on the plane
i.e. $2 + 3 + 2\alpha + \beta = 0$
or $2\alpha + \beta + 5 = 0$ ....(i)
Also normal to plane will be perpendicular to line,
$\therefore\ 3 \times 1 - 5 \times 3 + 2 \times (-\alpha) = 0$
$\Rightarrow\ \alpha = -6$
From equation (i) we have, $\beta = 7$
$\therefore\ (\alpha, \beta) = (-6, 7)$

26. **(b)** Let direction ratios of the line be $\langle a, b, c\rangle$, then

$2a - b + c = 0$
$a - b - 2c = 0$

i.e., $\frac{a}{3} = \frac{b}{5} = \frac{c}{-1}$

$\therefore$ direction ratios of the line are (3, 5, – 1)

Any point on the given line is $(2+\lambda, 2-\lambda, 3-2\lambda)$. It lies on the plane π if

$2(2+\lambda) - (2-\lambda) + (3-2\lambda) = 4$

i.e., $4 + 2\lambda - 2 + \lambda + 3 - 2\lambda = 4$

i.e., $\lambda = -1$

$\therefore$ the point of intersection of the line and the plane is (1, 3, 5)

$\therefore$ equation of the required line is $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-5}{-1}$

27. **(c)** Plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the centre of spheres

$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ respectively

center of spheres are (– 3, 4, 1) and (5, – 2, 1). Mid point of centres is (1, 1, 1).

Satisfying this in the equation of plane, we get

$2a - 3a + 4a + 6 = 0 \Rightarrow a = -2.$

28. **(a)** The plane passing through the intersection line of given planes is

$$(x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$$

or $\quad (1+\lambda)x + (2-\lambda)y + (3+\lambda)z + (-2-3\lambda) = 0$

Its distance from the point (3, 1, –1) is $\frac{2}{\sqrt{3}}$

$$\therefore \left|\frac{3(1+\lambda) + 1(2-\lambda) - 1(3+\lambda) + (-2-3\lambda)}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2 + 4\lambda + 14}}\right| = \frac{2}{\sqrt{3}}$$

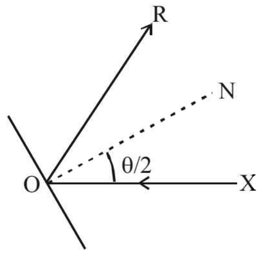
$$\Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2 \Rightarrow \lambda = -\frac{7}{2}$$

$\therefore$ Required equation of plane is

$$(x + 2y + 3z - 2) - \frac{7}{2}(x - y + z - 3) = 0$$

or $\quad 5x - 11y + z = 17$

29. **(d)**



Let the ray of light comes along x-axis and strikes the mirror at the origin.

Direction cosines of normal are

$\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ so. $\cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$

Let the reflected ray has direction cosines l, m, n then

$$\frac{l+1}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}} \Rightarrow l = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\frac{m+0}{2\cos\frac{\theta}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow m = -\frac{2}{3} \quad \frac{n+0}{2\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}} \Rightarrow n = \frac{2}{3}$$

30. **(a)** $\sin\theta = \left|\frac{2-3+2}{\sqrt{4+9+4}\sqrt{3}}\right| = \frac{1}{\sqrt{51}}$

Statement 1 is true, statement 2 is true by definition.